UC Berkeley
Economics 121
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Problem Set #3 – Suggested solutions

1 (a) Linear demand functions are of the form:

$Q = a + bP$
Note that $dQ/dP = b$

Recall the Lerner mark-up rule.

$$\frac{P-c}{P} = -\frac{1}{\varepsilon}$$

Recall also the definition of elasticity:

$$\varepsilon = \frac{P}{Q} * \frac{dQ}{dP}$$

Plugging in what we know for business-class consumers:

$$\frac{200-50}{200} = -\frac{1500}{200}\frac{dP_b}{dQ_b} \rightarrow -10 = \frac{dQ_b}{dP_b} = b_b$$

Substituting this into the linear demand formula:

$Q_b = a_b - 10P_b$

We know $Q_b = 1500$ and $P_b = 200$

So, $a_b = 10 P_b + Q_b = 2000 + 1500 = 3500$

The demand for business-class travelers is therefore:

$Q_b = 3500 - 10P_b$

Turning to economy-class consumers:

$$\frac{80-50}{80} = -\frac{6000}{80}\frac{dP_e}{dQ_e} \rightarrow -200 = \frac{dQ_e}{dP_e} = b_e$$

Again plugging this into the linear demand formula:

$Q_e = a_e - 200P_e$

$a_e = 6000 + 200(80) = 22000$

Then, the demand for economy-class consumers is:

$Q_e = 22000 - 200P_e$

(b) If the airline cannot price discriminate, it must maximize profits using a single P.

$$\pi = P*(Q_b + Q_e) - F - 50(Q_b + Q_e)$$
$$\pi = P*(3500 - 10P + 22000 - 200P) - F - 50(3500 - 10P + 22000 - 200P)$$
$$\pi = P*(25500 - 210P) - F - 175000 + 500P - 1100000 + 10000P$$

Maximize profit with respect to price.

$$\frac{d\pi}{dP} = 25500 - 420P + 500 + 10000 = 0$$
$$36000 = 420P$$
$$P = \$85.71$$

Calculate the quantities for both groups at this price. Remember also that we want to make sure that quantities are positive for both groups:

Q_b=3500-10P_b=3500-857.1=2,642.9
Q_e=22000-200P_e=22000-17142=4,858

(c) Recall the Lerner mark-up rule for oligopolies. More competitors make residual demand more elastic.

$$\frac{P-c}{P} = -\frac{1}{N\varepsilon}$$

So for the business travelers, who have access to more firms:

$$N\varepsilon = 10*\frac{P}{Q}*\frac{dQ}{dP} = \frac{10P}{3500-10P}*-10 = \frac{-10P}{350-P}$$

$$\frac{P-50}{P} = -\frac{(350-P)}{-10P} = \frac{350-P}{10P}$$

$$P - 50 = 35 - .1P \rightarrow 1.1P = 85 \rightarrow P = \$77.27$$

Q_b=3500-10(77.27)=3500-772.70=2727.3

For leisure travelers, the firms face the same elasticity as in part (a), so those prices and quantities are unchanged.

Note that the business travelers have less elastic demand. In the price discrimination example in part (a), this led to higher prices for business travelers. Here, because they are able to comparison shop, they are charged a lower price than leisure travelers.

2(a) The competitive price is p=mc=20. If the monopoly were constrained to the competitive price, it would have profits of -1800 (a loss equal to the fixed cost). It would therefore choose not to produce.

(b) Recall the Ramsey pricing formula:

$$\frac{p-c}{p}=-\frac{k}{\varepsilon}$$

This formula tells us that if a regulator wants to meet a breakeven constraint to encourage a firm to produce for both types of consumers, prices should be increased in proportion to the elasticities.

For two groups, set the ratio of the mark-ups equal to the ratio of demand elasticities (divide Ramsey pricing formula for group 1 by the Ramsey pricing formula for group 2).

$$\frac{(p_1-c)/p_1}{(p_2-c)/p_2}=\frac{\varepsilon_2}{\varepsilon_1}$$

Find the elasticity for each group:

$$\varepsilon_1=\frac{p_1}{q_1}*\frac{dq_1}{dp_1}=\frac{p_1}{100-p_1}*(-1)=\frac{-p_1}{100-p_1}$$
$$\varepsilon_2=\frac{p_2}{q_2}*\frac{dq_2}{dp_2}=\frac{p_2}{120-2p_2}*(-2)=\frac{-p_2}{60-p_2}$$

Plug into the ratio of the Ramsey prices:

$$\frac{(p_1-20)}{p_1}*\frac{p_2}{(p_2-20)}=\frac{-p_2}{(60-p_2)}*\frac{(100-p_1)}{-p_1}\rightarrow\frac{p_1-20}{p_2-20}=\frac{100-p_1}{60-p_2}$$

Simplify and rearrange:

$$p_1=2p_2-20$$

Now write out the firm's profit function:

$$\pi = p_1 q_1 + p_2 q_2 - 1800 - 20q_1 - 20q_2$$
$$\pi = p_1(100 - p_1) + p_2(120 - 2p_2) - 1800 - 20(100 - p_1) - 20(120 - 2p_2)$$
$$\pi = 100p_1 - p_1^2 + 120p_2 - 2p_2^2 - 1800 - 2000 + 20p_1 - 2400 + 40p_2$$
$$\pi = 120p_1 - p_1^2 + 160p_2 - 2p_2^2 - 6200$$

Substitute the derived relationship between p_1 and p_2:

$$\pi = 120(2p_2 - 20) - (2p_2 - 20)^2 + 160p_2 - 2p_2^2 - 6200$$
$$\pi = 240p_2 - 2400 - 4p_2^2 + 80p_2 - 400 + 160p_2 - 2p_2^2 - 6200$$
$$\pi = 480p_2 - 6p_2^2 - 9000$$
$$\pi = p_2^2 - 80p_2 + 1500 = 0$$

The last step follows from the breakeven constraint, which means that profits are set equal to zero.
Using the quadratic equation, we can factor the equation and find the value(s) of p_2.

$$\frac{80 \pm \sqrt{6400 - 6000}}{2} = 40 \pm 10$$

So either p_2=30 or p_2=50. This means that p_1=40 or p_1=80.

There are two possible sets of (p,q) pairs:
$(p_1,q_1)=(40,60)$; $(p_2,q_2)=(30,60)$
$(p_1,q_1)=(80,20)$; $(p_2,q_2)=(50,20)$

(c) While both sets of prices and quantities satisfy Ramsey pricing, keep in mind that Ramsey prices are set by a regulator. Presumably, a regulator would like to be as close to the efficient prices and quantities as possible. We found in (a) that the efficient price would be p=20. This would result in q_1=80, q_2=80. So, the regulator would prefer the first set of Ramsey prices that yield more output. That is, p_1=40 and p_2=30 (and $q_1=q_2$=60).