UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202B
Spring 2008
Y. Gorodnichenko/D. Romer

Problem Set 5
Due in lecture Thursday, February 28

1. Romer, Problem 3.3.

2. Knowledge accumulation may vary in a complicated way over time. This problem asks you to investigate one way that this might occur.

For simplicity, population is constant. Output at time t is given by $Y(t) = (1 - a_L)A(t)L$, where Y is output, a_L is the fraction of the population that is engaged in producing knowledge, A is knowledge, and L is population.

Knowledge accumulation is given by the function: $\dot{A}(t) = B_1 a_L L A(t)^\theta$ if $A < A^*$, $\dot{A}(t) = B_2 a_L L$ if $A \geq A^*$, where A^*, B_1, and B_2 are positive parameters, and where θ is a parameter that is assumed to be greater than 1. In addition, B_1 and B_2 are assumed to be such that $\dot{A}$ does not change discontinuously when A reaches A^*. This requires that $B_1 a_L L A^{*\theta} = B_2 a_L L$, which is equivalent to $B_2 = B_1 A^{*\theta}$.

The initial level of knowledge, A(0), is assumed to be greater than zero and less than A^*.

a. Consider the period when A is less than A^*.
   i. Define $g_A(t) \equiv \dot{A}(t)/A(t)$. What is $g_A(t)$ as a function of B_1, a_L, L, and A(t)?
   ii. Find an expression for $\dot{g}_A(t)$ as a function of $g_A(t)$ and θ.
   iii. Is $g_A(t)$ rising, falling, or constant over time?
b. Now consider the period when A is greater than or equal to A^*.
   i. What is $\dot{A}(t)$?
   ii. Is $g_A(t)$ rising, falling, or constant over time?
c. Combine your answers to (a) and (b) to:
   i. Sketch the path of the growth rate of output, $\dot{Y}(t)/Y(t)$, over time.
   ii. Sketch the path of the log of output, ln Y(t), over time.

3. Romer, Problem 3.12.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

4. Romer, Problem 3.2.

(OVER)

5. Consider the following variant of our model of R&D and growth. All of the notation is standard; R(t) denotes use of natural resources at time t, and a_R denotes the fraction of those resources that are used in the R&D sector.

$$Y(t) = A(t)[(1-a_L)L(t)]^{\beta}[(1-a_R)R(t)]^{1-\beta} \quad 0 < a_L < 1, \; 0 < a_R < 1, \; 0 < \beta < 1$$

$$\dot{L}(t) = nL(t) \qquad n > 0$$

$$\dot{R}(t) = -\mu R(t) \qquad \mu \geq 0$$

$$\dot{A}(t) = B[a_L L(t)]^{\gamma}[a_R R(t)]^{\phi} A(t)^{\theta} \quad B > 0, \; \gamma > 0, \; \phi > 0$$

Assume $\theta < 1$. A(0), L(0), and R(0) are all strictly positive.

a. Let $g_A(t) = \dot{A}(t)/A(t)$. Derive an expression for $\dot{g}_A(t)$ in terms of $g_A(t)$ and the parameters.

b. Sketch the function you found in part (a). For what values of g_A is $\dot{g}_A = 0$? For what parameter values and/or initial conditions does g_A converge to each of these values?

c. What is the growth rate of output per person on the balanced growth path as a function of the parameter values and/or initial conditions?

6. Romer, Problem 3.4.