Lecture 3

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Identifying Causality

Causal questions:

- What is the effect of taxes on consumption?
- What is the effect of alcohol on health?
- What is the effect of race on income?
- etc...

In order to answer these questions we need to know what happens when we treat (tax, drink, belong to race A) and when we don't treat.

From Angrist and Pischke (2009)

What is the effect of hospitalization on health outcomes? Let's think of healthy outcomes as high numbers and unhealthy outcomes as low numbers.

Let Y_i be the observed health outcome for person i. $D_i = 1$ if they went to the hospital and $D_i = 0$ if they didn't go.

Then the potential health outcome for individual i can be expressed as

$$\text{potential oucome} = \begin{cases} Y_{1i} & \text{if } D_i = 1 \\ Y_{0i} & \text{if } D_i = 0 \end{cases}$$

We can express the observed outcome:

$$Y_i = Y_{0i} + (Y_{1i} - Y_{0i}) D_i$$

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$Y_{1i} - Y_{0i}$ is the *causal effect* of going to the hospital.

Using our notation let's just compare average observed outcomes, i.e. we compare average outcomes of people who went to the hospital to average outcomes of people who didn't go:

$$E[Y_i|D_i = 1] - E[Y_i|D_i = 0] = E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 0]$$

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Adding zero:

$$E[Y_i|D_i = 1] - E[Y_i|D_i = 0] =$$
$$E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 1] + E[Y_{0i}|D_i = 1] - E[Y_{0i}|D_i = 0]$$

Let's examine this in detail to figure out what we have.

$$\underbrace{E[Y_i|D_i = 1] - E[Y_i|D_i = 0]}_{\text{Observed Difference}} =$$

$$E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 1]$$
$$+E[Y_{0i}|D_i = 1] - E[Y_{0i}|D_i = 0]$$

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$E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 1]$ is exactly what we want, but it's obscured.

"The Experimental Ideal"

We need to randomize D_i so that the correlation between D_i and the potential outcomes is zero. Mathematically, this means:

$$E[Y_{0i}|D_i = 1] = E[Y_{0i}|D_i = 0]$$

If we do this in our equation then we have

$$\begin{aligned} E[Y_i|D_i = 1] - E[Y_i|D_i = 0] &= E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 0] \\ &= E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 1] \\ &= E[Y_{1i} - Y_{0i}] \end{aligned}$$

Regression

Remember our expression for the observed outcome Y_i:

$$Y_i = Y_{0i} + (Y_{1i} - Y_{0i})D_i$$

We can reformulate this in terms of a regression:

$$Y_i = \underbrace{\alpha}_{E[Y_{0i}]} + \underbrace{\beta}_{Y_{1i}-Y_{0i}} D_i + \underbrace{\epsilon_i}_{Y_{0i}-E[Y_{0i}]}$$

If we evaluate this regression at $D_i = 0$ and $D_i = 1$ we get:

$$E[Y_i|D_i = 1] = \alpha + \beta + E[\epsilon_i|D_i = 1]$$
$$E[Y_i|D_i = 0] = \alpha + E[\epsilon_i|D_i = 0]$$

Rearranging:

$$E[Y_i|D_i = 1] - E[Y_i|D_i = 0] = \beta + \underbrace{E[\epsilon_i|D_i = 1] - E[\epsilon_i|D_i = 0]}_{\text{Selection bias}}$$

From your econometrics class: we need the error term and the regressor to be uncorrelated.

Review of OLS

Remember that for the linear regression model $Y = \beta X + \epsilon$ OLS gives you the solution the following minimization problem:

$$\min_{\beta} \sum_i (y_i - x_i\beta)^2$$

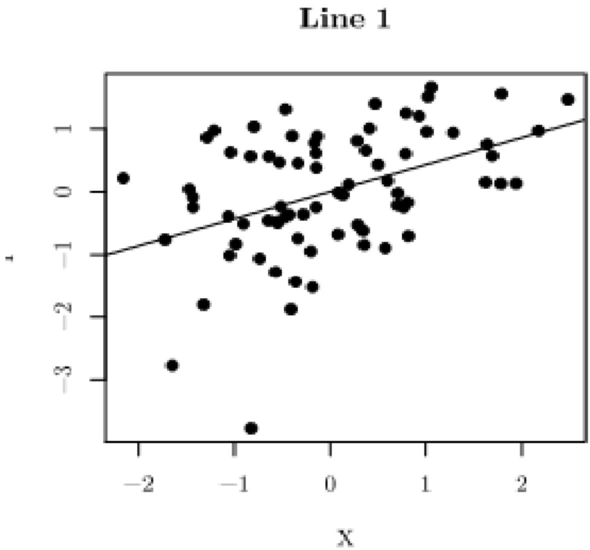
$$E[\epsilon] = 0$$

$$E[\epsilon X] = 0$$

If the moment conditions are satisfied then

$$\beta^{OLS} = \frac{cov(X, Y)}{var(X)} = \beta$$

Picture from C. Gibbons



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Parents of children in regular classrooms protested, so children were moved back after the kindergarten year.

Natural experiments might be more feasible:
Angrist and Lavy (1999) and Israeli class sizes.

But what if we can't run an experiment and can't find a natural one? How do we identify the treatment effect?

We have a problem in the regression model $Y = \beta X + \epsilon$ if there is a correlation between X and ϵ ($E[X\epsilon] \neq 0$).

$$Y = \beta X + \epsilon(X)$$

We want OLS to tell us $\frac{dY}{dX} = \beta$, instead it will tell us

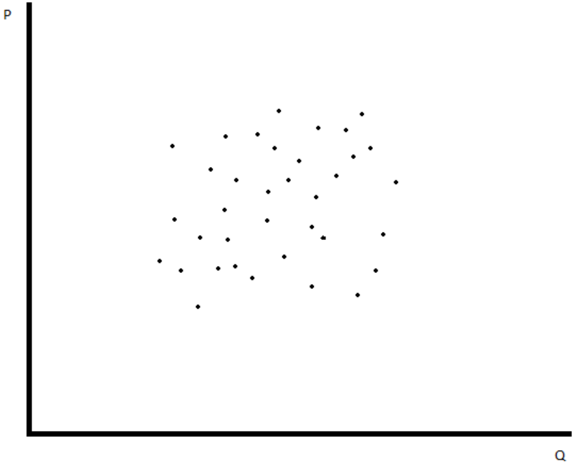
$$\frac{dY}{dX} = \beta + \frac{d\epsilon}{dX}$$

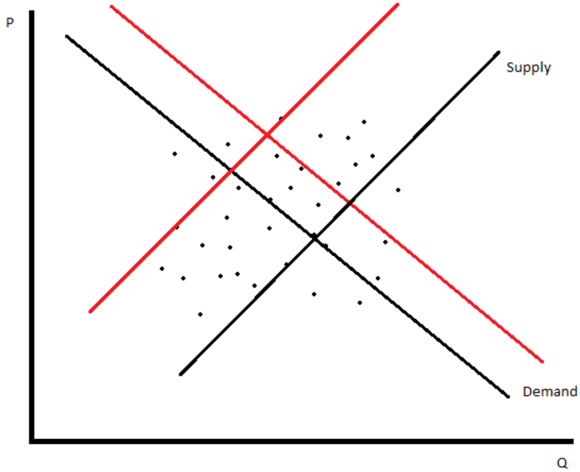
More accurately:

$$\beta^{OLS} = \beta + \frac{X'\epsilon}{X'X} = \beta + \frac{cov(X,\epsilon)}{var(X)} \tag{1}$$

It's double counting: direct effect through β and indirect effect through ϵ.

Supply and Demand




P
Supply
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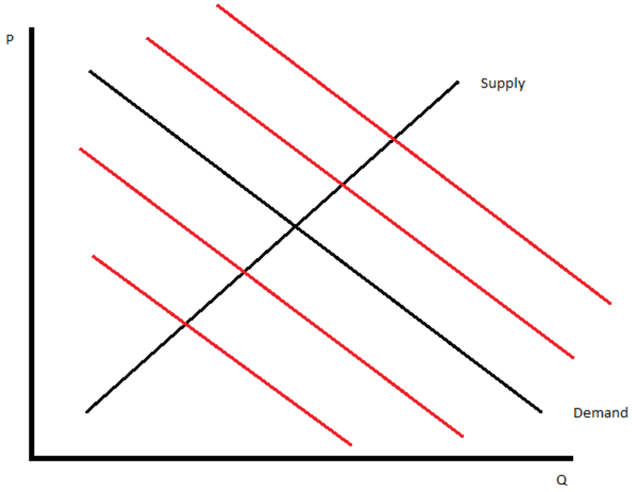
Demand Shifters



The demand shifters allow us to trace out the supply curve. The same applies for supply shifters. These are called instrumental variables. Formally, we are finding a variable Z such that:

$$E[Z\epsilon] = 0$$
$$E[ZX] \neq 0$$

Then we run two regressions:

$$X = \beta_Z^{OLS} Z + \epsilon_Z$$

This gives us $\hat{X} = \beta_Z Z$. Then we do:

$$Y = \beta \hat{X} + \epsilon$$

We have purified X of the part that was giving us trouble:

$$\beta^{IV} = \frac{Z'Y}{Z'X} + \frac{Z\epsilon}{Z'X} = \frac{Z'Y}{Z'X} + 0 \tag{2}$$

Example from Cameron and Trivedi (2005)

Want to estimate the effect of schooling on wages:

$$\log(w)_i = \alpha + \beta_1 \text{School}_i + \beta_2 \text{Exp}_i + X_i' \gamma + \epsilon_i$$

What is the problem with running this regression by OLS?

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Ideas?

We can (arguably) use proximity to a college as an instrumental variable for schooling and age as an instrumental variable for experience.

Table: Returns to Schooling

	OLS	IV
β_1 (School)	0.073	0.132
	[0.004]	[0.049]
R^2	0.304	0.207

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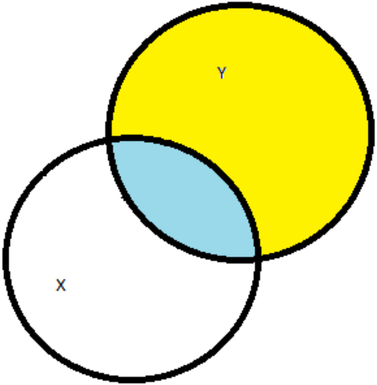
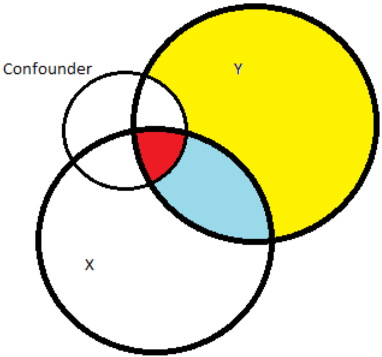
IV can help us, but it also has its own problems:

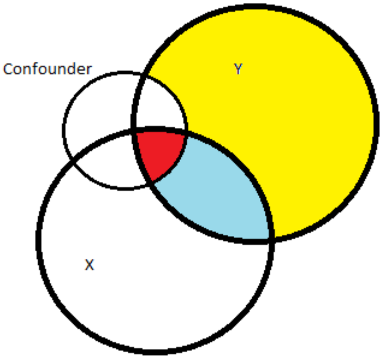
- Bigger standard errors.
- Explains less of the variation.
- A really weak instrument can contain very little information.

Graphic from Kennedy [2003])

Ordinary Least Squares (OLS) works here because it estimates the blue part.

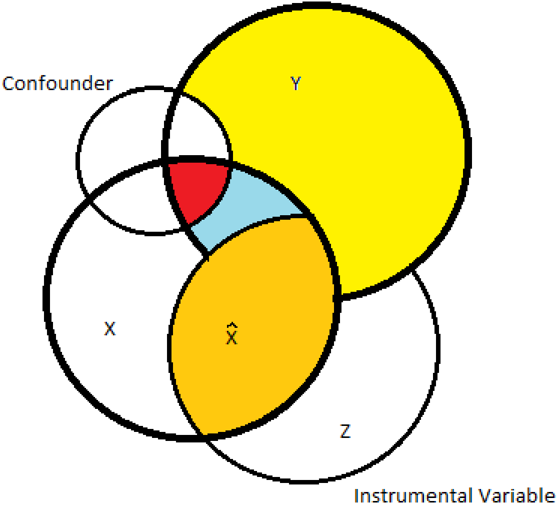



Confounder
Y
X

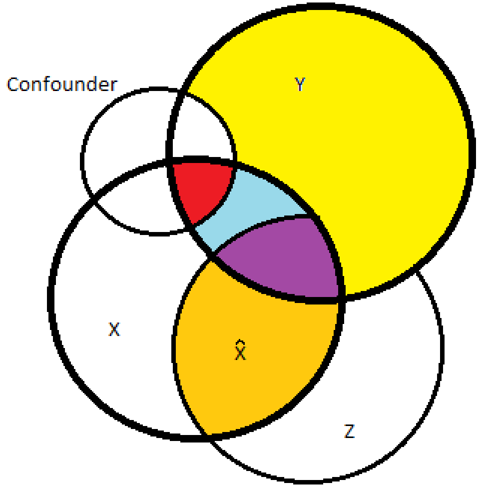


But here OLS will be inconsistent (we want blue without any red, but OLS estimates blue + red).

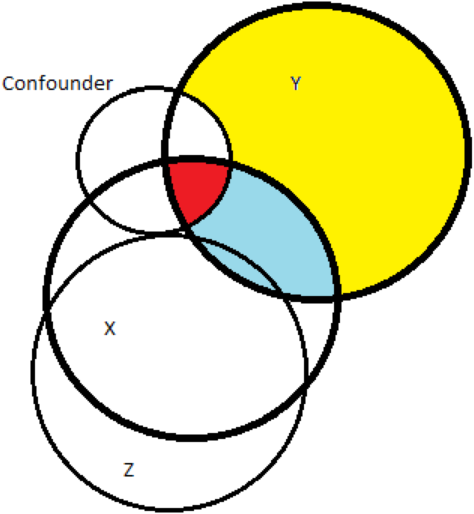
What if we have a third variable (Z) that is correlated with X, but not with the confounder?



Now if we estimate $Y = \beta\hat{X}$ we get purple. Free of red, but not exactly blue...



Good or bad IV?



Mathematically, the estimators for $Y = \beta X + \epsilon$

$$\beta^{OLS} = (X'X)^{-1}X'Y$$

and

$$\beta^{2SLS} = [X'Z(Z'Z)^{-1}Z'X]^{-1}X'Z(Z'Z)^{-1}Z'Y$$

When "just identified" this reduces to

$$\beta^{IV} = (Z'X)^{-1}Z'Y$$

In STATA type **reg Y X** to do OLS. Type **ivregress 2SLS Y X (instrument = Z)** to instrument Z with X.

For further reading:

- "A Guide to Econometrics" by Kennedy
- "Mostly Harmless Econometrics" by Angrist and Pischke
- "Introductory Econometrics" by Wooldridge
- "Microeconometrics" by Cameron and Trivedi