

# Bayesian Overconfidence\*

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## Abstract

We study three distinct measures of overconfidence: (1) *overestimation* of one's performance, (2) *overplacement* of one's performance relative to others, and (3) *overprecision* in one's belief about private signals. We show how a simple Bayesian model with uncertainty about task difficulty predicts overplacement and underestimation after unexpectedly easy tasks and underplacement and overestimation after unexpectedly difficult tasks. We also describe how these predictions are affected by overprecision. The predicted patterns of overconfidence are confirmed using a set of trivia quiz experiments. We conclude that no biases in judgement or other systematic errors are needed to explain observations of overconfidence.

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# 1 Introduction

It is often implied that overconfident beliefs are irrational because they are statistically incorrect. When Ola Svenson (1981) asked a sample of forty-one American college students with drivers licenses to rate their driving skill against the other members of the same sample, 92.7 percent reported themselves to be above the median. While this is statistically impossible, is it necessarily the case that some of these individuals hold irrational beliefs?<sup>1</sup> Are people erroneously biased toward believing that they are better than others, or is it possible that these inaccurate beliefs are fully rational and consistent with Bayes's rule given the available information?

In this paper, we carefully define three separate notions of overconfidence and explore a very simple model that makes clear predictions about the relationships between these types of overconfidence.<sup>2</sup> The model shows how individuals with incomplete information about a task's difficulty can exhibit overconfident beliefs in some situations and underconfident beliefs in others. In a new set of experiments we find behavior broadly consistent with the predictions of this simple model. This suggests that underlying biases in judgement are not necessary to explain many observations of overconfident behavior.

Our operative assumptions are that tasks have unknown difficulties and that agents believe that their performance (and the performance of others) is determined by the overall difficulty of the task plus an individual-specific component that quantifies individual performance net of average overall performance. After performing a task, each agent is asked to estimate her own performance and the performance of a randomly-selected other participant. The result of Bayesian inference in this setting is that, after an unexpectedly easy task, a subject will believe that she has out-performed her peers but will simultaneously un-

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<sup>1</sup>The driving example is somewhat problematic because driving quality is subjective and has multiple dimensions; however, the standard overconfidence results obtain using objective, numerical performance measures.

<sup>2</sup>The word "model" is perhaps an overstatement; our "model" is simply an application of Bayes's rule.

derestimate her own performance. When the task is unexpectedly difficult, she will believe her performance was worse than her peers but will also overestimate her own performance.

Critical to this result is the distinction between overconfidence about one's ranking relative to others and overconfidence about one's score relative to the true score. Much of the previous literature muddles these distinct concepts, which we label overestimation (overestimating one's own performance) and overplacement (overestimating one's rank relative to others). We also consider overprecision (beliefs that have greater precision than is warranted by the data), which is a third form of overconfidence that is typically found in models of stock market trading (see Terrance Odean (1999), for example). Using this terminology, the model predicts overplacement and underestimation after unexpectedly easy tasks and underplacement and overestimation after unexpectedly difficult tasks. Overprecision is predicted to be correlated with the other two forms of overconfidence, though the sign of the correlation is difficult to predict as it depends critically on the source of the overprecision; this is detailed in Section 2.3.

The intuition behind the model is straightforward: experiencing an unexpectedly good outcome implies that the task was somewhat easier than expected but also that you performed somewhat better than average. Thus, you predict that your competitors will also do well but that you have outperformed them by some degree. As an example, suppose every manager in a particular emerging industry agrees that the expected marginal cost of a new product is \$10 per unit. After production begins, however, each firm privately observes an actual marginal cost ranging from \$7 to \$9 – well below the common prior expectation. Each manager might conclude that their lower-than-expected cost was partly due to an incorrect prior estimate of \$10 but also partly due to his own firm's better-than-average ability at producing the product. Thus, it is possible that all managers simultaneously exhibit overplacement, believing that its costs are lower than the average competitor. Had the actual costs been higher than the prior estimate (ranging from \$11 to \$13, for example),

the result would reverse and all managers might exhibit underplacement, believing that its costs are higher than average. Thus, we can generally conclude that overplacement is more likely after unexpectedly easy tasks and underplacement is more likely after unexpectedly difficult tasks.

The logic for overestimation is similar: suppose now that firms build a prototype product before opening their production lines, and the cost of the prototype serves as an unbiased signal of the true production cost. If the range of prototype costs is lower than expected, managers might rationally conclude that their true production cost will lie somewhere between the prior estimate (\$10) and the observed prototype cost. Given any firm whose true production cost is \$8, we would expect that, on average, the firm has realized a prototype cost of \$8. But this firm's expectation about its true production cost would be higher – perhaps \$9 – because the firm's posterior expectation is a combination of the prior (\$10) and its observed data (\$8). An outsider who observes that true production costs (\$8) are lower than the prior estimate (\$10) will therefore observe that firms are, on average, overestimating their costs (at \$9). The result reverses for higher-than-expected true production costs. In general, agents are more likely to underestimate their ability when actual performance is better than previously expected and are more likely to overestimate their ability when performance is worse than expected.

Returning to the leading example of overconfident drivers, it may be that the typical driver finds driving to be easier than expected because crashes and citations are infrequent events. Since a driver observes her own driving record more completely than the record of anyone else, she may use the above Bayesian logic to conclude that driving is somewhat easy and also that she has also done somewhat better than the average driver. Although there are certainly other dimensions to this example that our simple argument ignores, the basic logic makes it clear that observing 93 percent of drivers ranking themselves above the median may be perfectly rational.

To examine the predictions of this model, we abstract away from the particulars of the competitive environment and directly examine individuals' beliefs about their own performances and the performances of others in an experimental setting where subjects participate in a sequence of trivia quizzes.<sup>3</sup> Our experimental results verify that overconfidence is closely linked to task difficulty; after easy tasks, subjects exhibit overplacement and underestimation. After difficult tasks, subjects exhibit underplacement and overestimation. We therefore confirm the predicted negative relationship between these two types of overconfidence. These results complement recent findings in psychology; Justin Kruger (1999) finds systematic underplacement on difficult tasks, and Don A. Moore and Tai Gyu Kim (2003) and Don A. Moore and Deborah A. Small (2007) document similar connections between overestimation, overplacement, and task difficulty.

From the perspective of our model, overconfidence is a "bias" only in the sense that beliefs do not match the actual distribution of outcomes. Agents' beliefs are consistent with Bayes's rule and are therefore optimal given the information available – the only way for agents to improve the accuracy of their beliefs is with more information about the distribution of performances. We therefore think of overconfidence as a *statistical* bias caused by incomplete information rather than a *behavioral* bias caused by systematic imperfections in judgement. In fact, no behavioral bias is needed to generate predictions in line with the statistical biases we observe in our experiment.

One notable difference between this and most other models of overconfidence is that this model does not predict overplacement or overestimation before agents experience the task; the managers' prior estimate of \$10 may be perfectly accurate (on average) given the initial information available. The models of Eric van den Steen (2004) and Luis Santos-

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<sup>3</sup>It may be that certain forms of competition will generate other behavioral biases that would interact with the observed overconfidence results. Thus, we study beliefs in the absence of competition as a first step in understanding the basic nature of overconfidence. Results from other studies that do incorporate competition (such as those cited below) can then be used to paint a more complete picture of the overconfidence phenomenon.

Pinto and Joel Sobel (2005), for example, assume each agent selects an action based on his own objective function and therefore concludes that others' decisions are suboptimal since they failed to maximize the same objective.<sup>4</sup> This bias in judgement leads agents to exhibit overplacement before performing the task, which is not observed in our experimental results. Furthermore, models such as these are designed to explain overplacement, overestimation, or overprecision, but can explain neither the *under*placement and *under*estimation we observe in our data nor the strong link with task difficulty.

Although we argue that there is no need to assume a bias in judgement to generate overconfidence, the effect may be exacerbated by other judgement biases that arise in certain settings. For example, in an experiment by Colin Camerer and Dan Lovallo (1999) subjects apparently fail to adjust for the fact that they self-selected into an experiment on trivia quizzes and are therefore facing competitors who are typically better at trivia questions than the general population. Since each erroneously expects his competitors to be average for the population, each exhibits overplacement prior to taking the task because of this error in judgement. In the treatments without self-selection, however, there is no significant evidence of overplacement prior to the task. Therefore, it is the error in recognizing self-selection that generates behavior consistent with overplacement and not an underlying behavioral bias towards overconfident beliefs.

Although our study also focuses on overconfidence in trivia quizzes, overconfident beliefs can have significant effects in a wide variety of settings. Many authors have argued that overconfidence is an economically meaningful phenomenon, causing excess market entry (see, e.g., Richard Roll (1986), Colin Camerer and Dan Lovallo (1999), or James G. March and Zur Shapira (1987)), increased volatility in financial markets (e.g., Terrance Odean (1998, 1999) or Kent D. Daniel, David Hirshleifer and Avanidhar Subrahmanyam (2001)), or excessive investment in capital (e.g., Ulrike Malmendier and Geoffrey Tate

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<sup>4</sup>We describe these models in more detail in Section 5.

(2005)). We do not dispute the importance of the statistical bias of overconfidence; our results imply only that behavioral biases are not necessary to explain its existence.

The theoretical environment is described in the following section. In Section 3 we discuss the design of our experiments. The results appear in Section 4. A more thorough review of the previous literature appears in Section 5, and the paper concludes with Section 6.

## 2 Bayesian Overconfidence

In this section we formally demonstrate how Bayesian inference can generate the observed patterns of overconfidence and underconfidence. In what follows, upper-case variables represent random variables and lower-case variables represent particular realizations of the corresponding random variable. We assume that each agent  $i$  performs an identical task in isolation and receives a numerical score, denoted  $x_i$ , which quantifies her performance. Each agent believes that each score  $x_i$  is a realization of  $X_i$ , and  $X_i$  is determined by

$$X_i = S + L_i, \tag{1}$$

where  $S$  is the overall expected score across agents (or, the *simplicity* of the task) and  $L_i$  is a mean-zero idiosyncratic component that determines the difference between  $i$ 's score and the overall average. We assume the mean of  $S$  exists and equals  $\mu$ .<sup>5</sup> As a simple mnemonic, we refer to  $L_i$  as agent  $i$ 's *luck*, but, depending on the application, it may include a variety of other idiosyncratic components such as  $i$ 's unknown task-specific ability level.<sup>6</sup> Assuming agent  $i$  has well-defined prior beliefs about the distributions of  $S$  and  $L_i$ , she can update those beliefs upon observing her realized score  $x_i$ . If agent  $i$  also has well-defined

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<sup>5</sup>For simplicity, we assume throughout that all random variables have well-defined means.

<sup>6</sup>We discuss the case where  $E[L_i]$  is non-zero in Subsection 2.4.2.

priors on  $L_j$  for some other agent  $j \neq i$ , her beliefs about  $X_j$  will also change as she observes  $x_i$  and updates her belief about  $S$ . In this way agent  $i$  may exhibit overplacement or underplacement with respect to others' scores after she observes her own score.

## 2.1 Overplacement

Under the above assumptions,  $i$ 's prior expectations of her own score and the score of another agent are  $E[X_i] = E[X_j] = \mu$ . After performing the task and observing  $x_i$ , she updates her beliefs about  $S$  and  $L_i$ . Since she does not observe  $x_j$  for any  $j \neq i$ , her beliefs about  $L_j$  remain unchanged, so that  $E[X_j|x_i] = E[S|x_i]$ . We say that  $i$  exhibits *overplacement* if  $E[X_j|x_i] < x_i$  and *underplacement* if  $E[X_j|x_i] > x_i$ . Note that overplacement and underplacement are properties of posterior beliefs with no necessary connection to actual scores; for example, it is possible for all agents to simultaneously exhibit overplacement even though some do and some do not outperform their peers.

Suppose an agent  $i$  who has never encountered the task before receives a score higher than expected ( $x_i > \mu$ ). She might infer that her high score was due to good luck ( $l_i > 0$ ) or a simpler-than-expected task ( $s > \mu$ ). If she attributes her high score entirely to the task's simplicity (*i.e.*, she believes  $E[S|x_i] = x_i$ ), then she will exhibit no overplacement because task simplicity affects all agents equally. If instead she attributes her high score at least partially to her own luck (*i.e.*, she believes  $E[S|x_i] < x_i$ ), then she will exhibit overplacement since  $E[X_j|x_i] = E[S|x_i] < x_i$ . Similarly, if  $x_i < \mu$  and she attributes her low score at least partially to luck, then she will exhibit underplacement. Thus, we expect overplacement after unexpectedly easy tasks and underplacement after unexpectedly difficult tasks.<sup>7</sup>

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<sup>7</sup>Since this theory includes no behavioral biases, it also applies in situations where some agent  $k$  observes  $x_i$  but not  $x_j$ . Thus, we predict that people also tend to exhibit the predicted patterns of overconfidence about their *friend's* performance when the friend's performance is observable but others' performances are not.

Whether or not we observe  $E[X_j|x_i] < x_i$  when  $x_i > \mu$  and  $E[X_j|x_i] > x_i$  when  $x_i < \mu$  depends on the belief distributions over  $S$  and  $L_i$ . If, for example, beliefs over  $S$  are uniformly distributed (and beliefs over  $L_i$  are not), then  $E[X_j|x_i] = E[S|x_i] = x_i$  for all  $x_i$  and no overplacement or underplacement is observed. If the belief distributions are such that

$$E[X_j|x_i] = E[S|x_i] = \alpha\mu + (1 - \alpha)x_i$$

for some  $\alpha > 0$ , then we must observe the required pattern of overplacement for every  $x_i$ .<sup>8</sup> The following examples highlight cases where  $E[S|x_i]$  takes this particular form.

**Example 1** Suppose that  $i$  believes that  $S \sim \mathcal{N}(\mu, \sigma_S^2)$  ( $S$  is distributed according to a normal distribution with mean  $\mu$  and variance  $\sigma_S^2$ ) and  $L_j \sim \mathcal{N}(0, \sigma_L^2)$  for each  $j$  (including  $i$ ). By Bayes's rule,  $E[X_j|x_i] = E[S|x_i] = \alpha\mu + (1 - \alpha)x_i$ , where  $\alpha = \sigma_L^2 / (\sigma_S^2 + \sigma_L^2)$ .<sup>9</sup>  $\square$

This example is in fact a special case of a more general theorem due to Persi Diaconis and Donald Ylvisaker (1979), who show that if the distribution of  $X_i$  given  $S$  is in the exponential family, then  $E[S|x_i]$  lies between  $\mu$  and  $x_i$  if and only if the prior on  $S$  is conjugate. Thus, we expect the predicted pattern of overplacement when  $X_i$  given  $S$  has a normal distribution with a normal prior on  $S$ , an exponential distribution with a gamma prior, a Pareto distribution with a Pareto prior, a poisson distribution with a gamma prior, a geometric distribution with a beta prior, and, as in the next example, when  $X_i$  has a binomial distribution with a beta prior.

**Example 2** In our experimental environment, subjects complete a sequence of 10-question quizzes. Suppose subjects believe their scores to be binomially distributed with parameter  $p$

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<sup>8</sup>Christopher P. Chambers and Paul J. Healy (2007) explore general conditions on belief distributions that guarantee posterior expectations of the form  $E[S|x_i] = \alpha\mu + (1 - \alpha)x_i$  for  $\alpha \in [0, 1]$  and, more generally, for  $\alpha \leq 1$ . For example, symmetry and quasiconcavity of the densities is sufficient for the result with  $\alpha \in [0, 1]$ .

<sup>9</sup>This is a familiar property of normal distributions; see Roger L. Berger (1980, p.127-8) for a derivation.

(meaning they expect to get each question correct with probability  $p$ ).<sup>10</sup> If  $p$  is an unknown parameter distributed according to a beta distribution with parameters  $\beta_1$  and  $\beta_2$  (so that  $\mu = 10 \beta_1 / (\beta_1 + \beta_2)$ ) then

$$E[X_j|x_i] = \left( \frac{\beta_1 + \beta_2}{\beta_1 + \beta_2 + 10} \right) \mu + \left( 1 - \frac{\beta_1 + \beta_2}{\beta_1 + \beta_2 + 10} \right) x_i.$$

Since the posterior mean for  $X_j$  lies between the prior mean and the observation of  $x_i$ , we predict the same pattern of overplacement as in example 1.<sup>11</sup>  $\square$

Not all beliefs on  $S$  and  $X_i$  generate this pattern of overplacement. As noted, a uniform prior on  $S$  results in  $E[S|x_i] = x_i$ , so agent  $i$  expects others to do exactly as well as she. If the prior is highly bimodal then  $E[X_j|x_i]$  might “overshoot”  $x_i$ . As an extreme example, if an agent’s prior belief is that  $S$  can only equal  $-8$  or  $8$  and that  $L_i$  can only range from  $-2$  to  $+2$ , then if she observes  $x_i = 6$  she knows that  $s = 8$  and so  $x_j \in [6, 10]$ , or  $x_j \geq x_i$ . If the belief on  $L_i$  is highly bimodal, the posterior might move in the *opposite* direction as the observed score. To see this, suppose now that  $L_i$  equals  $-8$  or  $8$ , each with probability one half, and that  $S$  ranges from  $-2$  to  $2$ . Now observing  $x_i = 6$  leads the agent to conclude that  $E[X_j|x_i] = s = -2$ . In other words, receiving a high score causes the agent to believe others will do *worse* than previously expected.<sup>12</sup>

## 2.2 Overestimation

In many situations, the act of performing a task does not perfectly reveal one’s performance.

A competitor in a judged competition such as figure skating typically has a strong signal

<sup>10</sup>This example assumes that subjects believe their success on each question is independent of success on all other questions *for a given p*. When  $p$  is unknown, however, independence fails since success on one question provides information about the probability of success on other questions.

<sup>11</sup>For a derivation of  $E[X_j|x_i]$ , see George Casella and Roger L. Berger (2002, p. 325).

<sup>12</sup>For a continuous example, suppose the density function on  $S$  is  $(1 - |x|/3)/3$  over  $[-3, 3]$  and  $L_i$  takes values of  $-2$  or  $2$ , each with probability one half. If  $x_i \in (-2, 2)$  then  $E[X_j|x_i] = -x_i$ , so the agent expects others’ scores to be exactly *opposite* of her own.

of her performance immediately after competing but will not know her score with certainty until the judges tally her scores and will not know her ranking relative to others until the competition is complete. We refer to the time before the individual competes as the *ex-ante* phase, where beliefs are determined entirely by priors. The time after the individual competes but before she learns her score is the *interim* phase, where her information is based only on an imperfect signal of her performance. Upon learning her own score she enters the *ex-post* phase, where she is fully informed about her own score, but knows nothing yet of her competitors' performances. After learning her competitors' scores, the *ex-post* phase is complete and she has full knowledge of the outcomes.

We model the interim phase by assuming that each agent  $i$  observes  $y_i$ , which is a noisy signal of her true score. The signal  $y_i$  is believed to be a realization of the random variable  $Y_i = x_i + E_i$ , where  $E_i$  is a mean-zero error term. Since we assume  $X_i = S + L_i$ , an agent who observes a draw  $y_i$  makes inferences about  $S$ ,  $L_i$ , and  $E_i$ . For example, a high value of  $y_i$  may lead her to conclude that  $S$  is relatively high but that  $L_i$  and  $E_i$  were positive as well. In this case, she will expect that she did better than average (because  $L_i$  is positive) but not as well as her signal indicated (because  $E_i$  is positive). If her signal is in fact accurate, then she has underestimated her actual performance. Formally, we say that  $i$  exhibits *overestimation* if  $E[X_i|y_i] > x_i$  and *underestimation* if  $E[X_i|y_i] < x_i$ .

In practice, we cannot observe agents' private signals, but we can observe the resulting distribution of  $X_i|y_i$ . The drawback of this approach is that this posterior distribution depends crucially on the unobservable signal, so that random noise in the draws of the signals will translate into noise in the observed posteriors on  $X_i$ . To avoid these difficulties, we integrate across all possible signals (or, average across all elicited beliefs) to calculate the expected value of  $E[X_i|y_i]$  when the true score ( $x_i$ ) is known. Formally, we can calculate  $E_{Y_i}[E[X_i|Y_i]|x_i]$  for any  $x_i$  and compare it against  $x_i$ . If this expected score is greater than  $x_i$ , we conclude that agents exhibit *overestimation in expectation*. If it is less than  $x_i$ ,

agents exhibit *underestimation in expectation*.

The following example shows how the results for overestimation can move in the opposite direction as those for overplacement; when a task is easier than expected, agents exhibit overplacement and *underestimation*. When a task is more difficult than expected, agents exhibit underplacement and overestimation. These predictions are summarized in Table 1.

**Example 1 (Continued)** Let  $Z_j = L_j + E_j$ . If we assume that  $E_j$  is also normally distributed with mean zero and variance  $\sigma_E^2$ , then  $Z_j \sim \mathcal{N}(0, \sigma_L^2 + \sigma_E^2)$ . Since  $Y_i = S + Z_i$ , we can apply Bayes's rule to see that  $E[S|y_i] = \hat{\alpha}\mu + (1 - \hat{\alpha})y_i$ , where  $\hat{\alpha} = (\sigma_L^2 + \sigma_E^2)/(\sigma_S^2 + \sigma_L^2 + \sigma_E^2)$ . Since  $E[X_j|y_i] = E[S|y_i]$ , it follows that  $i$ 's expectation of  $j$ 's score continues to lie between her prior expectation ( $\mu$ ) and her private signal ( $y_i$ ). Her expectation of her *own* score differs, however, because her signal also contains information about her own luck variable ( $L_i$ ). Formally, since  $Y_i = X_i + E_i$ ,  $E[X_i|y_i] = \bar{\alpha}\mu + (1 - \bar{\alpha})y_i$ , where  $\bar{\alpha} = \sigma_E^2/(\sigma_S^2 + \sigma_L^2 + \sigma_E^2)$ . Here,  $i$ 's expectation about her own score also lies between her prior expectation ( $\mu$ ) and her private signal ( $y_i$ ), but, since  $\bar{\alpha} < \hat{\alpha}$ , we have that either

$$y_i < E[X_i|y_i] < E[X_j|y_i] < \mu$$

or

$$\mu < E[X_j|y_i] < E[X_i|y_i] < y_i.$$

In other words,  $i$  displays overplacement after high signals and underplacement after low signals.

To evaluate overestimation note that, for this example,  $E_{Y_i}[E[X_i|Y_i]|x_i] = \bar{\alpha}\mu + (1 - \bar{\alpha})E[Y_i|x_i]$ , which equals  $\bar{\alpha}\mu + (1 - \bar{\alpha})x_i$ . Thus, agents' expected reports of  $E[X_i|y_i]$  lie strictly between  $\mu$  and  $x_i$ . If  $\mu < x_i$ , we observe underestimation in expectation, and if

Task Difficulty	Relative Performance	Absolute Performance
Easy	Overplacement	Underestimation
Difficult	Underplacement	Overestimation

Table 1: The key predictions of the Bayesian model.

$x_i < \mu$ , we observe overestimation in expectation. □

As with overplacement, the result that overestimation depends on the realization of task simplicity does not obtain with every combination of prior beliefs. With a uniform prior over  $X_i$ , for example, agents will fully update their expected score to the realized signal regardless of the actual simplicity of the task, generating no over- or underestimation.

### 2.3 Overprecision

If an agent’s beliefs about her own score has higher precision (lower variance) than her actual distribution of scores (taking a frequentist’s viewpoint), we say that she exhibits *overprecision*. Since our model of agents’ inferences operates only on subjective beliefs without assuming those beliefs are empirically accurate, the presence of overprecision will not qualitatively affect the above results on overplacement and overestimation; however, the *level* of precision will affect the magnitudes of these effects. For example, consider an agent who exhibits excessive precision in her estimates of her own score. If this overprecision on  $X_i$  stems from overprecision in her prior over  $S$ , then she attributes her high score more to her own performance (‘luck’)—and less to the task’s simplicity—than does someone with well-calibrated beliefs. Thus, she perceives less correlation between her own score and the scores of others, exacerbating the overplacement phenomenon when the task is easier than expected. On the other hand, if her overprecision is due to lower-than-warranted variance in  $L_i$ , she will perceive more correlation than actually exists and her overplacement will be

mitigated. In general, overprecision in  $S$  increases the magnitude of underestimation and overplacement after easy tasks and the magnitude of overestimation and underplacement after difficult tasks, while overprecision in  $L_i$  reduces these magnitudes.

In practice, we only observe beliefs over scores. Since we cannot differentiate overprecision in  $S$  and overprecision in  $L_i$ , we cannot use the theoretical links between overprecision and the other types of overconfidence to validate or falsify this model; we can only measure and describe the observed patterns of overprecision and how it correlates with overplacement and overestimation.

## 2.4 Modifications and Extensions

The above theoretical model is, by design, simple and unsophisticated. Several embellishments could be added to make the model better fit certain real-world environments. The main thrust of our argument remains unchanged, however, if these modifications do not alter the conclusion that  $E[X_j|x_i]$  and  $E[X_i|y_i]$  lie between  $\mu$  and  $x_i$  (at least, in expectation). In this section we detail three possible changes to the model and argue that these changes do not (necessarily) invalidate the main conclusions of the simple model.

### 2.4.1 Multiple-Dimensional Signals

Some tasks, such as exams, involve some uncertainty about the exact nature of the task that is revealed while the task is performed. According to the model above, a student taking an exam receives only a signal of how well she performed and can only make inferences about the test's true difficulty from that one signal. In some situations it may be appropriate to model the student as receiving a second signal that is directly related to the test difficulty. For example, discovering that a final exam's questions were taken from various homework problems assigned throughout the course may lead a student to increase her estimate of the

average score for the entire class, regardless of how she felt about her own performance.

We can model this possibility by assuming agents receive two signals while performing the task: an unbiased signal  $y_i$  of their actual performance and a second unbiased signal  $r_i$  of the task difficulty, where  $r_i$  is a realization of  $R_i = S + Q_i$  with  $E[Q_i] = 0$ . As before, we take expectations over the value of the signal since it is not observable to the experimenter. Thus, we calculate  $E_{Y_i, R_i}[E[S|Y_i, R_i]|x_i, s]$ . The following example demonstrates how the inclusion of this second signal does not qualitatively alter the above analysis. Intuitively, we expect that  $r_i$  equals  $\mu$  on average when prior beliefs are unbiased. In other words, the signal  $r_i$  is uninformative on average; its presence acts only to strengthen the prior belief, pulling the posterior means for  $X_i$  and  $X_j$  toward  $\mu$  on average. This does not change the prediction that these posterior means lie between  $\mu$  and  $x_i$ ; thus, the magnitude of the overplacement and overestimation effects may change, but the direction of the effects would not.

**Example 1 (Continued)** If  $R_i = S + Q_i$  with  $Q_i \sim \mathcal{N}(0, \sigma_L^2 + \sigma_E^2)$  then

$$E[S|r_i, y_i] = \frac{(\sigma_L^2 + \sigma_E^2)/2}{\sigma_S^2 + (\sigma_L^2 + \sigma_E^2)/2} \mu + \frac{\sigma_S^2}{\sigma_S^2 + (\sigma_L^2 + \sigma_E^2)/2} \left( \frac{r_i + y_i}{2} \right).$$

When taking the expectation of this expression over  $Y_i$  and  $R_i$ , we simply replace  $y_i$  with  $x_i$  and  $r_i$  with  $s$ . If prior beliefs are unbiased ( $\mu = s$  on average) then the expected posterior mean of  $X_j$  is

$$\left( \frac{\sigma_L^2 + \sigma_E^2 + \sigma_S^2}{\sigma_L^2 + \sigma_E^2 + 2\sigma_S^2} \right) \mu + \left( 1 - \frac{\sigma_L^2 + \sigma_E^2 + \sigma_S^2}{\sigma_L^2 + \sigma_E^2 + 2\sigma_S^2} \right) x_i.$$

Thus,  $i$ 's expectation of  $j$ 's score lies (on average) between her prior mean  $\mu$  and her actual score, leading to overplacement in expectation after high scores and underplacement in expectation after low scores.<sup>13</sup>

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<sup>13</sup>If  $r_i$  is substantially lower than  $\mu$  when  $\mu < x_i$  then it is possible that the posterior expectation drops below  $\mu$ . Similarly, if  $r_i$  is substantially greater than  $x_i$  then it is possible that the posterior expectation rises

## 2.4.2 Ability and Prior Overconfidence

The simple model does not incorporate the possibility of prior overconfidence since it is not needed to explain the patterns of overconfidence observed in the experimental data. In many situations, however, it may be more appropriate to assume agents' prior beliefs about their idiosyncratic component ( $L_i$ ) is not mean-zero, perhaps due to past experience or to a true behavioral bias. Such beliefs could be incorporated by assuming  $X_i = S + L_i + A_i$ , where  $A_i$  represents  $i$ 's prior ability level. This would be equivalent to  $X_i = S + \hat{L}_i + E[A_i]$ , where  $E[A_i]$  is the mean of  $A_i$  and  $\hat{L}_i$  has a mean of zero. The only changes in the analysis of this model (relative to the case where  $A_i = 0$ ) are that the 'luck' term may now have a larger variance (perhaps affecting the magnitude of overplacement and overestimation) and that the values of  $E[S|x_i]$  and  $E[S|y_i]$  (and, thus,  $E[X_j|x_i]$  and  $E[X_j|y_i]$ ) are shifted by  $E[A_i]$ . In other words, the effect of prior overconfidence is simply added to the results of the basic model; subjects' overplacement is *increased* after unexpectedly easy tasks and *reduced* after unexpectedly difficult tasks.

## 2.4.3 Non-Bayesian Overconfidence

The above mathematical arguments make generous application of Bayes's rule, but the results may also hold for agents whose updating process is not perfectly Bayesian. The only necessary component of the theory is the relative ordering of the prior mean, posterior expectation, and the observed score or signal. If a non-Bayesian subject exhibits the same ordering, then the resulting patterns of overprecision and overestimation will be the same as under Bayes's rule. For example, suppose an agent has normally-distributed priors as in Example 1, but her updating rule results in a posterior of the form  $E[X_j|x_i] = \hat{\alpha}\mu +$   

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above  $x_i$ . Thus, we can observe some individuals whose posterior expectations do not lie between  $\mu$  and  $x_i$ , but in expectation these opposing observations cancel out. In other words, the second signal adds noise to the data but does not change the expected conclusions.

$(1 - \hat{\alpha})x_i$ , where the actual weight  $\hat{\alpha}$  differs from the value  $\alpha$  required by Bayes's rule. As long as  $\hat{\alpha}$  remains between zero and one, the predictions of this section are essentially unchanged; only the magnitudes of overplacement and overestimation will change. Thus, the predictions can apply to Bayesians and non-Bayesians alike, so long as the posterior mean lands between the prior mean and the observed score or signal.

An important consequence of this observation is that our experiment is not a direct test of the details of the model; as with any experiment, we test only the *predictions* of the model. Since we do not impose specific assumptions about which distributions subjects use as their priors, we limit ourselves to examining directional and correlational predictions of the model rather than specific point estimates.<sup>14</sup> We therefore cannot reject any other model that generates qualitatively similar predictions. We are not aware, however, of any other model of overconfidence that predicts both *under*confidence and the observed correlations between overconfidence and task difficulty.

The fact that the same predictions can be generated by certain non-Bayesian behavior also means that our ability to generate the observed patterns of overconfidence using a theoretical model devoid of behavioral biases is quite robust; the result obtains as long as agents have incomplete information about task difficulty and form posterior expectations between prior expectations and received signals. In other words, the overconfidence result stems from the basic intuition of statistical inference rather than from various technical details of the model or the exact nature of Bayes's rule.

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<sup>14</sup>Although we can observe subjects' beliefs about  $X_i$ , beliefs about  $S$ ,  $L_i$ , and  $L_j$  would be needed to generate point predictions. In principle one could elicit beliefs over  $S$ ,  $L_i$ , and  $L_j$ , though this may lead subjects to dissect their predictions in ways that are not natural.

### 3 Experimental Design

Eighty-two undergraduate student participants were recruited from Carnegie Mellon University. Each participated on a computer terminal in the Center for Behavioral Decision Research laboratory. The experiment consisted of 18 rounds in which each participant completed a 10-item trivia quiz and reported various beliefs about their score and the scores of others.<sup>15</sup>

The timing of each round is broken into three phases. In the *ex-ante* phase, subjects know nothing of the content or difficulty of the upcoming quiz. After taking the quiz, subjects enter the *interim* phase in which they have experienced the quiz but do not yet know the correct answers, their score, or the scores of any other participants. In the *ex-post* phase subjects have seen the correct answers and know their own score but do not know the scores of others.

In each of the three phases, subjects are asked to submit a belief distribution about their own score on the quiz and a second distribution about the score of a randomly-selected previous participant (hereafter, RSPP). Subjects are not given any information about the RSPP, other than the fact that the RSPP completed the same 18 quizzes at some prior date.<sup>16</sup> Each probability distribution consisted of eleven probabilities, one for each of the possible scores (zero through ten). Subjects are shown eleven moveable horizontal bars to represent these eleven probabilities and can ‘drag’ each bar to represent the desired probability distribution.<sup>17</sup> Once a subject is satisfied with a particular distribution, she

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<sup>15</sup>The quiz questions and answers are available in the supplemental appendix, along with the mean, median, and variance of the scores for each quiz. To experience the computerized experimental environment, visit <http://cbdr.cmu.edu/roe> and log in using Participant ID 0000.

<sup>16</sup>In early sessions, data from pilot sessions was used as the source for the RSPP. Subjects were not explicitly informed about the pool of subjects from which the RSPP was drawn. Specifically, they were not told the number of subjects in the pool.

<sup>17</sup>Initial bar positions were randomly set each period. Moving one bar caused the ten other bars to adjust proportionally to their current length such that the sum of the bars was continuously equal to 100 percent. Subjects proceeded at their own pace and could spend as much time as needed adjusting these bars.

clicks a button to submit the reported distribution.

Specifically, the timing is as follows: Subjects in the *ex-ante* phase report a distribution for their own score followed by a distribution for the score of the RSPP. They then complete the 10-item trivia quiz. In the *interim* phase (before learning their own score), subjects again report a distribution for their own score and a distribution for the RSPP's score. Subjects are then shown the correct answers and grade their own quizzes.<sup>18</sup> Finally, in the *ex-post* phase each reports a distribution for the RSPP's score.

Subjects earn money from two sources on each quiz in each period. First, if the subject's percentile rank on the quiz is  $r \in [0, 1]$  then she earns  $\$25r$  for her performance.<sup>19</sup> Second, each subject receives payments based on the accuracy of each of her five reported distributions. This is calculated using a quadratic scoring rule. Specifically, if subject  $i$  reports a distribution for her score of  $\hat{p}_i = (\hat{p}_i(0), \hat{p}_i(1), \dots, \hat{p}_i(10))$  and earns an actual score of  $x_i$  on the quiz, then her payment for that report is

$$1 + 2\hat{p}_i(x_i) - \sum_{k=0}^{10} \hat{p}_i(k)^2.$$

An identical formula is used for reports about the distribution of the RSPP's score. This quadratic scoring rule pays between zero and two dollars per report and induces risk-neutral expected utility maximizers to reveal their beliefs truthfully (see, e.g., Reinhard Selten (1998)). Subjects were paid the sum of their earnings across the 18 rounds after the completion of the experiment.

In this setting, a subject can manipulate her quiz performance to increase the accuracy of her reported distributions. For example, a subject could intentionally score zero on the

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<sup>18</sup>Since we did not verify subjects' actual scores during the experiment, they could have incorrectly reported their actual earned score. Upon checking the quizzes after the experiment, we found no such instances of blatant misreports and very few instances of 'questionable' (misspelled or incomplete) answers being counted as correct. We did not remove these data from our analysis.

<sup>19</sup>For the sake of computing the percentile rank  $r$ , participants were counted as having scored better than half and worse than half of those who had obtained the same score.

quiz and predict a score of zero with certainty in both reports about her own score. Since subjects in practice were earning an average of \$12.18 on the quiz and \$2.39 on the two reports about their own score, and since scoring zero on the quiz would earn an average of \$2.54 on the quiz and \$4.00 on the two reports, only the most pessimistic subjects would find such a manipulation profitable. In practice, we do not observe these types of obvious manipulations with significant frequency.<sup>20</sup>

The 18 quizzes span six topics, each at three difficulty levels. The assignment of quizzes to the three difficulty levels (easy, medium, and difficult) was based on previous experience with the questions and on the subjective assessment of the authors.<sup>21</sup> The six topics were geography, movies, music, history, sports, and science. The 18 quizzes were randomly assigned to six blocks of three rounds each such that each block had one quiz of each difficulty level. The three difficulty levels were randomly ordered within each block, and the order in which each subject encountered the six blocks was randomized. This design allows for a relatively uniform distribution of quiz difficulty levels across the 18 rounds while making it difficult for a subject to predict the difficulty or subject matter of an upcoming quiz.<sup>22</sup>

## 4 Results

For each subject in each round we observe five probability distributions: The subject's ex-ante and interim beliefs about her own score, and her ex-ante, interim, and ex-post

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<sup>20</sup>Exact conditions on the beliefs necessary for manipulation to be profitable are explored in a working paper version of this manuscript, available from the authors upon request.

<sup>21</sup>Result 1 verifies that 'easy' quizzes were in fact easy, 'medium' quizzes were intermediate, and 'difficult' quizzes were difficult.

<sup>22</sup>It is true that a difficult quiz is the least likely to appear immediately after a difficult quiz, for example. Since the ordering of difficulty levels within each block is randomized, however, there will be no systematic effect on the results unless there is an interaction between quiz difficulty and within-block ordering effects. Note that if blocks were not used then a subject might encounter all six difficult questions long before the end of the quiz and could then (rationally) expect no additional difficult questions in that quiz.

beliefs about the score of the RSPP. We report the expected values of these distributions (averaged across all players and periods) for each quiz difficulty level in Table 2. Actual score averages appear in the table under the *ex-post* phase.

Before testing measures of overconfidence, we must first verify that our experimental design correctly incentivized subjects to reveal the data needed to compare the results to the predictions of the theory. For example, if subjects are manipulating their quiz performance to increase the accuracy of their predictions then stated beliefs will reflect expectations about manipulations—not true abilities—and the Bayesian model would not apply. Although small manipulations in performance would be difficult to detect, large manipulations are fairly obvious. Scores on easy quizzes averaged 8.86 (out of 10) with a standard deviation of 2.17, so a subject scoring zero or one is most likely “sandbagging” the quiz to make her performance more predictable. Of the 492 easy quizzes, we observe only 11 scores of zero or one.<sup>23</sup> Although these may represent true manipulations, they constitute only about 2 percent of the easy quiz data. Since these data would likely weaken the fit with the model predictions, we do not discard them in our analyses.

## 4.1 The Four Main Results

We now demonstrate four main results using the data: First, the quizzes are well calibrated in the sense that subjects score higher (and correctly believe they score higher) on easy quizzes and score lower (and correctly believe they score lower) on difficult quizzes. Second, subjects do not enter the experiment with significant levels of overplacement. Third, subjects exhibit overplacement on easy quizzes and underplacement on difficult quizzes.

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<sup>23</sup>These 11 low scores are due to 9 different subjects. Alternatively, we can check for manipulations by looking for extreme but correct ex-ante predictions. Subjects correctly reported an ex-ante expected score of zero or one in 21 out of 1476 quizzes and correctly reported an ex-ante expected score of nine or ten in 2 of 1476 quizzes. The latter can occur because subjects self-grade their quizzes and the accuracy of their grading is only checked after the experiment ends.

Difficulty	Phase	Distribution	Block						Overall	
			1	2	3	4	5	6		
Easy	Ex-Ante	Own Score	4.737 (0.20)	5.072 (0.18)	5.301 (0.15)	5.129 (0.17)	5.301 (0.17)	5.412 (0.19)	5.159 (0.07)	
		Other's Score	4.794 (0.16)	5.167 (0.14)	5.192 (0.14)	5.206 (0.12)	5.337 (0.12)	5.204 (0.10)	5.150 (0.05)	
	Interim	Own Score	8.407 (0.23)	8.921 (0.18)	8.808 (0.25)	8.677 (0.26)	8.395 (0.27)	8.657 (0.24)	8.644 (0.10)	
		Other's Score	8.018 (0.17)	8.286 (0.16)	8.300 (0.18)	8.426 (0.16)	8.219 (0.18)	8.306 (0.19)	8.259 (0.07)	
	Ex-Post	Actual Score	8.805 (0.23)	9.122 (0.17)	9.098 (0.23)	8.781 (0.26)	8.500 (0.28)	8.878 (0.25)	8.864 (0.10)	
		Other's Score	8.186 (0.18)	8.496 (0.17)	8.679 (0.17)	8.572 (0.17)	8.515 (0.17)	8.525 (0.17)	8.495 (0.07)	
	Medium	Ex-Ante	Own Score	5.197 (0.17)	5.413 (0.17)	5.224 (0.15)	5.267 (0.19)	5.247 (0.19)	5.482 (0.18)	5.305 (0.07)
			Other's Score	5.249 (0.16)	5.307 (0.15)	5.254 (0.14)	5.275 (0.13)	5.373 (0.15)	5.586 (0.14)	5.341 (0.06)
		Interim	Own Score	5.882 (0.32)	6.325 (0.32)	5.684 (0.36)	5.824 (0.35)	5.946 (0.35)	5.922 (0.35)	5.930 (0.14)
			Other's Score	6.007 (0.24)	6.284 (0.22)	5.964 (0.24)	6.276 (0.23)	6.063 (0.25)	6.312 (0.25)	6.151 (0.10)
		Ex-Post	Actual Score	5.963 (0.34)	6.183 (0.35)	5.573 (0.37)	5.720 (0.36)	5.927 (0.33)	5.476 (0.36)	5.807 (0.14)
			Other's Score	6.197 (0.23)	6.434 (0.20)	5.874 (0.26)	6.207 (0.25)	6.204 (0.23)	6.333 (0.24)	6.208 (0.10)
Difficult		Ex-Ante	Own Score	6.415 (0.19)	5.699 (0.18)	5.514 (0.15)	5.364 (0.17)	5.368 (0.19)	5.419 (0.20)	5.630 (0.08)
			Other's Score	6.467 (0.17)	5.645 (0.13)	5.386 (0.13)	5.250 (0.12)	5.365 (0.14)	5.377 (0.11)	5.582 (0.06)
		Interim	Own Score	1.688 (0.22)	1.560 (0.19)	1.452 (0.18)	1.370 (0.17)	1.407 (0.19)	1.542 (0.21)	1.503 (0.08)
			Other's Score	3.426 (0.23)	2.946 (0.20)	3.141 (0.23)	2.746 (0.20)	2.633 (0.20)	2.814 (0.21)	2.951 (0.09)
		Ex-Post	Actual Score	0.463 (0.10)	0.732 (0.13)	0.451 (0.09)	0.488 (0.10)	0.634 (0.12)	0.646 (0.11)	0.569 (0.04)
			Other's Score	2.834 (0.24)	2.542 (0.20)	2.441 (0.23)	2.049 (0.20)	2.049 (0.18)	2.205 (0.20)	2.353 (0.09)

Table 2: Averages (and standard errors) of expected values of reported belief distributions.

Result	1		2	3			4
Dependant Variable	Score	$E^1(\text{Self})$	$E^0(\text{Self})$ $-E^0(\text{Other})$	$E^1(\text{Self})$ $-E^1(\text{Other})$	Score	$E^1(\text{Self})$ $-E^2(\text{Other})$	$E^1(\text{Self})$ $-\text{Score}$
Easy	<b>8.864</b> (83.48)	<b>8.644</b> (79.68)	0.008 (0.14)	<b>0.385</b> (4.02)	<b>0.369</b> (3.58)	<b>-0.219</b> (-3.98)	
Medium	<b>5.925</b> (55.80)	<b>5.930</b> (54.67)	-0.036 (-0.58)	<b>-0.221</b> (-2.30)	<b>-0.284</b> (-2.76)	0.006 (0.10)	
Difficult	<b>0.693</b> (6.53)	<b>1.503</b> (13.86)	0.048 (0.78)	<b>-1.448</b> (-15.12)	<b>-1.660</b> (-16.14)	<b>0.810</b> (14.69)	

Table 3: Dummy variable regressions demonstrating the four main results. Superscripts indicate ex-ante expectations ( $E^0$ ), interim expectations ( $E^1$ ), or ex-post expectations ( $E^2$ ), and ‘Score’ refers to the subject’s own score. Bold-faced entries are significant at the 5% level.

Fourth, subjects exhibit underestimation on easy quizzes and overestimation on difficult quizzes. The first two results verify that the experimental setting is appropriate, and the last two results verify the predictions of the theory.

These results are each demonstrated by regressions whose estimates and standard errors appear in Table 3. In each regression an appropriate dependant variable is regressed against a full set of dummy variables indicating easy, medium, and difficult quizzes. Each quiz for each subject is treated as an independent observation in these regressions, for a total of 1,476 observations per regression. Each regression was also run including dummy variables for block effects and all interactions between blocks and difficulty levels, but fewer than five percent of these block and interaction estimates are significant at the five percent level, so we omit them from subsequent analysis.<sup>24</sup> Since blocks act as a proxy for time effects such as experience or learning, we can also conclude that overall performance and performances within each difficulty level are all stable across the 18 periods.

The first two regressions in Table 3 give the following result.

<sup>24</sup>The full regressions appear in the supplemental appendix. The significant block and interaction coefficients are Block 1  $\times$  Difficult and Block 5  $\times$  Easy in the regression of  $\text{Score} - E^2(\text{Other})$ , and Block 1  $\times$  Difficult in the regression of  $E^1(\text{Self}) - \text{Score}$ .

**Result 1** *Scores are high on easy quizzes, low on difficult quizzes, and slightly above the overall average on medium quizzes. Subjects correctly perceive these differences immediately after taking the quiz.*

This result is important in verifying that the three difficulty levels produce significantly different scores. If all quizzes produced similar scores, then the experiment would not provide a powerful test of the hard-easy effect predicted by the theory. It is clear from column 2 of Table 3 that in fact scores vary greatly by difficulty level. The average score across all quizzes is 5.16, while scores on easy quizzes are 8.86 points on average and the average score on difficult quizzes is 0.69. The average score on medium quizzes is 5.93, meaning that medium quizzes tend to be closer in performance to easy quizzes than difficult quizzes. These differences are all highly significant.<sup>25</sup> The median and mode are both 10 for easy quiz scores, 0 for difficult quiz scores, and 7 for medium quiz scores.

The regression in column 3 of Table 3 can be used to verify that subjects correctly perceive the differences in difficulty after taking the quiz. Subjects' expectations of their own score are 3.29 points higher than the overall average of 5.36 after an easy quiz and 3.86 points lower after a difficult quiz. These shifts are highly significant. Note also that the shifts in beliefs are slightly smaller than the shifts in actual scores. This is predicted by the model; after taking an easy quiz and receiving a positive (unbiased) signal about her own score, a Bayesian subject expects that her score is not as high as the received signal because high signals are more likely to contain positive errors. A symmetric argument applies to difficult quizzes.

**Result 2** *Subjects do not exhibit significant overplacement or underplacement before taking quizzes.*

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<sup>25</sup>Large-sample Mann-Whitney tests also confirm that the distribution of scores on medium quizzes is significantly different from that of difficult quizzes ( $z$ -stat = 22.83), and that scores on easy quizzes are significantly different from those on medium tests ( $z$  = 17.08).

A key difference between the theoretical model outlined above and the notion of overplacement as an intrinsic bias is that the above model assumes people are not necessarily overconfident *a priori* but become overconfident after positive experiences with the task at hand. When comparing first-period ex-ante expected scores of self versus the ex-ante expected scores of the RSPP we find that subjects on average report a higher expected score for themselves, though the difference is small and insignificant (Wilcoxon signed rank test  $p$ -value of 0.343). Of 82 subjects, 45 reported higher expectations about their own score than the RSPP. A simple binomial test cannot reject the hypothesis that subjects are as likely to exhibit overplacement as underplacement ( $p$ -value of 0.16). A regression of ex-ante overplacement on quiz difficulty (column 4 of Table 3) reveals no significant overplacement or underplacement for any difficulty level. Estimates are all insignificant if the same regression is run using lagged dummy variables, implying that subjects also do not exhibit significant overplacement before period  $t$  after taking an easy quiz in period  $t - 1$ .

**Result 3** *Subjects exhibit overplacement after easy quizzes and underplacement after difficult quizzes. This is true whether or not subjects actually scored better than the randomly-selected previous participant.*

The remaining three regressions from Table 3 (columns 5, 6, and 7) test the predictions of Table 1. We examine two measures of overplacement: interim overplacement and *ex-post* overplacement. In the first measure, subjects are uncertain about their own scores; in the second, they are not. The regression in column 5 indicates that subjects exhibit significant overplacement in the interim phase after an easy quiz and significant underplacement after a difficult or medium quiz. Specifically, subjects expect to out-perform the RSPP by an average of 0.39 points after an easy quiz but expect to be out-performed by an average of 1.45 points after difficult quizzes. The result is similar in the ex-post phase; subjects exhibit overplacement by an average of 0.37 points after easy quizzes and underplacement

Phase	Quiz Difficulty	Expected Ranking	Actual Ranking			Correct Beliefs
			Self < Other	Self = Other	Self > Other	
Interim	Easy	E[Self] < E[Other] - 1/2	15.2%	1.8%	0.6%	17.7%
		E[Self] ≈ E[Other]	7.5%	17.5%	12.8%	37.8%
		E[Self] > E[Other] + 1/2	7.5%	21.1%	15.9%	44.5%
	Medium	E[Self] < E[Other] - 1/2	27.4%	2.6%	7.9%	38.0%
		E[Self] ≈ E[Other]	8.9%	1.2%	13.0%	23.2%
		E[Self] > E[Other] + 1/2	9.3%	5.5%	24.0%	38.8%
	Difficult	E[Self] < E[Other] - 1/2	25.0%	27.4%	10.8%	63.2%
		E[Self] ≈ E[Other]	8.7%	8.3%	7.1%	24.2%
		E[Self] > E[Other] + 1/2	2.0%	3.0%	7.5%	12.6%
ExPost	Easy	E[Self] < E[Other] - 1/2	17.1%	1.4%	1.6%	20.1%
		E[Self] ≈ E[Other]	5.1%	18.5%	12.6%	36.2%
		E[Self] > E[Other] + 1/2	8.1%	20.5%	15.0%	43.7%
	Medium	E[Self] < E[Other] - 1/2	29.3%	2.4%	8.1%	39.8%
		E[Self] ≈ E[Other]	7.3%	2.0%	9.6%	18.9%
		E[Self] > E[Other] + 1/2	9.1%	4.9%	27.2%	41.3%
	Difficult	E[Self] < E[Other] - 1/2	28.0%	28.5%	11.8%	68.3%
		E[Self] ≈ E[Other]	6.7%	8.9%	6.5%	22.2%
		E[Self] > E[Other] + 1/2	1.0%	1.4%	7.1%	9.6%

Table 4: Frequency of subjects exhibiting various rankings of own score vs. other's score, compared to actual score rankings.

by an average of 1.66 points after difficult quizzes.

Recall that scores on medium quizzes (as well as the associated interim and ex-post expectations) are significantly greater than the overall average of 5.16. Since these quizzes are ‘slightly easy’, we should expect to observe some degree of overplacement in the interim and ex-post phases. According to Table 3, the *opposite* result obtains: subjects exhibit slight underplacement on medium quizzes at both the interim phase (by 0.22 points) and ex-post phase (by 0.28 points). In terms of the percentage of subjects exhibiting overplacement, however, there is no significant pattern in the data and overplacement occurs roughly as frequently as underplacement (see column 7 of Table 4). This indicates that the slight underplacement on medium quizzes stems from the fact that, in practice, the magnitude of underplacement is larger than the magnitude of overplacement. This might occur, for example, if subjects’ prior beliefs about the quiz difficulty are not symmetrically distributed.<sup>26</sup>

In the theory of Section 2, the link between quiz difficulty and overplacement stems from the assumption that the posterior expectation of others’ scores ( $E[X_j|x_i]$ ) lies between the prior mean ( $\mu$ ) and the realized score ( $x_i$ ). In practice, this ‘betweenness’ condition is satisfied in 64.8 percent of quizzes.<sup>27</sup> This condition is stronger than necessary; the predicted pattern for overplacement also obtains if  $E[X_j|x_i] < x_i$  when  $x_i > \mu$  and  $E[X_j|x_i] > x_i$  when  $x_i < \mu$ . This weaker sufficient condition is satisfied in 80.1 percent of quizzes in our data.<sup>28</sup>

Assuming  $E[X_j|x_i]$  is in fact a convex combination of the prior mean and the realized

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<sup>26</sup>Subjects in an extensive pilot study who were asked to compare their score against the *median* score of the previous participants (rather than a randomly-selected previous participant) exhibited overplacement after taking the same medium quizzes (by a significant 0.26 points in the interim phase and an insignificant 0.12 points in the ex-post phase). Qualitatively, all other results were the same between the two studies, suggesting that the results for medium quizzes are not particularly robust.

<sup>27</sup>This assumes  $\mu$  is the subjects’ prior expectation of their own score. Using subjects’ prior expectation of the RSPP’s score, betweenness is satisfied in 71.1 percent of the quizzes.

<sup>28</sup>Using subjects’ prior expectation of the RSPP’s score as  $\mu$ , the number increases to 84.4 percent.

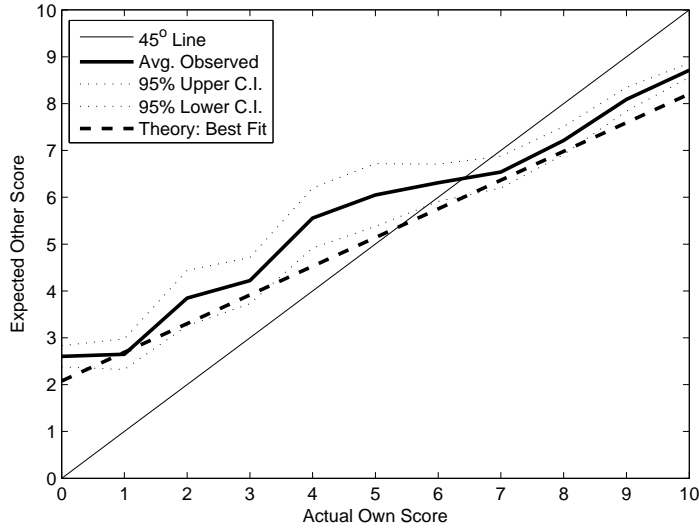


Figure I: Average reported expectation of others' scores versus own score.

score, a simple linear regression of the elicited values of  $E[X_j|x_i]$  against the observed values of  $x_i$  (with the constraint that  $E[X_j|\mu] = \mu$ , where  $\mu = 5.364$  is the average prior expectation of one's own score) provides an estimate for the best-fitting parameters of the theory. A simple least-squares regression indicates that  $E[X_j|x_i] = 0.387\mu + (1 - 0.387)x_i$  with a standard error of 0.012 on the coefficient. This line is plotted against the data in Figure I. Using the beta-binomial specification of Example 2, the resulting beta coefficients are  $(\beta_1, \beta_2) = (3.39, 2.93)$ , indicating a roughly symmetric prior over  $p$  with a mean of 0.5364 (since  $\mu = 5.364$ ) and a skewness of only  $-0.095$ .<sup>29</sup>

The definition of overplacement used in this paper requires only that subjects believe they will score higher than the RSPP; it does not require that this belief be incorrect. In Table 4, we separate the interim and *ex-post* data into those observations in which subjects actually did out-perform the RSPP and those in which they did not, allowing us to examine whether overplacement is generally consistent with actual outcomes. In the in-

<sup>29</sup>Clearly, a more accurate model would allow for parameter heterogeneity across subjects.

terim phase after easy quizzes, for example, 44.5 percent of subjects expected that they had outperformed the RSPP (with a tolerance of  $\pm 1/2$  since expectations are real-valued and actual scores are integer-valued). Of those subjects, only 35.6 percent were correct. Looking across all easy and difficult quizzes and both the interim and ex-post phases, the expected ranking predicted by Result 3 (overplacement for easy quizzes, underplacement for difficult quizzes) is the modal ranking. In each of those six cases, no more than 41.1 percent of the subjects had correct rankings of their expectations. Thus, the predicted overplacement/underplacement pattern is the modal observation even though these beliefs are inaccurate the majority of the time.

**Result 4** *Subjects exhibit underestimation after easy quizzes and overestimation after difficult quizzes.*

Our measure of overestimation is the difference between a subjects' expected score in the interim phase (after taking the quiz) and their actual score. The final regression from Table 3 confirms that, on average, subjects underestimate the score by 0.22 points after easy quizzes and overestimate their score by 0.81 points after difficult quizzes. Overestimation on medium quizzes is essentially absent.

Recall from Section 2 that the standard predictions may fail to hold when prior beliefs are excessively bimodal. In fact, scores on the quizzes are highly bimodal, with nearly half of all observed scores equal to zero or ten. As we discuss in the following Section, however, subjects' beliefs do not exhibit this extreme bimodality, even in later periods where subjects have experienced up to seventeen prior quizzes. In other words, the fit between the data and the predictions was apparently improved by the fact that subjects failed to recognize this bimodality.

## 4.2 Overprecision

Recall that overprecision occurs when agents' belief distributions have lower variance than the distribution of actual outcomes. We consider overprecision about one's own score and about others' scores at the *ex-ante* stage and overprecision about others' scores at the interim and *ex-post* stages. Any measurement of interim or *ex-post* overprecision about one's own score faces the problem that beliefs are conditional on private information ( $Y_i$  in our model), which cannot be observed. Thus, we cannot construct the appropriate empirical distribution that conditions on this information against which the reported beliefs should be compared.

The distribution of actual scores across all difficulty levels is highly bimodal, with 49.8% of all quiz scores equalling zero or ten.<sup>30</sup> On average, the combined weight subjects assigned to these two scores in their *ex-ante* distributions is 16.4% for their own score and 13.3% for the scores of others, both of which are less than the 18.2% weight assigned by a uniform distribution. By the final period, these average combined weights increase to only 24.7% and 23.1% for own and others' scores, respectively – still significantly below the true distribution.<sup>31</sup> Since subjects fail to recognize the bimodality of scores, their reported *ex-ante* variances (across all periods) are lower than the true variance by an average of 10.60 for their own scores and 9.99 for the scores of others.<sup>32</sup>

The overprecision of subjects' interim estimates of others' scores can be tested by comparing the variance of reported beliefs to the distribution of actual scores on that particular

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<sup>30</sup>This bimodality raises the issue of 'floor' and 'ceiling' effects in our design, where beliefs are simply truncated by the maximum and minimum possible score. Such an explanation would only partially explain our results, and Moore and Small (in press) see similar patterns of results without any upper or lower bounds on performance.

<sup>31</sup>Since we are examining *ex-ante* beliefs measured before the quiz questions were revealed, these distributions are not conditioning on quiz difficulty.

<sup>32</sup>Wilcoxon tests verify that these levels of overprecision are significantly different from each other ( $p < 0.001$ ) and greater than zero ( $p < 0.001$ ). These differences in variances drop to 9.16 and 8.67 in the final period, which are both significantly positive ( $p < 0.001$ ) but not significantly different ( $p = 0.298$ ).

quiz.<sup>33</sup> On average, the actual variance is 2.00 units larger than the variance of the reported beliefs. After subjects learn their own scores, this difference increases insignificantly to 2.18.<sup>34</sup> Both are significantly greater than zero, indicating the presence of overprecision in interim beliefs about others' scores.

Recall from Section 2.3 that overprecision should be correlated with the magnitudes of overplacement and overestimation, though the sign of this correlation cannot be predicted without knowing the underlying distributions for  $S$  and  $L_i$ . We explore this correlation in the data using Spearman rank-order correlation coefficients. From Table 5, the correlations between ex-ante overprecision and interim overplacement are all significantly negative, and the correlations between ex-ante overprecision and interim overestimation are all positive and insignificant.<sup>35</sup> The first result suggests that subjects' overprecision stems from overprecision in the idiosyncratic component of their score ('luck') rather than in the common component of their score ('simplicity'). Since subjects' private signals are comprised of the quiz difficulty, their individual luck, and the signal error, the second result indicates that the overprecision in luck must be offset by underprecision in the signal errors. In other words, these results suggest that subjects are overestimating the correlation between their scores and the scores of others, but underestimating the quality of their private signals about their own scores.

## 5 Previous Literature

A variety of other papers explore overconfidence under differing assumptions. Overconfidence has previously been modeled as stemming from other judgement biases (Matthew

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<sup>33</sup>Since these are interim beliefs, they are conditional on quiz difficulty.

<sup>34</sup>The  $p$ -value of the Wilcoxon-Mann-Whitney test is 0.149.

<sup>35</sup>Here, ex-ante overprecision is the variance of the distribution of all scores minus the variance of the subject's ex-ante distribution of her own score.

Difficulty	Overplacement	Overestimation
Easy	-0.137 (0.002)	0.073 (0.107)
Medium	-0.164 (< 0.001)	0.029 (0.517)
Difficult	-0.177 (< 0.001)	0.021 (0.645)

Table 5: Spearman correlation coefficients (and  $p$ -values) between ex-ante overprecision and interim overconfidence measures.

Rabin and Joel Schrag (1999)) or from rational choice when beliefs are flexible and overconfidence affects other interactions and motivations (e.g., Roland Benabou and Jean Tirole (2002), Juan D. Carillo and Thomas Mariotti (2000), or Oliver Compte and Andrew Postlewaite (2004)). Other research (March and Shapira (1987), Odean (1998), Daniel, Hirshleifer and Subrahmanyam (2001), and Malmendier & Tate (2005)) has demonstrated how overconfidence can lead to meaningful economic consequences. Our paper differs from these in that we assume no biases in judgement nor do we presume that overconfidence is beneficial in either the task at hand or future interactions; instead, we show how overconfidence (and underconfidence) can arise from Bayesian inference about others' performances after observing a signal of one's own performance.

Eric Van den Steen (2004) (hereafter VdS) provides a Bayesian model of overplacement driven by assuming heterogeneous priors. In his setup, agents choose their most-preferred action from a set  $A = \{a_1, a_2, \dots, a_n\}$ . Actions will either succeed or fail and each person  $i$  believes each action  $a_n$  will succeed with probability  $p_n^i$ . Suppose person 1 believes  $a_1$  has the highest probability of success and person 2 believes  $a_2$  has the highest probability of success. Clearly, person 1 picks  $a_1$  and person 2 picks  $a_2$ . If the two agents' priors are independent (meaning person  $i$  learns nothing by observing that person  $j$  chose  $a_j \neq a_i$ ), then each person will conclude that the other made an inferior choice. Thus, each will exhibit overplacement.

Luis Santos-Pinto and Joel Sobel (2005) (hereafter SP&S) provide a model that generalizes VdS in which agents choose the optimal skills to acquire in order to maximize their overall ability. The basic intuition of their model is captured by the following (simplified) example: Suppose person 1 and person 2 are trying to maximize different functions, denoted  $f_1(x)$  and  $f_2(x)$ , respectively. Think of  $x$  as a vector of skills and  $f_i(x)$  as  $i$ 's perception of his ability level at some task, given skills  $x$ .<sup>36</sup> Clearly, the optimal choices ( $x_1^*$  and  $x_2^*$ ) are likely to differ between the two agents, in which case we should expect that  $f_1(x_1^*) > f_1(x_2^*)$  and  $f_2(x_1^*) < f_2(x_2^*)$ . Thus, if person 1 evaluates person 2's choice using  $f_1$ , person 1 will conclude that he has made the better choice and therefore has the higher ability. In this way, both agents can exhibit overplacement.<sup>37</sup>

In both the VdS and SP&S models, overplacement stems from agents using their own objective function to compare their choices against the choices of others. By contrast, our model assumes all agents are attempting to maximize the same objective function (total quiz score) but maximization is imperfect and is more difficult in some tasks than in others. It is the simultaneous inference about the task's difficulty and one's own performance that leads to the overplacement and underplacement observed in the data.

A second difference between VdS and SP&S and the current model is that the former imply prior overplacement but the latter does not. Our experiments reveal no significant prior overplacement. Although the results of Camerer and Lovallo (1999) (henceforth C&L) are cited as evidence in favor of an overconfidence bias, they too find no strong evidence for *prior* overconfidence. In their setting, subjects choose whether or not to enter

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<sup>36</sup>To represent the VdS model as a special case of the SP&S environment, think of  $x$  as an  $n$ -vector with  $x_k = 1$  if action  $a_k$  is chosen and zero otherwise, and let  $f_i(x) = \sum_{k=1}^n x_k p_k^i$ .

<sup>37</sup>The full SP&S model is significantly more complex; agents aim to maximize  $f(x, \lambda_i)$  subject to  $x \in A(I_i)$  where  $\lambda_i$  and  $I_i$  are individual-specific parameters drawn from a known distribution. Person  $i$  then compares  $f(x_i^*, \lambda_i)$  against  $f(x_j^*, \lambda_i)$  and concludes that  $x_i^*$  was a (weakly) better choice than  $x_j^*$ . SP&S then derive conditions under which the fraction of individuals who believe they are in the top  $p$ -cile of ability levels in the population is (weakly) greater than  $p$ . This condition defines overplacement at the population level.

a market in which entrants' profitability depends on their assigned 'rankings'. They study three treatments: in the first, rankings are randomly chosen after the entry decisions are made. In the second and third, entrants are ranked based on their performance in a trivia quiz. Subjects in the third treatment are told before choosing to participate that their payoff will depend on their score on a trivia quiz, while subjects in the second treatment are not. Thus, the third treatment introduces a self-selection bias that is likely to favor higher scores on the quizzes.

C&L find that subjects over-enter (relative to the risk-neutral equilibrium prediction) in the self-selection treatment, apparently because they fail to recognize that their competitors also self-selected into the experiment.<sup>38</sup> Comparing the first two treatments reveals that subjects enter more frequently when rankings are quiz-based, but the aggregate level of entry is at or below the risk-neutral equilibrium prediction in both cases.<sup>39</sup> Thus, the C&L study highlights the role of competition (and beliefs about one's competitors) in generating overconfidence, but the lack of prior overconfidence in the absence of the self-selection bias is consistent with our observations.

Although we do not address the role of competition in the current study, the particular structure of incomplete information we assume has been used in other game theoretic models. In some cases, the results are indicative of the kind of 'rational overplacement' we describe. For example, Carl Shapiro (1986) assumes that firms in an oligopoly market have constant marginal costs and that each firm's marginal cost is drawn from a common prior distribution with imperfect positive correlation between firms. With little or no correlation, a firm with unexpectedly low marginal costs will conclude that its competitors' marginal

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<sup>38</sup>An alternative explanation is that those subjects who self-select into the experiment are those who have had unexpectedly high scores on previous trivia quizzes and consequently increased their expectation of their own trivia-quiz ability (see Section 2.4.2).

<sup>39</sup>Lower entry rates in the randomly-assigned rankings treatment may be attributable to risk-averse subjects (correctly) believing that payoffs in the quiz-based rankings treatment have a lower variance than in the randomly-assigned rankings treatment.

costs will not be as low as its own. In the symmetric Bayes-Nash equilibrium of the game, this low-cost firm will produce a fairly large quantity because it perceives a significant cost advantage ('overplacement'). If, on the other hand, costs are highly correlated, the low-cost firm believes other firms are likely to have similarly low costs, and so its equilibrium output is reduced.<sup>40</sup> In other words, low-cost firms produce more (and high-cost firms produce less) in exactly those situations where they exhibit a greater degree of overplacement (or underplacement).

## 6 Discussion

This paper accomplishes three goals. First, we clearly define various distinct notions of overconfidence that previous research has occasionally muddled. Second, we demonstrate that overconfidence can be predicted in a theoretical model without assuming any biases in judgement. This is achieved through a simple model of incomplete information regarding task difficulty. The model makes testable predictions about the correlations between the various notions of overconfidence and task difficulty. Finally, we confirm these testable predictions in an experimental study.

There have been a number of recent economic models that have attempted to explain how rational Bayesian agents could display overconfidence (for example, Benabou and Tirole (2002), Ronit Bodner and Drazen Prelec (2003), Rabin and Schrag (1999), or Van den Steen (2004)). Although overconfidence has been widely observed, none of these models can parsimoniously account for the evidence from the present experiment because they predict neither the systematic *under*confidence nor the correlations between overplacement, overestimation, and task difficulty observed in these results.

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<sup>40</sup>To see this from Shapiro's paper, simply compare the equilibrium with imperfect correlation ( $\rho < 1$ ) to the equilibrium with perfect correlation ( $\rho = 1$ ), which is the Nash equilibrium of the standard game with complete information.

We believe that the tendency for studies to focus on overconfidence (rather than underconfidence) may be attributable to methodology. For example, several studies examine individuals' beliefs about a single question, which confounds overestimation with overprecision since a more extreme probability estimate necessarily implies a lower variance (see, e.g., Joseph W. Alba and J. Wesley Hutchinson (2000) or Baruch Fischhoff *et al.* (1977)), making it impossible to determine the degree to which each is responsible for the result. These results, therefore, cannot provide unambiguous evidence for the existence of systematic overestimation. Our results lead us to speculate that these prior results may be more attributable to overprecision than to overestimation.

Additionally, overplacement and overestimation have not occurred in the same studies. Those studies in which people overestimate their absolute performance the most have tended to focus on contexts in which performance is low and success is rare (Peter Juslin *et al.* (2000), Malmendier and Tate (2005), or Neil D. Weinstein (1980)). Those studies in which people overplace their relative ranking the most have tended to focus on contexts in which performance is high and success is likely (College Board (1977), Kruger (1999), David M. Messick *et al.* (1985), or Ola Svenson, (1981)).

Although our model is Bayesian, there is ample reason to question whether people actually make judgments according to Bayes's rule. Under some circumstances, people appear to neglect priors (such as base rates), overweighting recent evidence (see, e.g., David M. Grether (1980, 1990)). Under other circumstances, people appear too conservative, overweighting priors and neglecting useful new evidence (e.g., Ward Edwards (1968) or Richard D. McKelvey and Talbot Page (1990)). Which of these errors people commit depends on the order and form in which they acquire information (e.g., Robin M. Hogarth and Hillel J. Einhorn (1992) or Gary L. Wells (1992)). What is important for our purposes here, however, is that although people are imperfect Bayesians, they rarely abandon Bayesian

logic completely.<sup>41</sup> Overweighting the prior or overweighting the data still leads posterior means to lie somewhere between the prior mean and the observed data, generating the same patterns of overconfidence and underconfidence predicted by our model and observed in our data.

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<sup>41</sup> See the discussion from Section 2.4.3.

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# Appendices

Not Intended for Publication

## **A Full Regression Results**

The complete regression results (including difficulty, block, and interaction effects) are provided in Table 6. These regressions omit one dummy variable and code the remaining dummies as negative one for the omitted category. This procedure allows the inclusion of a constant term, giving an estimate for the overall average, and guarantees that the treatment effects sum to zero, as in an ANOVA procedure. The significance results are equivalent to a regression with a full set of dummies and no constant term, as in the manuscript (see, for example, Neter *et al.* (1996, p. 696)), such as those provided in the manuscript.

## **B The Quizzes**

The eighteen trivia quizzes are shown in Table 7, with one quiz per page. The mean, median, and variance of scores on the quiz are shown for each quiz, along with the quiz ID number (1 through 18), the topic, and the difficulty level. Recall that quizzes were randomly placed into blocks with one difficulty level per block, and the order in which each subject encountered the six blocks was randomized. Therefore, the quiz ID numbers do not represent the order in which quizzes were shown to subjects.

Result	1		2	3		4
Dependant Variable	Score	$E^1(\text{Self})$	$E^0(\text{Self})$ $-E^0(\text{Other})$	$E^1(\text{Self})$ $-E^1(\text{Other})$	Score $-E^2(\text{Other})$	$E^1(\text{Self})$ $-Score$
Constant	<b>5.161</b> (84.06)	<b>5.359</b> (85.34)	0.007 (0.19)	<b>-0.428</b> (-7.72)	<b>-0.525</b> (-8.86)	<b>0.199</b> (6.26)
Easy	<b>3.703</b> (42.65)	<b>3.285</b> (36.99)	0.002 (0.03)	<b>0.813</b> (10.38)	<b>0.894</b> (10.66)	<b>-0.418</b> (-9.31)
Difficult	<b>-4.467</b> (-51.46)	<b>-3.856</b> (-43.42)	0.041 (0.82)	<b>-1.020</b> (-13.03)	<b>-1.135</b> (-13.54)	<b>0.611</b> (13.60)
Block 1	-0.043 (-0.31)	-0.034 (-0.24)	-0.061 (-0.76)	-0.064 (-0.51)	-0.096 (-0.73)	0.009 (0.13)
Block 2	<i>0.234</i> (1.70)	<i>0.243</i> (1.73)	0.015 (0.19)	0.191 (1.54)	0.096 (0.72)	0.009 (0.13)
Block 3	-0.055 (-0.40)	-0.045 (-0.32)	0.062 (0.77)	-0.059 (-0.48)	-0.034 (-0.25)	0.010 (0.15)
Block 4	-0.055 (-0.40)	-0.069 (-0.49)	0.003 (0.04)	-0.098 (-0.79)	0.021 (0.16)	-0.014 (-0.20)
Block 5	-0.055 (-0.40)	-0.110 (-0.78)	-0.060 (-0.75)	0.039 (0.31)	0.041 (0.31)	-0.055 (-0.77)
B1*Easy	-0.016 (-0.08)	-0.204 (-1.02)	-0.005 (-0.04)	0.068 (0.39)	<i>0.347</i> (1.85)	<i>-0.187</i> (-1.86)
B1*Diff.	-0.114 (-0.59)	0.218 (1.10)	-0.039 (-0.35)	-0.227 (-1.30)	<b>-0.541</b> (-2.89)	<b>0.332</b> (3.31)
B2*Easy	0.024 (0.13)	0.034 (0.17)	-0.119 (-1.05)	0.059 (0.34)	0.162 (0.87)	0.010 (0.10)
B2*Diff.	-0.183 (-0.94)	-0.186 (-0.94)	-0.009 (-0.08)	-0.130 (-0.74)	-0.233 (-1.25)	-0.003 (-0.03)
B3*Easy	0.289 (1.49)	0.209 (1.05)	0.039 (0.34)	0.182 (1.04)	0.084 (0.45)	-0.080 (-0.80)
B3*Diff.	-0.053 (-0.27)	-0.007 (-0.03)	0.018 (0.16)	-0.182 (-1.04)	-0.162 (-0.86)	0.046 (0.46)
B4*Easy	-0.028 (-0.15)	0.102 (0.51)	-0.088 (-0.78)	-0.036 (-0.21)	-0.182 (-0.97)	0.130 (1.30)
B4*Diff.	0.020 (0.10)	-0.064 (-0.32)	0.063 (0.56)	0.170 (0.97)	0.249 (1.33)	-0.085 (-0.84)
B5*Easy	-0.309 (-1.59)	-0.139 (-0.70)	0.016 (0.14)	-0.248 (-1.41)	<b>-0.425</b> (-2.27)	<i>0.170</i> (1.69)
B5*Diff.	0.081 (0.42)	0.014 (0.07)	0.015 (0.13)	0.183 (1.05)	0.289 (1.54)	-0.067 (-0.67)

Table 6: Dummy variable regressions (with block and interaction effects) demonstrating the four main results. Superscripts indicate *ex-ante* expectations ( $E^0$ ), *interim* expectations ( $E^1$ ), or *ex-post* expectations ( $E^2$ ), and ‘Score’ refers to the subject’s own score. Bold-faced entries are significant at the 5% level, italicized entries are significant at the 10% level.

**Quiz: 1**    **Topic:** Geography    **Difficulty:** Easy  
**Mean:** 8.817    **Median:** 10    **Variance:** 3.435

- 
- Question 1    What continent lies directly south of Europe?  
*Answer: Africa*
- Question 2    What African river (that empties into the Mediterranean sea through Egypt) is the longest river in the world?  
*Answer: Nile*
- Question 3    In what North American country is the city of Toronto located?  
*Answer: Canada*
- Question 4    The lowest temperature ever recorded on earth (-129.3F) occurred on what southern continent?  
*Answer: Antarctica*
- Question 5    In what western U.S. state is the Silicon Valley?  
*Answer: California*
- Question 6    Baghdad is the capital of what middle-eastern country?  
*Answer: Iraq*
- Question 7    The Golden Gate Bridge is located in which Californian city?  
*Answer: San Francisco*
- Question 8    What is the capital of the United States?  
*Answer: Washington D.C.*
- Question 9    On what continent is France located?  
*Answer: Europe*
- Question 10    What is the capital of and largest city in Japan?  
*Answer: Tokyo*

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Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz:</b> 2	<b>Topic:</b> Geography	<b>Difficulty:</b> Medium
<b>Mean:</b> 6.098	<b>Median:</b> 7	<b>Variance:</b> 7.299
Question 1	In what U.S. city does the best-known celebration of Mardi Gras take place? <i>Answer: New Orleans</i>	
Question 2	In what U.S. state is the ski-resort town of Aspen? <i>Answer: Colorado</i>	
Question 3	The “Ring of Fire” is located around what ocean? <i>Answer: Pacific</i>	
Question 4	In what U.S. state is Atlantic City located? <i>Answer: New Jersey</i>	
Question 5	The most famous “tea party” of the American Revolution took place in what city? <i>Answer: Boston</i>	
Question 6	The island of Honshu is part of what country? <i>Answer: Japan</i>	
Question 7	What is the name of the worlds largest coral reef, located off the coast of Australia? <i>Answer: Great Barrier Reef</i>	
Question 8	What is the highest mountain range in the world? <i>Answer: Himalayas</i>	
Question 9	What country is comprised of over 17,000 known islands? <i>Answer: Indonesia</i>	
Question 10	In what U.S. state is Disney World located? <i>Answer: Florida</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

**Quiz:** 3      **Topic:** Geography      **Difficulty:** Hard  
**Mean:** 0.683      **Median:** 0      **Variance:** 1.849

Question 1      What is South America's highest peak?

*Answer: Mt. Aconcagua*

Question 2      What is the capital of Australia?

*Answer: Canberra*

Question 3      What two South American countries are land-locked?

*Answer: Bolivia and Paraguay*

Question 4      What geographical area was once referred to as "Seward's Folly?"

*Answer: Alaska*

Question 5      What is the capital city of Uganda?

*Answer: Kampala*

Question 6      What Pacific island mountain claims to be the wettest spot on Earth?

*Answer: Mt. Waialeale (on the Hawaiian island of Kauai)*

Question 7      Sweden, Denmark, Poland, and Finland all border what sea?

*Answer: Baltic Sea*

Question 8      Bechuanaland was the colonial name of what country?

*Answer: Botswana*

Question 9      What is the capital of Turkey?

*Answer: Ankara*

Question 10      What two countries border Mexico to the south?

*Answer: Belize and Guatemala*

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Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz:</b> 4	<b>Topic:</b> Movies	<b>Difficulty:</b> Easy
<b>Mean:</b> 8.939	<b>Median:</b> 10	<b>Variance:</b> 5.12
Question 1	Leonardo DiCaprio and Kate Winslet starred in a 1997 film about the sinking of what famous ship after striking an iceberg on her maiden voyage? <i>Answer: Titanic</i>	
Question 2	Arnold Swartzeneger, current governor of California, is sometimes called the Governator a nickname poking fun at his role in what 1984 film? <i>Answer: Terminator</i>	
Question 3	Toby Maguire starred in two movies as what web-slinging super hero with spider powers? <i>Answer: Spiderman</i>	
Question 4	Cinderella, The Little Mermaid, Aladdin, and The Lion King are all films produced by what famous entertainment company? <i>Answer: Disney</i>	
Question 5	Keanu Reeves starred as Neo, a computer hacker, in what film trilogy about machines taking over the earth? <i>Answer: The Matrix</i>	
Question 6	What was the title of George Lucas's original 1977 science fiction film about Luke Skywalker and Darth Vader, set "Long ago, in a galaxy far, far away"? <i>Answer: Star Wars</i>	
Question 7	What recent film trilogy was based on J.R.R. Tolkeins novels about the quest of a hobbit named Frodo to destroy a ring? <i>Answer: Lord of the Rings</i>	
Question 8	The quotes Life is like a box of chocolates and Run, Forrest, run! are from what 1994 movie starring Tom Hanks? <i>Answer: Forrest Gump</i>	
Question 9	J.K. Rowling's books tell about a young wizard named Harry who goes to a school called Hogwarts. What is Harry's last name? <i>Answer: Potter</i>	
Question 10	What actor, formerly married to Nicole Kidman, starred in the films Rain Man, Minority Report, War of the Worlds, Mission Impossible, and Jerry Maguire? <i>Answer: Tom Cruise</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz: 5</b>	<b>Topic:</b> Movies	<b>Difficulty:</b> Medium
<b>Mean:</b> 6.293	<b>Median:</b> 7	<b>Variance:</b> 12.333
Question 1	Who starred as high school student Marty McFly in the hit science-fiction comedy “Back to the Future” (1985)? <i>Answer: Michael J. Fox</i>	
Question 2	Who wrote and directed Kill Bill Volumes 1 and 2? <i>Answer: Quentin Tarantino</i>	
Question 3	What actor plays an overweight college instructor & several other characters in The Nutty Professor? <i>Answer: Eddie Murphy</i>	
Question 4	Who was the first African American to win an Academy Award for best actress? <i>Answer: Halle Berry</i>	
Question 5	What actress co-starred with Tom Hanks in the movies Sleepless In Seattle, You’ve Got Mail, and Joe Versus the Volcano? <i>Answer: Meg Ryan</i>	
Question 6	Ben Affleck and Matt Damon won an Academy Award for writing the screenplay of what film, starring themselves and Robin Williams? <i>Answer: Good Will Hunting</i>	
Question 7	Mo, Larry, and Curly are a trio more commonly known by what name? <i>Answer: The Three Stooges</i>	
Question 8	What film about a female boxer, starring Hilary Swank and Clint Eastwood, won an Academy Award for best picture in 2005? <i>Answer: Million Dollar Baby</i>	
Question 9	Dorothy, Scarecrow, Tinman, and the Wicked Witch of the West are all characters from what movie? <i>Answer: The Wizard of Oz</i>	
Question 10	What was the full name of the cannibalistic main character in the film Silence of the Lambs? <i>Answer: Hannibal Lecter</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz: 6</b>	<b>Topic:</b> Movies	<b>Difficulty:</b> Hard
<b>Mean:</b> 0.561	<b>Median:</b> 0	<b>Variance:</b> 1.039
Question 1	Who is the only actress to have been nominated for the “Best Actress” Academy Award 13 times? <i>Answer: Meryl Streep</i>	
Question 2	Who played Mozart in the 1984 film “Amadeus”? <i>Answer: Tom Hulce</i>	
Question 3	Who is the father of actress Gwyneth Paltrow? <i>Answer: Bruce Paltrow</i>	
Question 4	In what film did actress Mae West say the line, “When I’m good, I’m very good, but when I’m bad I’m better”? <i>Answer: I’m No Angel</i>	
Question 5	What actress performed the voice of Bo Peep in the film Toy Story? <i>Answer: Annie Potts</i>	
Question 6	Who starred opposite Gene Wilder in the 1984 film “Woman in Red”? <i>Answer: Kelly LeBrock</i>	
Question 7	What 1984 film was the big-screen debut of actress Sarah Jessica Parker? <i>Answer: Footloose</i>	
Question 8	In Monty Python’s Holy Grail, the French soldier taunts King Arthur by telling him, “Your mother was a hamster and your father smelt of _____”? <i>Answer: Elderberries</i>	
Question 9	What actress played Ferris Bueller’s girlfriend in the 1986 film, “Ferris Bueller’s Day Off”? <i>Answer: Mia Sara</i>	
Question 10	What actor holds the record for having been nominated most frequently for the “Best Actor” Academy Award (9 times)? <i>Answer: Spencer Tracy</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz: 7</b>	<b>Topic:</b> Music	<b>Difficulty:</b> Easy
<b>Mean:</b> 8.317	<b>Median:</b> 10	<b>Variance:</b> 7.232
Question 1	John Lennon, Paul McCartney, George Harrison and Ringo Starr were the four members of what famous classic rock band? <i>Answer: The Beatles</i>	
Question 2	Janet Jackson and Latoya Jackson are sisters of what famous and eccentric pop star? <i>Answer: Michael Jackson</i>	
Question 3	What pop group, led by Bono, has created the following albums: The Joshua Tree, Rattle and Hum, Achtung Baby, Zooropa, and How to Dismantle an Atomic Bomb? <i>Answer: U2</i>	
Question 4	The pop star known for such songs as Like a Virgin, Material Girl, and Like a Prayer, goes by what single name (rather than a first and last name)? <i>Answer: Madonna</i>	
Question 5	What famous classical composer, who eventually went deaf, wrote 9 symphonies, the most famous of which is his 5th symphony? <i>Answer: Beethoven</i>	
Question 6	Which famous Britney of the pop music world recently married and had a baby with her back-up dancer? <i>Answer: Britney Spears</i>	
Question 7	Ashlee Simpson is the younger sister of what pop singer and star of the reality show Newlyweds on MTV? <i>Answer: Jessica Simpson</i>	
Question 8	Singer Celine Dion sang the hit song My heart will go on for the soundtrack of what 1997 film about the sinking of a famous ship? <i>Answer: Titanic</i>	
Question 9	Stevie Wonder and Ray Charles both wore sunglasses during performances and had what physical disability? <i>Answer: Blindness</i>	
Question 10	Sporty Spice, Baby Spice, and Posh Spice were members of what musical group? <i>Answer: Spice Girls</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz:</b> 8	<b>Topic:</b> Music	<b>Difficulty:</b> Medium
<b>Mean:</b> 6.415	<b>Median:</b> 8	<b>Variance:</b> 13.456
Question 1	What is the real name of the artist who once went by the pseudonyms “Puffy”, “Puff Daddy”, and “P. Diddy”? <i>Answer: Sean Combs</i>	
Question 2	What band was Justin Timberlake in? <i>Answer: N-Sync</i>	
Question 3	Who is considered The King of rock? <i>Answer: Elvis Presley</i>	
Question 4	Former pop star Paula Abdul is now a judge for what T.V. show? <i>Answer: American Idol</i>	
Question 5	Ozzy and Sharon Ozbourne took their music careers to television when they began a reality show about their family on what TV station? <i>Answer: MTV</i>	
Question 6	Jim Morrison (lead singer of The Doors), Elvis Presley, and Jimi Hendrix all died from what? <i>Answer: Drug overdoses</i>	
Question 7	Mick Jagger, at the age of 62, recently went on tour with what legendary rock band for which he sings lead vocals? <i>Answer: Rolling Stones</i>	
Question 8	What was the name of the famous 3-day music festival, held in 1969, which featured artists such as Bob Dylan, Janis Joplin, and Jimi Hendrix? <i>Answer: Woodstock</i>	
Question 9	Beyonce Knowles rejoined what musical group in 2003? <i>Answer: Destinys Child</i>	
Question 10	Gwen Stefani launched a successful solo career, but is also the lead singer of what band? <i>Answer: No Doubt</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz:</b> 9	<b>Topic:</b> Music	<b>Difficulty:</b> Hard
<b>Mean:</b> 0.451	<b>Median:</b> 0	<b>Variance:</b> 1.806
Question 1	On which album does actor William Shatner sing “Lucy in the Sky with Diamonds”? <i>Answer: The Transformed Man</i>	
Question 2	What is the real name of U2’s guitarist “The Edge”? <i>Answer: David Evans</i>	
Question 3	What, according to Billboard, was the top-ranked movie soundtrack album for the year 2004, at #6? <i>Answer: O Brother, Where Art Thou?</i>	
Question 4	What album ranks #1 on the best-selling albums of all time, just ahead of Michael Jackson’s “Thriller”? <i>Answer: The Eagles: Their Greatest Hits</i>	
Question 5	The first video played on MTV was “Video killed the radio star” by the Buggles. Who was the Buggles’ lead singer? <i>Answer: Mike Scott</i>	
Question 6	Former Texas gubernatorial candidate Kinky Friedman sang the song “Get your biscuits in the oven and your buns in the bed” with which band? <i>Answer: Kinky Friedman and the Texas Jewboys</i>	
Question 7	Who wrote the theme song to the television series “Hill Street Blues”? <i>Answer: Mike Post</i>	
Question 8	What was the name of the person who shot and killed Marvin Gaye one day before his 45th birthday? <i>Answer: Marvin Gaye, Senior (his father)</i>	
Question 9	What three famous musicians died together in a plane crash on February 3rd, 1959? <i>Answer: Buddy Holly, Ritchie Valens, and J.P. Richardson (“The Big Bopper”)</i>	
Question 10	The band “New Kids on the Block” included Donnie Wahlberg, Daniel Wood, Jordan Knight, and Jonathan Knight. Who was the fifth member until 1985? <i>Answer: Jamie Kelley</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz:</b> 10	<b>Topic:</b> History	<b>Difficulty:</b> Easy
<b>Mean:</b> 9.598	<b>Median:</b> 10	<b>Variance:</b> 0.861
Question 1	What U.S. president faced an impeachment trial to investigate his alleged affair with White House intern Monica Lewinsky? <i>Answer: Bill Clinton</i>	
Question 2	What explorer is credited with discovering America in 1492? <i>Answer: Christopher Columbus</i>	
Question 3	In 1776, the United States declared independence from what country? <i>Answer: Great Britain or England</i>	
Question 4	On what country did the United States drop atomic bombs on during World War 2? <i>Answer: Japan</i>	
Question 5	Who was the leader of Germany's Nazi party during World War 2? <i>Answer: Adolph Hitler</i>	
Question 6	On what date in 2001 were airplanes crashed into New York's World Trade Centers, destroying them? <i>Answer: September 11th</i>	
Question 7	What structure was built over 2000 years ago in China and stretches 4,163 miles? <i>Answer: The Great Wall of China</i>	
Question 8	At the beginning of what year, known as "Y2K" were computers expected to crash due to the "Millennium bug"? <i>Answer: 2000</i>	
Question 9	Who was elected President of the United States in 2004? <i>Answer: George W. Bush</i>	
Question 10	How many World Wars have there been? <i>Answer: Two</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz:</b> 11	<b>Topic:</b> History	<b>Difficulty:</b> Medium
<b>Mean:</b> 5.720	<b>Median:</b> 7	<b>Variance:</b> 8.97
Question 1	Who painted the Sistine Chapel? <i>Answer: Michelangelo</i>	
Question 2	The Red Scare was a fear of what political system? <i>Answer: Communist</i>	
Question 3	The Korean War was fought in which decade? <i>Answer: 1950s</i>	
Question 4	The New Deal is most closely associated with what U.S. president? <i>Answer: Franklin Delano Roosevelt</i>	
Question 5	Who was the Native American woman who accompanied Lewis and Clark on part of their expedition in the Pacific Northwest? <i>Answer: Sacajaweeja</i>	
Question 6	What period in European cultural history took place from the 14th to the 17th century, after the Dark Ages? <i>Answer: Renaissance</i>	
Question 7	What famous believer in non-violent protest led India to freedom from the rule of Great Britain? <i>Answer: Mohandas "Mahatma" Gandhi</i>	
Question 8	The Spanish Conquistadors were soldiers who conquered land for Spain and spread what religion? <i>Answer: Christianity</i>	
Question 9	The Italian village of Pompeii was destroyed in 79 AD by what type of natural disaster? <i>Answer: Volcano</i>	
Question 10	Elliot Ness and the Untouchables are associated with the pursuit of what prohibition-era Chicago mafia figure? <i>Answer: Al Capone</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz:</b> 12	<b>Topic:</b> History	<b>Difficulty:</b> Hard
<b>Mean:</b> 1.207	<b>Median:</b> 1	<b>Variance:</b> 2.759
Question 1	In what year did Martin Luther post his 95 theses? <i>Answer: 1517</i>	
Question 2	In what year did Nigeria gain its independence from Great Britain? <i>Answer: 1960</i>	
Question 3	What former First Lady is credited with having coined the phrase “Absence makes the heart grow fonder”? <i>Answer: Eleanor Roosevelt</i>	
Question 4	In the story of the Trojan War, what son of King Priam dies in battle with the Greek warrior Achilles? <i>Answer: Hector</i>	
Question 5	In what year did Abraham Lincoln deliver the Gettysburg Address? <i>Answer: 1863</i>	
Question 6	In what year was Norwegian Viking Erik the Red lead the first European settlement of North America? <i>Answer: 986</i>	
Question 7	The stock market crash of 1929, remembered as the beginning of the Great Depression, occurred most dramatically on what day? <i>Answer: “Black” Monday (October 28, 1929)</i>	
Question 8	During the Second Punic War, what Carthaginian general led his army on a famous crossing of the Alps? <i>Answer: Hannibal</i>	
Question 9	What former U.S. vice president killed Alexander Hamilton in a duel? <i>Answer: Aaron Burr</i>	
Question 10	John Adams and Thomas Jefferson, the 2nd and 3rd Presidents of the United States, both died on what day (date and year)? <i>Answer: July 4, 1826</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz:</b> 13	<b>Topic:</b> Sports	<b>Difficulty:</b> Easy
<b>Mean:</b> 8.537	<b>Median:</b> 10	<b>Variance:</b> 7.289
Question 1	What does N.F.L. stand for ? <i>Answer: National Football League</i>	
Question 2	The Tour de France, which Lance Armstrong has won 7 times, is the premier competition in what sport? <i>Answer: Cycling</i>	
Question 3	What sport did Babe Ruth play? <i>Answer: Baseball</i>	
Question 4	What state do the Yankees play for? <i>Answer: New York</i>	
Question 5	What does WNBA stand for? <i>Answer: Women's National Basketball Association</i>	
Question 6	What Chicago Bulls guard wore number 23 and led his team to 6 championships in the 1990's? (Hint: his last name is Jordan) <i>Answer: Michael Jordan</i>	
Question 7	Ervin Magic Johnson plays what sport? <i>Answer: Basketball</i>	
Question 8	Mohammed Ali and Mike Tyson have each held the Heavyweight Championship Title in what sport? <i>Answer: Boxing</i>	
Question 9	What sport has been most famously played by Tiger Woods, Jack Nicklaus, and Arnold Plamer? <i>Answer: Golf</i>	
Question 10	In baseball, what is it called when a player hits a ball over the outfield wall and into the stands? <i>Answer: A home run</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz:</b> 14	<b>Topic:</b> Sports	<b>Difficulty:</b> Medium
<b>Mean:</b> 4.598	<b>Median:</b> 4	<b>Variance:</b> 9.182
Question 1	March Madness refers to what college sport's tournament? <i>Answer: Basketball</i>	
Question 2	What team did Larry Bird play for? <i>Answer: Celtics</i>	
Question 3	Which golf tournament's winner is traditionally awarded a green jacket? <i>Answer: Masters</i>	
Question 4	What sport does Sheryl Swoopes play? <i>Answer: Basketball</i>	
Question 5	What sport does Oscar De LaHoya participate in? <i>Answer: Boxing</i>	
Question 6	What city's NFL team is known as the Cowboys? <i>Answer: Dallas</i>	
Question 7	Who holds the NHL record for the most goals in a season? <i>Answer: Wayne Gretzky</i>	
Question 8	How many times was Michael Jordan MVP of the NBA Finals? <i>Answer: 6</i>	
Question 9	What is it called when a race horse wins the Kentucky Derby, the Preakness Stakes, and the Belmont Stakes? <i>Answer: Triple Crown</i>	
Question 10	What country's team won the 2002 World Cup? <i>Answer: Brazil</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz: 15</b>	<b>Topic:</b> Sports	<b>Difficulty:</b> Hard
<b>Mean:</b> 0.195	<b>Median:</b> 0	<b>Variance:</b> 0.159
Question 1	Who is the only player to have scored 61 points in an NCAA Basketball Tournament game? <i>Answer: Austin Carr</i>	
Question 2	Who was the first MVP of the NBA? <i>Answer: Bob Pettit</i>	
Question 3	What college won the first Women's NCAA Basketball Tournament? <i>Answer: Louisiana Tech</i>	
Question 4	Who was the first NHL player to win the Conn Smythe Trophy? <i>Answer: Jean Beliveau</i>	
Question 5	Which College or University has won the most football bowl games? <i>Answer: University of Alabama</i>	
Question 6	Who won the first Super Bowl? <i>Answer: Green Bay Packers</i>	
Question 7	Who was the only unseeded man to win the Wimbledon singles title? <i>Answer: Boris Becker</i>	
Question 8	Who was the first jockey to ride 7,000 winners? <i>Answer: Willie Shoemaker</i>	
Question 9	Who won the Tour de France in 1997? <i>Answer: Jan Ullrich</i>	
Question 10	Who is the only hockey player to score at least 50 goals in 9 consecutive seasons? <i>Answer: Mike Bossy</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

**Quiz:** 16      **Topic:** Science      **Difficulty:** Easy  
**Mean:** 8.976      **Median:** 10      **Variance:** 3.629

- 
- Question 1      Where in the human body is the cerebellum located?  
*Answer: Brain (or Head)*
- Question 2      The average length of a human pregnancy is how many months?  
*Answer: 9*
- Question 3      Where in the human body is food digested?  
*Answer: the stomach (or intestines)*
- Question 4      What is the smallest prime number greater than 3?  
*Answer: 5*
- Question 5      What is the chemical symbol for water?  
*Answer: H<sub>2</sub>O*
- Question 6      The theory of evolution by means of natural selection is attributed to whom?  
*Answer: Darwin*
- Question 7      The \_\_\_\_\_ is the star at the center of our solar system.  
*Answer: Sun*
- Question 8      An octopus has how many arms?  
*Answer: 8*
- Question 9      What is the major pumping organ of the human circulatory system?  
*Answer: Heart*
- Question 10      Carnivores are animals that eat meat, while herbivores are animals that eat what?  
*Answer: Plants (or Vegetation)*
- 

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz: 17</b>	<b>Topic: Science</b>	<b>Difficulty: Medium</b>
<b>Mean: 6.427</b>	<b>Median: 7</b>	<b>Variance: 8.001</b>
Question 1	What is the name for a group of lions? <i>Answer: pride</i>	
Question 2	What are the only mammals that truly fly? <i>Answer: bats</i>	
Question 3	Antibiotics kill what class of pathogen? <i>Answer: bacteria</i>	
Question 4	What African predator is the fastest land animal? <i>Answer: cheetah</i>	
Question 5	Deoxyribonucleic acid is better known as what? <i>Answer: DNA</i>	
Question 6	What is the largest species of whale? <i>Answer: blue whale</i>	
Question 7	What durable substance give human ears and noses their shapes? <i>Answer: cartilage</i>	
Question 8	What is normal human body temperature? <i>Answer: Roughly 98 F or 37 C</i>	
Question 9	Dobermans, schnauzers, and alsations are all breeds of what kind of animal? <i>Answer: Dog</i>	
Question 10	How many meters are there in a kilometer? <i>Answer: 1000</i>	

Table 7: The 18 quizzes used in the experiment. (Continued on the next page.)

<b>Quiz:</b> 18	<b>Topic:</b> Science	<b>Difficulty:</b> Hard
<b>Mean:</b> 0.939	<b>Median:</b> 1	<b>Variance:</b> 1.268
Question 1	Who is credited with inventing the wristwatch in 1904? <i>Answer: Louis Cartier</i>	
Question 2	Laudanum is a form of what drug? <i>Answer: opium</i>	
Question 3	The psychoactive ingredient in marijuana is THC. What does THC stand for? <i>Answer: delta-9-Tetrahydrocannabinol</i>	
Question 4	What chemical element has the atomic number five? <i>Answer: Boron</i>	
Question 5	The study of the structural and functional changes in cells, tissues and organs that underlie disease is called what? <i>Answer: Pathology</i>	
Question 6	What does the suffix "itis" mean? <i>Answer: Inflammation</i>	
Question 7	The bilby, bandicoot, and quokka are all representatives of what mammalian subclass? <i>Answer: Marsupials</i>	
Question 8	Which one of the 50 United States is the only one never to have experienced an earthquake? <i>Answer: North Dakota</i>	
Question 9	What evolutionary biologist wrote, "Creation science has not entered the curriculum for a reason so simple and so basic that we often forget to mention it: because it is false."? <i>Answer: Stephen Jay Gould</i>	
Question 10	What is the single most diverse phylum within the animal kingdom? <i>Answer: Arthropoda (arthropods, including crustaceans, insects, and spiders)</i>	

Table 7: The 18 quizzes used in the experiment.