Lecture 1: Economic Growth

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August 31, 2011

Goal: Build a model of economic growth. Use it to discuss differences in income and growth rates.

Robert Solow, "A Contribution to the Theory of Economic Growth" (1956)

These slides follow Jones (2002) closely.

Production Function: how inputs become outputs

- Y: Output (cars, movies, BBQ,...)
- K: Capital (bulldozers, factory buildings, computers,...)
- L: Labor (engineers, servers, teachers,...)

$$Y = K^{\alpha} L^{1-\alpha},$$
$$0 \leq \alpha \leq 1$$

Constant returns to scale:

$$aY = (aK)^{\alpha}(aL)^{1-\alpha}$$

We want to deal with per capita or per worker variables.

$$y = \frac{Y}{L}$$

This gives us the per worker production function

$$y = \frac{K^{\alpha} L^{1-\alpha}}{L} = k^{\alpha}$$

Diminishing returns to capital per worker.

Capital Accumulation

Agents invest/save some of their income:

$$\underbrace{\dot{K}}_{\text{Change in capital}} = \underbrace{sY}_{\text{savings}} - \underbrace{dK}_{\text{depreciation}} \tag{1}$$

Notice that savings and depreciation are constant.

L grows at rate n:

$$\frac{\dot{L}}{L} = n$$

Define capital per worker k:

$$k \equiv \frac{K}{L} \Rightarrow$$
$$\log(k) = \log(K) - \log(L) \Rightarrow$$
$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}.$$

Now we can derive the evolution of capital per worker.

$$
\begin{aligned}
\frac{\dot{k}}{k} &= \frac{\dot{K}}{K} - \frac{\dot{L}}{L} \\
&= \frac{sY - dK}{K} - n \\
&= \frac{sY/L}{K/L} - d - n \\
&\Rightarrow \\
\dot{k} &= sy - (d+n)k
\end{aligned}
$$

Now we can solve the model for the steady state $\dot{k} = 0$.

Analytically:

$$k^* = \left(\frac{s}{n+d}\right)^{\frac{1}{1-\alpha}}$$

The production function $y = k^\alpha$ implies

$$y^* = \left(\frac{s}{n+d}\right)^{\frac{\alpha}{1-\alpha}}$$

Higher savings implies higher income per worker.

Solow Diagram

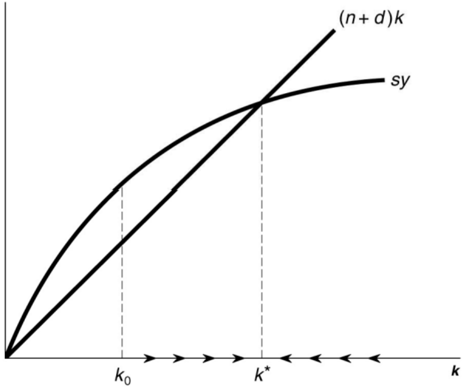

FIGURE 2.2 THE BASIC SOLOW DIAGRAM

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Transition Dynamics

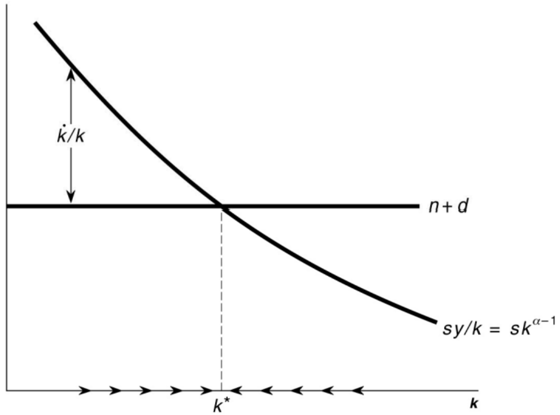

FIGURE 2.8 **TRANSITION DYNAMICS**

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Consumption and Income Dynamics

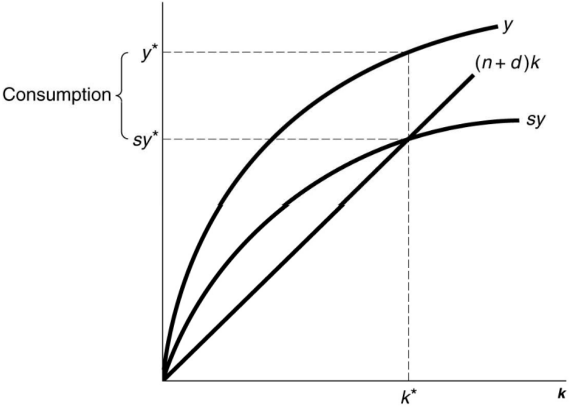


FIGURE 2.3 **THE SOLOW DIAGRAM AND THE PRODUCTION FUNCTION** *Economic Growth*, 2nd Edition

Increase in Savings

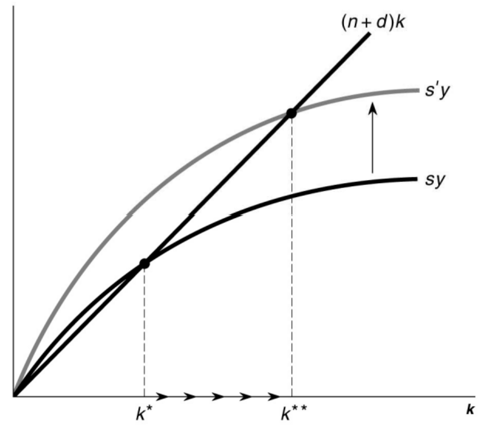
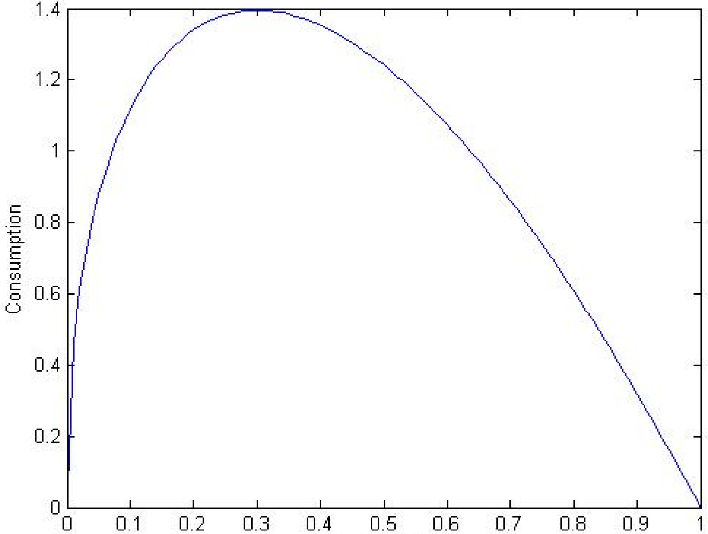


FIGURE 2.4 AN INCREASE IN THE INVESTMENT RATE

Consumption and Savings (n = .01, d = .05, α = .3)

$c = (1 - s)y$



Increase in Population

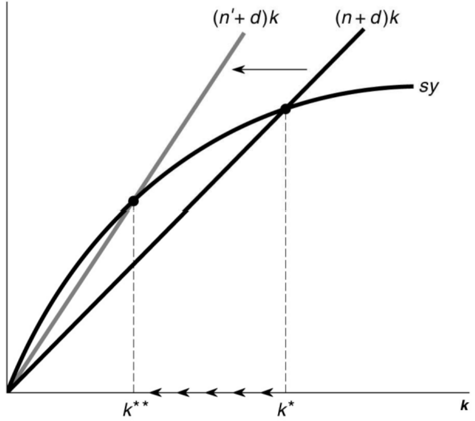


FIGURE 2.5 AN INCREASE IN POPULATION GROWTH

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Taking Stock

Solow model predicts:

- Countries that save more are wealthier.

Investment

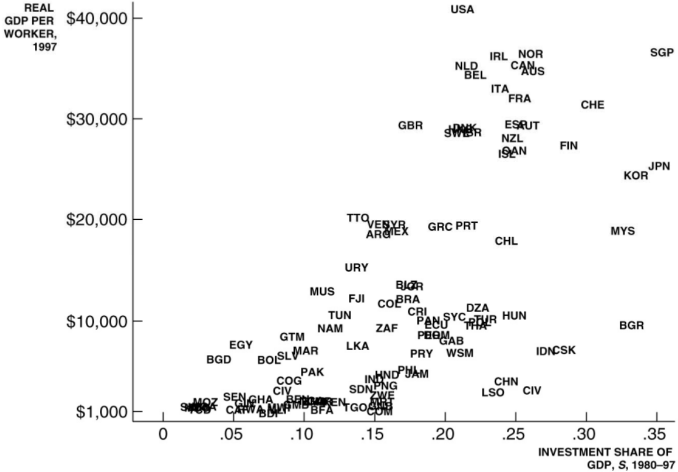

FIGURE 2.6 GDP PER WORKER VERSUS THE INVESTMENT RATE

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Taking Stock

Solow model predicts:

- Countries that save more are wealthier.
- Countries with higher population growth rates are poorer (capital widening).

Population

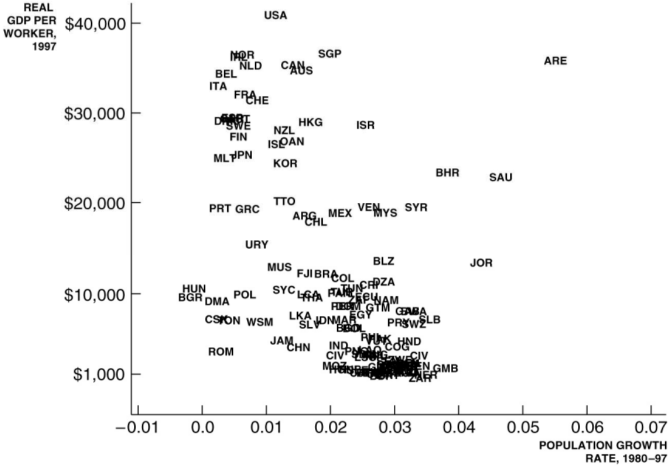

FIGURE 2.7 GDP PER WORKER VERSUS POPULATION GROWTH RATES

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Taking Stock

Solow model predicts:

- Countries that save more are wealthier.
- Countries with higher population growth rates are poorer (capital widening).
- No long run growth per capita!

What can we do to fix this prediction?

Taking Stock

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- No long run growth per capita!

What can we do to fix this prediction?

Technology.

Technology

Assume growth in technology A:

$$\frac{\dot{A}}{A} = g$$

Change the production function:

$$Y = K^{\alpha}(AL)^{1-\alpha}$$

Harrod-neutral technology. Efficiency of labor increases. This technology is *exogenous*.

Capital accumulation equation is unchanged:

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - d$$

Output per worker:

$$y = k^{\alpha} A^{1-\alpha}$$

$$\frac{\dot{y}}{y} = \alpha\frac{\dot{k}}{k} + (1-\alpha)\frac{\dot{A}}{A}$$

Balanced Growth Path

We want to look at a situation where growth rates are constant.

- Capital accumulation tells us that the growth rate of capital is constant only when Y/K is constant.
- This means output Y and capital K must grow at the same rate.
- Which implies that y and k also grow at the same rate.

Let $g_y = g_k$ be the growth rate for capital and output.

$$\begin{aligned}
\frac{\dot{y}}{y} &= \alpha\frac{\dot{k}}{k} + (1-\alpha)\frac{\dot{A}}{A} \\
g_y - \alpha g_k &= (1-\alpha)g \\
(1-\alpha)g_y &= (1-\alpha)g \\
g_y &= g
\end{aligned}$$

Output and capital grow at the rate of technological progress.

Define $\tilde{k} \equiv K/AL \equiv k/A$. This is the capital - technology ratio.

$$\tilde{y} = \tilde{k}^{\alpha}$$

Capital accumulation (you should construct it as an exercise):

$$\dot{\tilde{k}} = s\tilde{y} - (n + g + d)\tilde{k}$$

We can solve for $\tilde{k}$ as before and find steady state output per worker y^*:

$$y^* = A\left(\frac{s}{n+g+d}\right)^{\frac{\alpha}{1-\alpha}}$$

Now we have growth in output per worker.
What happens now when we increase the savings rate?

Increase in Savings Rate

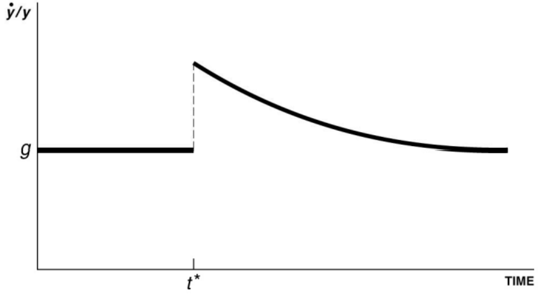

FIGURE 2.12 THE EFFECT OF AN INCREASE IN INVESTMENT ON GROWTH *Economic Growth*, 2nd Edition

Increase in Savings Rate

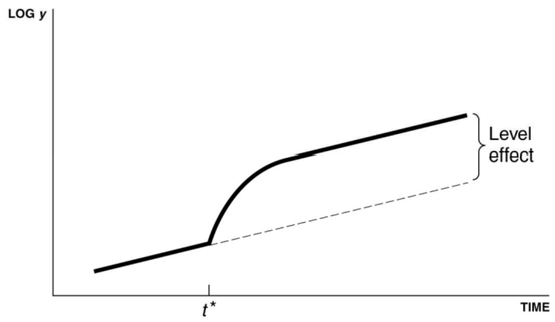

FIGURE 2.13 THE EFFECT OF AN INCREASE IN INVESTMENT ON y

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Unresolved issues:

- What is technology and where does it come from? (See Romer paper.)
- Can technology really explain all differences? (See Lucas paper.)
- What factors are missing?

Additional reading:

- "Introduction to Economic Growth" by Charles I. Jones
- "Advanced Macroeconomics" by David Romer
- Hall and Jones, "Why Do Some Countries Produce So Much More Output per Worker than Others?" (1999)
- Lucas, "Why Doesn't Capital Flow from Rich to Poorer Countries?" (1990)
- Romer, "Endogenous Technological Change" (1990)