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Economics 121
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Problem Set #2 - Suggested Solutions.
1.i.
In this problem, we are extending the usual Hotelling line – so that now it runs from [-a, 1+a]. Firms are located at 0 and 1. Unless there is a large price difference, consumers in the tails buy from the closest firm. We will consider only the case in which the consumers in the tails buy from the closer firm, so the firms compete only for the consumers in the (0,1) range. Note that for large enough cost difference, however, there is a discontinuity of demand – the firm with the lower cost (price plus transportation costs) will take the whole market.

In the figure below, the point d indicates the location of the consumer that is under competition. Transportation costs are given by td.

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  |----------|------------------|--------------------|----------|
-a           0                  d                    1        1+a
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We can write out the total cost to the consumer in the middle range:
If a consumer in the middle purchases from Firm 1, pays: p_0+td
If a consumer in the middle purchases from Firm 2, pays: $p_1+t(1-d)$
Consumers in [-a,0] will buy from Firm 0 unless there is a very large price difference, i.e., that p_0 plus transportation costs is greater than p_1 plus (higher) transportation costs. Consumers in [1,1+a] will similarly buy from Firm 1.

To find the consumer who is indifferent between the two firms:
$p_0+td=p_1+t(1-d)$

Solving for d:
$d=(1/2) + (p_1 – p_0/2t)$

Now think about maximizing profits of Firm 0.

$\pi= (p_0-c)*a + (p_0-c)*[(1/2)+(p_1 – p_0)/2t]$
$\pi= a\, p_0 – ca + (p_0-c)/2 + (p_0\, p_1/2t) – (p_0\text{^}2/2t) - (c\, p_1/2t) + (c\, p_0/2t)$

$d\pi/d\, p_0= a + ½ + p_1/2t – 2\, p_0/2t + c/2t = 0$
$2ta + t + p_1 – 2\, p_0 + c =0$
$2\, p_0 = 2ta + t + p_1+c$
By symmetry,
$2\, p_1 = 2ta + t + p_0 + c$
and, $p_0= p_1=p$

Substituting and adding together,
$4p = 4ta + 2t + 2p + 2c$
$p = 2ta + t + c$

Note that the price is now higher by the amount 2ta – the transportation cost to cover the extra distance of a on either end. Price is increasing in both a and t – a greater number of captive consumers and higher transportation costs (or equivalently, more differentiation), imply a higher price.

As in class, another way to solve once we have $d=(1/2) + (p_1 - p_0)/(2t)$ is to use residual demand elasticity and the Lerner equation. Since the *slope* of demand (for the firm at 0, say) is -1/(2t), and in symmetric equilibrium the quantity for each firm is now a + (1/2), the elasticity of residual demand is p/[t(2a+1)]. So the Lerner equation tells us that $(p - c)/p = t(2a+1)/p$, or $p = c + (2a+1)t$.

Note that the case a=0 confirms the calculations we did in class.

ii. Since consumers pay more under this framework, and relative to the case discussed in class, the additional price *2at* exceeds the additional transportation cost, which is at most *at*, it follows that consumers have greater incentive to credibly ignore differentiation. A central purchasing agency would thus be more attractive in this set-up.

2. a. This is a surprisingly high elasticity, especially for a durable good like cars. In 1955, 45% change in output only led to a 6% change in price. (Arc) Elasticity = 45/6 = 7.5.
b. Output increased a lot (45%), while prices fell only a little bit (6%). So, if profits fell while output increased and prices fell a small amount, this implies that firms were pricing close to marginal cost. If not, you would expect a large change in output accompanied by a small change in price to increase profits. Specifically, if p is the "normal" price and Q the "normal" quantity, we have: $(0.94p - c)*(1.45Q) < (p - c)Q$. Dividing both sides by Q and doing some arithmetic yields $1.363p - 1.45c < p - c$, or $0.45c > 0.363p$, or $c > 0.8p$.
c. If MC, AC < p and price is close to marginal cost, then there are not significant economies of scale in the industry. Scale economies are increasing in the ratio of AC/MC.
d. The key difference is that cars are a durable good. You would expect that a durable good would be more sensitive to price changes in nearby years. Ignoring 1955, the trend of automobile output given in "Car Wars" is surprisingly steady. It looks a bit more like what you would expect for a non-durable good, such as airline travel. You would expect that if prices were low (and output was high) in one year that the output in the following year would be abnormally low (since everyone thinking about buying a car in 1956 was lured into buying a car in 1955 instead). For airline travel, you might expect that a temporary decrease in price would (mostly) expand rather than shift demand for that year.

3. $p = A - Q$, N firms in the market
firm-specific unit costs, ci
$\bar{c} = (1/N)\Sigma c_i$

$Q = \Sigma q_i$

a.
First, without loss of generality, look at the profit function for firm 1:
$\pi_I = pq_1 - c_1q_1$
$\pi_I = (A\text{-}Q)\, q_1 - c_1\, q_1 = A\, q_1 - q_1\Sigma\, q_i - c_1q_1$

Maximize with respect to q_1.
$d\pi_I /d\, q_1 = A - 2\, q_1 - \Sigma\, q_i + q_1 - c_1 = 0$
$q_1 = A - Q - c_1$

That is Firm 1's quantity. Now add up $q_1 + q_2 + \ldots + q_n$

$\Sigma\, q_i = NA - NQ - \Sigma c_i$
$Q = NA - NQ - \Sigma\, c_i$
$Q/N = A - Q - (1/N)\Sigma c_i = A - Q - \bar{c}$

$$Q = \frac{N(A-\bar{c})}{(1+N)}$$

Then plug this back into the price equation:

$$p = A - Q = A - \frac{N(A-\bar{c})}{(1+N)}$$

$$p = \frac{A + N\bar{c}}{(1+N)}$$

Notice (not strictly part of answering the question, but worth noticing) that as N gets big, this says price will be near cost; for N=1 it gives well known monopoly price formula. Notice also that "near cost" in that last sentence means near $\bar{c}$. The extremely alert student will already be asking how firms with noticeably above-average unit costs can survive…see part (c) below!

b. Average cost of production = total cost/Q
Firm's total cost of production = $TC_i = c_iq_i$
Industry total cost of production =$TC = \Sigma\, TC_i$

$\Sigma\, TC_i = \Sigma c_i\, (A\text{-}Q\text{-}c_i) = \Sigma(Ac_i - Qc_i - c_i^2)$

$TC = \Sigma Ac_i - \Sigma Qc_i - \Sigma c_i^2)$

Recall, $\text{var}[c] = \frac{1}{N}\sum(c_i - \bar{c})^2$

Expanding, we find that

$$\text{var}[c] = \frac{1}{N}\sum c_i^2 - \sum 2c_i\bar{c} + N\bar{c}^2$$

Rearranging and substituting $\sum c_i = N\bar{c}$:

$$\sum c_i^2 = N\,\text{var}[c] + N\bar{c}^2$$

Substitute this into above total cost equation:

$$TC = A\sum c_i - Q\sum c_i - N\,\text{var}[c] + N\bar{c}^2$$

Again substituting $\sum c_i = N\bar{c}$ and dividing by Q, to express as average cost:

$$\frac{TC}{Q} = \frac{AN\bar{c} - QN\bar{c} - N\,\text{var}[c] + N\bar{c}^2}{Q} = \frac{AN\bar{c} - N\,\text{var}[c] + N\bar{c}^2}{Q} - N\bar{c}$$

Recall that Q = $\frac{N(A-\bar{c})}{(1+N)}$ (from part a.) and substitute:

$$\frac{TC}{Q} = (AN\bar{c} - N\,\text{var}[c] + N\bar{c}^2)\frac{(1+N)}{N(A-\bar{c})} - N\bar{c}$$

Cancelling out, rearranging and combining:

$$\frac{TC}{Q} = \frac{A\bar{c} - \bar{c}^2 - \text{var}[c](N+1)}{A-\bar{c}} = \bar{c} - \frac{(N+1)\,\text{var}[c]}{A-\bar{c}}$$

By the way, we can also say the following (not asked in the question) about average production cost in Cournot equilibrium with nonlinear demand. Average cost of production is $\sum (q_i/Q)c_i = \sum s_i c_i$ where s_i is firm i's market share. Remembering that each firm's residual demand elasticity in Cournot is the market demand elasticity divided by its share, we have

$$(p - c_i)/p = s_i/\varepsilon$$

Multiplying through by p and by s_i and adding up for all firms gives us

$$\sum s_i(p - c_i) = p\sum s_i^2/\varepsilon\,.$$

Since the shares add up to 1, and remembering the definition of the Herfindahl index H, this is

$$p - \sum s_i c_i = pH / \varepsilon$$

so the average cost of production is equal to $p(1 - H / \varepsilon)$. This would give an alternative route to answer the question here, though not necessarily a simpler one given that both p and market demand elasticity are functions of all the c's.

c. If firms shut down, it will be the high cost firms that do so. So, the theoretical average cost of production is higher than the actual average cost of production, given that firms can decide not to produce. Firm will exit the market if firm AC (=c_i) exceeds MR (=p).

Exit if:

$$p = \frac{A + N\bar{c}}{A - \bar{c}} < c_i$$

$$(c_i - \bar{c})N + c_i > A$$

$$(N+1)c_i > N\bar{c} + A,$$

which says that the firm's unit cost exceeds a weighted average of all (including its own) cost draws together with the demand intercept (note that the demand intercept is the cost level above which even a monopoly would exit). A minute's thinking about how averages work should tell you that this means the same as saying that the firm's unit cost exceeds a weighted average of *other* firms' cost draws and the demand intercept.

d. Undifferentiated Bertrand competition assigns all production to the lowest cost firm, so the average cost of production will be lower than under Cournot (given varying c_i's).

Bertrand pricing gives the second-lowest cost as price. If the cost of the second-lowest firm is below the Cournot price then the Bertrand price will be lower. It is tempting to say that if the second-lowest cost level is above the Cournot price then the Bertrand price would be higher. But actually the only way that could happen (since then only the lowest-cost firm would be producing in Cournot equilibrium) is if the monopoly price, for the lowest-cost firm, is below the second-lowest cost: what we described in class as a *drastic* cost difference. And in that case, of course, the lowest-cost firm picks its monopoly price and other firms stay out, both in Bertrand and in Cournot.