Power Laws and the Granular Origins of Aggregate Fluctuations

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Abstract

If firm sizes have a small dispersion, idiosyncratic firm-level shocks lead to negligible aggregate fluctuations. This has led economists to appeal to macroeconomic (sectoral or aggregate shocks) shocks to explain aggregate fluctuations. However, the empirical distribution of firms is fat-tailed. This paper shows how, in a world with fat-tailed firm size distribution, idiosyncratic firm-level fluctuations aggregate up to non-trivial aggregate fluctuations. We illustrate why and how this happens, and contend that aggreage fluctuations come in large part from idiosyncratic shocks to firms. We show empirically that idiosyncratic volatility is indeed large enough to account for GDP volatility.This "granular" hypothesis suggests new directions for macroeconomic research, in particular that macroeconomic questions will be clarified by looking at the behavior of large firms. This mechanism might be useful understanding the fluctuations of many aggregate quantities, such as business cycle fluctuations, inventories, inflation, short or long run movements in productivity, and the current account.

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1 Introduction

This paper proposes a simple origin for aggregate shocks. It will develop the view that a large part of aggregate shocks comes from idiosyncratic shocks to individual firms. It will also argue that this approach sheds light on a number of issues that are difficult to address in models that postulate aggregate shocks. Though economy-wide shocks (inflation, wars, policy shocks) are no doubt important, they have a difficulty explaining most fluctuations. Often, the explanation for quarter-to-quarter jumps of aggregate quantities is elusive. On the other hand, there is a host of anecdotal evidence for important idiosyncratic shocks. For instance, the McKinsey Institute (2001) estimates that in 1995-1999, 1/6 of the increase in productivity growth of the whole U.S. economy was due to one firm, Wal-Mart[1]. Likewise, shocks to GDP may stem from a variety of events such as a success by Nokia, the difficulties of a Japanese bank, new sales by Boeing, a new chip by Intel, and a downsizing at Nestlé.

Idiosyncratic shocks aggregate to non-trivial shocks, because modern economies have many large firms. For instance, in the US, the sales of the top 20 firms account for about 20% of total US GDP. In Japan, the top 10 firms account for 35% of the exports. There is a systematic structure in this high concentration. The firm size distribution can be described by a power law (Ijiri and Simon 1977, Okuyama et al. 1999, Axtell 2001). Economies with a power law distribution have a host of small firms, and a few very large ones. This structure will be useful to derive tractable, clean models.

This hypothesis, that idiosyncratic shocks generate aggregate shocks, offers a microfoundation for the "aggregate shocks" of real business cycle models. Hence real business cycle shocks are not, at heart, mysterious "aggregate productivity shocks". Rather they are well-defined shocks to individual firms[2]. This view sheds lights on a number of issues, such as the dependence of the amplitude of GDP fluctuations with GDP level, the microeconomic composition of GDP, the distribution of GDP and firm-level fluctuation.

Some of the mathematics will be involved, so it is useful to highlight the main argument. First, a result based on Hulten (1978) shows that, if firm

[1] In their interesting study, McKinsey (2001) seek to understand why U.S. productivity growth increased from 1.5% to 2.8% per year in the second half of the 1990s. Also see Lewis (2004).

[2] These shocks can propagate to the rest of the economy. There is a very large literature on these "propagation mechanisms". This paper focuses on the original shocks, not their propagation.

i has a productivity shock $d\pi_i$, those shocks are i.i.d., then GDP moves by: is:

$$\sigma_{GDP} = h_S \sigma_\pi \tag{1}$$

where h_S is the Sales herfindahl of the economy:

$$h_S = \left(\sum_{i=1}^{N} \left(\frac{\text{Sales}_{it}}{\text{GDP}_t} \right)^2 \right)^{1/2}$$

and σ_π is the standard deviation of the i.i.d. productivity shocks. Second, microeconomic volatility is very large. We find that, even for large firms, the volatility of productivity is $\sigma_\pi = 20\%/\text{year}$. Third, as countries have large firms the sales herfindahl h_S is high. For instance, for the U.S. in 2002, it is $h_S = 6.2\%$. Using (1), we predict a GDP volatility equal to: $\sigma_{GDP} = 20\% \cdot 6.2\% = 1.2\%$. This is the order of magnitude of business cycle fluctuations. Using non-US data leads to even larger business cycle fluctuations.

We will also show how demand linkages such as Long and Plosser (1982)'s generate an amount of comovement among firms that ressembles the one of business cycles. Hence, firm level shocks create both non-trivial aggregate fluctuations, but also comovement. We have all the ingredients we need for a business cycle.

The main theoretical contribution is to break the curse of $1/\sqrt{N}$ diversification. A simple diversification argument shows that, in an economy with N firms with independent shocks, aggregate fluctuations should have a size proportional to $1/\sqrt{N}$. Given modern economies can have millions of firms, this suggests that those idiosyncratic fluctuations will be negligible. Horvath (1998,2000) and Dupor (1999) discuss ways out of this problem based on the sparsity of the input ouput matrix. We offer a simple alternative solution. When firm size is power law distributed, then conditions under which one derives the central limit theorem break down, and other mathematics (due to Paul Lévy) apply. In the central case of Zipf's law, aggregate volatility scales like $1/\ln N$, rather than $1/\sqrt{N}$. The draconian $1/\sqrt{N}$ diversification is replaced by a much milder one that goes in $1/\ln N$. Diversification effects due to country size will be quite small in practice. Section 5 provide gathers the empirical evidence on this, and is very congruent with the model.

We will present the argument with several degrees of sophistication. Section 2 develops a simple model that can be calibrated. Section 3 shows that empirically, our effects are large enough. It also examines the model's predictions about the shape of the fluctuations of the growth rate of firms and countries. Section 4 revisits how demand linkages can in turn create comovements. Section 5 and discusses some extensions.

1.1 Related literature

1.1.1 Macroeconomics

A few papers have proposed way to generate macro shocks from purely micro shocks. A pioneering paper is Jovanovic (1987), which we discuss in section 2.2. It relies on an extremely large multiplier M that has an order of magnitude of 1000 – the square root of the number of firms in the economy. This high multiplier has proved an obstacle of the Jovanovic model by macroeconomists. Different routes were explored by very innovative papers, Durlauf (1993) and Bak et al. (1993). Durlauf (1993) generates macroeconomic uncertainty with idiosyncratic shocks and local interactions between firms. The action comes from the non-linear interactions between firms, while in our paper the core comes from the skewed distribution of firms. Durlauf's model is analytically difficult, and we suspect that embedding our power law distributed firm in his models could be quite interesting. This is difficult to do at this point. Bak et al. (1993) explore self-organizing criticality [3]. While we have much sympathy for their approach (which is very different from ours), their model generates fluctuations that are probably "too fat tailed": they have a power law exponent of 1/3, so that fluctuations don't even have a mean, much less a variance. Nirei (2003) proposes an elaborate model whose spirit is related to Bak et al. 1993, and finds fluctuations with a power law exponent 1/2.

Long and Plosser (1983) worked out the view that sectoral (rather than firm) shocks might account for GDP fluctuations. As their model has a small number of sectors, those shocks can be viewed as mini aggregate shocks. Horvath (1998, 2000) and Conley and Dupor (2003) explore this hypothesis further. They find that sector-specific shock are an important source of aggregate volatility. Studies disaggree somewhat on the share of sector specific shocks, aggregate shocks, and complementarities. Shea (2002) quantifies that complementarities play a major role in aggregate business cycle fluctuations. Caballero, Engel and Haltiwanger find that aggregate shocks are important (1997), while Horvath (1998) find that sector-specific shocks go a long way to explain aggregate disturbances. Finally Horvath (1998,2000) and Dupor (1999) debate about whether N sector can have a volatility that does not decay in $N^{-1/2}$. We find an alternative solution

[3] Also see the pedagogical version in Scheinkman and Woodford (1994).

to their debate. This solution, formalized in Proposition 2, is that firm size distribution is very skewed[4], that a few large sectors will dominate the economy. Also, we propose that thinking about firm might be a useful way to think about the world. Many "industry shocks" originate in the decision of one large firms (Toyota, WalMart, IBM) to introduce a radical innovation. The shocks are also easier to explain: they are the fruit of R&D efforts, and bets on the organization of production.

1.1.2 Some relation social power laws

A growing number of economic variables appear to follow power laws. The earliest is the distribution of incomes (Pareto, 1896). Many power laws have an exponent 1, i.e. they follow Zipf's law. A number of economic systems appear to follow Zipf's law: cities (Zipf 1949, Gabaix and Ioannides 2004), firms (Axtell 2001, Okuyama et al. 2003), mutual funds (Gabaix, Reuter and Ramalho 2003), web sites (Barabasi and Albert 1999). Gabaix (1999) provides an explanation and a survey of the literature. Stock market fluctuations also follow power laws. Intriguingly, the exponent is typically either 3 or 3/2. Gabaix et al. (2003, 2004) survey and propose an explanation for a series of puzzling facts on the distribution of stock market returns. They base there explanation on the power law distribution of large traders. This is analogous to the way this paper bases GDP fluctuations on a power law distribution of large firms.

2 The essence of the idea

2.1 A simple "islands" economy

To illustrate the idea, we will consider a very simple economy, composed of N firms that are independent islands with no feedback[5]. In this economy there are only idiosyncratic shocks to firms. We study its aggregate volatility. We call this volatility the GDP volatility coming from idiosyncratic shocks, σ_{GDP}. Say that firm i produces S_{it}. In a year t, it has a growth rate:

$$\frac{\Delta S_{i,t+1}}{S_{i,t}} = \frac{S_{i,t+1} - S_{it}}{S_{it}} = \sigma_i \varepsilon_{i,t+1} \tag{2}$$

[4] Canals et al. (2004) find that this is particularly true for the exports, whose distribution are extremely skewed. For instance, they find that the root-Herfindahl of exports is about 50%.

[5] Appendix A fleshes out such a model. In the next version of the paper, we will propose a general equilibrium model. The conclusions do not change, but the economics are less transparent.

where σ_i is firm i's volatility and the $\varepsilon_{i,t+1}$ are independent random variables with mean 0 and variance 1. Total GDP is:

$$Y_t = \sum_{i=1}^{N} S_{it} \tag{3}$$

and GDP growth is:

$$\frac{\Delta Y_{t+1}}{Y_t} = \frac{1}{Y_t} \sum_{i=1}^{N} \Delta S_{i,t+1} = \sum_{i=1}^{N} \sigma_i \frac{S_{it}}{Y_t} \varepsilon_{i,t+1}.$$

As the shocks $\varepsilon_{i,t+1}$ are uncorrelated, the variance of GDP growth is:

$$\sigma_{GDP}^2 = var \frac{\Delta Y_{t+1}}{Y_t} = \sum_{i=1}^{N} \sigma_i^2 \left(\frac{S_{it}}{Y_t} \right)^2.$$

The volatility of GDP fluctuations coming from the idiosyncratic micro shocks are

$$\sigma_{GDP} = \left(\sum_{i=1}^{N} \sigma_i^2 \cdot \left(\frac{S_{it}}{Y_t} \right)^2 \right)^{1/2}. \tag{4}$$

Hence the variance of GDP, σ_{GDP}^2, is the weighted sum of the variance σ_i^2 of idiosyncratic shocks with weights equal to $\left(\frac{S_{it}}{Y_t} \right)^2$, the squared share of output that firm i accounts for. We shall use equation (4) throughout the paper.

If the firms all have the same volatility $\sigma_i = \sigma$, we get the following simple identity:

$$\sigma_{GDP} = \sigma h \tag{5}$$

with

$$h = \left[\sum_{i=1}^{N} \left(\frac{S_{it}}{Y_t} \right)^2 \right]^{1/2}. \tag{6}$$

h is the square root of the Herfindahl of the economy. For simplicity, we will call it the "herfindahl" of the economy.

In the body of this paper, we will work with the "bare-bones" model (2)-(3). This can be viewed as the linearization of a host of richer models. We present such a model in Appendix A. Our arguments will apply if feedback mechanisms are added, as we do in section 6.1. We take advantage of the high tractability and portability of the simple model.

2.2 The $1/\sqrt{N}$ argument for the appeal to aggregate shocks

First, we briefly recall the reason why macroeconomics usually appeals to common (or at least sector-wide) aggregate shocks. With a large number of firms N, one could expect the sum of their σ_{GDP} shocks to be vanishingly small. Indeed, take firms of initially identical size equal to $1/N$ of GDP, and identical standard deviation $\sigma_i = \sigma$. Then (5)-(6) gives:

$$\sigma_{GDP} = \frac{\sigma}{\sqrt{N}}.$$

To get an idea of the order of magnitude delivered by this view, we take an estimate of firm volatility $\sigma = 20\%$ from Appendix B, and consider an economy with $N = 10^6$ firms[6]. We get

$$\sigma_{GDP} = \frac{\sigma}{\sqrt{N}} = \frac{20\%}{10^3} = 0.02\% \text{ per year.}$$

This theoretical annual GDP volatility of 0.02% is just too small to account for the empirically measured size of macroeconomic fluctuations. This is why economists typically[7] appeal to aggregate shocks. We will see that in fact this argument will fail, because large firms in modern economies have a size much bigger than $1/N$. Before we do that, we show that more general modeling assumptions predict a $1/\sqrt{N}$ scaling.

Proposition 1 *Consider an islands economy with N firms whose sizes are drawn from a distribution with finite variance. Suppose that they all have the same volatility σ. Then the economy's GDP volatility is:*

$$\sigma_{GDP} = \frac{E\left[S^2\right]^{1/2}}{E\left[S\right]} \frac{\sigma}{\sqrt{N}}.$$

Proposition 1 should be contrasted to Proposition 2 below. Its proof is in Appendix D.

We will now show how a different model of the size distribution of firms leads to dramatically different results.

[6] Axtell (2001) reports that in 1997 there were 5.5 million firms in the United States.

[7] One way around this has been taken by Jovanovic (1987), who observes that when the multiplier is very large ($1/(1-\lambda) = M \sim \sqrt{N}$, so $1-\lambda \sim 1/\sqrt{N}$), we get non-vanishing aggregate fluctuations. The problem is that empirically, such a large multiplier (of order of magnitude $\sqrt{N} \sim 10^3$) is very implausible: the impact of government purchases or trade shocks, for instance, would be much higher than we observe. Hence most economists do not see the "extremely large multiplier" route as plausible.

2.3 The $1/\sqrt{N}$ argument breaks down with power law firms

2.3.1 Empirical evidence shows that the distribution of firms has fat tails

A long literature establishes that the distribution of firm sizes (sales, assets, or number of employees give the same results) is very skewed. A good model parametrization is a power law distribution:

$$P(S > x) = ax^{-\zeta}. \tag{7}$$

for $x > a^{1/\zeta}$. To estimate this, it is useful to take the density:

$$f(x) = \frac{\zeta a}{x^{\zeta+1}}$$

and its logarithm:

$$\ln f(x) = -(\zeta + 1)\ln x + C \tag{8}$$

where C is a constant. A long literature has estimated the size distribution of firms, but typically the sample would include only firms listed in the stock market. Axtell (2001) breaks new ground by using the Census, which lists all the U.S. firms.

We reproduce his[8] plot of (8) in Figure 1. The horizontal axis shows $\ln x$, where x is the size of a firm in number of employees. The vertical axis shows the log of the fraction of firms with size x, $\ln f(x)$. One expects to see a straight line in the region where (8) holds, and indeed the Figure shows a very nice fit. An OLS fit of (8) yields an $R^2 = 0.992$, and a slope $= -2.059$, with a standard error of 0.054. This yields an estimate of $\zeta = 1.059 \pm 0.054$.

In the rest of the paper we will often take the approximation $\zeta = 1$, the "Zipf" value. This value ($\zeta \simeq 1$) is often found in the social sciences, for instance in the size of cities (Zipf 1949), and the in the amount of assets under management of mutual funds (Gabaix, Ramalho and Reuter 2003). The origins of this distribution are becoming better understood (see Gabaix (1999), and Gabaix and Ioannides (2004) for a survey of various candidate explanations).

The power law distribution (7) has fat tails, and thus produces some very large firms. We look at the implications for GDP fluctuations in the next section.

[8] Okuyama et al. (1999) also find that $\zeta \simeq 1$ for Japanese firms.

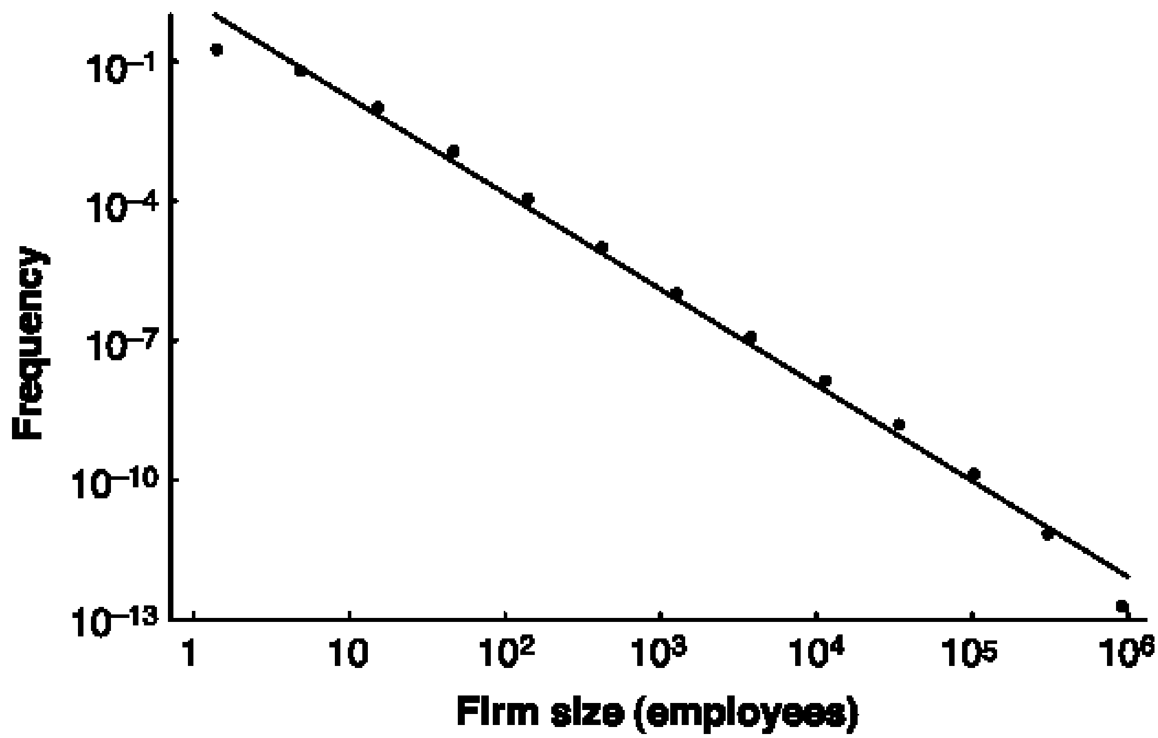

Figure 1: Log frequency $\ln f(S)$ vs log size $\ln S$ of U.S. firm sizes (by number of employees) for 1997. OLS fit gives a slope of 2.059 (s.e.= 0.054; R^2 =0.992). This corresponds to a frequency $f(S) \sim S^{-2.059}$, i.e. a power law distribution with exponent $\zeta = 1.059$. This is very close to Zipf's law, which says that $\zeta = 1$. Source: Axtell (2001).

2.3.2 GDP volatility when the volatility of a firm does not depend on of its size

Proposition 1 does not address what happens when the variance of sizes is infinite. More precisely, the empirical distributions we find, with power laws $\zeta < 2$, have infinite variance. The next Proposition examines what happens in that case of a "fat tailed" distribution of firms. Its proof is in Appendix D.

Proposition 2 *Consider an islands economy with N firms that have power law distributions (7) with exponent $\zeta \in [1, 2)$ and volatility σ. Then its GDP volatility is:*

$$\begin{aligned}\sigma_{GDP} &= \frac{v_\zeta}{\ln N}\sigma \text{ for } \zeta = 1 \\ \sigma_{GDP} &= \frac{v_\zeta}{N^{1-\frac{1}{\zeta}}}\sigma \text{ for } 1 < \zeta < 2\end{aligned}$$

where v_ζ is a random variable that is independent of N and σ.

The main conclusion is that if firms have fat tails, σ_{GDP} decreases as $N^{-\beta}$ for $0 \leq \beta < 1/2$, and thus decays much slowler than $N^{-1/2}$. In the Zipf limit $\zeta = 1$, we get $\beta = 0$, and the decay is barely perceptible[9].

2.3.3 GDP volatility when the volatility of a firm depends on its size

This section completes the theoretical picture, but in the first reading we recommend the reader skip to section 3.

We just understood the benchmark case where all firms have the same volatility σ. We now turn to the case where the volatility decreases with size, which seems to be the case empirically. We will examine the functional form suggested by the empirical discussion in section 5.1

$$\sigma^{\text{Firm}}(S) = kS^{-\alpha} \tag{9}$$

for $\alpha \geq 0$.

[9] If there are N identical firms, $1/h_N^2 = N$. So $1/h_N^2$ reveals the "effective" number of firms in the economy, for diversification purposes. So, in a Zipfian world (where $\zeta = 1$), the effective number of firms is not N but $(\ln N)^2$. For $1 < \zeta < 2$, the effective number of firms scales as $N^{2-2/\zeta}$. This notion of the "effective" number of firms is important as long as diversification plays a role, as is the case in Caballero and Engle (2003) and the present paper.

Proposition 3 *Take an islands economy with N firms that have power law distributions $P(S > x) = x^{-\zeta}$ for $\zeta \in [1, \infty]$. Assume that the volatility of a firm of size S is*

$$\sigma^{Firm}(S) = kS^{-\alpha} \tag{10}$$

for some $\alpha \geq 0$. Then, GDP fluctuations have the form:

$$\frac{\Delta Y_t}{Y_t} = kN^{-\alpha'} g_t \tag{11}$$

with

$$\alpha' = \min\left(\frac{1}{2}, \frac{\alpha + \zeta - 1}{\zeta}\right) \tag{12}$$

and g_t is a symmetrical Lévy stable distribution with exponent min $\{\zeta/(1-\alpha), 2\}$.

In particular, the volatility $\sigma(S)$ of GDP decreases in a power law fashion as a function of its size S [10]:

$$\sigma^{\text{GDP}}(S) \sim S^{-\alpha'}. \tag{13}$$

Corollary 4 *(Similar scaling of firms and countries). For $\zeta = 1$ and $\alpha \leq 1/2$, we have $\alpha' \simeq \alpha$, i.e. firms and countries should see their volatility scale with a similar exponent:*

$$\sigma^{Firms}(S) \sim \sigma^{GDP}(S) \sim S^{-\alpha}$$

In section 5.1, we will present some evidence that the above prediction holds. The above Propositions indicate that the volatility could decay very slowly with size. In the next section we examine whether these effects are large enough.

3 Empirical evidence

3.1 Firm-level volatility

Most estimations of firm-level volatility find very large volatilities σ, with an order of magnitude $\sigma = 30\%$ to $\sigma = 50\%$ per year. Appendix B reviews the evidence. For instance, the volatility of firm size is a very large 40% of year. Much of the work has been done on the median firm, rather than

[10] In this paper, $f(S) \sim g(S)$ for some functions f, g, means that the ratio $f(S)/g(S)$ tends, for large S, to a positive real number. So f and g have the same scaling "up to a constant real factor".

on large firms. But large firms also have large volatility. For instance, with Comin and Mulani (2003) we consider the following very simple measure of firm produtivity, π_{it}, defined as the sales over the number of workers. We consider the top 20 firms in 1980, and compute σ_i to be the standard deviation of $\ln \pi_{i,t+1} / \ln \pi_{it}$. We find that the average σ_i of the top 20 firms is: $\sigma = 20\%$. This is slightly less high than the median firms, which makes sense.

In what follows we will use an estimate of $\sigma = 20\%$ per year for firm level volality.

3.2 Induced volatility

If firm i has a Hicks-neutral productivity growth $d\pi_i$, then an important theorem by Hulten (1978) shows that the increase in GDP is:

$$\frac{d\text{GDP}}{\text{GDP}} = \sum_i \frac{\text{Sales of firm } i}{\text{GDP}} d\pi_i \tag{14}$$

The weights add up to more than 1. This reflects the fact that productivity growth in a firm generate an increase in the social value of all the inputs it uses. The firms' sales are the proper statistics for that social value[11]. For clarity, Appendix E shows a simple proof of Hulten's theorem. It shows that it holds under weaker condition's that Hulten's original conditions[12].

Suppose productivity shocks $d\pi_i$ are i.i.d. with standard deviation σ_π. Then, the variance of productivity growth is:

$$var \frac{d\text{GDP}}{\text{GDP}} = \sum_i \left(\frac{\text{Sales of firm } i}{\text{GDP}} \right)^2 var\left(d\pi_i\right)$$

so

$$\sigma_{GDP} = h_S \sigma_\pi \tag{15}$$

where h_S is the Sales herfindahl:

$$h_S = \left(\sum_{i=1}^{N} \left(\frac{\text{Sales}_{it}}{\text{GDP}_t} \right)^2 \right)^{1/2}. \tag{16}$$

[11] This mechanism can be seen in detail in Long and Plosser (1982). Hulten (1978)'s result, however, is more general.

[12] In particular it shows that Hulten's theorem holds even if factors are not reallocated right after the shock.

Hulten's theorem allows us to simplify a lot the analysis. For the total volatility, one does not need to know the details of the input-output matrix. The sales herfindah is the sufficient statistics.

We consider the following employment herfindahl:

$$h_W = \left(\sum_{i=1}^{N} \left(\frac{\text{Workforce}_{it}}{\text{Total workforce}_t} \right)^2 \right)^{1/2} \tag{17}$$

It is less theoretically motivated, but it is useful as a robustness check.

We get our herfindahls from Acemoglu, Johnson and Mitton (2004), who analyze the Dun and Bradstreet data. This data has a good coverage of the major firms for many countries. It is not without problems, but at least it provides an order of magnitude for the empirical values of the herfindahls.

		All Countries	Rich Countries	USA
Sales herfindahl	h_S	22.0	26.6	6.1
Workers herfindahl	h_W	3.8	4.0	1.2
GDP volatility induced by idiosyncratic firm-level shocks	$\sigma_{GDP} = \sigma h_S$	4.4	5.2	1.2

Table 1: Sales herfindahl h_S and Workforce herfindahl h_W (Eqs.16–17) in the year 2002. Units are %.
Rich countries are the countries with GDP per capita greater than $13,000. For the induced GDP volatility, we use take $\sigma_{GDP} = \sigma h_S$, with a firm-level volatility $\sigma = 20\%$. See Eq. 15. Source: Acemoglu, Johnson and Mitton (2004) for the international data, and Compustat for the USA data.

As seen above, a good estimate for the firm-level volatiltiy is $\sigma = 20\%$. Table 1 displays the results. We see that the sales herfindahl h_S is quite large: $h_S = 22\%$ for all countries, and $h_S = 6.1\%$ for the USA. By Eq. 15 this corresponds to a GDP volatility

$$\sigma_{GDP} = 20\% \times 6.1\% = 1.2\%$$

for the USA, and $\sigma_{GDP} = 20\% \times 22\% = 4.4\%$ for a typical countries. This is very much in the order of magnitude of GDP fluctuations. As shown in Section 6.1, feedback mechanisms can increase this estimate. We conclude that *idiosyncratic volatility is quantitatively large enough to explain macroeconomic volatility.*

4 Enriching the model with Long Plosser demand linkages

The above calibration showed that idiosyncratic shock can account to a large aggregate volatility. We provide here some detail about the comovement they imply. Shea (2002) present a series of models that generate comovement. We take his "instantaneous" version of the Long Plosser (1982) model. There are N firms. The representative consumer has utility: $U = \sum_{i=1}^{N} \theta_i \ln C_i$.Firm i produces Q_i with L_i units of labor, and X_{ik} inputs from firm k. The production function is Cobb-Douglas:

$$Q_i = \lambda_i \exp\left(b \ln L_i + \sum_k \phi_k \ln X_{ik}\right)$$

with $1 = b + \sum_k \phi_k$. The clearing constraints are $Q_i = C_i + \sum_k X_{ki}$ and $L = \sum_i L_i$, where L is the fixed labor supply. We assume that firms behave competitively[13].

The analysis is standard. The economic importance of firms is captured by

$$\gamma_i = \frac{\text{Sales of firm } i}{\text{GDP}} = \frac{p_i Q_i}{\text{GDP}} = \frac{\phi_i}{b} + \theta_i$$

while its share of value added is $L_i/L = b\gamma_i$.

Let hats note log changes, i.e. $\widehat{Z} = dZ/Z$. If firm i has a productivity shock $\widehat{\lambda}_i$, then Eq. 14 indicates that GDP increases by:

$$\widehat{Y} = \sum_i \gamma_i \widehat{\lambda}_i. \tag{18}$$

while the production of firm i increases by:

$$\widehat{Q}_i = \widehat{C}_i = \widehat{\lambda}_i + (1-b)\,\widehat{Y} \tag{19}$$

The term $\widehat{Y}$ generates a comovement of firms that we will analyze shortly.

[13] This is to simplify the analysis. Firms could be competitive because markets are contestable. Otherwise, our "firms" can be interpreted as "sectors". There is some debate about the size of markups. Basu and Fernald (1997) find markups less than 10%, while other studies find higher markups, and much of macroeconomics uses on zero markups.

We now analyze further the mechanism. If all the firm have productivity growth $\widehat{\lambda}_i = \widehat{\lambda}\%$ then GDP growth will be

$$\begin{aligned}\widehat{Y} &= \widehat{\lambda}\sum_i \gamma_i = \widehat{\lambda}\sum_i \frac{\phi_i}{b} + \theta_i \\ &= \widehat{\lambda}\left(\frac{1-b}{b} + 1\right) = \frac{1}{b}\widehat{\lambda}.\end{aligned}$$

So $1/b$ is the "multiplier" of productivity shocks: a uniform 1% increase in productivity translates into a $1/b\%$ increase in GDP.

How big is $1/b$ empirically? A simple measure is to observe that:

$$\frac{h_S}{h_W} = 1/b$$

Alternatively, $b = h_W/h_S$ is the ratio of value added to sales of a typical firm.

		All Countries	Rich Countries	USA
Sales herfindahl	h_S	22.0%	26.6%	6.1%
Workers herfindahl	h_W	3.8%	4.0%	1.2%
Ratio of value added to sales b	h_W/h_S	0.17	0.15	0.20

Table 2: Sales herfindahl h_S and Workforce herfindahl h_W (Eqs.16–17) in the year 2002.
Rich countries are the countries with GDP per capita greater than \$13,000. Source: Acemoglu, Johnson and Mitton (2004) for the international data, and Compustat for the USA data.

The conservative estimate is the U.S. one, which gives $b = 0.20$. This translates into a "productivity multiplier" $1/b = 5$.

Another way to measure b is to observe that it is 1 minus the share of intermediate inputs ("materials") in the production function. This data is more difficult to get. For the U.S., the Jorgensen, Gollop and Fraumeni (1987) data, updated in 1996, gives $b = 0.50$.[14] So we conclude that b is between 0.15 and 0.5.

This allows us to quantify better intensity of the comovement. Shea (2002) proposes a useful measure of comovement. If, by a statistical or

[14] Susanto Basu kindly provided this number.

mental procedure, we removed the common component of firms, instead of (19), firm level volatility would be $\widehat{Q}_i = \widehat{\lambda}_i$. GDP increase would be:

$$\widehat{Y}_{\text{No Cov}} = \sum_i \frac{\text{Value added of firm } i}{\text{GDP}} \widehat{Q_i}$$

But with the Long Plosser demand linkages, GDP increase is:

$$\widehat{Y} = \sum_i \frac{\text{Sales of firm } i}{\text{GDP}} \widehat{Q_i}$$

We have

$$\widehat{Y}_{\text{No Cov}} = b\widehat{Y}$$

So the ratio of GDP variance attributed to comovements is:

$$1 - \frac{var\left(\widehat{Y}_{\text{No Cov}}\right)}{var\widehat{Y}} = 1 - b^2 \tag{20}$$

This is the type of ratio that Shea calculates. He finds that 80% to 96% of the variance is due to complementarities. We compare this to what the model predicts. If $b = 0.2$ (resp. if $b = 0.5$), then $1-b^2 = 96\%$ (resp. 75%) of comovements are attributable to complementarities. We conclude that Long Plosser demand linkages generate enough realistically high comovements between firms.

This section shows that, to analyze the size of complementarities, it is enough (under some conditions) to work with the herfindahls of the economy. One does not need to know the details of the input output matrix. When we the empirical values for the value added to sales ratio b, we find that the complementarities generates by demand linkages indeed generate a large enough comovement across firms. We conclude that our "granular" hypothesis, when augmented by the Long Plosser model, generates both plausible aggregate fluctuations and comovements between firms.

5 Evidence on scalings and distributions

The reader may skip this section in the first reading. This section examines the model's predictions for the scaling of country level and firm level quantities.

5.1 Scaling of firm-level volatility

Here we summarize some evidence for the scaling of the growth rate of firms (9) and the scaling of GDP growth (13). It has been discussed in a series of papers by Stanley et al. (1996), Amaral et al. (1997), Canning et al. (1998) and Lee et al. (1998). In a nutshell, firms and countries have identical, non-trivial, scaling of growth rates. Stanley et al. (1996) and Amaral et al. (1997) study how the volatility of the growth rate of firms changes with size[15] S . To do this, one divides the firms in a number of bins of sizes S, calculate the standard deviation of the growth rate of their sales $\sigma(S)$, and plots $\ln \sigma(S)$ vs $\ln S$. One finds a roughly affine shape, displayed in Figure 2:

$$\ln \sigma^{\text{firms}}(S) = -\alpha \ln S + \beta. \tag{21}$$

Exponentiation gives (9). A firm of size S has volatility proportional to $S^{-\alpha}$ with $\alpha = 0.15$. This means that large firms have a smaller proportional standard deviation than small firms, but this diversification effect is weaker than would happen if a firm of size S was composed of S independent units of size 1, which would predict $\alpha = 1/2$.

Canning et al. (1999) do the same analyses for country growth rates and find[16] that countries with a GDP of size S also have a volatility of size $S^{-\alpha'}$, with $\alpha' = 0.15$. The two graphs are plotted in Figure 2. The slopes are indeed very similar, and statistical tests reported in Canning et al. (1998) say that one cannot reject the null that $\alpha = \alpha'$. This is particularly interesting in light of Proposition 3 and Corollary 4, which say that this should be the case if Zipf's law holds[17].

One important caveat is in order. The above estimate of α, the scaling exponent of firms, is likely biased upwards. The reason is that it is estimated only with firms in Compustat, i.e. listed in the stock market. For a given size, a firm that is highly volatile is more likely to be in Compustat than a less volatile firm. This effect is weaker for big firms. This implies that the

[15] The measure of size can be assets, sales, or number of employees. Those three measures give similar results.

[16] Another way to see their result is to regress:

$$\begin{aligned} \ln \sigma_i &= -\alpha \ln Y_i + \beta \ln \text{GDP/Capita} + \gamma \text{Openness} \\ &\quad + \delta \text{Gvt share of GDP} + \text{constant} \end{aligned}$$

where σ_i is the standard deviation of $\ln Y_{it}/Y_{it-1}$ and Y_i the mean of the Y_{it}. We run this over the top 90% of the countries to avoid the tiniest countries, and find that $\alpha = .15$ with a standard deviation of .015.

[17] Acemoglu and Zilibotti (1997) propose a different mechanism by which large countries are more diversified and have a smaller volatility.

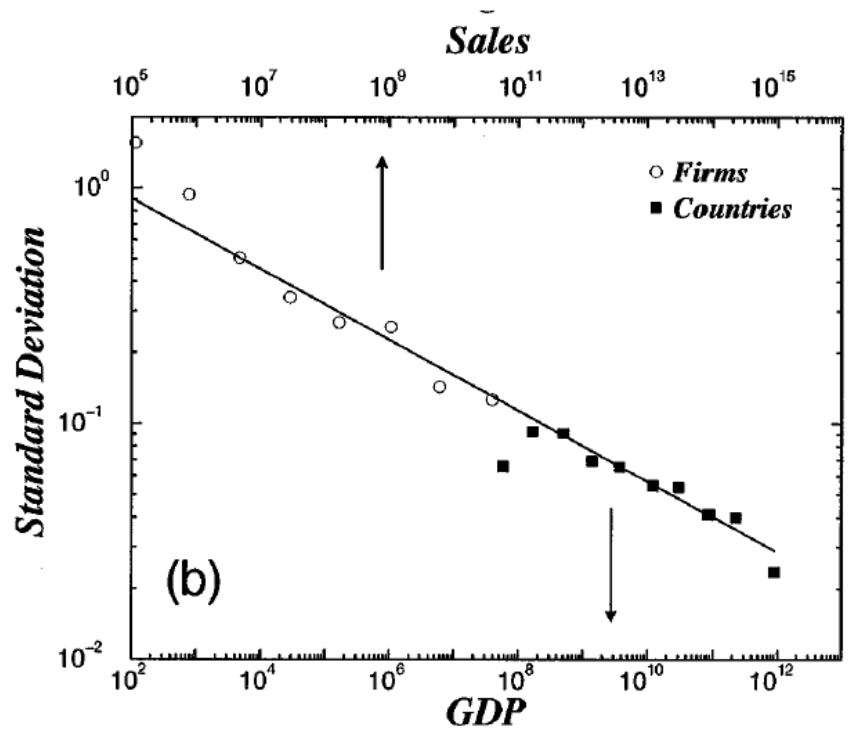

Figure 2: Standard deviation of the distribution of annual growth rates (log log axes). Note that $\sigma(S)$ decays with size S with the same exponent for both countries and firms, as $\sigma(S) \sim S^{-\alpha}$, with $\alpha = .15$. The size is measured in sales for the companies (top axis) and in GDP for the countries (bottom axis). The firm data are taken from the Compustat from 1974, the GDP data from Summers and Heston (1991). Source: Lee et al. (1998).

value of α measured in a sample composed only of firms in Compustat will be larger than the true empirical value. So, the empirical value we find is more likely to be an upper bound on the true α rather than the true value. The best way to estimate the true value of α would be to run a regression (21) on a sample that includes all firms, not just firms listed in Compustat (Census data, for example lists more firms). It is possible, indeed, that the best value is $\alpha = 0$, as random growth models have long postulated. More research is needed to assess this.

5.2 The distribution of fluctuations in firms and GDP growth

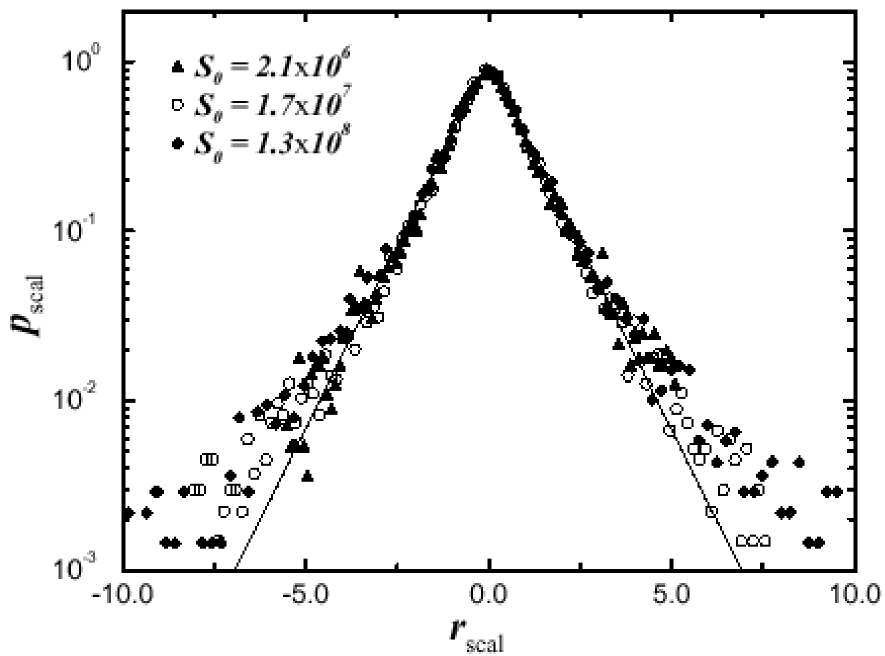

Figure 3: Empirical distribution of the fluctuation of firm sizes. The shape is very similar to that of the Levy distribution predicted by the model (see Figure 4 below). Source: Amaral et al. (1997).

This section examines the prediction of Proposition 3 for the distribution of fluctuations in firms and GDP.

5.2.1 Fluctuations without border effects: Empirical evidence on a Lévy distribution of firms' fluctuations

One can reinterpret Proposition 3 by interpreting a large "firm" as a "country" made up of smaller entities. If those entities follow a power law distribution, then Proposition 3 applies and predicts that the fluctuations of the growth rate $\Delta \ln S_{it}$, once rescaled by $S_{it}^{-\alpha}$, will follow a Levy distribution with exponent $min\{\zeta/(1-\alpha), 2\}$. Amaral et al. (1997) and Canning et al. (1998) plot this empirical distribution, and we reproduce their finding in Figure 3.

We next compare this graph to Proposition 3's prediction – a symmetrical Lévy distribution with exponent $1/(1-\alpha)$ and $\alpha = 0.15$. Figure 4 draws

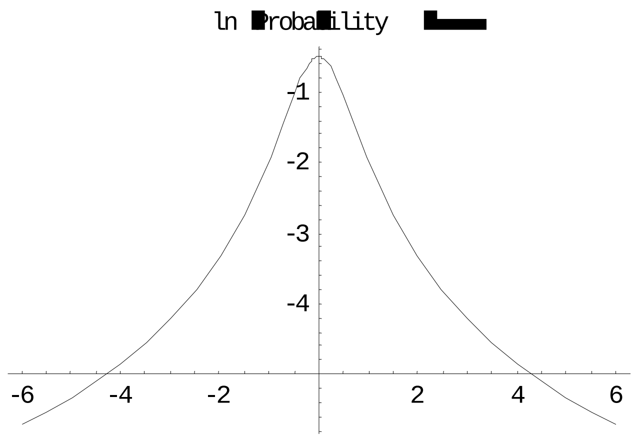

Figure 4: Log of a symmetrical Levy distribution with an exponent of $1/(1-\alpha)$, with $\alpha = 0.15$.

this distribution ($\ln p(x)$ vs x).

We see that the shapes are both much fatter than a Gaussian. We now investigate the best fist, assuming that the growth rate follows a symmetrical Lévy distribution with exponent β. The Gaussian benchmark corresponds to $\beta = 2$.

Calling g_{it} the growth rate of firm i in year t, we transform $\gamma_{it} = A_t g_{it} + B_t$ such that for all t's, $E[\gamma_{it}] = 0$ and $\text{Median}(|\gamma_{it}|) = 1$. We plot the distribution of γ_{it}, which is strikingly close to a Lévy with exponent $1/(1-\alpha)$. There are some deviations, for very large $|\gamma|$. Hypothesizing that for $|\gamma_{it}| \leq \overline{\gamma}$, γ_{it} follows a Lévy with exponent β, we estimate β by maximum likelihood. We take $\overline{\gamma} = 10$. As $P(|\gamma_{it}| \leq \overline{\gamma}) = 0.99$ empirically, this means that we fit the 99% of the points. We do this for each year separately, which give us a series of β's. We find:

$$\begin{aligned} \text{Mean of } \beta &= 1.28 \\ \text{Standard deviation of } \beta_t &= 0.11 \\ \sigma(\beta)/(\text{Number of years})^{1/2} &= 0.016. \end{aligned}$$

Empirically, we conclude that $\beta = 1.28$ with a standard deviation of 0.016. The prediction is $1/(1-\alpha) = 1.18$ for $\alpha = 0.15$. Thus, the empirical data is fairly close to the theoretical prediction.

5.2.2 Fluctuations with border effects: Distribution of GDP growth

Theory The above theory needs to be amended slightly for GDP, because typically the largest firm in a country only accounts for a small fraction (say couple of percentage points) of a country's GDP. We speculate that this is because of antitrust concerns.

We now modify the analysis to incorporate this fact. The payoff will be a better prediction of the shape of GDP fluctuations. We adopt the following representation. If we have a country with N firms, the size of firms S_i are drawn from a power law with exponent $\zeta = 1+\varepsilon$, but with bounded support $[1, mN]$. The density is assumed to be a power law with an exponent ζ in $[1, mN]$, i.e.:

$$f(S) = \frac{\zeta}{1-(mN)^{-\zeta}} S^{-\zeta-1}.$$

The total size is $Y = \sum_{i=1}^{N} S_i$. We can also establish the distribution of the fluctuations in Y.

Proposition 5 *If the subcomponents cannot have a size bigger than mN, for some finite m, we have, given the standard deviation σ_i of a country, that the fluctuations are normal*

$$\frac{\Delta Y}{Y} =^d Y^{-\alpha} V^{1/2} u$$

where u is a normal variable. In particular, if $m < \infty$, all moments are finite. Given only the size Y of the country, the fluctuations have the density:

$$f_{m,\alpha}(g) = \int_0^\infty e^{-\psi_{m,2-2\alpha}\left(k^2/2\right)} \cos(kg) \frac{dk}{\pi} \tag{22}$$

and all the moments are finite. We call this distribution a "modified Levy distribution". In the limit $m \to \infty$, this distribution tends to a symmetrical Lévy distribution with an exponent of $1/(1-\alpha)$. In the limit $m \to 0$, this distribution tends to a Gaussian.

The proof is in Appendix D.

We find a new "universal" distribution that does not depend on the details of the shocks to the individual firms. This is analogous to the fact that in the central limit theorem the limiting distribution does not depend on the details of the distribution of the initial shocks.

We make a few observations on our modified Levy distribution. When $m \to \infty$, there are no restrictions on the support of the subunits, and we get the the Lévy $1/(1-\alpha)$ distribution predicted by Proposition 3. When $m \to 0$, even the largest firms are small (they are bounded above by $mY/E[S]$). Since the total variance is the sum of lots of small variances, the central limit theorem applies, and hence the fluctuations are Gaussian. The proof shows that their order of magnitude is $m^{1/2-\alpha}$.

To calibrate the value of m, we observe that a typical value for the size of the top firm is 2% [give source]. The size of the largest firms in the model is $m/E[S]$ times Y. So for the calibrations we can take $m = 2\% \cdot E[S] = 0.5$ with $E[S] = 25$ employees. Numerical simulations indicate that the resulting distribution is quite close to the theoretical limit $m \to \infty$ in the relevant domain, so that we get a Lévy distribution with parameter $1/(1-\alpha)$.

Empirical evidence The empirical distribution is plotted in Figure 5. Figure 6 shows the corresponding theoretical plot for the distribution of growth rates. We see that the two distributions are pretty close. (A formal measure of the distance will be put in the next iteration of the paper).

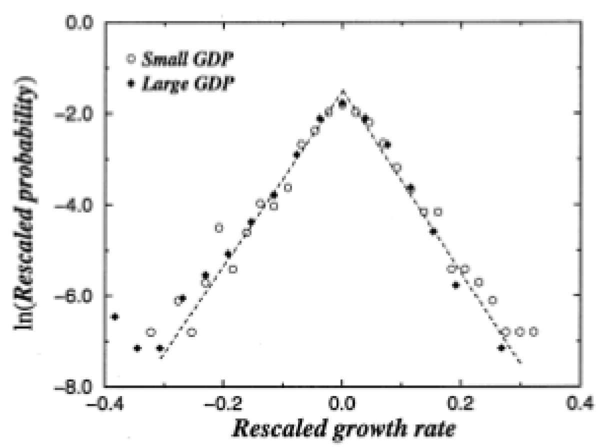

Figure 5: Empirical distribution of GDP fluctuations. Source: Canning et al. (1998)

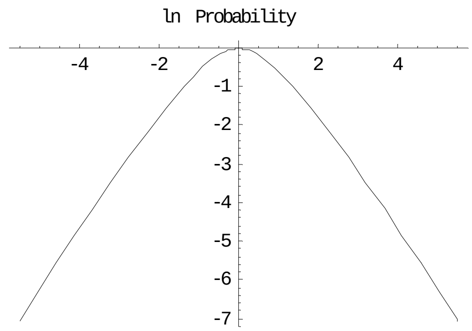

Figure 6: ln(Probability of a growth rate g) vs g under the null of the modified Levy distribution predicted by the model (with parameters $2-2\alpha = 1.7$ and $m = 1$).

6 Discussion

6.1 Extension of the model with feedback

The previous sections established the main theoretical results. Having calibration and greater descriptive realism in mind, we modify the model into:

$$\frac{\Delta S_{it+1}}{S_{it}} = \lambda \frac{\Delta S_t}{S_t} + v S_{i,t-1}^{-\alpha} u_{it}. \tag{23}$$

The interpretation of the $\lambda \Delta S_t / S_t$ term is that there is a feedback effect of past aggregate fluctuations ($\Delta S_{t+1}/S_t$) onto new decisions of firm i. This leads to a "multiplier" of shocks: a shock to firm j affects firm i in the next period. This feedback could come from a variety of sources, among them the Long-Plosser (1983) production demand type, Keynesian "aggregate demand" effects, or via expectations (consumers, or businesses, see the other firms are doing very well, so they have more optimistic expectations and spend or invest more).

We allow firm specific shocks to be autocorrelated in an AR(1) manner:

$$u_{it} = \sum_{s \geq 0} \delta^s \varepsilon_{i,t-s}$$

where the ε_{it} are i.i.d. Aggregate fluctuations are:

$$\Delta Y_t = \sum_{i=1}^{N} \Delta S_{it} = \lambda \Delta Y_{t-1} + v \sum_{i=1}^{N} S_i^{1-\alpha} \sum_{s \geq 0} \delta^s \varepsilon_{i,t-s}$$

thus, with L the lag operator ($Lx_t = x_{t-1}$ for a random process x_t) :

$$\frac{\Delta Y_t}{Y^{1-\alpha}} = \frac{v}{Y^{1-\alpha}} \sum_{i=1}^{N} S_i^{1-\alpha} (1-\lambda L)^{-1} (1-\delta L)^{-1} \varepsilon_{it}$$

and if we look at the fluctuations sampled at horizon H (for instance, if the underlying unit of action is the quarter, and we look at yearly fluctuations, $H = 4$), defining:

$$\begin{aligned} \Delta S_t^{(H)} &= S_t - S_{t-H} \\ &= \left(1 + L + ... + L^{H-1}\right) \Delta S_t \end{aligned}$$

we get:

$$\frac{\Delta S_t^{(H)}}{S^{1-\alpha}} = \frac{v}{S^{1-\alpha}} \sum_{i=1}^{N} S_i^{1-\alpha} \eta_{it}$$

defining

$$\eta_{it} = \left(1 + L + ... + L^{H-1}\right) (1-\lambda L)^{-1} (1-\delta L)^{-1} \varepsilon_{it}.$$

So the essence of this algebra is that, like in the simple case of section 2, we can represent:

$$\frac{\Delta S_t^{(H)}}{S^{1-\alpha}} = v\sigma g_t \tag{24}$$

with only with a messier expression for σ :

$$\sigma^2 = var \left(1 + L + ... + L^{H-1}\right) (1-\lambda L)^{-1} (1-\delta L)^{-1} \varepsilon_{it}.$$

The main points from (24) are the following: we get that $\Delta S/S$ has fluctuations with the shape of g, it scales like $S^{-\alpha'}$, and (as is classic in the literature) the feedback λ can considerably increase the variance of aggregate fluctuations.

Given that the volatility of a firm is $var \left(1 + L + ... + L^{H-1}\right) (1-\delta L)^{-1} \varepsilon_{it}$, the ratio

$$M = \left[\frac{var \left(1 + L + ... + L^{H-1}\right) (1-\lambda L)^{-1} (1-\delta L)^{-1} \varepsilon_{it}}{var \left(1 + L + ... + L^{H-1}\right) (1-\delta L)^{-1} \varepsilon_{it}} \right]^{1/2} \tag{25}$$

plays the role of a "volatility multiplier". Indeed, we have:

$$\sigma_{GDP} = M\sigma_{\text{Micro}}, \text{ with} \tag{26}$$

$$\sigma_{\text{Micro}}^2 := \sum_i \sigma_i^2 \left(\frac{S_i}{Y}\right)^2 \tag{27}$$

where σ_i is the volatility of firm i, and S_i is size as a fraction of total GDP.

For $H = 1$, $\delta = 0$, we have

$$M = \frac{1}{\sqrt{1 - \lambda^2}}.$$

For $H \gg 1/(1 - \lambda)$, $\delta = 0$, (no autocorrelation of shocks, but essentially all the propagation via $\lambda \Delta S_t / S_t$ happens within a period) we have:

$$M = \frac{1}{1 - \lambda}.$$

As mentioned above, the nature of the feedback leading to the multiplier could be very diverse. We do not want to take a stand here on the various "amplification mechanisms" proposed in macroeconomic research. We summarize their reduced form here by M. Given our earlier Compustat calibration $\sigma_{Micro} = 1.3\%$, it is not difficult to generate a fluctuation $\sigma_{GDP} = M \sigma_{Micro}$ of an empirical order of magnitude around 2%. We only need a multiplier close to 1.5.

7 Conclusion

There are clearly "macroeconomic" shocks: monetary policy shocks, policy shocks, trade (e.g. exchange rate) shocks, and possibly aggregate productivity shocks. However, is it possible that, though they are the most visible ones, they are not the major contributors to GDP fluctuations. The present paper lays down the theoretical possibility that idiosyncratic shocks are an important, and possibly the major, part of the origins of business cycle fluctuations.

It may be worthwhile to contemplate the possible consequences of the hypothesis that idiosyncratic shocks to large firms are an important determinant of the volatility of aggregate quantities.

First, one may understand the origins of fluctuations better: they do not come from mysterious "aggregate productivity shocks," but from concretely observable shocks to the large players, such as Wal-Mart, Intel, and Nokia.

Second, these shocks to large firms, initially independent of the rest of the economy, will offer a rich source of shocks for VARs and impulse response studies – the real-side equivalent of the "Romer and Romer" shocks for monetary economics. For instance, a strike, or the tenure of a new CEO could be a source of for a macroeconomic shocks plausibly independent from the rest of the economy.

Third, this gives a new theoretical angle for the propagation of fluctuations: For instance, if Wal-Mart innovates, its competitors may suffer in the short term, but then scramble to catch-up. This creates rich industry-level dynamics (that are already actively studied in IO) which my work implies should be very useful for studying macroeconomic fluctuations since they allows us to trace the dynamics of productivity shocks.

Fourth, this could explain the reason why people, in practice, do not know "the state of the economy" – i.e. the level of productivity, in the RBC language. In our view, this is because "the state of the economy" depends on the behavior (productivity and investment behavior, among others) of many large firms. So the integration is not easy, and no readily accessible single number can summarize this state. This could offer a new and relevant mechanism for the dynamics of "animal spirits".

Finally this mechanism might explain a large part of the volatility of many aggregate quantities such as inventories, inflation, short or long run movements in productivity, and the current account. The latter is explored in a companion paper, Canals et al. (2004).

8 Appendix A: A simple model illustrating the "islands" economy

The paper presents a mechanism that emerges from a variety of economic structures. Here we present one possible type of model that generates the mechanism. Markets are competitive. Firm i has a capital K_{it}. It invests in a technology with random productivity A_{it} such that $E\left[A_{it}\right]$ is constant across i's and

$$\sigma\left(A_{it}\right)=bK_{ti}^{-\alpha}. \tag{28}$$

A variety of mechanisms (e.g. Amaral et al. (1998), Sutton (2001)) can generate the microeconomic scaling presented in equation (28). These mechanisms typically assume that firms of size S are made up of N smaller units, with $N\sim S^{\alpha/2}$, which generates (**??**) and (28). Capital is fully reinvested, so that:

$$K_{i,t+1}=A_{i,t+1}K_{t}. \tag{29}$$

GDP is simply:

$$Y_{t}=\sum_{i}A_{i,t}K_{t-1}.$$

Adding labor does not change the conclusion of this paper. Suppose that the production function is $A_{i,t}F\left(K_{ti},L_{ti}\right)$, with constant returns to scale. Risk neutral firms maximize

$$\max_{L_{it}}E\left[A_{it}\right]F\left(K_{ti},L_{ti}\right)-w_{t}L_{ti}.$$

The quantity of labor chosen L_{ti}will be $L_{it}=\lambda_{t}K_{it}$, for a factor of proportionality λ, so that we will have:

$$K_{i,t+1}=A_{i,t+1}F\left(K_{it},\lambda_{t}K_{it}\right)-w_{t}\lambda_{t}K_{it}=\left(A_{i,t+1}F\left(1,\lambda_{t}\right)-w_{t}\lambda_{t}\right)K_{it}.$$

The equation of motion follows the same structure as (29), with random productivity:

$$A_{it}'=A_{i,t+1}F\left(1,\lambda_{t}\right)-w_{t}\lambda_{t}.$$

GDP is

$$Y_{t}=\sum_{i}A_{i,t}K_{t-1}F\left(1,\lambda_{t-1}\right)$$

so that it evolves as the stochastic sum in the paper.

9 Appendix B: Evidence on firm-level volatility

9.1 Idiosyncratic volatility is very large

Our data on idiosyncratic volatility come from Compustat. For large firms it is likely that Compustat is very representative, as it includes most of these firms. For small firms, Compustat may not be fully representative, just like the stock market may not represent of all firms[18]. It is still the best dataset we have so far. Future studies using Census data will give us much better data.

As firms can die, some choices have to be made on which firms are included in the sample. Luckily, various specifications give very similar results. The simplest exercise is to follow a set a firm for a extended amount of time. To have good statistics, we need many firms, and thus fairly recent data as the Compustat coverage has been growing. Thus, we use all firms for all years from 1980 to 2002. The results do not depend at all on the starting year,1980. Yet, a much earlier starting date would yield too few firms while a much later one would yield too few years. We remove foreign firms and we use reports on sales (data12: sales(net) in MM$). Alternative measures give similar results, as indeed they are proportional in the medium run. We deflate sales using BEA Implicit Price Deflators for Gross Domestic Product (year 2000=100). Thus we have observations from 6155 firms (21016 if we don't remove firms absent in 1980) from 1980 to 2002. This adds up to 76926 (186075 if we don't remove firms absent in 1980) data-points (year-firm) on sales, and 69743 (159660 if we don't remove firms absent in 1980) data-points (year-firm) on the growth rate $g_{it} = \ln(S_{it}/S_{it-1})$.

The raw standard deviation of the g_{it} is 0.442. This means that the standard deviation of the sales of firms in Compustat is 44.2% a year – a very high number. This number is a bit smaller for large firms, according to (**??**). The average standard deviation is a very similar number, 0.462.

Simple standard deviation	$stddev(g_{it}) = 0.442$
Average standard deviation	$\sqrt{E[\sigma_i^2]} = 0.462$
Absolute deviation	$absdev_{tot} = E[\lvert g_{it} - E[g_{it}]\rvert] = 0.204$
Interquartile range	$IQR_{tot}(g_{it}) = 0.193$

Table: Statistics on the dispersion of growth rates $g_{it} = \ln(S_{it}/S_{it-1})$, where S_{it} are the sales of firm i at time t.

[18] Indeed this "selection bias" creates an upward bias in the measurement of β in the microeconomic scaling law (**??**).

As firms' growth rates have fat tails, the variance might not be a very robust estimator. We look at two robust estimators of deviation. The interquartile range is the value of the 75% percentile minus the 25% percentile of growth rates. It is equal to 0.193. For a Gaussian with standard deviation σ, the interquartile range is 0.675σ, so if the distribution was Gaussian we would infer a standard deviation 0.193/0.675= 0.285. Again, this is a very high dispersion. Our last measure of dispersion is the absolute deviation, which gives 0.204. If the distribution is Gaussian, we would infer a standard deviation of $0.203\sqrt{\pi/2} = 0.254$. As the distribution has tails fatter than a Gaussian, the true standard deviation is higher than those last two values.

We conclude from this analysis that indeed, the typical standard deviation of the growth rate of firm in Compustat is very high, with a point estimate of 0.44% per year, which is robust to a variety of other measures of dispersion.

It is clear that this must be accounted for by idiosyncratic shocks, as the standard deviation of macroeconomic quantities such as GDP growth is much lower. To verify this formally, we run the following regression with fixed effects and AR(1) noise:

$$\begin{aligned} g_{it} &= \alpha_i + f_t + \varepsilon_{it} \\ \varepsilon_{it} &= \rho \varepsilon_{it-1} + u_{it} \end{aligned}$$

where u_{it} is i.i.d. with mean zero.

We find a standard deviation

$$\begin{aligned} \sigma(f_t) &= 0.044 \\ \sigma(\varepsilon_{it}) &= 0.400 \\ \sigma(g_{it} - \alpha_i) &= 0.402. \end{aligned}$$

Hence aggregate shocks account for only 1.25% $(= \sigma(f_t)^2 / \sigma(g_{it} - \alpha_i)^2)$ of the variance of firm growth rate. Likewise, the correlation of the growth rate between two random firms is only $\rho = 0.012$.

9.2 Microeconomic scaling

The scaling law says that a unit of size S, in a year t, will have a standard deviation:

$$\sigma(S,t) = \text{standard deviation}\left(\ln S_{t+1} - \ln S_t \mid S_t = S\right) = b_t S^{-\alpha_t}. \tag{30}$$

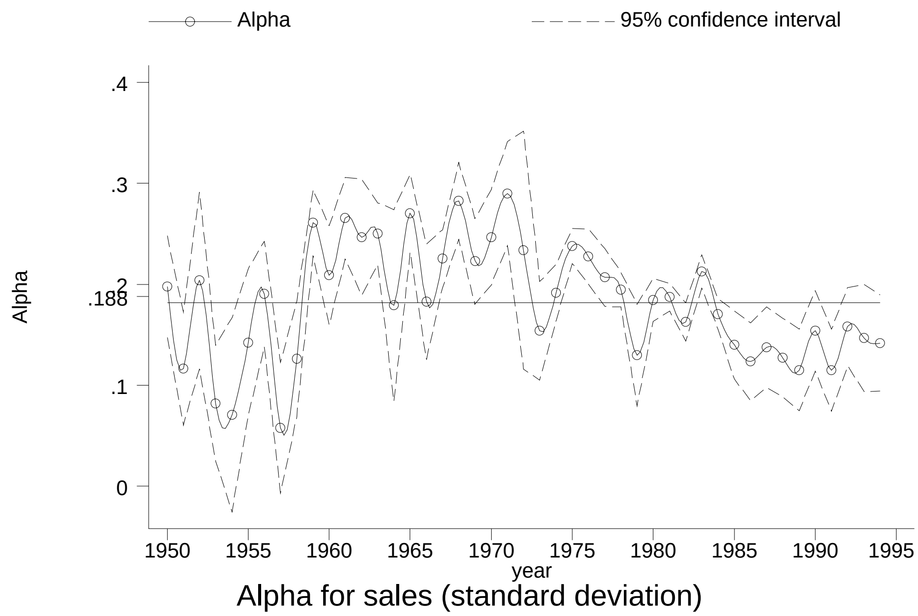

Figure 7: Time series of the scaling exponent α_t for the growth of sales. For each year t we estimate the scaling exponent α_t such that $\sigma(g_t \mid S_t = S) \sim S^{-\alpha_t}$.

Amaral et al. (1997) present evidence for the scaling law for a particular year t. Here we extend their empirical analysis.

We first proceed with size as a measure of sales. We estimate α_t for each year and plot in the resulting values of α_t in Figure 7. We show here that α_t has remained fairly constant throughout the years. Its mean value is 0.188.

Interestingly, the coefficient b_t has increased over the year.

We have estimated α for the firms in different SIC 1-digit codes. The coefficient is constant across 1-digit industries.

10 Appendix C: Lévy's theorem

The basic theorem can be found in most probability textbooks, e.g. Durrett, (1996, p.153).

Theorem 6 *(Levy, Gnedenko-Kolmogorov). Suppose that* $x_1, x_2, ...$ *are i.i.d. with a distribution that satisfies:*

(i) $\lim_{x\to\infty} P(x_1 > x)/P(|x_1| > x) = \theta \in [0,1]$
(ii) $P(|x_1| > x) = x^{-\zeta} L(x)$
with $\zeta \in (0,2)$ *and* $L(x)$ *slowly varying*[19]*. Let* $s_n = \sum_{i=1}^{n} x_i$*, and*

$$a_n = \inf\{x : P(|x_1| > x) \le 1/n\} \text{ and } b_n = nE\left[x_1 1_{|x_1|\le a_n}\right]$$

As $n \to \infty$*,* $(s_n - b_n)/a_n \to^d Y$ *where* Y *is a Lévy distribution with exponent* ζ*.*

In practice, for a power law distribution $P(x_1 > x) = (x/x_0)^{-\zeta}$,

$$a_n = x_0 n^{-1/\zeta}. \tag{31}$$

A symmetrical Lévy distribution with exponent $\zeta \in (0,2]$ has the distribution:

$$\lambda(x,\zeta) = \frac{1}{\pi}\int_0^\infty e^{-k^\zeta}\cos(kx)\,dk \tag{32}$$

and the cumulative:

$$\Lambda(x,\zeta) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty e^{-k^\zeta}\frac{\sin(kx)}{k}dk. \tag{33}$$

For $\zeta = 2$, a Levy distribution is a Gaussian. For $\zeta < 2$, the distribution has power law tail with exponent ζ. Unfortunately, there are no closed form formulae for λ and Λ except in the case $\zeta = 1$ (Cauchy distribution) and $\zeta = 2$ (normal distribution).

11 Appendix D: Longer derivations

11.1 Proof of Proposition 1

Because of $\sigma_{GDP} = \sigma h$, we examine h.

$$N^{1/2}h = \frac{\left(N^{-1}\sum_{i=1}^{N} S_i^2\right)^{1/2}}{N^{-1}\sum_{i=1}^{N} S_i}$$

[19] $L(x)$ is said to be slowly varying (e.g. Embrechts *et al.* 1997, p.564) if

$$\lim_{x\to\infty} L(tx)/L(x) = 1 \text{ for all } t > 0.$$

Prototypical examples are $L = a$ and $L(x) = a\ln x$ for a non-zero constant a.

The law of large numbers ensures that

$$N^{-1}\sum_{i=1}^{N} S_i^2 \quad \rightarrow \quad ^{a.s.} E\left[S^2\right]$$
$$N^{-1}\sum_{i=1}^{N} S_i^2 \quad \rightarrow \quad ^{a.s.} E\left[S\right]$$

and we can conclude: $N^{1/2}h \rightarrow E\left[S^2\right]^{1/2}/E\left[S\right].$

11.2 Proof of Proposition 2

Because of $\sigma_{GDP} = \sigma h$, we examine h.

$$h = \frac{\left(\sum_{i=1}^{N} S_i^2\right)^{1/2}}{\sum_{i=1}^{N} S_i} \tag{34}$$

We treat the cases where $\zeta > 1$ and $\zeta = 1$ separately.

Case A: $1 < \zeta \leq 2$. By the law of large numbers,

$$N^{-1}\sum_{i=1}^{N} S_i \rightarrow E\left[S\right].$$

However, S_i^2 has power law exponent $\zeta/2 < 1$, as shown by:

$$P\left(S^2 > x\right) = P\left(S > x^{1/2}\right) = a\left(x^{1/2}\right)^{-\zeta} = ax^{-\zeta/2}.$$

So to handle the numerator of (34), we use Lévy's Theorem from Appendix A. This implies:

$$N^{-2/\zeta}\sum_{i=1}^{N} S_i^2 \rightarrow u$$

where u is a Levy distributed random variable with exponent $\zeta/2$. So

$$N^{1-1/\zeta}h = \frac{\left(N^{-2/\zeta}\sum_{i=1}^{N} S_i^2\right)^{1/2}}{N^{-1}\sum_{i=1}^{N} S_i} \rightarrow^{d} \frac{u^{1/2}}{E\left[S\right]}.$$

Case B: $\zeta = 1$. Some more care is required, because $E\left[S\right] = \infty$. We use Theorem 6, which gives $b_n = n\ln n$, hence:

$$N^{-1}\left(\sum_{i=1}^{N} S_i - N\ln N\right) \rightarrow^{d} g$$

where g is a Levy with exponent 1. We conclude:

$$\ln N \cdot h \to^d \frac{u^{1/2}}{g}.$$

11.3 Proof of Proposition 3

As $\Delta S_i / S_i = \nu S_i^{-\alpha} u_i$:

$$\frac{\Delta Y_{t+1}}{Y_t} = \frac{\sum_{i=1}^N \Delta S_{it}}{Y_t} = v \frac{\sum_{i=1}^N S_i^{1-\alpha} u_{it}}{\sum_{i=1}^N S_i}. \tag{35}$$

By the law of large numbers:

$$N^{-1} Y_t = N^{-1} \sum_{i=1}^N S_i \to \overline{S}.$$

To tackle the numerator, we observe that $S_i^{1-\alpha}$ has power law tails with exponent $\zeta' = \zeta / (1-\alpha)$. We need to consider two cases.

If $\zeta' < 2$, $x_i = S_i^{1-\alpha} u_i$, which has power law tails with exponent ζ', and by Levy's theorem:

$$N^{-1/\zeta'} \Delta Y_t = N^{-1/\zeta'} \sum_{i=1}^N S_i^{1-\alpha} u_{it} \to^d g$$

where g is a Levy with exponent ζ'.

If $\zeta' \geq 2$, $S_i^{1-\alpha} u_i$ has finite variance, and $N^{-1/2} \Delta Y_t \to^d g$, where g is a Gaussian.

We conclude that in both cases:

$$N^{-\max(1/2, 1/\zeta')} \Delta Y_t \to^d g$$

for a distribution g. So

$$N^{1-\max(1/2,1/\zeta')} \frac{\Delta Y_{t+1}}{Y_t} \to^d \frac{g}{\overline{S}} g.$$

We conclude that the Proposition holds, with

$$\begin{aligned} \alpha' &= 1 - \max\left(1/2, 1/\zeta'\right) = 1 + \min\left(-1/2, -1/\zeta'\right) \\ &= \min\left(1/2, 1 - 1/\zeta'\right) = \min\left(1/2, 1 - \frac{1-\alpha}{\zeta}\right). \end{aligned}$$

12 Proof of Proposition 5

We start by stating:

Proposition 7 *If the subcomponents cannot have a size bigger than mN, for some finite m, the variance of Y scales as:*

$$\sigma_Y^2 \sim Y^{-2\alpha} V$$

where V is a random variable whose Laplace transform is:

$$L^V(k) := E\left[e^{-kV}\right] = e^{-\psi_{m,2-2\alpha}(k)}$$

where $\psi(k)$ is defined in (39)–(40). In the limit $m \to \infty$, V is a totally positive Lévy distribution with exponent $1/(2-2\alpha)$.

In particular, all the moments are finite. Indeed, one can easily calculate the cumulants of V (the κ_i such that $-\ln L^V(k) = \sum \kappa_i k^i / i!$) and find:

$$\kappa_i(V) = \frac{m^{\gamma i - 1}}{\gamma i - 1}.$$

Recall that the 4 first cumulants $(\kappa_i)_{i=1\ldots4}$ are respectively $\langle V \rangle, varV, \left\langle (V - \langle V \rangle)^3 \right\rangle$, and $\left\langle (V - \langle V \rangle)^4 \right\rangle - 3varV$; i.e. the mean, variance, skewness and excess kurtosis.

We define:

$$V_N := \frac{1}{N^{2-2\alpha}} \sum_{i=1}^{N} S_i^{2-2\alpha} \tag{36}$$

where S_i is drawn from the above distribution. We study V_N in the limit of large $N's$. We know from the analysis above, that for $m = \infty$, V_N tends to a Lévy distribution with exponent $1/(2-2\alpha)$.We study its behavior for $m < \infty$. The tool of choice is the Laplace transform (using $\zeta = 1 + \varepsilon \simeq 1$)

$$\begin{aligned} L^{V_N}(k) \quad &: \quad = E\left[e^{-kV_N}\right] = E\left[\exp \frac{-k}{N^{2-2\alpha}} \sum_{i=1}^{N} S_i^{2-2\alpha}\right] \\ &= \quad E\left[\exp \frac{-k}{N^\gamma} S_i^\gamma\right]^N \quad \text{with} \\ \gamma \quad &: \quad = 2 - 2\alpha. \end{aligned} \tag{37}$$

Now

$$
\begin{aligned}
H \quad : \quad &= E\left[\exp \frac{-k}{N^{\gamma}} S_i^{\gamma}\right] = \int_1^{mN} \frac{\zeta}{1-(mN)^{-\zeta}} S^{-\zeta-1} \exp\left(\frac{-k}{N^{\gamma}} S_i^{\gamma}\right) dS \\
&= \frac{1}{1-(mN)^{-1}} \int_1^{mN} \exp\left(\frac{-k}{N^{\gamma}} S_i^{\gamma}\right) \frac{dS}{S^2} \\
&= \frac{1}{1-(mN)^{-1}} N^{-1} \int_{N^{-\gamma}}^{m^{\gamma}} \frac{\exp(-kt)}{\gamma t^{1+1/\gamma}} dt \text{ by the change in variables } S = N t^{1/\gamma}.
\end{aligned}
$$

Note that as $N \to \infty$,

$$
H \sim N^{-1} \int_{N^{-\gamma}}^{m^{\gamma}} \frac{dt}{\gamma t^{1+1/\gamma}} \sim 1.
$$

So we use (verifying that $H(k=0) = 1$)

$$
\begin{aligned}
H - 1 &= N^{-1} \int_{N^{-\gamma}}^{m^{\gamma}} \frac{\exp(-kt) - 1}{\gamma t^{1+1/\gamma}} dt + o\left(\frac{1}{N}\right) \\
&= -\frac{1}{N} \psi(k) + o\left(\frac{1}{N}\right) \qquad (38)
\end{aligned}
$$

with the new function:

$$
\psi_{m,\gamma}(k) := \int_0^{m^{\gamma}} \frac{1 - \exp(-kt)}{\gamma t^{1+1/\gamma}} dt \qquad (39)
$$

which has a closed form in terms of the Gamma function (analytically continued for $a < 0$). With $\Gamma(a, z) := \int_0^z e^{-t} t^{a-1} dt$ we have:

$$
\psi_{m,\gamma}(k) = -\frac{k^{1/\gamma}}{\gamma} \Gamma\left(-\frac{1}{\gamma}, k\, m^{\gamma}\right) - m. \qquad (40)
$$

Finally, expressions (37) and (38) give, in the limit of large N's:

$$
\begin{aligned}
\ln L^{V_N}(k) &= N \ln H = N \ln\left(1 - \frac{1}{N}\psi(k) + o\left(\frac{1}{N}\right)\right) \\
&= -\psi(k) + o(1).
\end{aligned}
$$

Thus V_N converges in distribution to a well-defined random variable V, whose Laplace transform is:

$$
L^V(k) = e^{-\psi(k)}. \qquad (41)
$$

We can also establish the distribution of the fluctuations in Y.

$\frac{\Delta Y}{Y} =^d Y^{-\alpha} V^{1/2} u$ from above. Thus the Fourier transform of the fluctuations is:

$$F(k) = E\left[e^{-ikV^{1/2}u}\right] = E\left[e^{-k^2V}/2\right] = e^{-\psi\left(k^2/2\right)}$$

so taking the inverse Fourier transform we get (22).

When $m \to \infty$,

$$\psi_{m,\gamma=2-2\alpha}\left(k^2/2\right) \to \int_0^\infty \frac{1-\exp\left(-k^2/2t\right)}{\gamma t^{1+1/\gamma}} dt = \frac{k^{2/\gamma}\Gamma\left(-1/\gamma\right)}{2^{1/\gamma}\gamma} = bk^{1/(1-\alpha)}$$

for some b. The characteristic function is that of a symmetric Lévy distribution.

When $m \to 0$,

$$\begin{aligned}\psi_{m,\gamma}(k) &= \int_0^{m^\gamma} \frac{1-\exp(-kt)}{\gamma t^{1+1/\gamma}} dt \sim \int_0^{m^\gamma} \frac{kt}{\gamma t^{1+1/\gamma}} dt \\ &= \frac{m^{\gamma-1}}{\gamma-1} k = \frac{m^{1-2\alpha}}{1-2\alpha} k\end{aligned}$$

so that $\psi_{m,\gamma}\left(k^2/2\right) \sim \frac{m^{1-2\alpha}}{1-2\alpha} k^2/2$, which shows that $\Delta Y/Y \left(\frac{m^{1-2\alpha}}{1-2\alpha}\right)^{-1/2}$ tends to a standard Gaussian distribution.

13 Appendix E: Hulten's theorem

For clarity, we will here rederive and extend Hulten (1978)'s result, which says that a productivity shock $d\pi_i$ to firm i causes an increase in GDP equal to:

$$\text{GDP growth} = \sum_i \frac{\text{Sales of firm } i}{\text{GDP}} d\pi_i.$$

There are N firms. Firm i produces good i, and uses a quantity X_{ij} is intermediary inputs from firm j. It also uses L_i units of labor, K_i units of capital. It has productivity π_i. If production is: $Q_i = F^i\left(I_{i1}, ..., I_{iN}, L_i, K_i, \pi_i\right)$.

The representative agent consumer C_i of good i, and has a utility function is $U\left(C_1, ..., C_N\right)$. Production of firm i serves as consumption, and intermediary inputs, so: $Q_i = C_i + \sum_k X_{ki}$The optimum in this economy reads:

$$\max_{C_i, X_{ik}, L_i, K_i} U\left(C_1, ..., C_N\right) \text{ s.t.}$$

$$\begin{aligned} C_i + \sum_k X_{ki} &= F^i\left(X_{i1}, ..., X_{iN}, L_i, K_i, \lambda_i\right) \\ \sum_i L_i &= L, \sum_i K_i = K \end{aligned}$$

The Lagragian is:

$$W = U\left(C_1, ..., C_N\right) + \sum_i p_i \left[F^i\left(X_{i1}, ..., X_{iN}, L_i, K_i, \lambda_i\right) - C_i - \sum_k X_{ki}\right] + w\left[L - \sum_i L_i\right] + r\left[K - \sum_i \right. \tag{42}$$

Assume marginal cost pricing[20]. GDP is this economy is

$$Y = wL + rK = \sum_i p_i C_i$$

So the value added of firm i is $wL_i + rK_i$, while its sales are: $p_i Q_i$.

Suppose that technological progress is Hicks-neutral productivity, so that $F^i\left(X_{i1}, ..., X_{iN}, L_i, K_i, \pi_i\right) = e^{\pi_i} G^i\left(X_{i1}, ..., X_{iN}, L_i, K_i\right)$. Say that each firm i has a shock $d\pi_i$ to productivity. Then, GDP changes by:

$$\begin{aligned} \frac{dW}{W} &= \frac{1}{W} d\left\{U\left(C_1, ..., C_N\right) + \sum_i p_i\left[e^{\pi_i} G^i\left(X_{i1}, ..., X_{iN}, L_i, K_i, \lambda_i\right) - C_i - \sum_k X_{ki}\right] + w\left[\sum_i L_i - \right.\right. \\ &= \frac{1}{W}\sum_i p_i\left[e^{\pi_i} G^i\left(X_{i1}, ..., X_{iN}, L_i, K_i, \lambda_i\right) d\pi_i\right] \\ &= \sum_i \frac{\text{Sales of firm } i}{\text{GDP}} d\pi_i \end{aligned}$$

which is Eq. 14.

This marginalist analysis shows that Hulten's theorem holds even if, after the shock, the capital, labor, and material inputs are not reallocated. This is a simple consequence of the envelope's theorem, and can alternatively be seen in Eq. 43. Hence Hulten's result holds also in a world with frictions to labor, capital, and or intermediate inputs.

[20] See Basu (XXX) for the analysis with imperfect competition.

References

[1] Acemoglu, Daron, Simon Johnson, and Todd Mitton (2004) "Reassessing Entry Barriers", mimeo.

[2] Acemoglu, Daron, and Fabrizio Zilibotti, "Was Prometheus Unbound by Chance? Risk, Diversification and Growth", August 1997, *Journal of Political Economy*, volume 105, pp. 709-751.

[3] Amaral, L. A. N. , S. V. Buldyrev, S. Havlin, H. Leschhorn, P. Maass, M. A. Salinger, H. E. Stanley, and M. H. R. Stanley, "Scaling behavior in economics: I. Empirical results for company growth", *J. Phys. (France) I* 7, 621-633 (1997).

[4] Axtell, Robert (2001) "Zipf distribution of U.S. firm sizes", *Science* 293, p.1818-20.

[5] Bak, P., K. Chen, José Scheinkman and M. Woodford (1993) "Aggregate Fluctuations from Independent Sectoral Shocks: Self-Organized Criticality in a Model of Production and Inventory Dynamics", *Ricerche Economiche*, 47, p.3-30.

[6] Barabási, A.-L. and R. Albert, "Emergence of Scaling in Random Networks" *Science* 286, 509-12 (1999)

[7] Basu, Susanto and Fernald, John G, 1997. "Returns to Scale in U.S. Production: Estimates and Implications," Journal of Political Economy, Vol. 105 (2) pp. 249-83.

[8] Bertrand, Marianne and Antoinette Schoar "Managing with Style: The Effect of Managers on Firm Policies," *The Quarterly Journal of Economics*, November 2003.

[9] Blanchard, Olivier and Simon Johnson (2001) "The Long and Large Decline in U.S. Output Volatility" *Brookings Papers on Economic Activity.*

[10] Buldyrev, S. V. , L. A. N. Amaral, S. Havlin, H. Leschhorn, P. Maass, M. A. Salinger, H. E. Stanley, and M. H. R. Stanley "Scaling behavior in economics: II. Modeling of company growth", *J. Phys. (France) I* 7, 635-650 (1997).

[11] Caballero, Ricardo and Eduardo M.R.A. Engel, "Adjustment is Much Slower than You Think" (2003), NBER WP.

[12] Caballero, Ricardo J., Eduardo M.R.A. Engel, and John C. Haltiwanger, "Aggregate Employment Dynamics: Building from Microeconomic Evidence", *American Economic Review*, 87 (1), March 1997, 115–137.

[13] Canals, Claudia, Xavier Gabaix, Josep Vilarrubia, David Weinstein (2004), "Can macroeconomics explain trade balance fluctuations?", Columbia and MIT mimeo.

[14] Canning, D. , L. A. N. Amaral, Y. Lee, M. Meyer, and H. E. Stanley (1998), "Scaling the volatility of GDP growth rates", *Economics Letters* 60, 335-341.

[15] Chaney, Thomas; Gabaix, Xavier and Philippon, Thomas. "The evolution of microeconomic and macroeconomic volatility," Mimeo, MIT, 2003.

[16] Comin, Diego, and Mulani (2003). "Diverging Trends in Macro and Micro Volatility: Facts," NYU mimeo.

[17] Conley, Timothy and Bill Dupor "A spatial analysis of sectoral complementarity", *Journal of Political Economy*, vol. 111(2), p.311-52.

[18] Davis, Steven, John C. Haltiwanger, and Scott Schuh (1996) "Job Creation and Destruction", MIT Press.

[19] Domar, Evsey D. (1961). "On the Measurement of Technical Change." *Economic Journal* 71: 710-729.

[20] Dupor, William, "Aggregation and irrelevance in multi-sector models", *Journal of Monetary Economics* 43 (1999) 391–409

[21] Durlauf, Steven (1993) "Non ergodic economic growth", *Review of Economic Studies*, vol. 60(2), p.349-66.

[22] Gabaix, Xavier (1999) "Zipf's law for cities: An explanation", *Quarterly Journal of Economics* 114 (3), August 1999, pp.739-67.

[23] Gabaix, Xavier, Parameswaran Gopikrishnan, Vasiliki Plerou and H. Eugene Stanley, "A theory of power law distributions in financial market fluctuations," *Nature*, 423 (2003), 267–230.

[24] Gabaix, Xavier, Parameswaran Gopikrishnan, Vasiliki Plerou and H. Eugene Stanley, "A theory of large fluctuations in stock market activity," (2004) mimeo.

[25] Gabaix, Xavier and Yannis Ioannides, "The evolution of the city size distributions", in *Handbook of Urban and Regional Economics*, 4 (2004) V. Henderson and J.-F. Thisse eds., Chapter 53, p. 2341-2378.

[26] Gibrat, R., *Les inégalités économiques*, (Paris, France: Librairie du Recueil Sirey, 1931).

[27] Gnedenko, B.V., and Kolmogorov, A.N. (1954) "Limit Distributions for Sums of Independent Random Variables", Reading, MA: Addison-Wesley Press.

[28] Head, A.C., 1995. "Country size, aggregate fluctuations, and international risk sharing". *Canadian Journal of Economics* 28 (4b), 1096–1119.

[29] Horvath, M.T.K., 1998. "Cyclicality and sectoral linkages: aggregate fluctuations from sectoral shocks". *Review of Economic Dynamics* 1, 781-808.

[30] Horvath, Michael "Sectoral shocks and aggregate fluctuations" 2000, *Journal of Monetary Economics* Vol. 45 (1) pp. 69-106.

[31] Horvath, M.T.K., Verbrugge, R.J., 1999. Shocks and sectoral interactions: an empirical investigation. Manuscript, Stanford University.

[32] Hulten, Charles(1978) "Growth accounting with intermediary inputs", vol. 45(3) *Review of Economic Studies*, p.511-18..

[33] Ijiri, Y. and Simon, H., *Skew distributions and the sizes of business firms*, (Amsterdam, Netherlands: North-Holland, 1977).

[34] Jones, Charles I. "The Shape of Production Functions and the Direction of Technical Change", Berkeley mimeo.

[35] Jorgenson, Dale W., Frank M. Gollop and Barbara M. Fraumeni (1987) "Productivity and U.S. Economic Growth" Harvard University Press, Cambridge, MA.

[36] Jovanovic, B. (1987) "Micro Shocks and Aggregate Risk," *Quarterly Journal of Economics*, 102, 395-409.

[37] Kim, C.-J. and C.R. Nelson (1999), "Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-Switching Model of the Business Cycle," *The Review of Economics and Statistics* 81, 608 – 616.

[38] Klette, Tor Jakob and Zvi Griliches "Empirical Patterns of Firm Growth and R&D Investment: A Quality Ladder Model Interpretation", NBER 6753.

[39] Kray, Aart and Jaume Ventura (2001) "Comparative advantage and the cross-section of business cycles", NBER WP 8104.

[40] Lamont, Owen, "Cash Flow and Investment:Evidence from Internal Capital Market," *Journal of Finance*, 52, 83-109, 1997

[41] Lee, Y. , L. A. N. Amaral, M. Meyer, D. Canning, and H. E. Stanley, "Universal features in the growth dynamics of complex organizations", *Phys. Rev. Lett.* 81, 3275-3278 (1998).

[42] Lewis, William, *The Power of Productivity*, U. Chicago Press, 2004.

[43] Long, John and Plosser, Charles "Real Business Cycles", JPE, 1983, p.49–69.

[44] Luttmer, Erzo G.J. (2004) "The Size Distribution of Firms in an Economy with Fixed and Entry Costs", Minneapolis Federal Reserve Bank WP.

[45] McConnell, M.M and G. Perez-Quiros, (2000), "Output Fluctuations in the United States: What has Changed Since the Early 1980's" *American Economic Review*, Vol. 90, No. 5, 1464-1476.

[46] McKinsey Global Institute (2001), "Productivity in the United States", October. `http://www.mckinsey.com/knowledge/mgi/reports/productivity.asp`.

[47] Long, John and Charles Plosser (1983), "Real business cycles", *Journal of Political Economy* 91, 1(Feb.), 39-69.

[48] Nirei, Makoto (2003). "Threshold Behavior and Aggregate Critical Fluctuations", Santa Fe mimeo.

[49] Okuyama, K., M. Takayasu and Hideki Takayasu, "Zipf's Law in Income Distribution of Companies," *Physica A*, 269 (1999), 125-131.

[50] Pareto, V., *Cours d'Economie Politique*, (Geneva, Switzerland: Droz, 1896).

[51] Philippon, Thomas "Corporate Governance and Aggregate Volatility" (2002), NYU mimeo.

[52] Scharfstein, David S., “The Dark Side of Internal Capital Markets II: Evidence from Diversified Conglomerates”

[53] Shea, John (2002) “Complementarities and Comovements”, Journal of Money, Credit and Banking, vol. 42(2), p. 412–433.

[54] Shin, Hyun-Han and René Stulz, “Are Internal Capital Markets Efficient?” *Quarterly Journal of Economics*, 113, 531-553, 1998.

[55] Stanley, M. H. R. , L. A. N. Amaral, S. V. Buldyrev, S. Havlin, H. Leschhorn, P. Maass, M. A. Salinger, and H. E. Stanley, “Scaling behaviour in the growth of companies”, *Nature* 379, 804-806 (1996).

[56] Stock, James and Mark Watson, “Has the Business Cycle Changed and Why?” *NBER Macroeconomics Annual*, Mark Gerler and Kenneth Rogoff ed. (2003), MIT Press.

[57] Stock, James and Mark Watson, “Understanding Changes in International Business Cycle Dynamics” (2004), NBER WP 9859.

[58] Summers, R., Heston, A., 1991. “The Penn World Tables (Mark 5): An expanded set of international comparisons, 1950–1988”. *Quarterly Journal of Economics* 106, 327–368.

[59] Sutton, John “The Variance of Firm Growth Rates: The ‘Scaling’ Puzzle”, *Physica A*, 2003.

[60] Weron, Alexander and R.Weron (1995) “Computer simulation of Lévy stable variables and processes”, Lecture Notes in Physics 457, 379-392, Springer.

[61] Wyart, Matthieu and Jean-Philippe Bouchaud, “Statistical models for company growth”, *Physica A* (2003).