

219B – Spring 2010  
Problem set on Market Response – Due on April 20

**Question #1 (Behavioral IO – Contract Design and Self-Control)**

This Question elaborates on the DellaVigna-Malmendier (QJE, 2004) paper. Assume that consumers have preferences  $(\beta, \hat{\beta}, \delta)$  and they are interested in consuming an investment good which yields a payoff of  $-c$  at  $t = 1$  and a delayed payoff of  $b > 0$  at  $t = 2$ . At  $t = 0$ ,  $c$  is unknown, with a distribution  $F(c)$ ; the realization  $c$  is realized at  $t = 1$ , before the consumer decides whether to consume the good. A monopolistic firm produces such investment goods for a marginal cost  $a$  (paid at  $t = 1$ ) and intends to sell them to the consumer using a two-part tariff:  $L$  (paid at  $t = 1$ ) is the lump-sum fee and  $p$  (also paid at  $t = 1$ ) is the per-usage fee. The consumer alternative option yields a utility  $\bar{u}$ , realized at  $t = 1$ . The firm offers a contract  $(L, p)$  to the consumer at  $t = 0$  and the consumer accepts it or rejects it also at  $t = 0$ . At  $t = 1$ , the consumer (if she accepted the contract) decides whether to consume the good.

a) Under what condition for  $c$  the consumer *actually* consumes at  $t = 1$  (assuming that she signs the contract)? Under what condition for  $c$  the consumer *expects* to consume at  $t = 0$ ? Under what condition for  $c$  the consumer *would like* to consume at  $t = 1$ , as of  $t = 0$ ? Relate to the notions of self-control and naiveté.

b) Write down the maximization problem for the monopolist at  $t = 0$ . The monopolist maximizes profits subject to the Individual Rationality constraint for the agent. The profits are determined by two elements: (i) the lump-sum fee  $L$ , and (ii) the price  $p$  net of the marginal cost  $a$  of a visit if David attends. (Remember: The firm is aware of the self-control problems of the agent) Solve for  $L$  from the IR constraint and substitute it into the maximization problem.

c) Derive the first-order condition and derive an expression for  $p^*$ . [Hint: You may need the rule  $\frac{\partial}{\partial x} \left( \int_{g(x)}^{f(x)} h(x, z) dz \right) = \frac{\partial f(x)}{\partial x} h(x, f(x)) - \frac{\partial g(x)}{\partial x} h(x, g(x)) + \int_{g(x)}^{f(x)} \frac{\partial h(x, z)}{\partial x} dz$ ]

d) What type of pricing for  $p^*$  do you get for exponential agents ( $\beta = \hat{\beta} = 1$ )? Provide intuition on this result.

e) What type of pricing for  $p^*$  do you get for sophisticated agents ( $\beta < \hat{\beta} = 1$ )? Provide intuition on this result, commenting on the magnitude of  $p^*$ .

f) What type of pricing for  $p^*$  do you get for fully naive agents ( $\beta < \hat{\beta} = 1$ )? Provide intuition on this result.

g) Comment on/dispute the following assertion: “Present-biased preferences explain the prevalence of contracts with no payment per visit ( $p = 0$ ) in the health-club industry.”

h) Now re-interpret the results in points (e)-(f) under the assumption that  $b < 0$ . That is, the good is a leisure good, with immediate benefits and delayed costs  $b$ . (Notice that we are

stretching the notation and  $-c$ , which will be generally positive, is the immediate benefit.) What pricing is predicted under the leisure good case? Interpret.

i) So far we assumed homogeneity of consumers. Assume now that there are two groups of consumers. A share  $\mu$  of consumers are fully naive with  $\hat{\beta} = 1$ , while a share  $1 - \mu$  of consumers are exponential ( $\beta = \hat{\beta} = 1$ ). The two consumers have the same  $\delta$  and the same cost distribution  $F(c)$ . Set-up the firm maximization problem. [Hint: Argue that these consumers choose the same contract]

j) Derive the first-order conditions and solve for  $p^*$  for the case in point i). Compare the solution to the solutions that you derived in points d) (for exponentials) and f) (for naives).

k) Consider now the case of perfect competition, reverting back to the assumption of homogeneity among consumers. Instead of having just one company, there are multiple firms competing on the contracts, as in a Bertrand model. A way to solve the case of perfect competition is to maximize the perceived utility of consumers, subject to a condition that the firm profits equal zero. (Since we know that in equilibrium, profits will equal zero in a Bertrand-type competition). Set up this problem.

l) Solve for  $p_{PC}^*$  and compare to the  $p^*$  that you derived above. How does the optimal contract  $(L^*, p^*)$  differ under perfect competition and monopoly?

m) Going back to the monopoly case above, how would the problem change if the firm cannot offer a two-part tariff, but only a price  $p$ . Does self-control still matter in the determination of prices? Discuss.

n) Briefly, a discussion of welfare. What are the welfare effects of the contracts for the naive and the sophisticated present-biased agents? Compare their welfare at time 0 in equilibrium to the welfare of an agent with the same  $\delta$  but  $\beta = \hat{\beta} = 1$ .

## Question #2 (Behavioral Finance – Noise Traders)

This Question elaborates on the DeLong, Shleifer, Summers, Waldman (*JPE* 1990) paper. The idea is to consider what happens to asset prices when a share of the traders have irrational expectations about future dividends. These traders in the literature are called noise traders. Consider the set-up of DeLong, Shleifer, Summers, Waldman (*JPE* 1990), which I summarize here. There is a share  $\mu$  of noise traders,  $(1 - \mu)$  of arbitrageurs. The arbitrageurs are risk averse and have a short horizon, that is, they have to sell the shares at the end of period to consumer. Formally, consider an OLG model where in period 1 the agents have initial endowment and trade, and in Period 2 they consume. There are two assets with identical dividend  $r$ : a safe asset with perfectly elastic supply, whose price we will set to 1 (numeraire), and an unsafe asset in inelastic supply (1 unit) and a price  $p$  that is determined by supply and demand. We denote the demand for unsafe asset:  $\lambda^a$  and  $\lambda^n$ . The investors have CARA utility function  $U(w) = -e^{-2(\gamma w)}$  with  $w$  being the wealth in Period 2, which is what the investor consumes. Compared to the arbitrageurs, the noise traders believe that in period  $t$  the asset will have higher return  $\rho_t$ .

a) Assume that the wealth  $w$  is distributed  $N(\bar{w}, \sigma_w^2)$ . Show that maximizing  $EU(w)$  is equivalent to maximizing  $\bar{w} - \gamma\sigma_w^2$ , that is, the problem reduces to one of mean-variance optimization.

b) Show that arbitrageurs maximize the problem

$$\max(w_t - \lambda_t^a p_t)(1 + r) + \lambda_t^a (E_t[p_{t+1}] + r) - \gamma (\lambda_t^a)^2 \text{Var}_t(p_{t+1}).$$

Derive the first order condition and solve for  $\lambda_t^{a*}$ .

c) Show that noise traders maximize the problem

$$\max(w_t - \lambda_t^n p_t)(1 + r) + \lambda_t^n (E_t[p_{t+1}] + \rho_t + r) - \gamma (\lambda_t^n)^2 \text{Var}_t(p_{t+1}).$$

Derive the first order condition and solve for  $\lambda_t^{n*}$ .

d) Discuss how the optimal demand of the risky asset will depend on the expected returns ( $r + E_t[p_{t+1}] - (1 + r)p_t$ ), on risk aversion ( $\gamma$ ), on the variance of returns ( $\text{Var}_t(p_{t+1})$ ), and on the overestimation  $\rho_t$ .

e) Under what conditions noise traders hold more of the risky asset than arbitrageurs do?

f) To solve for the price  $p_t$ , we impose the market-clearing condition  $\lambda^n \mu + \lambda^a (1 - \mu) = 1$ . Use this condition to solve for  $p_t$  as a function of  $E_t[p_{t+1}]$ ,  $\text{Var}_t(p_{t+1})$ , and the other parameters.

g) To solve for the equilibrium, assume that the average price is not time-varying (that is,  $E_t[p_t] = E_t[p_{t+1}] = E[p]$ ), and take expectations on the right and left of the expression for  $p_t$ . Solve for  $E[p]$ , and substitute into the expression for  $p_t$ . Now, use this expression to

compute  $Var [p_t]$ . Finally, substitute the expression for  $Var [p_t]$  in the updated expression for  $p_t$ . In the end, you should obtain

$$p_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1+r} + \frac{\mu\rho^*}{r} - \frac{2\gamma\mu^2\sigma_\rho^2}{r(1+r)^2}. \quad (1)$$

h) Analyze how the price  $p$  responds to an increase in  $\mu$ , in  $\rho_t$ , in  $\rho^*$ , in  $\gamma$ , and in  $\sigma_\rho^2$ . For each of these terms provide intuition.

i) In light of expression (1), comment on the following statement: ‘Biases of investors do not matter in financial markets because they do not affect prices’. What are the key assumptions in the set-up driving this result?

j) (Extra credit) The returns for traders of group  $j$  ( $j = a, n$ ) are  $R^j = (w_t - \lambda_t^n p_t)(1+r) + \lambda_t^n (p_{t+1} + r) - w$ . Straightforwardly, this implies that  $\Delta R = R^n - R^a = (\lambda_t^n - \lambda_t^a) (p_{t+1} + r - p_t (1+r))$ . Solve that  $E(\Delta R | \rho_t)$ , that is, the expected return to noise traders relative to arbitrageurs conditional on  $\rho_t$ , is

$$E(\Delta R | \rho_t) = \rho_t - \frac{(1+r)^2 \rho_t^2}{2\gamma\mu\sigma_\rho^2} \quad (2)$$

k) Using (2), discuss whether it is possible that noise traders outperform in expectations the arbitrageurs, and under what conditions.

l) What is the intuition for why noise traders may outperform in expectations the arbitrageurs?

m) Does your answer in (l) imply that noise traders can achieve a higher expected utility than arbitrageurs? (Note: I intend when utility is evaluated with the actual returns, not with naive expectations that noise traders have)