A New Micro Model of Exchange Rate Dynamics

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Abstract

This paper bridges the new macro and microstructure approaches by addressing how currency markets aggregate information in general equilibrium. The model departs from new macro by including dispersed information and financial intermediaries. It departs from microstructure by identifying real activities where dispersed information originates, as well as the technology by which this information is subsequently aggregated and impounded in the exchange rate. Financial intermediaries are consolidated with consumers, in the spirit of the "yeoman farmer" consolidation of production and consumption in new macro models. Results include: (1) exchange rate movements without public news, (2) order flow effects on exchange rates that persist, and (3) a structural understanding of why order flow explains exchange rates at higher frequencies better than macro variables, whereas at lower frequencies macro variables predominate. We also identify an efficiency loss that arises when dispersed information and constant relative risk aversion interact: less-than-full information about the distribution of wealth is a source of noise that leads to informational inefficiency.

Keywords: Exchange Rate Dynamics, Dispersed Information, FX Trading.

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Introduction

"[T]he papers in this collection should convince one of the difficulty of constructing a general equilibrium model of the exchange rate. A successful combination of microstructure theory and macroeconomic theory appears to be out of reach at this stage."

Frankel, Galli, and Giovannini (1996, p.12)

This paper addresses a new disconnect puzzle: the distressing disconnect between the two micro-founded approaches to exchange rates that emerged in the 1990s. These are the new open-economy macro approach (henceforth "new macro") and the microstructure approach. New macro modeling is general equilibrium, rich in welfare analysis, but thin on the microeconomics of financial markets and the information environments in which they operate. The microstructure approach, in contrast, has the microeconomics of financial markets at its center, at the cost of relying on partial equilibrium (and rather stylized) analysis. This paper seeks to integrate the microstructure and new macro approaches into what we term a "new micro" approach.[2] Specifically, the model embeds the micro-foundations of currency-relevant information in a dynamic general-equilibrium (GE) setting.

The macro features of our model are standard. There are two countries populated by consumers who have utility defined over a basket of home and foreign goods. Consumers have access to two financial assets, home and foreign currency deposits, which pay interest monthly and can be used to purchase consumption goods in the same currency. Consumers also control a domestic production process subject to exogenous productivity shocks (which differ, home versus foreign). We introduce the international aspect of the model via the information structure. Specifically, bits of fundamental information available to individuals lead their individual currency trades to be slightly more correlated with (unobserved) shocks to home productivity. When aggregated to the country level, this slightly higher correlation becomes a distinct information differential, allowing trades initiated by home agents to convey superior information about home shocks.[3] This information structure differentiates the macro side of our model from the new macro literature.

The micro features of the model are closely related to microstructure models of asset trade in which financial intermediaries act as marketmakers who provide two-way prices. We introduce this provision of liquidity by assuming that all agents engage in both consumption and marketmaking.[4] This consolidates the activities of households with that of financial institutions in a way similar in spirit to the "yeoman farmer" consolidation of production and consumption decisions in new macro models. The consolidation

[2] Though analysis in new micro relies heavily on the theory of microstructure finance, it does not draw uniformly from the modeling approaches within microstructure, nor does it address the same questions, hence the need for a different label. The modeling approach in microstructure finance that does play a central role in new micro is the information approach (versus the inventory and industrial organization approaches). For questions, new micro is oriented toward macro phenomena, whereas microstructure finance is oriented toward micro phenomena (such as institution design, regulation, individual behavior, and partial-equilibrium price determination).

[3] Note the contrast of this dispersed information setting from one of concentrated, or "insider" information. The model is thus less concerned with strategic exploitation of large information advantages at the individual level. Strategic exploitation of large information advantages may be important at certain times for fixed-rate currencies (see, e.g., Corsetti et al. 2001), but are less important for the everyday functioning of major floating-rate currencies (even, arguably, in rare instances of central bank intervention).

[4] Note the emphasis here on liquidity provision that is private, in contrast to the public provision of liquidity (in the form of central banks) at the center of the monetary approach to exchange rates.

greatly facilitates integration of elements from the microstructure approach into a dynamic GE setting.[5] In particular, it ensures that the objectives of financial-market participants are exactly aligned with those of consumers. All trading is therefore consistent with expected utility maximization; noise traders, behavioral traders, and other non-rational agent types are absent.

This paper belongs to a theoretical literature that emerged recently to address why exchange rate changes are so well explained empirically by signed transaction flows (e.g., R^2 statistics in the 40-80 percent range for a host of major currencies; see Evans and Lyons 2002b). For example, the model of Hau and Rey (2002) addresses the empirical significance of transaction flows by introducing two key elements: a central role for cross-border equity flows and a private supply of foreign exchange that is price elastic. The latter means that cross-border equity flows affect exchange rates via induced currency transactions. In a nutshell, their focus for understanding currency-market developments is on innovations in equity markets, a substantial departure from the traditional asset approach which emphasizes instead the importance of bond markets. Their focus is not on information aggregation as ours is here (no information aggregation takes place in their model). A second paper along this theoretical line is Bacchetta and van Wincoop (2002), which does explicitly address how transaction flows relate to information aggregation. Their trading model is a rational-expectations model (in the spirit of Grossman and Stiglitz 1980). An important finding in that paper is that greater dispersion of information across agents can lead to greater price impact from non-fundamental trades (resulting from rational confusion of non-fundamental trades for fundamental trades). Our modeling departs from theirs in two main ways. First, our GE setting extends "upstream" in the information process in that it specifies the structural source of the information that currency markets need to aggregate (i.e., the underlying economic activities that produce it.) Second, marketmaking in our model aligns closely with actual institutions, producing implications that map directly into transactions data. A third recent paper, Devereux and Engel (2002), shares both our GE approach and a role for marketmakers. Marketmakers in their model are explicitly non-rational, however, so the reason their trades affect price is quite different than in our model.[6]

From the above it is clear that currency markets' ability to process information is a central theme so let us address it in more detail. The type of information we have in mind is information that is dispersed throughout the economy and aggregated by markets, as opposed to official institutions. Examples include the heterogeneous micro-level activity that, when aggregated, produces measures like output, money demand, inflation, consumption preferences, and risk preferences. For some of these measures official aggregations exist, but publication trails the underlying activity by 1-4 months (not to mention noise as reflected in subsequent revisions), leaving much room for market-based aggregation to precede publication. For other key macro variables, such as realized risk preferences and money demands, official aggregations of the underlying micro-level activity do not exist, leaving the full task to market-based aggregation. In traditional macro modeling of exchange rates, information that needs to be aggregated by markets is not admitted. Instead,

[5] To non-macro readers this type of consolidation is surely unfamiliar. The assumption facilitates GE analysis because the agent population remains defined over a single continuum, and differences along that continuum arise as parsimoniously as possible to capture the model's essential features.

[6] For example, in a risk neutral setting the trades of marketmakers in the Devereux and Engel (2002) model would not affect price since they can do so only by affecting expected returns (i.e., risk premia; see also Jeanne and Rose 2002). In contrast, trades of marketmakers in our model would still affect price under risk neutrality because they do so by affecting expected cash flows.

relevant information is either symmetric economy-wide, or, in some models, asymmetrically assigned to a single agent—the central bank. As an empirical matter, however, most information that exchange rates need to impound is certainly originating as dispersed, micro-level bits. In addition, there is now strong empirical evidence that this dispersed information is indeed being impounded in exchange rates before ever being symmetrized through official aggregation.[7] Understanding the nature of this information problem and how it is solved remains a significant challenge.

So what do we learn from modeling currency trade in a GE setting? The question is important: the shift from partial to GE modeling involves much technical complexity and, for most people, a drop in economic transparency. The following two paragraphs address this question in more detail.

A first lesson from modeling currency trade in a GE setting is that the information problem noted above is a good deal more nuanced than suggested by past partial-equilibrium analysis. For example, the model clarifies that even if the timing of individuals' receipt of information is exogenous, the timing of impounding that information in price is endogenous. This is because the signals within the market that lead to that impounding correspond to participants' equilibrium actions. Naturally, then, the model is able to characterize when order flow should be especially informative, dollar for dollar, and when less so. In effect, the dynamics of the model create a constant tension between strong and semi-strong form efficiency. (Strong form efficiency means that prices impound all information public and dispersed, whereas semi-strong efficiency means that prices impound only public information.) This tension is the difference between the union of all individuals' information sets and the smaller set that includes public information only. Finally, relative to partial-equilibrium models, the information structure of the GE model provides needed clarity on why transaction-flow effects on exchange rates should persist, and, importantly, whether that persistence applies to real exchange rates or only to nominal rates.[8]

A second lesson from GE modeling of price discovery in currency markets is that real decisions are affected, and in ways not considered in either new macro or microstructure models. For example, our model clarifies the channels through which intermediation in currency markets affects consumption and intertemporal consumption hedging. The basic intuition for why these channels are operative is that innovations in agents' learning from currency-market activity are correlated with other things they care about (e.g., real output). Decision-making about real choices will be conditioned on information that is generally less that the union of individuals' information sets, which naturally leads to effects on real allocations. Yet, this is optimal: one cannot wait for a full resolution of uncertainty because informationally the economy is always

[7] This evidence is from both micro studies of individual price setters and macro studies of price setting marketwide. See, e.g., Lyons (1995), Payne (1999), Rime (2001), Evans (2002), Covrig and Melvin (2002), Froot and Ramadorai (2002), and Evans and Lyons (2002a,b). Among other things, these papers show that actual flows of signed transactions (order flow) and demand are not the same: that order flow includes an information dimension beyond the pure quantity concept of demand is clear from, for example: (1) findings that order flow has different effects on price, dollar for dollar, depending on the institution type behind it and (2) findings that order flow in one currency market has price effects in other currency markets, despite not occurring in those markets. As a theoretical matter, the two are obviously distinct: demand moves price without transactions being necessary, whereas order flow necessarily involves transactions (i.e., it is signed transaction flow, where the sign is determined from the direction of the non-quoting counterparty).

[8] With respect to the information conveyed by flows, it is important to distinguish order flows from portfolio flows. Order flows–by tracking the initiating side of transactions–are a theoretically sound way to distinguish *shifts* in demand curves from *movements along* demand curves. Informationally, the two are different: there is news in curve shifts but no news in price-induced movements along known curves (the latter representing a type of feedback trading). For portfolio flows, theory provides little guidance on which flows in the aggregate mix reflect the news, i.e., the demand-curve shifts. We return to this when discussing implications of the model in section 5.

in "transition" (i.e., there is always some dispersed information that remains unaggregated).

The GE environment we study has a number of complicating features. It includes a large number of risk averse consumers with heterogenous information who make consumption, investment and trading decisions with incomplete markets. Furthermore, some information about the state of the economy is only learned by consumers endogenously as they trade. To analyze the model we therefore need to solve each consumer's inference and decision problems jointly. For this purpose, we extend the log-linear approximation techniques developed by Campbell and Viceira (2002) to our setting. These techniques allow us to derive analytic approximations for consumers' consumption, investment, and trading decisions at a point in time given a conjecture about (i) the equilibrium exchange and interest rate processes (which are generally not i.i.d.) and (ii) the information available to each consumer. The complication arises from the need to show that implications of these decisions are consistent with market clearing, and that the conjectured information available to each consumer is supported by inference based on observations of market activity. An important aspect of this solution procedure is that it accounts for consumer's risk aversion when characterizing optimal decisions. As a result, risks associated with incomplete knowledge about the state of the economy influence the consumption, investment, and trading decisions, which, in turn, affect the inferences consumers draw from market observation. To our knowledge, this is the first paper to solve a GE model with this combination of risk-averse decision-making, heterogeneous information, and endogenous learning.

The remainder of the paper is organized as follows. Section 1 presents some over-arching characteristics of the model. Details of the model are laid out formally in Section 2. Section 3 describes the process of solving for equilibrium. Section 4 studies the equilibrium for two nested specifications of the productivity process (nesting the specifications helps clarify the process by which dispersed information is aggregated and impounded). In Section 5 we address various implications of interest (e.g., volatility, trading volume, departures from fundamentals, announcement effects). Section 6 concludes.

1 Theoretical Overview

Before presenting specifics of the model, there are three overarching characteristics that warrant attention. The first of these is the above-noted consolidation of consumers and financial intermediaries into households. Whereas a focus of new macro models is richer micro-foundations on the the economy's supply side, hence the consolidation in those models of consumers with producers, our focus is instead richer micro-foundations in the area of financial intermediation. In particular, we focus on how financial markets achieve economy-wide risk sharing in a setting of heterogeneous information. In actual markets this process takes time and involves financial institutions in a non-trivial way, hence the value of embedding the process in a rich and dynamic setting. The consolidation recognizes that consumption depends both on learned information about future consumption opportunities and on the evolution of consumption risks (the latter being affected within the process of market-wide risk sharing). In effect, our specification of financial intermediation represents a risk-sharing and learning "technology." In so doing the two are intimately linked, and also intimately linked to consumption decisions. Households recognize that economy-wide sharing of concentrated risks is not instantaneous and not costless.

The second overarching characteristic of the model is its "simultaneous trade" design (see, e.g., Lyons 1997). The simultaneous trade design itself embeds two important and distinct features. The first is simultaneous actions, in the sense that trading at any point in time occurs simultaneously throughout the economy (in the spirit of simultaneous-move games in game theory). In essence, this assumption imposes a constraint on the information available for making trading decisions because simultaneous moves cannot be conditioned on one another. More concretely, one cannot condition on concurrent trading intentions of other agents in the economy at the time one chooses to trade. We find this an inherently realistic assumption relative to that made in, for example, Walrasian models of trade in which all concurrent trades are conditioned on the information conveyed by all other trades. In effect, the Walrasian clearing mechanism makes heroic assumptions about the transparency of market activity and the information available to price setters.[9] Our simultaneous-move framework is a convenient way to relax the polar extreme represented by the Walrasian framework.[10]

Another important feature embedded in the simultaneous trade design is that quoted prices are single prices, not bid-ask spreads. For the type of macro-level analysis we are doing here, bid-ask spreads enter as a nuisance parameter, hence our choice to omit them. One objection to this suppression of spreads is that intermediaries no longer have an incentive to quote two-way prices (the spread being their compensation). As a matter of modeling, there is a simple fix to restore this incentive that involves spreads but does not alter the basic process of learning from trades. That fix is quite general in the sense that it allows intermediaries to quote a separate bid-ask spread for every possible trade quantity (i.e., a schedule relating every possible trade quantity from minus infinity–customer sale–to plus infinity–customer buy–to a specific price). Each intermediary's schedule would reflect an upward sloping willingness to supply foreign exchange as a function of the single incoming trade. This would not alter the basic process of learning from trades because each individual intermediary's quoted schedule would be conditioned only on the single incoming trade, i.e., there is no feasible way to condition the transaction price on the realization of *all* other concurrently realized trades. Under this specification too, then, individual transaction prices would not embed the Walrasian level of economy-wide information (and, as in the specification we do adopt, prices may not embed this level of information even with long lags).

The third overarching characteristic of the model is the long-run real exchange rate, specifically, the channels through which the long-run real rate is affected by order flow (i.e., information conveyed by trading). One important channel is via household investment decisions. Investment decisions are affected by productive capabilities, which are in turn time-varying and correlated with realized transaction flows in foreign exchange. If production technology is non-linear, then there is a strong channel through which trading information affects the aggregate capital stock and thereby the long-run real exchange rate. In the simple model we present here, production technology is linear, which, among other things, implies that returns to real capital are exogenous. This simplifying assumption limits the degree to which investment decisions affect long run exchange rate dynamics. We make this technology assumption for tractability, not because we are convinced

[9] Another unfortunate feature of Walrasian mechanisms is that agents never take positions that they intend in the future to liquidate (because all trades are conditioned on all concurrent trading information). Among other things, this produces counterfactual predictions about how liquidity is provided in financial markets: transitory position-taking is a deep property of liquidity provision, and is important for understanding how trade quantities (i.e., realized order flow) maps into price changes.

[10] One could also take an intermediate road and assume that financial transactions at any "point" in time are executed sequentially. In this case, early trades would share the feature of all trades in our set-up in that they could not condition on information revealed in later trades (whereas later trades under the sequential set-up could condition on early trades). This alternative would produce the same qualitative constraint on the information set available for setting prices, but in a more awkward way.

this channel is inoperative or unimportant. There is a second channel through which order flow can affect the long-run exchange rate that we do not address in this paper, but which may be fruitful in future analysis. Specifically, if exchange rates "shocks" that arise in learning from order flow are accommodated by a monetary authority, then such shocks can feed into aggregate price levels. Note that this channel pertains more to the long-run nominal exchange rate than to the long-run real rate.

To summarize, the model is designed to focus on information effects on price that persist, not on "microstructure effects", i.e., transitory price effects from marketmaker inventory management and bouncing between bid and ask prices. From a macro perspective, these microstructure effects are second order. Moreover, our focus is on clarifying the transmission mechanism–the GE process by which information is impounded in price–not on a particular structural interpretation of the driving fundamentals (in our model, productivity). For example, we could just as easily set up the model with a different real shock as the fundamental driver, or with a nominal shock as the fundamental driver (e.g., assuming that individuals' trades are correlated with unobserved shocks to home money demand). Finally, for those interested in integrating sticky goods prices and imperfectly competitive firms, these two key features of open-macro modeling could be introduced in the usual way. We chose the most streamlined structure possible to highlight the new information dimension we are addressing.

2 The Model

2.1 Environment

The world is populated by a continuum of infinitely-lived consumers indexed by $z \in [0,1]$ who are evenly split between the home country (i.e., for $z \in [0,1/2)$) and foreign country ($z \in [1/2,1]$). For concreteness we shall refer to the home country as the US and the foreign country as the UK. Preferences for the z'th consumer are given by:

$$\mathsf{U}_{t,z} = \mathsf{E}_{t,z} \sum_{i=0}^{\infty} \beta^i U(C_{t+i,z}, \hat{C}_{t+i,z}) \tag{1}$$

where $1 > \beta > 0$ is the subjective discount factor, and $U(.)$ is a concave sub-utility function. All consumers have identical preferences over the consumption of US goods $C_{t,z}$ and UK goods $\hat{C}_{t,z}$. $\mathsf{E}_{t,z}$ denotes expectations conditioned on consumer $z's$ information set at time t, $\Omega_{t,z}$. Expectations conditioned on a common time t information set (i.e., $\Omega_t \equiv \cap_{z\in[0,1]}\Omega_{t,z}$) will be denoted by E_t.

Decision-making in the model takes place at two frequencies. Consumption-savings decisions take place at a lower frequency than financial decision-making (where the latter includes determination of asset prices and reallocation of portfolios via trading). To implement this idea we split each "month" t into four periods. Consumption-savings decisions are made "monthly" while financial decisions are made periodically within the month. As will become clear, the use of the term "month" is nothing more than a convenient label. The economic intuition developed by the model is exactly the same if we replaced "month" t by some other consumption-relevant period. That said, let us now describe the structure of the model by considering the "monthly" sequence of events.

Period 1: Consumers begin the month with their holdings of US and UK currency deposits, $B^1_{t,z}$ and $\hat{B}^1_{t,z}$

and domestic capital: $K_{t,z}$ for US consumers, and $\hat{K}_{t,z}$ for UK consumers. Each consumer then quotes a spot price (\$/£) $S^1_{t,z}$ at which he is willing to buy or sell any amount of foreign currency (£s). These quotes are observable to all consumers.[11]

Period 2: Each consumer z chooses the amount of foreign currency, $T^2_{t,z}$, he wishes to purchase (negative values for sales) by initiating a trade with other consumers (the sum of which constitutes order flow for the period). Trading is simultaneous, trading with multiple partners is feasible, and trades are divided equally among agents offering the same quote. Once these transactions have taken place, consumer z's deposits at the start of period 3 are given by

$$\begin{aligned} B^3_{t,z} &= B^1_{t,z} + S^1_t T^2_{t,z*} - S^1_t T^2_{t,z}, \\ \hat{B}^3_{t,z} &= \hat{B}^1_{t,z} + T^2_{t,z} - T^2_{t,z*}, \end{aligned}$$

where $T^2_{t,z*}$ denotes the incoming foreign currency orders from other consumers trading at z's quoted price. S^1_t is the period-1 spot rate quote at which z purchases pounds. In equilibrium, this will be the spot rate quoted by all consumers (i.e., $S^1_t = S^1_{t,z}$) for reasons we explain below. Notice that period-3 currency holdings depend not only on the transactions initiated by z, (i.e., $T^2_{t,z}$) but also on the transactions initiated by other consumers $T^2_{t,z*}$. An important assumption of our model is that the choice of $T^2_{t,z}$ by consumer z, cannot be conditioned on $T^2_{t,z*}$ because period-2 trading takes place simultaneously. Consequently, though consumers target their desired allocation across dollar and pound assets, resulting allocations include a stochastic component from the arrival of unexpected orders from others.

Period 3: All consumers again quote a spot price and also a pair of one-month interest rates for dollar and pound deposits.[12] The spot quote, $S^3_{t,z}$, is good for a purchase or sale of any amount of pounds, while the interest rates, $R_{t,z}$ and $\hat{R}_{t,z}$ indicate the rates at which the consumer is willing to borrow or lend one-month in dollars and pounds respectively. As in period 1, all quotes are publicly observable.

Period 4: In period 4, consumers choose a second round of foreign currency purchases (if there remain motives for further intra-month trade).[13] They also choose their real allocations: consumption of US and UK goods and real investment expenditures. After US consumers z have chosen their consumption of US and UK goods, $C_{t,z}$ and $\hat{C}_{t,z}$, their foreign currency purchases $T^4_{t,z}$, and their real investment $I_{t,z}$, the resulting

[11] It will be clear below that consumers in this model have both speculative and non-speculative motives for trading (the non-speculative motive arising from the need to facilitate periodic consumption and investment). That these motives are not purely speculative obviates concern about so-called "no trade" results (i.e., the theorem proposed by Milgrom and Stokey 1982, that if I know that your only motive for trade with me is superior information, then I would never want to trade with you at any price at which you want to trade).

[12] Deposit rates are not set in every period because interest is assumed to accrue at the monthly frequency only. As a qualitative matter, abstracting from intra-month interest misses little in the context of the world's major currencies, all of which are generally characterized by relatively low inflation and low nominal interest rates.

[13] That motives for further currency trade within the month will indeed remain is one of the model's important properties. It addresses the question of why agents would want to trade at such high frequencies.

capital and deposit holdings in period 1 of month $t+1$ are:

$$\begin{aligned} B^1_{t+1,z} &= R_t(B^3_{t,z} + S^3_t T^4_{t,z*} - S^3_t T^4_{t,z} + C_{t,z*} - I_{t,z}), \\ \hat{B}^1_{t+1,z} &= \hat{R}_t(\hat{B}^3_{t,z} + T^4_{t,z} - T^4_{t,z*} - \hat{C}_{t,z}), \\ K_{t+1,z} &= R^k_{t+1}\left(K_{t,z} - C_{t,z} - C_{t,z*} + I_{t,z}\right) \end{aligned}$$

where R_t and $\hat{R}_t$ are the dollar and pound interest rates that are quoted by all consumers in equilibrium (i.e., $R_{t,z} = R_t$, and $\hat{R}_{t,z} = \hat{R}_t$ for all z, as shown below). R^k_t is the one-month return on capital. At the end of period-4 trading, the US capital stock is equal to $K_{t,z} - C_{t,z} - C_{t,z*} + I_{t,z}$. We assume that this capital stock is augmented by monthly production, $Y_{t+1,z}$, according to a linear production technology:

$$Y_{t+1,z} = A_{t+1}\left(K_{t,z} - C_{t,z} - C_{t,z*} + I_{t,z}\right)$$

where A_{t+1} is a productivity shock. For simplicity we ignore depreciation so the one-month return on capital is $R^k_{t+1} = (1 + A_{t+1})$. As in period 2 trading, actual bond holdings also depend on the actions of other consumers. In particular, orders for foreign currency and US goods received from other consumers , (i.e., , $T^4_{t,z*}$ and $C_{t,z*}$; more on the latter later) affect the stocks of bonds and capital available next month. The dynamics of the bond holdings and capital held by UK consumers is similarly determined by

$$\begin{aligned} B^1_{t+1,z} &= R_t(B^3_{t,z} + S^3_t T^4_{t,z*} - S^3_t T^4_{t,z} - C_{t,z}), \\ \hat{B}^1_{t+1,z} &= \hat{R}_t(\hat{B}^3_{t,z} + T^4_{t,z} - T^4_{t,z*} + \hat{C}_{t,z*} - \hat{I}_{t,z}), \\ \hat{K}_{t+1,z} &= \hat{R}^k_{t+1}\left(\hat{K}_{t,z} - \hat{C}_{t,z} - \hat{C}_{t,z*} + \hat{I}_{t,z}\right) \end{aligned}$$

The monthly return on UK capital is $\hat{R}^k_{t+1} = 1 + \hat{A}_{t+1}$ where $\hat{A}_{t+1}$ denotes UK productivity (i.e., $\hat{Y}_{t+1,z} = \hat{A}_{t+1}(\hat{K}_{t,z} - \hat{C}_{t,z} - \hat{C}_{t,z*} + \hat{I}_{t,z})$). As in period 2, trading is simultaneous and independent so UK consumers cannot condition their consumption, investment or currency orders on the decisions of US consumers, and vice versa.

2.2 Decision-Making

Consumers make two types of decisions: consumption-savings decisions and financial pricing (quoting) decisions. The former are familiar from standard macro models, but the latter are new. By quoting spot prices and interest rates at which they stand ready to trade, consumers are taking on the liquidity-providing role of financial intermediaries. Specifically, the quote problem facing consumers in periods 1 and 3 is identical to that facing a dealer in a simultaneous trading model (see, for example, Lyons 1997, Rime 2001, Evans and Lyons 2002a,). We therefore draw on this literature to determine how quotes are set.

Equilibrium quotes have two properties: (i) they must be consistent with market clearing, and (ii) they are a function of public information only. The latter property is important to the information transmission role of flow so let us address it more fully. With this property, unanticipated flow can only be impounded into price when it is realized and publicly observed. This lies at the opposite pole of the information assumptions underlying Walrasian (or Rational Expectations) mechanisms in which the market price at a given time impounds information in *every* trade occurring at that time. The Walrasian mechanism is akin to assuming that all trades are publicly observable and conditioned on one another, which is obviously counter-factual in most markets, including FX. (In the jargon of microstructure, the FX market is not a centralized auction with full transparency, but is instead a decentralized dealer market that is relatively opaque.) As noted in the previous section, what is really necessary for the intertemporal transmission role of flow is that at least some flow information is not impounded in price at the time of execution. That quotes are conditioned only on public information insures this, and goes a bit further to simplify the analytics.

We should add, though, that quotes being conditioned only on public information is not an assumption, but a result. Put differently, we make other assumptions that are sufficient for this outcome (drawing from the simultaneous-trade references above). Those assumptions are (1) that actions within any given quoting or trading period are simultaneous and independent, (2) that quotes are a single price good for any size, and (3) that trading with multiple market-makers is feasible.[14] The resulting solution to the quote problem facing consumer z in periods $j = \{1, 3\}$ will be a quote $S^j_{t,z} = S^j_t$ where S^j_t is a function of public information Ω^j_t (determined below). Similarly, the period-3 interest rate quotes are given by $R_{t,z} = R_t$ and $\hat{R}_{t,z} = \hat{R}_t$ where R_t and $\hat{R}_t$ are functions of Ω^3_t. To understand why these quotes represent a Nash equilibrium, consider a market-maker who is pondering whether to depart from this public-information price by quoting a weighted average of public information and his own private information. Any price that deviates from other prices would attract unbounded arbitrage trade flows, and therefore could not possibly represent an equilibrium. Instead, it is optimal for market-makers to quote the same price as others (which means the price is necessarily conditioned on public information), and then exploit their private information by initiating trades at other market-makers' prices. (In some models, market-makers can only establish desired positions by setting price to attract incoming trades, which is not the case here since they always have the option of initiating outgoing trades.)

Next we turn to the consumption and portfolio choices made in periods 2 and 4. Let $W^j_{t,z}$ denote the wealth of individual z at the beginning of period j in month t. This comprises the value of home and foreign bond holdings and domestic capital:

$$\begin{aligned} W^2_{t,z} &\equiv B^1_{t,z} + S^1_t \hat{B}^1_{t,z} + K_{t,z} + S^1_t \hat{K}_{t,z} \\ W^4_{t,z} &\equiv B^3_{t,z} + S^3_t \hat{B}^3_{t,z} + K_{t,z} + S^3_t \hat{K}_{t,z} \end{aligned}$$

Notice that wealth is valued in dollars using the equilibrium spot rate quoted in the period before trading takes place.

In period 2 consumers initiate transactions, (i.e., choose $T^2_{t,z}$) to achieve an optimal allocation of their wealth between dollar and pound assets. Because trading takes place simultaneously, the choice of $T^2_{t,z}$ cannot be conditioned on the orders they receive from others, $T^2_{t,z*}$. Instead, consumers must choose $T^2_{t,z}$ based on the expected order flow, $\mathsf{E}^2_{t,z} T^2_{t,z*}$. (Hereafter we use $\mathsf{E}^j_{t,z}$ to denote expectations conditioned on information available to individual z at the beginning of period j in month t). We formalize this idea by assuming that

[14] As noted, it is also true that the assumption of no spreads is not necessary, though it greatly facilitates the analytics. Specifically, each trader-consumer's quote could be a schedule of prices, one for each incoming order quantity from minus infinity to plus infinity, as long as that schedule is conditioned only on the incoming order, as opposed to the realization of all other orders in the market (i.e., the quoting trader can protect against information contained in the single incoming trade).

$T^{2}_{t,z}$ is chosen to achieve a desired portfolio allocation at the end of period-2 trading conditioned on $\mathsf{E}^{2}_{t,z}T^{2}_{t,z*}$.

Let $J^{2}_{z}(W^{2}_{t,z})$ and $J^{4}_{z}(W^{4}_{t,z})$ denote the value functions for consumer z at the beginning of periods 2 and 4. $T^{2}_{t,z}$ is determined as the solution to the dynamic programming problem

$$J^{2}_{z}(W^{2}_{t,z}) = \max_{\lambda_{t,z}} \mathsf{E}^{2}_{t,z}\left[J^{4}_{z}(W^{4}_{t,z})\right], \tag{2}$$

s.t.

$$W^{4}_{t,z} = H^{3}_{t,z}W^{2}_{t,z}, \tag{3}$$

where

$$\begin{aligned}
H^{3}_{t,z} &\equiv \left(1+\left(\frac{S^{3}_{t}}{S^{1}_{t}}-1\right)(\lambda_{t,z}-\xi_{t})\right),\\
\lambda_{t,z} &\equiv \frac{S^{1}_{t}\left(\hat{B}^{1}_{t,z}+\hat{K}_{t,z}+T^{2}_{t,z}-\mathsf{E}^{2}_{t,z}T^{2}_{t,z*}\right)}{W^{2}_{t,z}},\\
\xi_{t} &\equiv \frac{S^{1}_{t}(T^{2}_{t,z*}-\mathsf{E}^{2}_{t,z}T^{2}_{t,z*})}{W^{2}_{t,z}}.
\end{aligned}$$

The parameter $\lambda_{t,z}$ is a key parameter. It identifies the target fraction of wealth consumers wish to hold in pounds, given their expectations about incoming orders they will receive during trading, $\mathsf{E}^{2}_{t,z}T^{2}_{t,z*}$. (Actual orders, $T^{2}_{t,z}$, are determined from the optimal value of $\lambda_{t,z}$ given $\mathsf{E}^{2}_{t,z}T^{2}_{t,z*}$, $\hat{B}^{1}_{t,z}+\hat{Y}_{t,z}+\hat{K}_{t,z}$ and $W^{2}_{t,z}$). $H^{3}_{t,z}$ identifies the within-month return on wealth (i.e., between periods 1 and 3). This depends on the rate of appreciation in the pound and the actual faction of wealth held in foreign deposits at the end of period-2 trading. The latter term is $\lambda_{t,z}-\xi_{t}$ where ξ_{t} represents the effect of filling unexpected pound orders from other consumers (a shock). This means that the return on wealth, $H^{3}_{t,z}$, is subject to two sources of uncertainty: uncertainty about the future spot rate S^{3}_{t}, and uncertainty about order flow in the form of trades initiated by other consumers.

In period 4, consumers choose consumption of US and UK goods, foreign currency orders and investment expenditures. Let $\alpha_{t,z}$ and $\gamma_{t,z}$ denote the desired fractions of wealth held in pounds and domestic capital respectively:

$$\begin{aligned}
\alpha_{t,z} &\equiv \frac{S^{3}_{t}\hat{K}_{t,z}+S^{3}_{t}\hat{B}^{3}_{t,z}+S^{3}_{t}\left(T^{4}_{t,z}-\mathsf{E}^{4}_{t,z}T^{4}_{t,z*}\right)-S^{3}_{t}\hat{C}_{t,z}}{W^{4}_{t,z}},\\
\gamma_{t,z} &\equiv \begin{cases} \dfrac{K_{t,z}+I_{t,z}-C_{t,z}-\mathsf{E}^{4}_{t,z}C_{t,z*}}{W^{4}_{t,z}} & z<1/2,\\[2ex] \dfrac{\hat{K}_{t,z}+\hat{I}_{t,z}-\hat{C}_{t,z}-\mathsf{E}^{4}_{t,z}\hat{C}_{t,z*}}{W^{4}_{t,z}} & z\geq 1/2,\end{cases}
\end{aligned}$$

The period-4 problem can now be written as

$$J^{4}_{z}(W^{4}_{t,z}) = \max_{\left\{C_{t,z},\hat{C}_{t,z},\alpha_{t,z},\gamma_{t,z}\right\}} \left\{U(\hat{C}_{t,z},C_{t,z})+\beta\mathsf{E}^{4}_{t,z}\left[J^{2}_{z}(W^{2}_{t+1,z})\right]\right\}, \tag{4}$$

s.t.

$$W^{2}_{t+1,z} = R_{t}H^{1}_{t+1,z}W^{4}_{t,z}-R_{t}\left(C_{t,z}+S^{3}_{t}\hat{C}_{t,z}\right), \tag{5}$$

where

$$H^1_{t+1,z} = \begin{cases} 1 + \left(\frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t} - 1\right)(\alpha_{t,z} - \varsigma_t) + \left(\frac{R^k_{t+1}}{R_t} - 1\right)(\gamma_{t,z} - \zeta_t) & z < 1/2 \\ \\ 1 + \left(\frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t} - 1\right)(\alpha_{t,z} - \varsigma_t) + \left(\frac{S^1_{t+1}\hat{R}^k_{t+1}}{S^3_t R_t} - \frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t}\right)\left(\gamma_{t,z} - \hat{\zeta}_t\right) & z \geq 1/2 \end{cases}.$$

with $R^k_{t+1} \equiv 1 + A_{t+1}$, and $\hat{R}^k_{t+1} = 1 + \hat{A}_{t+1}$.

$H^1_{t+1,z}$ is the excess return on wealth (measured relative to the dollar one-month rate R_t). As above, realized returns depend on the actual faction of wealth held in pounds $\alpha_{t,z} - \varsigma_{t,z}$, where $\varsigma_t \equiv S^3_t(T^4_{t,z*} - \mathsf{E}^4_{t,z}T^4_{t,z*})/W^4_{t,z}$ represents the effects of unexpected currency orders. Monthly returns also depend on the fraction of wealth held in the form of capital. For the US case this is given by $\gamma_{t,z} - \zeta_{t,z}$, where $\zeta_{t,z} \equiv \left(C_{t,z*} - \mathsf{E}^4_{t,z}C_{t,z*}\right)/W^4_{t,z}$ identifies the effects of unexpected demand for US goods (i.e. US exports).[15] In the UK case, the fraction is $\gamma_{t,z} - \hat{\zeta}_{t,z}$, where $\hat{\zeta}_{t,z} \equiv \left(\hat{C}_{t,z*} - \mathsf{E}^4_{t,z}\hat{C}_{t,z*}\right)/W^4_{t,z}$. Monthly returns are therefore subject to four sources of uncertainty: uncertainty about future spot rates (i.e., S^1_{t+1}) that affects bond returns; uncertainty about future productivity that affects the return on capital; uncertainty about currency orders; and uncertainty about export demand.

The first order conditions governing consumption and portfolio choice (i.e., $C_{t,z}, \hat{C}_{t,z}, \lambda_{t,z}, \alpha_{t,z}$) take the same form for both US and UK consumers

$$\hat{C}_{t,z} \quad : \quad U_{\hat{c}}(\hat{C}_{t,z}, C_{t,z}) = \beta R_t S^3_t \mathsf{E}^4_{t,z}\left[V_{t+1,z}H^3_{t+1,z}\right], \tag{6}$$

$$C_{t,z} \quad : \quad U_c(\hat{C}_{t,z}, C_{t,z}) = \beta R_t \mathsf{E}^4_{t,z}\left[V_{t+1,z}H^3_{t+1,z}\right], \tag{7}$$

$$\lambda_{t,z} \quad : \quad 0 = \mathsf{E}^2_{t,z}\left[V_{t,z}\left(\frac{S^3_t}{S^1_t} - 1\right)\right], \tag{8}$$

$$\alpha_{t,z} \quad : \quad 0 = \mathsf{E}^4_{t,z}\left[V_{t+1,z}H^3_{t+1,z}\left(\frac{S^1_{t+1}R}{S^3_t R_t} - 1\right)\right], \tag{9}$$

where $V_{t,z} \equiv dJ^4_z(W^4_{t,z})/dW^4_{t,z}$ is the marginal utility of wealth. The first order conditions governing real investment (i.e. $\gamma_{t,z}$) differ between US and UK consumers and are given by

$$\gamma_{t,z<1/2} \quad : \quad 0 = \mathsf{E}^4_{t,z}\left[V_{t+1,z}H^3_{t+1,z}\left(\frac{R^k_{t+1}}{R_t} - 1\right)\right], \tag{10}$$

$$\gamma_{t,z\geq 1/2} \quad : \quad 0 = \mathsf{E}^4_{t,z}\left[V_{t+1,z}H^3_{t+1,z}\left(\frac{S^1_{t+1}\hat{R}^k_{t+1}}{S^3_t R_t} - 1\right)\right]. \tag{11}$$

To further characterize the form of optimal consumption, portfolio and investment decisions, we need to identify the marginal utility of wealth. This is implicitly defined by the recursion

$$V_{t,z} = \beta R_t \mathsf{E}^4_{t,z}\left[V_{t+1,z}H^3_{t+1,z}H^1_{t+1,z}\right]. \tag{12}$$

In a standard macro model where consumers provide no liquidity provision, equations (8) - (12) together

[15] When superior information about home-country income is not symmetrized by month's end, the residual uncertainty is manifested as a shock to export demand.

imply that $V_{t,z} = U_c(\hat{C}_{t,z}, C_{t,z})$. The first order conditions can then be rewritten in familiar form using the marginal rate of substitution. This is not generally the case in our model. As we shall show, $V_{t,z}$ can diverge from the marginal utility of consumption because unexpected currency and export orders affect portfolio returns.

2.3 Market Clearing

Market clearing in the currency market requires that the dollar value of pound orders initiated equals the dollar value of pound orders received:

$$\int T^j_{t,z} dz = \int T^j_{t,z^*} dz^*,$$

for $j = \{2, 4\}$.

We assume that dollar and pound deposits are in zero net supply so that aggregate deposit holdings at the start of periods 3 and 1 are given by

$$\int B^3_{t,z} dz = 0, \qquad \int \hat{B}^3_{t,z} dz = 0, \tag{13}$$

$$\int B^1_{t+1,z} dz = 0, \qquad \int \hat{B}^1_{t+1,z} dz = 0. \tag{14}$$

Combining these conditions with the budget constraints for dollar and pound deposits implies that both US and UK investment expenditures must equal zero if the bond and goods markets are to clear.[16] The reason is that both currency and goods market transactions only affect the distribution of deposits not their aggregate level. This means that any investment expenditures must be financed by an increase aggregate deposit holdings, an implication that is inconsistent with market clearing. The implications of market clearing for the dynamics of capital are therefore represented by

$$K_{t+1,z} = R^k_t K_{t,z} - \int C_{t,z} dz, \tag{15}$$

$$\hat{K}_{t+1,z} = \hat{R}^k_t \hat{K}_{t,z} - \int \hat{C}_{t,z} dz. \tag{16}$$

3 Equilibrium

An equilibrium in this model is described by: (i) a set of quote functions (that define the relationship between public information and both spot rates and interest rates) that clear markets given the consumption, investment and portfolio choices of consumers; and (ii) a set of consumption, investment and portfolio decision rules that maximize expected utility given the spot and interest rates and the exogenous productivity processes. In this section we describe how the equilibrium is constructed given particular specifications for utility and the productivity processes.

[16] Though this feature of the model appears rather special, it is not driving our results.

3.1 Utility and Productivity

We assume that the sub-utility function of both US and UK consumers takes the log form:

$$U(\hat{C}_{t,z}, C_{t,z}) = \tfrac{1}{2}\ln \hat{C}_{t,z} + \tfrac{1}{2}\ln C_{t,z}.$$

This assumption simplifies consumers decision-making and allows us to focus more easily on the novel aspects of the model.

The international aspect of our model becomes apparent with the specification of the productivity processes. In particular, the key feature that differentiates US from UK consumers in our model is the comparative advantage they have in acquiring information on local productivity. This information advantage creates an environment where dispersed information exists about the current and future returns to capital across the world. We examine below how this dispersed information becomes aggregated into exchange rates and interest rates via trading. Our focus is thus on information transmission process rather than the underlying source of the dispersed information. In a more general model, a comparative local advantage in information acquisition could also apply to monetary policy (in the form of superior local information about the path for future interest rates), or fiscal policy (in the form of superior tax rate forecasts). Our analysis could be readily extended to an environment where dispersed information originates from productivity and other sources.

To frame the information structure, we characterize the exogenous productivity processes in terms of the log returns on real capital:

$$\ln R_t^k \equiv r_t^k = r + \theta(e_{t-1} - \hat{e}_{t-1}) + u_t + e_t \tag{17}$$

$$\ln \hat{R}_t^k \equiv \hat{r}_t^k = r + \theta(\hat{e}_{t-1} - e_{t-1}) + \hat{u}_t + \hat{e}_t \tag{18}$$

with

$$\begin{pmatrix} u_t \\ \hat{u}_t \\ e_t \\ \hat{e}_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} \sigma_u^2 & \rho\sigma_u^2 & 0 & 0 \\ \rho\sigma_u^2 & \sigma_u^2 & 0 & 0 \\ 0 & 0 & \sigma_e^2 & 0 \\ 0 & 0 & 0 & \sigma_e^2 \end{pmatrix}\right)$$

Log capital returns here include two random components beyond the constant r: a transitory component u_t ($\hat{u}_t$) and a persistent component e_t ($\hat{e}_t$). The transitory component u_t ($\hat{u}_t$) is a one-month effect on US (UK) returns with cross-country correlation ρ. Unlike u_t ($\hat{u}_t$), the random variable e_t ($\hat{e}_t$) is contemporaneously independent across countries, but gives rise to an intertemporal impact that depends on this component's cross-country differential from the previous period.

These return specifications are not meant as precise empirical representations. They are instead chosen to facilitate our analysis of how exchange rates respond to primitive assumptions about each consumer's information. That said, we consider it uncontroverial that capital returns should include both transitory and persistent components. For the analysis below, we examine information structures in which US consumers observe their home shocks $\{u_t, e_t\}$ in period 1 of month t, whereas UK consumers observe their home shocks $\{\hat{u}_t, \hat{e}_t\}$. In this setting, dispersed information exists inter-nationally but not intra-nationally. (One can think of intra-national information as having been aggregated "in the background." This is a common feature of the equilibria we study below.) Allowing information to be dispersed at both the intra- and inter-national levels is an interesting but complex undertaking that we leave for future work. Similarly, the returns specifications might also be extended to deal with components that follow general moving average processes. The specifications in (17) and (18) highlight the theoretical consequences of dispersed information in the simplest possible way.

3.2 Log Approximations

To facilitate finding the optimal consumption, investment and currency trading decisions of US and UK consumers we make use of log linear approximations to the budget constraints and first order conditions.[17] Combining (3) and (5) the monthly budget constraint is approximated by

$$\Delta w^4_{t+1,z} \cong r_t + h^3_{t+1,z} + \ln(1-\mu) + \frac{1}{1-\mu} h^1_{t+1,z} - \frac{\mu}{1-\mu}\delta_{t,z}, \tag{19}$$

where lowercase letters denote natural logs and $\delta_{t,z} \equiv c_{t,z} - w^4_{t,z} - \ln(\mu/2)$ is the log consumption wealth ratio. μ is a positive constant equal to the steady state value of $2C_{t,z}/W^4_{t,z}$. $h^1_{t,z}$ and $h^3_{t,z}$ are the log excess returns on the wealth of consumer z realized respectively in periods 1 and 3 in month t. Using the definitions of $H^1_{t,z}$ and $H^3_{t,z}$ represented above, we approximate the within-month returns by

$$h^3_{t,z} \cong \lambda_{t,z}\left(s^3_t - s^1_t\right) + \tfrac{1}{2}\lambda_{t,z}\left(1-\lambda_{t,z}\right)\mathsf{V}^2_{t,z}\left(s^3_t\right) - \mathsf{CV}^2_{t,z}\left(s^3_t, \xi_t\right), \tag{20}$$

where $\mathsf{V}^j_{t,z}$ and $\mathsf{CV}^j_{t,z}$ denote the variance and covariance conditioned on consumer $z's$ information at the start of period j in month t. This approximation is similar to those adopted by Campbell and Viceira (2002) and is based on a second order approximation that holds exactly in continuous time when the change in spot rates and unexpected order flow follow Wiener processes. Monthly returns are approximated in a similar fashion. For US consumers (i.e. $z < 1/2$) we use

$$\begin{aligned} h^1_{t+1,z} \cong\ & \alpha_{t,z}\left(s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right) + \gamma_{t,z}\left(r^k_{t+1} - r_t\right) + \tfrac{1}{2}\alpha_{t,z}\left(1-\alpha_{t,z}\right)\mathsf{V}^4_{t,z}\left(s^1_{t+1}\right) \\ & + \tfrac{1}{2}\gamma_{t,z}\left(1-\gamma_{t,z}\right)\mathsf{V}^4_{t,z}\left(r^k_{t+1}\right) - \alpha_{t,z}\gamma_{t,z}\mathsf{CV}^4_{t,z}\left(s^1_{t+1}, r^k_{t+1}\right) \\ & - \mathsf{CV}^4_{t,z}\left(s^1_{t+1}, \varsigma_t\right) - \mathsf{CV}^4_{t,z}\left(r^k_{t+1}, \zeta_t\right), \end{aligned} \tag{21}$$

and for UK consumers ($z \geq 1/2$)

$$\begin{aligned} h^1_{t+1,z} \cong\ & \alpha_{t,z}\left(s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right) + \gamma_{t,z}\left(\hat{r}^k_{t+1} - \hat{r}_t\right) + \tfrac{1}{2}\left(\alpha_{t,z} - \gamma_{t,z}\right)\left(1 - \left(\alpha_{t,z} - \gamma_{t,z}\right)\right)\mathsf{V}^4_{t,z}\left(s^1_{t+1}\right) \\ & + \tfrac{1}{2}\gamma_{t,z}\left(1-\gamma_{t,z}\right)\mathsf{V}^4_{t,z}\left(\hat{r}^k_{t+1} + s^1_{t+1}\right) - \left(\alpha_{t,z} - \gamma_{t,z}\right)\gamma_{t,z}\mathsf{CV}^4_{t,z}\left(s^1_{t+1}, \hat{r}^k_{t+1} + s^1_{t+1}\right) \\ & - \mathsf{CV}^4_{t,z}\left(s^1_{t+1}, \varsigma_t\right) - \mathsf{CV}^4_{t,z}\left(r^k_{t+1}, \hat{\zeta}_t\right). \end{aligned} \tag{22}$$

[17] Complete derivations are contained in the appendix.

Notice that unexpected order flows and export demand affect returns through the last covariance terms shown in each equation. These terms represent the effects of non-diversifiable risk that arises from liquidity provision. Unexpected currency orders and export orders during period 2 and 4 trading represent a source of risk that consumers cannot fully hedge.

To derive our log approximations to the first order conditions, we combine the log linearized versions of equations (6) - (12) and our assumption of log utility to obtain

$$v_{t,z} = -c_{t,z} - \phi_{t,z}, \tag{23}$$

where $\phi_z \equiv \mathsf{CV}^4_{t,z}\left(s^1_{t+1}, \varsigma_t\right) + \mathsf{CV}^4_{t,z}\left(r^k_{t+1}, \zeta_t\right)$ for $z < 1/2$ (US consumers), and $\phi_z \equiv \mathsf{CV}^4_{t,z}\left(s^1_{t+1}, \varsigma_t\right)$ $+\mathsf{CV}^4_{t,z}\left(\hat{r}^k_{t+1}, \hat{\zeta}_t\right)$ for $z \geq 1/2$ (UK consumers). In the absence of unexpected period-4 currency orders and export demand, the shocks ς_t, ζ_t and $\hat{\zeta}_t$ are zero and the (log) marginal utility of wealth equals the marginal utility of consumption. When these shocks are present and correlated with the future spot rate, and/or returns on capital, the return on wealth is exposed to these sources of systematic risk that may push up or down the log return on wealth according to the sign of the covariance terms. As we shall see, the covariance between currency orders and the future spot rate, $\mathsf{CV}^4_{t,z}\left(s^1_{t+1}, \varsigma_{t,z}\right)$, will differ from zero when period-4 currency trading provides information relevant to the setting of future spot rates. Thus, the transmission of price-relevant information via trading can push a wedge, $\phi_{t,z}$, between the marginal utilities of wealth and consumption.

Substituting for $v_{t,z}$ in the log linearized versions of (6) - (11) gives the following linearized first order conditions:

$$\lambda_{t,z} \quad : \quad \mathsf{E}^2_{t,z} s^3_t - s^1_t + \tfrac{1}{2}\mathsf{V}^2_{t,z}\left(s^3_t\right) = \mathsf{CV}^2_{t,z}\left(c_{t,z} + \phi_{t,z}, s^3_t\right), \tag{24}$$

$$\alpha_{t,z} \quad : \quad \mathsf{E}^4_{t,z}\left[s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right] + \tfrac{1}{2}\mathsf{V}^4_{t,z}\left(s^1_{t+1}\right) = \mathsf{CV}^4_{t,z}\left(c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}, s^1_{t+1}\right), \tag{25}$$

$$c_{t,z} \quad : \quad \ln\beta + r_t = \mathsf{E}^4_{t,z}\left[\Delta c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}\right] - \tfrac{1}{2}\mathsf{V}^4_{t,z}\left(c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}\right), \tag{26}$$

$$\hat{c}_{t,z} \quad : \quad c_{t,z} = s^3_t + \hat{c}_{t,z}, \tag{27}$$

for both US and UK consumers. The linearized versions of (10) and (11) are

$$\gamma_{t,z<1/2} \quad : \quad \mathsf{E}^4_{t,z}\left[r^k_{t+1} - r_t\right] + \frac{1}{2}\mathsf{V}^4_{t,z}\left(r^k_{t+1}\right) = \mathsf{CV}^4_{t,z}\left(c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}, r^k_{t+1}\right), \tag{28}$$

$$\begin{aligned}\gamma_{t,z\geq 1/2} \quad : \quad & \mathsf{E}^4_{t,z}\left[\hat{r}^k_{t+1} + s^1_{t+1} - s^3_t - r_t\right] + \frac{1}{2}\mathsf{V}^4_{t,z}\left(\hat{r}^k_{t+1} + s^1_{t+1}\right) = \\ & \mathsf{CV}^4_{t,z}\left(c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}, \hat{r}^k_{t+1} + s^1_{t+1}\right).\end{aligned} \tag{29}$$

Notice that presence of liquidity provision in the model only affect the first order conditions characterizing consumer behavior through the $\phi_{t,z}$ terms. When combined with the linearized budget constraint, these equations allow us to find analytic approximations for the solution to the optimizations problems facing consumers at the beginning of period 2 and 4 (i.e. expressions for $\lambda_{t,z}$, $\alpha_{t,z}$, $\gamma_{t,z}, c_{t,z}$ and $\hat{c}_{t,z}$) given the r^k_t and r^k_t processes, and the equilibrium dynamics of spot exchange rates and interest rates (determined below).

We also utilized log linear approximations to the capital stock dynamics implied by market clearing in (15) and (16):

$$
\begin{aligned}
k_{t+1} - k_t &\cong r^k_{t+1} + \ln(1-\mu) - \frac{\mu}{2(1-\mu)}\left(s^3_t + \hat{k}_t - k_t + \int \delta_{t,z} dz\right), && (30)\\
\hat{k}_{t+1} - \hat{k}_t &\cong \hat{r}^k_{t+1} + \ln(1-\mu) - \frac{\mu}{2(1-\mu)}\left(k_t - s^3_t - \hat{k}_t + \int \delta_{t,z} dz\right). && (31)
\end{aligned}
$$

In deriving these equations we have assumed that deposit holdings always represent a small fraction of consumer wealth. This condition is met trivially in the steady state because both US and UK consumers hold all their wealth in the form of domestic capital. The accuracy of these approximations will deteriorate if consumers accumulate substantial financial assets/liabilities relative to their capital holdings when away from the steady state (see Appendix for a further discussion).

3.3 Solution Method

We solve for equilibrium using a guess and verify method with the following five steps:

1. We make a conjecture about the information available to consumers at each point in time. This involves specifying what information consumers receive directly and what they learn from observing trading.
2. Based on this information structure, we then guess the form of equilibrium quote functions for spot rates and interest rates in periods 1 and 3 noting that quotes can only be a function of common information.
3. We use the log linearized first order conditions and budget constraint to approximate consumers' optimal consumption, investment and currency choices given the spot and interest rates from step 2.
4. We check that consumer choices for consumption, investment and currency holdings clear markets.
5. We verify that the conjectured information structure (from step 1) can be supported by an inference problem based on exogenous information available to each consumer, and their observations of quotes and trading activity.

4 Dispersed Information Results

Recall that the capital returns processes follow:

$$
\begin{aligned}
\ln R^k_t &\equiv r^k_t = r + \theta(e_{t-1} - \hat{e}_{t-1}) + u_t + e_t\\
\ln \hat{R}^k_t &\equiv \hat{r}^k_t = r + \theta(\hat{e}_{t-1} - e_{t-1}) + \hat{u}_t + \hat{e}_t
\end{aligned}
$$

with

$$\begin{pmatrix} u_t \\ \hat{u}_t \\ e_t \\ \hat{e}_t \end{pmatrix} \sim i.d.N \left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \begin{pmatrix} \sigma_u^2 & \rho\sigma_u^2 & 0 & 0 \\ \rho\sigma_u^2 & \sigma_u^2 & 0 & 0 \\ 0 & 0 & \sigma_e^2 & 0 \\ 0 & 0 & 0 & \sigma_e^2 \end{pmatrix} \right)$$

All of our analysis below is based on the following key information assumptions: (i) US consumers all observe the realization of their home shocks $\{u_t, e_t\}$ in period 1 of month t, UK consumers all observe the realizations of their home shocks $(\hat{u}_t, \hat{e}_t)$ in period 1 of month t, and all consumers in both countries observed the realized values of log capital returns from the previous month, r_{t-1}^k and $\hat{r}_{t-1}^k$, when they are publicly announced in period 1 of month t.

4.1 Version 1: $\rho = -1$

The equilibrium FX process is:

$$\begin{aligned} \Delta s_t^3 &= 2\theta\nabla e_{t-1} + \nabla u_t + \nabla e_t \\ s_{t+1}^1 - s_t^3 &= \nabla u_{t+1} + 2\theta\nabla e_t \\ s_{t+1}^3 - s_{t+1}^1 &= \nabla e_{t+1} \\ r_t &= r + \theta\nabla e_t \\ \hat{r}_t &= r - \theta\nabla e_t \end{aligned}$$

where the notation ∇u_t denotes $(\hat{u}_t - u_t)$.

Properties of this equilibrium:

- The shocks u_t and $\hat{u}_t$ have an immediate and one-to-one effect on the period-1 spot rates because they are common-knowledge (perfectly negatively correlated).
- Aggregation of dispersed information about e_t and $\hat{e}_t$ is complete by the end of period-2 trading.
- The announcement of r_{t-1}^k and $\hat{r}_{t-1}^k$ in period 1 of month t, has no impact on the exchange rate because the announcement contains no new information relative to that contained in the common information set of consumers.
- The within-month exchange rate change $s_t^3 - s_t^1$ will be correlated with unexpected order flow arising from period-2 trading.
- Because information aggregation is complete, there is no unexpected order flow arising from period-4 trading.
- Note that UIP holds:

$$\mathsf{E}_{t,z}^3[s_{t+1}^1 - s_t^3] = r_t - \hat{r}_t$$

$$s_{t+1}^1 - s_t^3 = r_t - \hat{r}_t + \nabla u_{t+1}$$

There is no risk premium because with log utility there is no hedging demand for FX between t:4 and t+1:1 (where t:4 denotes period 4 of month t).

4.2 Version 2: $\rho < -1$

Now we drop the assumption that one component of returns is perfectly negatively correlated internationally. Naturally this prevents the u_t and $\hat{u}_t$ shocks from having an immediate, one-to-one effect on the period-1 spot rates (since both are not contained in the set of public information at the time of quoting). Now signal processing involves disentangling the two types of capital-return shocks. This equilibrium is much more complex than that in version 1 and displays several features not found in standard models. The equilibrium FX process is now:

$$\begin{aligned} s_{t+1}^1 - s_t^3 &= \kappa_e \nabla e_t + \kappa_u \nabla u_t \\ s_t^3 - s_t^1 &= \psi \xi_t \\ r_t &= r + \varphi_e \nabla e_t + \varphi_u \nabla u_t \\ \hat{r}_t &= r - \hat{\varphi}_e \nabla e_t - \hat{\varphi}_u \nabla u_t \end{aligned}$$

where π_x, ψ φ_x and $\hat{\varphi}_e$ are coefficients determined below.

To understand the origins of these dynamics, we start with the market-clearing conditions. From the dynamics of domestic and foreign capital we have that:

$$\nabla k_{t+1} = s_t^3 + \frac{1}{1-\mu}(\nabla k_t - s_t^3) + \nabla r_{t+1}^k$$

Now if $s_t^3 = E_t^3 \nabla k_t$ (as we will establish), this equation implies that:

$$\Delta s_{t+1}^3 = E_{t+1}^3 \nabla r_{t+1}^k + \frac{1}{1-\mu} E_{t+1}^3 (\nabla k_t - E_t^3 \nabla k_t)$$

Further if $s_t^1 = E_t^3 s_t^3$, then $s_{t+1}^1 = E_{t+1}^1 \nabla k_{t+1}$ and:

$$s_{t+1}^1 - s_t^3 = E_{t+1}^1 \nabla r_{t+1}^k + \frac{1}{1-\mu} E_{t+1}^1 (\nabla k_t - E_t^3 \nabla k_t)$$

This implies that:

$$s_{t+1}^3 - s_{t+1}^1 = \left(E_{t+1}^3 - E_{t+1}^1\right) \nabla r_{t+1}^k + \frac{1}{1-\mu}\left(E_{t+1}^3 - E_{t+1}^1\right)(\nabla k_t - E_t^3 \nabla k_t)$$

Consider the first term $\left(E_{t+1}^3 - E_{t+1}^1\right) \nabla r_{t+1}^k$. What do all consumers learn about returns between periods 1 and 3? Well the only new information augmenting the common information set comes via unexpected order flow from period-2 trading, ξ_t. So we conjecture that:

$$E^3_{t+1}\nabla r^k_{t+1} = E^1_{t+1}\nabla r^k_{t+1} + \psi\xi_t.$$

Next we need to determine whether ξ_t will indeed provide information about returns. For this to be the case $s^3_{t+1} - s^1_{t+1}$ must be forecastable based on the private information held by consumers. Consumers will then have an incentive to trade, and in so doing some of their private information will be revealed via order flow. This is exactly the same information aggregation process that is at work in version 1 of the model. However, here there is one crucial difference. In equilibrium, observations on order flow are not sufficient for all consumers to learn the complete structure of international returns. In other words, US consumers cannot precisely infer the values of $\hat{u}_t$ and $\hat{e}_t$ from their observation of ξ_t and their private knowledge of u_t and e_t. This means that the aggregation of dispersed information is incomplete at the end of period-2 trading.

Two important implications follow from this incomplete aggregation result. First, the international distribution of capital, ∇k_t is not common knowledge by the start of period 3. In other words, the current state of fundamentals is not common knowledge. Period-3 spot rates are therefore not equal to their "fundamental value" (i.e., $S^3_t \neq \nabla k_t$). Second, incomplete information aggregation implies that consumers still have an incentive to trade in period 4 because they have superior information about the future behavior of returns. Thus, incomplete aggregation will lead order flow to be correlated with exchange rate changes over several periods, even though no new private information has become available to individual consumers. Rather the sequence of order flows is symptomatic of a prolonged information aggregation process because complete aggregation cannot be accomplished within a singe trading period. (This is an important feature of the model because we observe that spot rate changes and order flow are contemporaneously correlated over long periods of time. Our model show that it is not necessary for new dispersed information to be continually arriving across the economy to sustain the observed persistence of the correlation.)

Uncertainty about the state of fundamentals affects the dynamics of spot rates via $\nabla k_t - E^3_t\nabla k_t$. In version 1, this term is always zero because the complete state of the economy is common knowledge by period 3 of month t. In this version the lack of common knowledge concerning fundamentals contributes to the dynamics of spot rates between $t:3$ and $t+1:1$ via $E^1_{t+1}(\nabla k_t - E^3_t\nabla k_t) = E^1_{t+1}\nabla k_t - E^3_t\nabla k_t$. This term will differ from zero for two reasons. First, as consumers trade in period 4 they will reveal private information about fundamentals. And, just like period-2 trading, this information augments the common information set via order flow. In other words, the "market" is continuing to learn about fundamentals from order flow because information aggregation was incomplete in period-2 trading. The second reason for $E^1_{t+1}\nabla k_t - E^3_t\nabla k_t$ to differ from zero is that the announcement of r^k_t and $\hat{r}^k_t$ in period 1 of month $t+1$ provides information on ∇k_t that was not common knowledge in $t:3$. Notice that the announcement contains no information that was previously unknown to all consumers because US (UK) consumers knew r^k_t ($\hat{r}^k_t$) back in $t:1$. Rather the announcement makes public information that was dispersed across the economy. In the case of our model, the information contained in the announcement is sufficient for all consumers to learn the true state of fundamentals so $E^1_{t+1}\nabla k_t = \nabla k_t$.Thus, the announcement brings to an end the process of "market" learning via order flow in period 2 and 4 trade. Clearly, if the announcement contained information on $r^k_{t-\tau}$ and $\hat{r}^k_{t-\tau}$ for $\tau > 0$, then the market learning process could continue for 2τ periods of trading. Our specification for announcements curtails the learning process for the sake of clarity.

The key to understanding the dynamics of spot rates in our model resides in the role played by period-2 order flow. As in version 1, order flow conveys information about fundamentals dispersed across the economy. The difference is that order flow now only provides an imprecise signal to consumers about foreign fundamentals. To see why this is so, we start with the definition:

$$\xi_t = \int \lambda_{t,z} \frac{W^2_{t,z}}{W^2_t} dz - \frac{S^1_t \hat{K}_{t+1}}{W^2_t}$$

where $W^2_t = \int W^2_{t,z} dz = S^1_t \hat{K}_t + K_t$ by market clearing. Substituting in this definition gives:

$$\begin{aligned} \xi_t \quad = \quad & \frac{\lambda_{t,\mathsf{US}} W^2_{t,\mathsf{US}}}{\left(\exp\left(s^1_t - \nabla k_t\right) + 1\right) K_t} + \frac{\lambda_{t,\mathsf{UK}} W^2_{t,\mathsf{UK}}}{\left(\exp\left(\nabla k_t - s^1_t\right) + 1\right) S^1_t \hat{K}_t} \\ & - \frac{1}{\left(\exp\left(\nabla k_t - s^1_t\right) + 1\right)} \end{aligned}$$

Linearizing this expression around the steady state (where $\lambda_{t,z} = 1/2$, $W^2_{t,\mathsf{US}} = K_t, W^2_{t,\mathsf{UK}} = \hat{K}_t$ and $S^1_t = K_t/\hat{K}_t$) we find:

$$\xi_t \cong \frac{1}{4}(\lambda_{t,\mathsf{US}} - \frac{1}{2}) + \frac{1}{4}(\lambda_{t,\mathsf{UK}} - \frac{1}{2}) + \frac{1}{4}(w^2_{t,\mathsf{US}} - k_t) + \frac{1}{4}\left(w^2_{t,\mathsf{UK}} - s^1_t - \hat{k}_t\right) - \frac{1}{4}\left(s^1_t - \nabla k_t\right)$$

which can be combined with the linearized market clearing condition, $w^2_{t,\mathsf{US}} - k_t \cong s^1_t + \hat{k}_t - w^2_{t,\mathsf{UK}}$, to give:

$$\xi_t \cong \frac{1}{4}(\lambda_{t,\mathsf{US}} - \frac{1}{2}) + \frac{1}{4}(\lambda_{t,\mathsf{UK}} - \frac{1}{2}) - \frac{1}{4}\left(s^1_t - \nabla k_t\right).$$

We make use of this approximation to formally solve the inference problem facing consumers at the end of period-2 trading (when order flow is observed). However, examination of the definition of unexpected order flow ξ_t, makes clear why it may not convey sufficient information to make the current state of fundamentals common knowledge. In particular, the definition shows ξ_t to be a function of the desired portfolio shares $\lambda_{t,z}$ and the distribution of wealth $W^2_{t,z}/W^2_t$. This means that from the perspective of a US consumer, say, order flow conveys information on both $\lambda_{t,\mathsf{UK}}$ and $W^2_{t,\mathsf{UK}}$.The former will have been chosen by UK consumers as the solution to their period-2 portfolio allocation problem. As such, $\lambda_{t,\mathsf{UK}}$ will depend on expected foreign exchange returns (and risk). By contrast, $W^2_{t,\mathsf{UK}}$ is a state variable that reflects the effects of past consumption, trading and returns on UK capital. As a result, both $\lambda_{t,\mathsf{UK}}$ and $W^2_{t,\mathsf{UK}}$ will be functions of elements in $\Omega^2_{t,\mathsf{UK}}$, the private information set of UK consumers. In version 1 of the model consumers try to infer the value of $\hat{e}_t$ from their observation on order flow. This is a simple inference problem because $\lambda_{t,\mathsf{UK}}$ and $W^2_{t,\mathsf{UK}}$ are both functions of $\hat{e}_t$ and elements of Ω^3_{t-1}. US consumers can therefore precisely determine the value of $\hat{e}_t$ from their observation of order flow and prior information. In version 2, $\lambda_{t,\mathsf{UK}}$ and $W^2_{t,\mathsf{UK}}$ are both functions of $\hat{e}_t, \hat{u}_t$ and elements of Ω^3_{t-1}. This means that ξ_t and the elements of Ω^3_{t-1} can only provide imprecise estimates of $\hat{e}_t$, and $\hat{u}_t$ to US consumers.

To demonstrate this formally, we begin by noting that month t fundamentals are common knowledge after the month $t+1$ announcement. Hence,

$$\begin{aligned} s^3_{t+1} - s^1_{t+1} &= \left(\mathsf{E}^3_{t+1} - \mathsf{E}^1_{t+1}\right) \nabla r^k_{t+1} + \frac{1}{1-\mu} \left(\mathsf{E}^3_{t+1} - \mathsf{E}^1_{t+1}\right) (\nabla k_t - \mathsf{E}^3_t \nabla k_t) \\ &= \left(\mathsf{E}^3_{t+1} - \mathsf{E}^1_{t+1}\right) \nabla r^k_{t+1} \\ &= \mathsf{E}[\nabla e_{t+1} + \nabla u_{t+1} | \xi_t] \end{aligned}$$

because $\nabla k_t - E^3_t \nabla k_t$ is common knowledge by t+1:1.Similarly,

$$\begin{aligned} s^1_{t+1} - \nabla k_{t+1} &= s^1_{t+1} - s^3_t - \frac{1}{1-\mu} \left(\nabla k_t - \mathsf{E}^3_t \nabla k_t\right) - \nabla r^k_{t+1} \\ &= \left(\mathsf{E}^1_{t+1} - 1\right) \nabla r^k_{t+1} + \frac{1}{1-\mu} \left(\mathsf{E}^1_{t+1} - 1\right) (\nabla k_t - \mathsf{E}^3_t \nabla k_t) \\ &= -\nabla e_{t+1} - \nabla u_{t+1} \end{aligned}$$

where, in the last line we have made use of the fact that $\mathsf{E}^1_{t+1} \nabla r^k_{t+1} = 2\theta \nabla e_t$.

Next, we posit that the solutions to the period-2 portfolio problem facing US and UK consumers take the form:

$$\begin{aligned} \lambda_{t,\mathsf{US}} &= \lambda + \lambda_e e_t + \lambda_u u_t \\ \lambda_{t,\mathsf{UK}} &= \hat{\lambda} - \lambda_e \hat{e}_t - \lambda_u \hat{u}_t \end{aligned}$$

Substituting for $s^3_t - s^1_t$ and $\lambda_{t,z}$ into the approximation for order flow gives:

$$\xi_t \cong \frac{1}{4}(\lambda_e + 1)\nabla e_t + \frac{1}{4}(\lambda_u + 1)\nabla u_t.$$

Now consider the estimates of ∇e_t and ∇u_t conditioned on observing ξ_t. Using the fact that ∇e_t and ∇u_t are normally distributed, we have:

$$\begin{aligned} \mathsf{E}\left[\nabla e_t | \xi_t\right] &= \frac{(\lambda_e + 1)\sigma^2_e}{2\mathsf{V}\left(\xi_t\right)} \xi_t = \frac{4(\lambda_e + 1)\sigma^2_e}{(\lambda_e + 1)^2 \sigma^2_e + (\lambda_u + 1)^2 (1-\rho)\sigma^2_u} \xi_t \\ \mathsf{E}\left[\nabla u_t | \xi_t\right] &= \frac{(\lambda_u + 1)(1-\rho)\sigma^2_u}{2\mathsf{V}\left(\xi_t\right)} \xi_t = \frac{4(\lambda_u + 1)(1-\rho)\sigma^2_u}{(\lambda_e + 1)^2 \sigma^2_e + (\lambda_u + 1)^2 (1-\rho)\sigma^2_u} \xi_t \end{aligned}$$

Hence

$$\begin{aligned} s^3_t - s^1_t &= \frac{4(\lambda_e + 1)\sigma^2_e + 4(\lambda_u + 1)(1-\rho)\sigma^2_u}{(\lambda_e + 1)^2 \sigma^2_e + (\lambda_u + 1)^2 (1-\rho)\sigma^2_u} \xi_t \\ &= \psi \xi_t \end{aligned}$$

The last step is to find the values of λ_e and λ_u. From the first order conditions we have

$$\lambda_{t,\mathsf{US}} = \frac{1}{2} + \frac{\mathsf{E}^2_{t,\mathsf{US}}\left[s^3_t - s^1_t\right]}{\mathsf{V}^2_{t,\mathsf{US}}\left(s^3_t\right)}$$

Now,

$$\begin{aligned}\mathsf{E}^2_{t,\mathsf{US}}\left[s^3_t - s^1_t\right] &= \frac{\psi}{4}(\lambda_e+1)\mathsf{E}^2_{t,\mathsf{US}}\nabla e_t + \frac{\psi}{4}(\lambda_u+1)\mathsf{E}^2_{t,\mathsf{US}}\nabla u_t \\ & \frac{\psi}{4}(\lambda_e+1)e_t + (\lambda_u+1)(1-\rho)u_t\end{aligned}$$

and

$$\mathsf{V}^2_{t,\mathsf{US}}\left(s^3_t\right) = \left(\frac{\psi}{4}\right)^2\left((\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho^2)\sigma_u^2\right).$$

So putting this all together we get,

$$\begin{aligned}\lambda_{t,\mathsf{US}} &= \frac{1}{2} + \frac{4(\lambda_e+1)}{\psi\left[(\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho^2)\sigma_u^2\right]}e_t \\ &+ \frac{4(\lambda_u+1)(1-\rho)}{\psi\left[(\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho^2)\sigma_u^2\right]}u_t\end{aligned}$$

Similarly, in the case of UK consumers:

$$\begin{aligned}\lambda_{t,\mathsf{UK}} &= \frac{1}{2} + \frac{\mathsf{E}^2_{t,\mathsf{UK}}\left[s^3_t - s^1_t\right]}{\mathsf{V}^2_{t,\mathsf{UK}}\left(s^3_t\right)} \\ &= \frac{1}{2} - \frac{4(\lambda_e+1)}{\psi[(\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho^2)\sigma_u^2]}\hat{e}_t \\ &- \frac{4(\lambda_u+1)(1-\rho)}{\psi[(\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho^2)\sigma_u^2]}\hat{u}_t\end{aligned}$$

Thus, the optimal portfolio shares take the form we guessed above. We are now left to find the values of λ_e and λ_u that satisfy:

$$\lambda_e = \frac{(\lambda_e+1)\left((\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho)\sigma_u^2\right)}{((\lambda_e+1)\sigma_e^2 + (\lambda_u+1)(1-\rho)\sigma_u^2)\left((\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho^2)\sigma_u^2\right)}$$

and

$$\lambda_u = \frac{(\lambda_u+1)(1-\rho)\left((\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho)\sigma_u^2\right)}{((\lambda_e+1)\sigma_e^2 + (\lambda_u+1)(1-\rho)\sigma_u^2)\left((\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho^2)\sigma_u^2\right)}$$

Consider some simple cases: If $\sigma_u^2 = 0$, then $\lambda_e = 1/\sigma_e^2$ and $\psi = \frac{4}{(\lambda_e+1)}$ so

$$\begin{aligned}s^3_t - s^1_t &\cong \frac{\psi}{4}(\lambda_e+1)\nabla e_t \\ &= \nabla e_t\end{aligned}$$

and we get the same equation as in version 1. Similarly, if $\rho = 1$, then $\lambda_e = 1/\sigma_e^2$ and $\lambda_u = 0$, so $\psi = \frac{4}{(\lambda_e+1)}$ and

$$s^3_t - s^1_t = \nabla e_t.$$

In both cases, order flow provides enough information about fundamentals for all consumers to learn their value. (It should also be clear that $s_t^3 - s_t^1 = \nabla e_t + \nabla u_t$ if $\rho = 0$.)

It does not appear possible to solve for λ_e and λ_u when $1 > \rho > -1$ with $\rho \neq 0$, and $\sigma_u^2 > 0$. We can show however that

$$\lambda_e = \frac{1}{\sigma_e^2 + (1-\rho^2)\sigma_u^2}, \lambda_u = \frac{1-\rho}{\sigma_e^2 + (1-\rho^2)\sigma_u^2}$$

do not solve the equations above. Since these are the values for λ_e and λ_u implied by the first order conditions to the period-2 portfolio problem if the spot rate process were $s_t^3 - s_t^1 = \nabla e_t + \nabla u_t$, the equilibrium spot rate process must take the form of

$$s_t^3 - s_t^1 = (1-\pi_e)\nabla e_t + (1-\pi_u)\nabla u_t$$

where

$$\begin{aligned}
\pi_e &= \frac{(\lambda_u+1)(1-\rho)(\lambda_u-\lambda_e)\sigma_u^2}{(\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho)\sigma_u^2} \neq 0 \\
\pi_u &= \frac{(\lambda_e+1)(\lambda_e-\lambda_u)\sigma_e^2}{(\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho)\sigma_u^2} \neq 0
\end{aligned}$$

As an illustration, consider the case where $\sigma_u^2 = \sigma_e^2 = 1$ and $\rho = -0.1$. Solving the equations above we find that $\lambda_u = 0.56748$,and $\lambda_e = 0.490\,59$.[18] These values imply that

$$s_t^3 - s_t^1 = 0.973\,09\nabla e_t + 1.0233\nabla u_t$$

so $\pi_e > 0$ and $\pi_u < 0$. Alternative if $\rho = -0.9$, we find that

$$s_t^3 - s_t^1 = 0.24614\nabla e_t + 1.0896\nabla u_t.$$

Deriving the remain elements of the solution is straightforward. Recall that

$$s_{t+1}^1 - s_t^3 = E_{t+1}^1\nabla r_{t+1}^k + \frac{1}{1-\mu}E_{t+1}^1(\nabla k_t - E_t^3\nabla k_t)$$

We need to identify the common information set in $t+1:1$. In particular, we need to show that ∇e_t and ∇u_t

[18] The equations to be solved are now

$$\begin{aligned}
\lambda_e &= \frac{(\lambda_e+1)\left((\lambda_e+1)^2 + (\lambda_u+1)^2(1-\rho)\right)}{((\lambda_e+1)+(\lambda_u+1)(1-\rho))\left((\lambda_e+1)^2 + (\lambda_u+1)^2(1-\rho^2)\right)} \\
\lambda_u &= \frac{(\lambda_u+1)(1-\rho)\left((\lambda_e+1)^2 + (\lambda_u+1)^2(1-\rho)\right)}{((\lambda_e+1)+(\lambda_u+1)(1-\rho))\left((\lambda_e+1)^2 + (\lambda_u+1)^2(1-\rho^2)\right)}
\end{aligned}$$

With the solution in hand we find

$$\begin{aligned}
\psi &= \frac{4(\lambda_e+1) + 4(\lambda_u+1)(1-\rho)}{(\lambda_e+1)^2 + (\lambda_u+1)^2(1-\rho)} \\
s_t^3 - s_t^1 &= \frac{\psi}{4}(\lambda_e+1)\nabla e_t + \frac{\psi}{4}(\lambda_u+1)\nabla u_t
\end{aligned}$$

are common knowledge after the month $t+1$ announcement is made. To establish this we conjecture that the following information structures for US and consumers can be supported by inferences from announcements and order flows:

$$\begin{array}{ll}
\Omega^1_{t,\mathsf{US}} = \left\{e_t, u_t, \hat{e}_{t-1}, \hat{u}_{t-1}, \zeta_{t-1}, \varsigma_{t-1} \cup \Omega^4_{t-1,\mathsf{US}}\right\} & \Omega^1_{t,\mathsf{US}} = \left\{\hat{e}_t, \hat{u}_t, e_{t-1}, u_{t-1}, \zeta_{t-1}, \hat{\varsigma}_{t-1} \cup \Omega^4_{t-1,\mathsf{UK}}\right\} \\
\Omega^2_{t,\mathsf{US}} = \left\{s^1_t \cup \Omega^1_{t,\mathsf{US}}\right\} & \Omega^2_{t,\mathsf{UK}} = \left\{s^1_t \cup \Omega^1_{t,\mathsf{UK}}\right\} \\
\Omega^3_{t,\mathsf{US}} = \left\{\xi_t \cup \Omega^2_{t,\mathsf{US}}\right\} & \Omega^3_{t,\mathsf{UK}} = \left\{\xi_t \cup \Omega^2_{t,\mathsf{UK}}\right\} \\
\Omega^4_{t,\mathsf{US}} = \left\{s^3_t \cup \Omega^3_{t,\mathsf{US}}\right\} & \Omega^4_{t,\mathsf{UK}} = \left\{s^3_t \cup \Omega^3_{t,\mathsf{UKS}}\right\}
\end{array}$$

Together these information sets imply that common information evolves according to

$$\begin{aligned}
\Omega^1_t &= \left\{e_{t-1}, u_{t-1}, \hat{e}_{t-1}, \hat{u}_{t-1} \cup \Omega^4_{t-1}\right\} \\
\Omega^2_t &= \Omega^1_t \\
\Omega^3_t &= \left\{\xi_t \cup \Omega^2_t\right\} \\
\Omega^4_t &= \Omega^3_t
\end{aligned}$$

The key feature here is that US (UK) consumers learn the values of u_t and e_t ($\hat{u}_t$ and $\hat{e}_t$) after the announcement concerning the values of r^k_{t-1} and $\hat{r}^k_{t-1}$ is made in period 1 of month $t+1$. To see this is possible, rewrite the equations of period-2 order flow and the foreign return on capital as:

$$\begin{aligned}
\chi^3_{t,\mathsf{US}} &\equiv \xi_t - \frac{1}{4}(\lambda_e+1)e_t - \frac{1}{4}(\lambda_e+1)u_t = -\frac{1}{4}(\lambda_e+1)\hat{e}_t - \frac{1}{4}(\lambda_u+1)\hat{u}_t \\
\chi^1_{t,+1,\mathsf{US}} &\equiv \hat{r}^k_t - r + \theta(e_{t-1} - \hat{e}_{t-1}) = \hat{e}_t + \hat{u}_t.
\end{aligned}$$

$\chi^3_{t,\mathsf{US}}$ and $\chi^1_{t,+1,\mathsf{US}}$ provide two signals on the values of $\hat{e}_t$ and $\hat{u}_t$ that can be constructed from information available to US consumers after the announcement is made in month $t+1$ (i.e., $\left\{\chi^3_{t,\mathsf{US}}, \chi^1_{t,+1,\mathsf{US}}\right\} \in \Omega^1_{t+1,\mathsf{US}}$). Combining these equations we find that:

$$\begin{aligned}
\hat{e}_t &= \frac{4}{(\lambda_u - \lambda_e)}\chi^3_{t,\mathsf{US}} + \frac{(\lambda_u+1)}{(\lambda_u - \lambda_e)}\chi^1_{t,+1,\mathsf{US}} \\
\hat{u}_t &= \frac{(\lambda_e+1)}{(\lambda_e - \lambda_u)}\chi^1_{t,+1,\mathsf{US}} - \frac{4}{(\lambda_u - \lambda_e)}\chi^3_{t,\mathsf{US}}
\end{aligned}$$

Similarly, UK consumers can combing their observation on period-2 order flow with the information in the announcement in $t+1:1$ to precisely infer the values of e_t and u_t.

We can now identify the terms on the right hand side of the equation for $s^1_{t+1} - s^3_t$. The first term is straightforward:

$$\begin{aligned}
E^1_{t+1}\nabla r^k_{t+1} &= E^1_{t+1}\left[2\theta\nabla e_t + \nabla e_{t+1} + \nabla u_{t+1}\right] \\
&= 2\theta\nabla e_t
\end{aligned}$$

For the second we guess and verify that:

$$\nabla k_t - E^3_t\nabla k_t = \pi_e\nabla e_t + \pi_u\nabla u_t.$$

Since ∇e_t and ∇u_t are common knowledge by $t+1:1$, this guess implies that:

$$E^1_{t+1}(\nabla k_t - E^3_t \nabla k_t) = \pi_e \nabla e_t + \pi_u \nabla u_t.$$

Hence the dynamics for spot rates is given by:

$$\begin{aligned} s^1_{t+1} - s^3_t &= 2\theta \nabla e_t + \frac{\pi_e}{1-\mu}\nabla e_t + \frac{\pi_u}{1-\mu}\nabla u_t \\ &= \kappa_e \nabla e_t + \kappa_u \nabla u_t \end{aligned}$$

Now to verify that the guess for $\nabla k_t - E^3_t \nabla k_t$ is correct, we combine these dynamics with the equation for $s^3_{t+1} - s^1_{t+1}$, to give

$$\Delta s^3_{t+1} = (1-\pi_e)\nabla e_{t+1} + (1-\pi_u)\nabla u_{t+1} + 2\theta \nabla e_t + \frac{\pi_e}{1-\mu}\nabla e_t + \frac{\pi_u}{1-\mu}\nabla u_t.$$

Next we use the market-clearing dynamics of capital to write

$$\nabla k_{t+1} - s^3_{t+1} = \frac{1}{1-\mu}(\nabla k_{t-1} - s^3_t) + \nabla r^k_{t+1} - \Delta s^3_{t+1}$$

Substituting for capital returns and Δs^3_{t+1} using the equation above gives,

$$\nabla k_{t+1} - s^3_{t+1} = \frac{1}{1-\mu}(\nabla k_{t-1} - s^3_t) + \pi_e \nabla e_{t+1} + \pi_u \nabla u_{t+1} - \frac{\pi_e}{1-\mu}\nabla e_t - \frac{\pi_u}{1-\mu}\nabla u_t$$

which in turn implies that:

$$\nabla k_t - s^3_t = \pi_e \nabla e_t + \pi_u \nabla u_t$$

Finally, since $s^3_t = E^3_t \nabla k_t$ this equation confirms our guess about $\nabla k_t - E^3_t \nabla k_t$ above.

The only part of the solution that remains are the equations for US and UK interest rates. These rates are set in period-3 based on common information according to:

$$\begin{aligned} r_t &= E^3_t r^k_{t+1} = \theta E\left[\nabla e_t | \xi_t\right] \\ &= \theta \psi \xi_t \\ &= \theta \frac{\psi}{4}(\lambda_e + 1)\nabla e_t + \theta \frac{\psi}{4}(\lambda_u + 1)\nabla u_t \end{aligned}$$

In the case of UK rates

$$\begin{aligned} \hat{r}_t &= E^3_t \hat{r}^k_{t+1} = -\theta E\left[\nabla e_t | \xi_t\right] \\ &= -\theta \psi \xi_t \\ &= -\theta \frac{\psi}{4}(\lambda_e + 1)\nabla e_t - \theta \frac{\psi}{4}(\lambda_u + 1)\nabla u_t \end{aligned}$$

To close this section, note that if this model were to include dispersed information about productivity (or other fundamentals) in *future* months, then exchange rates would impound information about these future paths before their realization, leading to an even stronger result that at higher frequencies order flow would explain exchange-rate changes better than macro variables, whereas at lower frequencies macro variables would predominate (a consequence of order flow anticipating long-horizon macro paths that are, on average, realized).

5 Implications

In this section we study five important implications of our model: (1) departures of exchange rates from fundamentals, (2) exchange rate volatility, (3) responses to public announcements, (4) order flows versus portfolio flows, and (5) trading volume. We include the fourth of these because these two flow concepts differ in terms of how information is aggregated in trading. For trading volume, we examine implications of our model for the composition of FX trade. The huge volume of foreign exchange transactions compared to international real trade remains a significant puzzle, so it is natural to ask whether the presence of dispersed information casts new light on the issue.

5.1 Deviations from Fundamentals.

When there is common knowledge about the complete state of the economy by period 3, version 1 shows that the spot rate is given by $s_t^3 = \nabla k_t$. We can therefore think of ∇k_t as identifying common knowledge "fundamentals". In version 3, the spot rate differs from the level implied by common knowledge fundamental because there is incomplete information aggregation in period-2 trading. In particular, our solution for the equilibrium spot rate implies that:

$$s_t^3 = \nabla k_t - \pi_e \nabla e_t - \pi_u \nabla u_t.$$

It is also worth noting that this gap between the spot rate and “its fundamental level” affects the behavior of fundamentals. Returning once again to the market-clearing dynamics for capital we see that:

$$\begin{aligned}
\nabla k_{t+1} &= \nabla k_t + \nabla r_{t+1}^k - \frac{\mu}{1-\mu}\left(s_t^3 - \nabla k_t\right) \\
&= \nabla k_t + \nabla r_{t+1}^k + \frac{\mu}{1-\mu}\left(\pi_e \nabla e_t + \pi_u \nabla u_t\right) \\
&= \nabla k_t + \nabla u_{t+1} + \nabla e_{t+1} + 2\theta \nabla e_t + \frac{\mu}{1-\mu}\left(\pi_e \nabla e_t + \pi_u \nabla u_t\right) \\
&= \nabla k_t + \left(2\theta + \frac{\mu}{1-\mu}\pi_e\right)\nabla e_t + \frac{\mu}{1-\mu}\pi_u \nabla u_t + \nabla u_{t+1} + \nabla e_{t+1}
\end{aligned}$$

The economic intuition behind this result is straightforward. Deviations between last month’s spot rate and its fundamental level, affect the international distribution of wealth. This, in turn, affects exports in both the US and UK thereby influencing the rate of capital accumulation in both counties. In this way, past exchange rates affect the current level of “fundamentals”. Notice also, that the effects deviations are not transitory. Even though the value of fundamentals becomes common knowledge in version 2 with just a one month lag, the effects of a deviation on the level of fundamentals can persist indefinitely. Intuitively, although consumers learn about their past “consumption mistakes” resulting from spot quotes based on incomplete information, they never have the incentive to undo their effects going forward.

5.2 Volatility

In version 1, the volatility of monthly exchange rate changes is pinned down by the variance of the capital return differential:

$$\mathsf{V}(\Delta s^3_{t+1}) = \mathsf{V}(\nabla r^k_{t+1})$$

In version 2, the monthly change in the exchange rate is given by:

$$\Delta s^3_{t+1} = \nabla r^k_{t+1} + \frac{\pi_e}{1-\mu}\nabla e_t + \frac{\pi_u}{1-\mu}\nabla u_t - \pi_e \nabla e_{t+1} - \pi_u \nabla u_{t+1}$$

Thus:

$$\begin{aligned}
\mathsf{V}(\Delta s^3_{t+1}) - \mathsf{V}(\nabla r^k_{t+1}) &= \left(\frac{\pi_e}{1-\mu} + \pi_e\right)^2 2\sigma^2_e + \left(\frac{\pi_u}{1-\mu} + \pi_u\right)^2 2(1-\rho)\sigma^2_u \\
&\quad + 2\mathsf{CV}\left(2\theta\nabla e_t + \nabla e_{t+1} + u_{t+1}, \frac{\pi_e}{1-\mu}\nabla e_t + \frac{\pi_u}{1-\mu}\nabla u_t - \pi_e\nabla e_{t+1} - \pi_u\nabla u_{t+1}\right) \\
&= \left(\frac{\pi_e}{1-\mu} + \pi_e\right)^2 2\sigma^2_e + \left(\frac{\pi_u}{1-\mu} + \pi_u\right)^2 2(1-\rho)\sigma^2_u \\
&\quad + 4\pi_e\left(\frac{1+\mu}{1-\mu}\right)\sigma^2_e - 4\pi_u(1-\rho)\sigma^2_u
\end{aligned}$$

If $\pi_u < 0$, as our numerical examples suggest (with $\rho < 0$), then the right hand side of this equation is unambiguously positive. Under these circumstances, we get greater volatility in the month depreciation rate than we would see when the exchange rate is equal to common knowledge fundamentals. To see why this happens consider the exchange rate equation derived in the last set of notes:

$$\Delta s^3_{t+1} = E^3_{t+1}\nabla r^k_{t+1} + \frac{1}{1-\mu}E^3_{t+1}(\nabla k_t - E^3_t \nabla k_t)$$

Hence

$$\begin{aligned}
\mathsf{V}(\Delta s^3_{t+1}) - \mathsf{V}(\nabla r^k_{t+1}) &= \mathsf{V}(E^3_{t+1}\nabla r^k_{t+1}) - \mathsf{V}(\nabla r^k_{t+1}) \\
&\quad + \frac{1}{(1-\mu)^2}\mathsf{V}\left(E^3_{t+1}[\nabla k_t - E^3_t\nabla k_t]\right) \\
&\quad + \frac{2}{1-\mu}\mathsf{CV}\left(E^3_{t+1}\nabla r^k_{t+1}, E^3_{t+1}(\nabla k_t - E^3_t\nabla k_t)\right)
\end{aligned}$$

Now $\mathsf{V}(E^3_{t+1}\nabla r^k_{t+1}) - \mathsf{V}(\nabla r^k_{t+1}) < 0$ (from the definition of a variance), so the first term suggests that the lack of common knowledge should reduce volatility. But, as the equation shows, this argument over looks the effects of consumers learning about past states of the economy. In version 2, $E^3_{t+1}\nabla k_t = \nabla k_t$ so the terms on the second and third lines become:

$$+\frac{1}{(1-\mu)^2}\mathsf{V}\left(\nabla k_t - E^3_t\nabla k_t\right) + \frac{2}{1-\mu}\mathsf{CV}\left(E^3_{t+1}\nabla r^k_{t+1}, \nabla k_t - E^3_t\nabla k_t\right)$$

Clearly the first term is positive because it is proportion to the variance of forecast errors for fundamentals. The second term will also be positive when consumers use the information learned about the past fundamentals to estimate the current return on capital.

5.3 Response of spot rates to announcements

In version 1, announcements have no impact on spot rates because the information contained in the announcement is already aggregated into Ω_t^3 via period-2 trading. Thus we would have a situation where there was no contemporaneous correlation between the change in spot rates and announcements.

In version 2, there is a contemporaneous correlation because the announcement contains information that has not been previously aggregated. This information takes two forms: The first is information about the state of fundamentals last period (i.e. $\nabla k_t - \mathsf{E}_t^3 \nabla k_t$). The second is information about the persistent component of the capital return differential (i.e. $2\theta\nabla e_t$). Thus announcements can have contemporaneous affects on exchange rates even when they only contain information that is already dispersed across consumers in the economy. The reason why announcements appear to account for so little of the variance in spot rates empirically maybe that most the information they contain has already been aggregated up via trading between the time it was first learned by (some) consumers and the time of the announcement.

5.4 Portfolio Shifts

In our model, interdealer flows are the central flow concept in terms of facilitating information aggregation. At the same time, consumer portfolios are shifting over time, so it is worthwhile asking whether these consumer-level portfolio flows are also useful for understanding how dispersed information is aggregated. Since the answer to this question is quite subtle, we begin with a simple example.

Suppose a researcher has data on the asset positions of all consumers. As such, she can track aggregate holdings of dollar and pound deposits period-by-period, $B_t^j \equiv \int B_{t,z}^j dz$, and $\hat{B}_t^j \equiv \int \hat{B}_{t,z}^j dz$ for $j = \{1, ..., 4\}$. Would changes in B_t^j and/or $\hat{B}_t^j$ be correlated with exchange rate innovations arising from the aggregation of dispersed information? The answer is no. Changes in aggregate holdings are determined solely by asset supply via the requirement of market clearing and so are unrelated to the information transmission mechanism driving the exchange rate. This is readily apparent in our model because market clearing requires that $B_t^j = \hat{B}_t^j = 0$ every period.

In practice a researcher will not have access to data on all asset holdings in the economy so the issue becomes whether data on a subset of asset holdings can be usefully employed. To examine this we need to study how asset positions change at the micro level. Consider the change in a US consumer's holdings of pound deposits between periods 1 and 3:

$$\hat{B}_{t,\mathsf{US}}^3 - \hat{B}_{t,\mathsf{US}}^1 = \left(\lambda_{t,\mathsf{US}} \frac{W_{t:\mathsf{US}}^1}{S_t^1} - \hat{B}_{t,\mathsf{US}}^1 \right) - \left(T_{t,z^*}^2 - \mathsf{E}_{t:\mathsf{US}}^2 T_{t,z^*}^2 \right) \tag{33}$$

The first term on the right identifies the desired increase in the foreign asset position. Notice that this term depends on the private forecast of returns, $\mathsf{E}_{t:\mathsf{US}}^2 \left[s_t^3 - s_t^1 \right]$, via the optimal choice of $\lambda_{t,\mathsf{US}}$, and so may embody consumers private information $\Omega_{t,\mathsf{US}}^2$. The second term identifies the effects of unexpected incoming foreign exchange orders from other consumers. This term plays a central role in our model because it acts as the medium for the transmission of new information to the consumer. Thus, equation (33) depicts the change in asset position as a noisy signal of the unexpected order flow that carries information (the "noise" here is represented by the desired change in foreign asset position). With this perspective, it is clear that a change in asset holdings need not be associated with information aggregation. A consumer could want to change their foreign asset holdings even when there is no dispersed information in the economy. Under these circumstances, incoming orders can be perfectly predicted so the second term in (33) vanishes. And, as a result, there need not be any relation between the change in asset holdings and the exchange rate.

The relation between changes in asset holdings and exchange rates in the presence of dispersed information is more complex. In this case the change in asset holdings signal the arrival of new information to the consumer, but this need not imply that changes in the exchange rate and asset holdings are contemporaneously correlated. The reason is that information transmitted to each consumer via unexpected order flow only becomes embedded in the new exchange rate if it augments the common information set. This always happens in our model because news to US consumers in $T^{2}_{t,z^*} - \mathsf{E}^{2}_{t:\mathsf{US}} T^{2}_{t,z^*}$ is already known to UK consumers and vice versa. In general, however, there is no guarantee that the information received by each consumer during trade augments the common information set and so becomes immediately embedded in the exchange rate.

To summarize, the information aggregation that drives exchange rates here changes the **distribution** of asset holdings (across US and UK consumers). But going in reverse–i.e., inferring information from changes in that distribution–is difficult. At the subset-of-consumers level, changes in holdings may be informative, but only to the extent that the subset captures those distribution changes that are relevant. At the individual-consumer level, changes in holdings at are a quite noisy estimate of the information in trades, so even when information aggregation is taking place these individual changes in holdings will not be strongly correlated with exchange rate changes.

5.5 Trading Composition

Our model provides interesting perspectives on the determinants of currency trading. In particular, the model allows us to decompose order flows into three components: a transactions component related to the need to finance the purchase of foreign goods with foreign currency, a speculative component related to private information concerning the expected return on foreign currency, and a hedging component related to the expected arrival of currency orders from other consumers. These three components are readily identified by rearranging the definition of $\alpha_{t,z}$:

$$T^{4}_{t,z} = \hat{C}_{t,z} + \left(\alpha_{t,z} \frac{W^{4}_{t,z}}{S^{3}_{t}} - \hat{K}_{t,z} - \hat{B}^{3}_{t,z} \right) + \mathsf{E}^{4}_{t,z} T^{4}_{t,z} \tag{34}$$

The first term on the right shows the transactions component of period-4 foreign currency purchases. As one would expect, the effect is one-to-one. The second term identifies the desired increase in holdings of pound assets. The speculative demand for foreign assets contributes to this term via the choice of $\alpha_{t,z}$ which depends, in turn, on the expected excess return on bonds and domestic capital. The third term identifies the effect of expected currency orders from other dealers.

Equation (34) has two noteworthy implications in terms of the volume of currency trading. First, transactions in international goods and services ($\hat{C}_{t,z}$ in our model) may account for an empirically insignificant amount of FX trading even if there are no sizable shifts in desired portfolio holdings. Rather, trades may be driven almost exclusively by the expectation of incoming orders, $\mathsf{E}^4_{t,z}T^4_{t,z}$. Such a situation is analogous to "hot potato" trading; a phenomenon where risky inventories are passed between dealers in the process of wider risk sharing. In this model, consumers rationally anticipate incoming orders generated by unwanted inventories rather than simply waiting for their arrival.

The second implication of (34) for trading volume arises from the role played by dispersed information in determining the speculative demand for foreign assets. In model versions 1 and 2, information has been completely aggregated by period 3 so that spot rates and interest rates embody all available information about future capital returns. Under these circumstances $\alpha_{t,z}$ is a constant, so period-4 trade is not driven by changes in the speculative demand for foreign assets. Version 3 of the model is a more realistic, however, and in that version dispersed information still exists in period 3. As a result, spot and interest rates do not embody all the information some consumers know about future capital returns. Under these circumstances, $\alpha_{t,z}$ varies through time and across consumers as they speculate on the basis of their private information. Hence, dispersed information contributes to the variability of the speculative component, thereby contributing to trading volume.

6 Conclusion

This paper is certainly not the last word on bridging the gap between the new macro and microstructure approaches. Other structural assumptions can be made (e.g., allowing learning to extend over many "months"). Different questions can be addressed. With respect to exchange rates, it remains clear that new macro models need to find more traction in the data. At the same time, microstructure modeling needs a richer placement within the underlying real economy if it is to realize its potential in addressing macro phenomena. It is precisely this joint need that motivates us to write a paper like this, one which (we hope) helps establish a dialogue.

What have we learned? One broad lesson from GE modeling of currency trade is that the information problem faced by the foreign exchange market is more nuanced than suggested by past microstructure analysis. Even if individuals receive information via an exogenous process, the timing of when that information is impounded in price is endogenous because signals correspond to participants' equilibrium actions. The dynamics of the model create a constant tension between strong and semi-strong form efficiency. Finally, relative to microstructure models, the information structure of the GE model provides needed clarity on why transaction effects on exchange rates should persist, and, importantly, whether that persistence applies to real exchange rates or only to nominal rates.

A second broad lesson from GE modeling is that currency price discovery affects real decisions in ways not considered in either new macro or microstructure models. For example, our model clarifies the channels through which financial intermediation in currency markets affects consumption and intertemporal consumption hedging. As we show, innovations in agents' learning from currency-market activity are correlated with other things they care about (e.g., real output). Decision-making about real choices is conditioned on information that is generally less that the union of individuals' information sets, which naturally leads to effects on real allocations.

Specific results from the model include the following. First, the model produces exchange rate movements without public news. This is important empirically: analysis of macro announcements has never accounted for even 10 percent of total exchange rate variation. Second, order flow effects on exchange rates were shown to persist. If order flows are conveying dispersed information about permanent components of capital returns (or about any permanent component of fundamentals), then the exchange rate effects should be permanent. Third, the model provides a structural understanding of why order flow explains exchange rates at higher frequencies better than macro variables, whereas at lower frequencies macro variables predominate. Indeed, order flow is the proximate driver of exchange rate changes. Only when the macro variables that order flow is forecasting have been realized will macro variables themselves have traction in the data. Fourth, combining dispersed information and constant relative risk aversion leads to a form of informational inefficiency, which arises because incomplete information about the distribution of wealth is itself a source of noise.

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A Appendix

A.1 Optimization Problems

To derived the budget constraint in (3), we use the definitions of $\lambda_{t,z}$ and ξ_t together with the intraday dynamics of US and UK bonds to obtain

$$\begin{aligned} S_t^3\left(\hat{B}_t^3+\hat{K}_{t,z}\right) &= \tfrac{S_t^3}{S_t^1}(\lambda_{t,z}-\xi_t)W_{t,z}^2, \\ B_t^3 &= \left[1-(\lambda_{t,z}-\xi_t)\right]W_t^2-K_{t,z}. \end{aligned}$$

(Note that consumers only hold domestic capital so that $K_{t,z}=0$ for $z\geq 1/2$, and $\hat{K}_{t,z}=0$ for $z<1/2$.) Substituting these expressions into the definition of $W_{t,z}^4$, gives (3):

$$W_{t,z}^4=\left(1+\left(\frac{S_{t+1}^3}{S_t^1}-1\right)(\lambda_{t,z}-\xi_t)\right)W_{t,z}^2.$$

Let $\varsigma_t\equiv S_t^3(T_{t,z*}^4-\mathsf{E}_{t,z}^4T_{t,z*}^4)/W_{t,z}^4$, $\zeta_t\equiv\left(C_{t,z*}-\mathsf{E}_{t,z}^4C_{t,z*}\right)/W_{t,z}^4$ and $\hat{\zeta}_t\equiv\left(\hat{C}_{t,z*}-\mathsf{E}_{t,z}^4\hat{C}_{t,z*}\right)/W_{t,z}^4$ respectively denote unexpected order flow, US export demand, and UK export demand measured relative to period-4 wealth. Then using the definitions of $\alpha_{t,z}$, and $\gamma_{t,z}$ together with the overnight dynamics of bonds and capital for US consumers we obtain:

$$\begin{aligned} S_{t+1}\hat{B}_{t+1}^1 &= \frac{S_{t+1}^1\hat{R}_t}{S_t^3}(\alpha_{t,z}-\varsigma_t)W_t^4, \\ B_{t+1}^1 &= R_t\left[1-(\alpha_{t,z}-\varsigma_t)\right]W_{t,z}^4-R_t\left(C_{t,z}+S_t^3\hat{C}_{t,z}\right)-R_t(\gamma_{t,z}-\zeta_{t,z})W_{t,z}^4, \\ K_{t+1,z} &= R_{t+1}^k(\gamma_{t,z}-\zeta_{t,z})W_{t,z}^4. \end{aligned}$$

Substituting these expressions into the definition of $W_{t+1,z}^2$, gives the US version of (5):

$$W_{t+1,z}^2=R_t\left(1+\left(\frac{S_{t+1}^1\hat{R}_t}{S_t^3R_t}-1\right)(\alpha_{t,z}-\varsigma_t)+\left(\frac{R_{t+1}^k}{R_t}-1\right)(\gamma_{t,z}-\zeta_t)\right)W_{t,z}^4-R_t\left(C_{t,z}+S_t^3\hat{C}_{t,z}\right).$$

In the case of UK consumers, we have

$$\begin{aligned} S_{t+1}\hat{B}_{t+1}^1 &= \frac{S_{t+1}^1\hat{R}_t}{S_t^3}\left[(\alpha_{t,z}-\varsigma_t)-(\gamma_{t,z}-\hat{\zeta}_{t,z})\right]W_t^4, \\ B_{t+1}^1 &= R_t\left[1-(\alpha_{t,z}-\varsigma_t)\right]W_{t,z}^4-R_t\left(C_{t,z}+S_t^3\hat{C}_{t,z}\right), \\ S_t^3\hat{K}_{t+1,z} &= \hat{R}_{t+1}^k(\gamma_{t,z}-\zeta_{t,z})W_{t,z}^4. \end{aligned}$$

Substituting these expressions into the definition of $W_{t+1,z}^2$, gives

$$W_{t+1,z}^2=R_t\left(1+\left(\frac{S_{t+1}^1\hat{R}_t}{S_t^3R_t}-1\right)(\alpha_{t,z}-\varsigma_t)+\left(\frac{S_{t+1}^1\hat{R}_{t+1}^k}{S_t^3R_t}-\frac{S_{t+1}^1\hat{R}_t}{S_t^3R_t}\right)\left(\gamma_{t,z}-\hat{\zeta}_t\right)\right)W_{t,z}^4-R_t\left(C_{t,z}+S_t^3\hat{C}_{t,z}\right),$$

which is the UK version of (5).

The first order and envelope conditions from the period-2 optimization problem are

$$0 = \mathsf{E}^2_{t,z}\left[\mathcal{D}J^4_z\left(W^4_{t,z}\right)\left(\frac{S^3_t}{S^1_t}-1\right)\right], \tag{A1}$$

$$\mathcal{D}J^2_z(W^2_{t,z}) = \mathsf{E}^2_{t,z}\left[\mathcal{D}J^4_z(W^4_{t,z})H^3_t\right], \tag{A2}$$

where $\mathcal{D}J_z(.)$ denotes the derivative of $J_z(.)$. The first order conditions for $C_{t,z}$, $\hat{C}_{t,z}$, and $\lambda_{t,z}$ in the period-4 problem take the same form for US and UK consumers:

$$\lambda_{t,z} \quad : \quad 0 = \mathsf{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})\left(\frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t}-1\right)\right], \tag{A3}$$

$$C_{t,z} \quad : \quad U_c(\hat{C}_t, C_t) = R_t\beta\mathsf{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})\right], \tag{A4}$$

$$\hat{C}_{t,z} \quad : \quad U_{\hat{c}}(\hat{C}_t, C_t) = R_t\beta S^3_t\mathsf{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})\right]. \tag{A5}$$

The first order conditions for $\gamma_{t,z}$ differ:

$$\gamma_{t,z<1/2} \quad : \quad 0 = \mathsf{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})\left(\frac{R^k_{t+1}}{R_t}-1\right)\right], \tag{A6}$$

$$\gamma_{t,z\geq 1/2} \quad : \quad 0 = \mathsf{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})R_t\left(\frac{S^1_{t+1}\hat{R}^k_{t+1}}{S^3_t R_t}-\frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t}\right)\right] \tag{A7}$$

The envelope condition for US and UK consumers is

$$\mathcal{D}J^4_z(W^4_{t,z}) = \beta R_t\mathsf{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})H^1_{t+1,z}\right]. \tag{A8}$$

Equations (6) - (12) are obtained by combining (A1) - (A8) with $V_{t,z} \equiv \mathcal{D}J^4_z(W^4_{t,z})$.

A.2 Market Clearing Conditions

For any variable X, let $X_{t,\mathsf{US}}$ denote $X_{t,z}$ for $z < 1/2$, and $X_{t,\mathsf{UK}} = X_{t,z}$ for $z \geq 1/2$. Market clearing in US bonds in period 1 of day $t+1$ implies that

$$(B^3_{t,\mathsf{US}} + S^3_t T^4_{t,z*} - S^3_t T^4_{t,\mathsf{US}} + C_{t,\mathsf{UK}} - I_{t,\mathsf{US}}) + (B^3_{t,\mathsf{UK}} + S^3_t T^4_{t,z*} - S^3_t T^4_{t,\mathsf{UK}} - C_{t,\mathsf{UK}}) = 0.$$

With bond market clearing in period 3, this condition further simplifies to

$$S^3_t T^4_{t,z*} - S^3_t T^4_{t,\mathsf{US}} + S^3_t T^4_{t,z*} - S^3_t T^4_{t,\mathsf{UK}} - I_{t,\mathsf{US}} = 0.$$

Since market clearing in currency markets implies that $\int T^j_{t,z}dz = \int T^j_{t,z^*}dz^*$, this condition implies that $I_{t,\mathsf{US}} = 0$. Imposing this restriction on the overnight dynamics of US capital gives (15). Similarly, market clearing in the UK bond markets implies that

$$\begin{aligned}
0 &= (\hat{B}^1_{t,\mathsf{US}} + T^4_{t,\mathsf{US}} - T^4_{t,z*} - \hat{C}_{t,\mathsf{US}}) + (\hat{B}^1_{t,\mathsf{UK}} + T^4_{t,\mathsf{UK}} - T^4_{t,z*} + \hat{C}_{t,\mathsf{US}} - \hat{I}_{t,\mathsf{US}}) \\
&= T^4_{t,\mathsf{US}} - T^4_{t,z*} + T^4_{t,\mathsf{UK}} - T^4_{t,z*} - \hat{I}_{t,\mathsf{US}} \\
&= -\hat{I}_{t,\mathsf{US}}
\end{aligned}$$

Imposing $\hat{I}_{t,\mathsf{UK}} = 0$ on the overnight dynamics of UK capital gives (16).

A.3 Log Approximations

To approximate log portfolio returns we make use of a second order approximation similar to one employed by Campbell and Viceira (2002). Both $h^1_{t,z} \equiv \ln H^1_{t,z}$ and $h^3_{t,z} \equiv \ln H^3_{t,z}$ can be expressed as

$$h^j_{t,z} = \ln\left(1 + \left(e^x - 1\right)\left(a - u\right) + \left(e^y - 1\right)\left(b - w\right)\right)$$

where x, y, u and w are random variables that are zero in the steady state. Taking a second order Taylor series approximation to $h^j_{t,z}$ around this point gives

$$h^j_{t,z} \cong ax + by + \frac{1}{2}\left(a - a^2\right)x^2 + \frac{1}{2}(b - b^2)y^2 - abxy - xu - yw.$$

The final step is to replace x^2, y^2, xy, xu and yw by their respective moments:

$$h^j_{t,z} \cong ax + by + \frac{1}{2}\left(a - a^2\right)\mathsf{V}(x) + \frac{1}{2}(b - b^2)\mathsf{V}(y) - ab\mathsf{CV}\left(x, y\right) - \mathsf{CV}\left(x, u\right) - \mathsf{CV}\left(y, w\right)$$

Campbell and Viceira (2002) show that the approximation error associated with this expression disappears in the limit when x, y, u and w represent realizations of continuous time diffusion processes.

Applying this approximation to the definitions of $\ln H^1_{t+1,z}$ and $\ln H^3_{t,z}$ yields equations (20), (21) and (22). In deriving the solution of the model it is useful to write the latter two equations as:

$$h^1_{t+1,z} = \omega'_{t,z} x_{t+1,z} + \frac{1}{2}\omega'_{t,z}\Lambda_z - \frac{1}{2}\omega'_{t,z}\Sigma_z\omega_{t,z} - \phi_{t,z}, \tag{A9}$$

where $\Sigma_z \equiv \mathsf{V}^4_{t,z}\left(x_{t+1,z}\right)$, and $\Lambda_z \equiv \mathrm{diag}(\Sigma_z)$ with

$$\begin{aligned}
\omega'_{t,z} &\equiv \begin{bmatrix} \alpha_{t,z} & \gamma_{t,z} \end{bmatrix}, \\
x_{t+1,z} &\equiv \begin{bmatrix} s^1_{t+1} - s^3_t + \hat{r}_t - r_t & r^k_{t+1} - r_t \end{bmatrix}, \\
\phi_{t,z} &\equiv \mathsf{CV}^4_{t,\mathsf{US}}\left(s^1_{t+1}, \varsigma_{t,z}\right) + \mathsf{CV}^4_{t,\mathsf{US}}\left(r^k_{t+1}, \zeta_t\right),
\end{aligned}$$

for $z < 1/2$ (i.e. US consumers), and

$$\begin{aligned}
\omega'_{t,z} &\equiv \begin{bmatrix} \alpha_{t,z} - \gamma_{t,z} & \gamma_{t,z} \end{bmatrix}, \\
x_{t+1,z} &\equiv \begin{bmatrix} s^1_{t+1} - s^3_t + \hat{r}_t - r_t & \hat{r}^k_{t+1} + s^1_{t+1} - s^3_t - r_t \end{bmatrix}, \\
\phi_{t,z} &\equiv \mathsf{CV}^4_{t,z}\left(s^1_{t+1}, \varsigma_{t,z}\right) + \mathsf{CV}^4_{t,z}\left(\hat{r}^k_{t+1}, \hat{\zeta}_t\right),
\end{aligned}$$

for $z \geq 1/2$.

A.4 Marginal Utility of Wealth

To derive the relationship between the marginal utility of wealth and the marginal utility of consumption for US consumers, we first combine (A2)- (A4) and (A8):

$$\begin{aligned}
0 &= \mathsf{E}^4_{t,z}\left[V_{t+1,z} H^3_{t+1,z}\left(\frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t} - 1\right)\right] \\
0 &= \mathsf{E}^4_{t,z}\left[V_{t+1,z} H^3_{t+1,z}\left(\frac{R^k_{t+1}}{R_t} - 1\right)\right] \\
U_c(\hat{C}_{t,z}, C_{t,z}) &= \beta R_t \mathsf{E}^4_{t,z}\left[V_{t+1,z} H^3_{t+1,z}\right] \\
V_{t,z} &= \beta R_t \mathsf{E}^4_{t,z}\left[V_{t+1,z} H^3_{t+1,z} H^1_{t+1,z}\right]
\end{aligned}$$

Log linearizing these equations, with $U_c(\hat{C}_{t,z}, C_{t,z}) = \frac{1}{2} C^{-1}_{t,z}$ we find

$$\mathsf{E}^4_{t,z}\left[s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right] = -\mathsf{CV}^4_{t,z}\left(v_{t+1} + h^3_{t+1,z}, s^1_{t+1}\right) - \tfrac{1}{2}\mathsf{V}^4_{t,z}\left(s^1_{t+1}\right), \tag{A10}$$

$$\mathsf{E}^4_{t,z}\left[r^k_{t+1} - r_t\right] = -\mathsf{CV}^4_{t,z}\left(v_{t+1} + h^3_{t+1,z}, r^k_{t+1}\right) - \tfrac{1}{2}\mathsf{V}^4_{t,z}\left(r^k_{t+1}\right), \tag{A11}$$

$$c_t + \ln\beta + r_t = -\mathsf{E}^4_{t,z}\left[v_{t+1,z} + h^3_{t+1,z}\right] - \tfrac{1}{2}\mathsf{V}^4_{t,z}\left(v_{t+1,z} + h^3_{t+1,z}\right), \tag{A12}$$

$$v_{t,z} - \ln\beta - r_t = \mathsf{E}^4_{t,z}\left[v_{t+1,z} + h^3_{t+1,z} + h^1_{t+1}\right] + \tfrac{1}{2}\mathsf{V}^4_{t,z}\left(v_{t+1,z} + h^3_{t+1,z} + h^1_{t+1}\right). \tag{A13}$$

Stacking (A10) and (A11), and combining (A12) and (A13) and substituting for h^1_{t+1} gives

$$\mathsf{E}^4_{t,z}\left[x_{t+1,z}\right] + \frac{1}{2}\Lambda_z = -\mathsf{CV}^4_{t,z}\left(x_{t+1,z}, v_{t+1,z} + h^3_{t+1,z}\right), \tag{A14}$$

$$v_{t,z} + c_t + \phi_{t,z} = \omega'_{t,z}\mathsf{E}^4_{t,z}\left[x_{t+1,z}\right] + \frac{1}{2}\omega'_{t,z}\Lambda_z + \omega'_{t,z}\mathsf{CV}^4_{t,z}\left(x_{t+1,z}, v_{t+1,z} + h^3_{t+1,z}\right). \tag{A15}$$

Combining these expressions we obtain equation (23). In the case of UK consumers, we work with log linearized versions of (A2), (A3), (A5) and (A8):

$$\begin{aligned}
\mathsf{E}^4_{t,z}\left[s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right] &= -\mathsf{CV}^4_{t,z}\left(v_{t+1} + h^3_{t+1,z}, s^1_{t+1}\right) - \tfrac{1}{2}\mathsf{V}^4_{t,z}\left(s^1_{t+1}\right), \\
\mathsf{E}^4_{t,z}\left[\hat{r}^k_{t+1} + s^1_{t+1} - s^3_t - r_t\right] &= -\mathsf{CV}^4_{t,z}\left(v_{t+1} + h^3_{t+1,z}, r^k_{t+1}\right) - \tfrac{1}{2}\mathsf{V}^4_{t,z}\left(\hat{r}^k_{t+1} + s^1_{t+1}\right), \\
c_t + \ln\beta + r_t &= -\mathsf{E}^4_{t,z}\left[v_{t+1,z} + h^3_{t+1,z}\right] - \tfrac{1}{2}\mathsf{V}^4_{t,z}\left(v_{t+1,z} + h^3_{t+1,z}\right), \\
v_{t,z} - \ln\beta - r_t &= \mathsf{E}^4_{t,z}\left[v_{t+1,z} + h^3_{t+1,z} + h^1_{t+1}\right] + \tfrac{1}{2}\mathsf{V}^4_{t,z}\left(v_{t+1,z} + h^3_{t+1,z} + h^1_{t+1}\right).
\end{aligned}$$

Proceeding as before with our approximation for $h_{t+1,z}$ for $z \geq 1/2$, gives (A14) and (A15). Hence, equation (23) holds for UK consumers.

A.5 Dynamics of Capital

The dynamics of US capital can be written as

$$\frac{K_{t+1}}{K_t} = R^k_{t+1}\left(1 - \frac{C_{t,\mathsf{US}}W^4_{t,\mathsf{US}}}{W^4_{t,\mathsf{US}}K_t} - \frac{C_{t,\mathsf{UK}}W^4_{t,\mathsf{UK}}}{W^4_{t,\mathsf{UK}}K_t}\right).$$

Log linearizing the this equation gives

$$k_{t+1} - k_t \cong r^k_{t+1} + \ln(1-\mu) - \frac{\mu}{2(1-\mu)}\left(w^4_{t,\mathsf{US}} - k_t + \delta_{t,\mathsf{US}}\right) - \frac{\mu}{2(1-\mu)}\left(w^4_{t,\mathsf{UK}} - k_t + \delta_{t,\mathsf{UK}}\right).$$

Now bond market clearing implies that $K_t + S^3_t\hat{K}_t = W^4_{t,\mathsf{US}} + W^4_{t,\mathsf{UK}}$ so

$$w^4_{t,\mathsf{US}} - k_t = \ln\left(1 + \frac{S^3_t\hat{K}_t}{K_t} - \frac{W^4_{t,\mathsf{UK}}}{K_t}\right) \cong s^3_t + \hat{k}_t - k_t - (w^4_{t,\mathsf{UK}} - k_t).$$

Combining these equations gives (30). The approximate dynamics of UK capital in a similar manner. Bond market clearing implies that

$$\begin{aligned}\frac{\hat{K}_{t+1}}{\hat{K}_t} &= \hat{R}^k_{t+1}\left(1 - \frac{\hat{C}_{t,\mathsf{US}}W^4_{t,\mathsf{US}}}{W^4_{t,\mathsf{US}}\hat{K}_t} - \frac{\hat{C}_{t,\mathsf{UK}}W^4_{t,\mathsf{UK}}}{W^4_{t,\mathsf{UK}}\hat{K}_t}\right)\\ &= \hat{R}^k_{t+1}\left(1 - \frac{C_{t,\mathsf{US}}W^4_{t,us}}{W^4_{t,\mathsf{US}}S^3_t\hat{K}_t} - \frac{C_{t,\mathsf{UK}}W^4_{t,UK}}{W^4_{t,\mathsf{UK}}S^3_t\hat{K}^3_t}\right)\end{aligned}$$

where the second line follows from the fact that the first order conditions for consumption imply that $C_{t,z} = S^3_t\hat{C}_{t,z}$ for all z. Log linearizing this equation gives (31).

A.6 Quote Determination

The exchange rate dynamics derived in versions 1 and 2 assume that the quote setting rules

$$s^3_t = \mathsf{E}^3_t\nabla k_t \tag{A16}$$
$$s^1_t = \mathsf{E}^1_T\nabla k_t \tag{A17}$$

are consistent with market clearing given the consumption, investment and portfolio choices of consumers. To show that this is indeed the case we combine the capital accumulation dynamics implied by market clearing;

$$\begin{aligned}\Delta k_{t+1} &= \ln(1-\mu) + r^k_{t+1} - \frac{\mu}{2(1-\mu)}\left(s^3_t - \nabla k_t + \int \delta_{t,z} dz\right) \\ \Delta \hat{k}_{t+1} &= \ln(1-\mu) + \hat{r}^k_{t+1} - \frac{\mu}{2(1-\mu)}\left(\nabla k_t - s^3_t + \int \delta_{t,z} dz\right)\end{aligned}$$

to give

$$s^3_{t+1} - \nabla k_{t+1} = \frac{1}{1-\mu}\left(s^3_t - \nabla k_t\right) + \Delta s^3_{t+1} - \nabla r^k_{t+1} \tag{A18}$$

Next we take conditional expectations on both sides of this equation

$$\mathsf{E}^3_t\left[s^3_{t+1} - \nabla k_{t+1}\right] = \frac{1}{1-\mu}\left(s^3_t - \mathsf{E}^3_t \nabla k_t\right) + \mathsf{E}^3_t\left[\Delta s^3_{t+1} - \nabla r^k_{t+1}\right]$$

By iterated expectations, the left hand side of this equation is equal to $\mathsf{E}^3_t\left[s^3_{t+1} - \mathsf{E}^3_{t+1}\nabla k_{t+1}\right]$. Substituting this expression on the left and iterating forward gives

$$s^3_t = \mathsf{E}^3_t \nabla k_t - \mathsf{E}^3_t \sum_{i=1}^{\infty}(1-\mu)^i\left(\Delta s^3_{t+i} - \nabla r^k_{t+i}\right) \tag{A19}$$

This equation embodies the restrictions implied by market clearing on the determination of the period-3 spot rate. We now verify that the equation simplifies to $s^3_t = \mathsf{E}^3_t \nabla k_t$ in versions 1 and 2 of our model

In version 1, we assume that the initial distribution of capital is common knowledge (i.e. $\nabla k_0 \in \Omega^3_0$), and show that by induction that that $\nabla k_t \in \Omega^3_t$ for $t > 0$. If $\nabla k_t \in \Omega^3_t$, then $s^3_t = \mathsf{E}^3_t \nabla k_t = \nabla k_t$. With log utility $\delta_{t,z} = 0$, and the capital accumulation equations become

$$\begin{aligned}\Delta k_{t+1} &= \ln(1-\mu) + r^k_{t+1} \\ \Delta \hat{k}_{t+1} &= \ln(1-\mu) + \hat{r}^k_{t+1}\end{aligned}$$

Since r^k_{t+1} and $\hat{r}^k_{t+1}$ are common knowledge by $t+1:3$, these equations confirm that k_{t+1} and $\hat{k}_{t+1}$ are also common knowledge. Thus, we have shown that if $s^3_t = \mathsf{E}^3_t \nabla k_t$ with $\nabla k_t \in \Omega^3_t$, then fact that $\nabla r^k_{t+1} \in \Omega^3_{t+1}$ implies, via the implications of market clearing, that $\nabla k_{t+1} \in \Omega^3_{t+1}$. Next, we need to show that $s^3_t = \nabla k_t$ satisfies the market clearing condition in (A19). To do this, we note that the capital accumulation equations imply $\Delta k_{t+1} - \Delta \hat{k}_{t+1} = \nabla r^k_{t+1}$. Combining this expression with the identity $\nabla k_{t+1} - \nabla k_t \equiv \Delta k_{t+1} - \Delta \hat{k}_{t+1}$ and the proposed quote equation $s^3_t = \nabla k_t$, gives $s^3_{t+1} - s^3_t = \nabla r^k_{t+1}$. Combining this expression with (A19) gives $s^3_t = \mathsf{E}^3_t \nabla k_t$. We have therefore established that the quote equation (A16) is indeed consistent with market clearing given the information available to consumers in version 1.

In version 2, $\nabla k_t \notin \Omega^3_t$ so period-3 quotes are set as $s^3_t = \mathsf{E}^3_t \nabla k_t$. For this to be consistent with market clearing we need to show that $\mathsf{E}^3_t\left(\Delta s^3_{t+i} - \nabla r^k_{t+i}\right) = 0$ for $i > 0$. Consider the $i = 1$ case. From the proposed solution for spot rates we have

$$\begin{aligned}
\mathsf{E}_t^3[\Delta s_{t+1}^3 - \nabla r_{t+1}^k] &= \mathsf{E}_t^3\left[\frac{\pi_e}{1-\mu}\nabla e_t + \frac{\pi_u}{1-\mu}\nabla u_t - \pi_e \nabla e_{t+1} - \pi_u \nabla u_{t+1}\right] \\
&= \mathsf{E}_t^3\left[\frac{\pi_e}{1-\mu}\nabla e_t + \frac{\pi_u}{1-\mu}\nabla u_t\right] \\
&= \frac{1}{1-\mu}\mathsf{E}\left[\pi_e \nabla e_t + \pi_u \nabla u_t | \xi_t\right] \\
&= \frac{1}{1-\mu}\left(\frac{4(\lambda_u+1)(1-\rho)(\lambda_u-\lambda_e)4(\lambda_e+1)4\sigma_u^2\sigma_e^2}{\left[(\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho)\sigma_u^2\right]^2}\right)\xi_t \\
&\quad + \frac{1}{1-\mu}\left(\frac{4(\lambda_u+1)(1-\rho)(\lambda_e-\lambda_u)4(\lambda_e+1)4\sigma_u^2\sigma_e^2}{\left[(\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho)\sigma_u^2\right]^2}\right)\xi_t \\
&= 0
\end{aligned}$$

where we've made use of the definitions of π_e and π_u and the fact that

$$\begin{aligned}
\mathsf{E}\left[\nabla e_t | \xi_t\right] &= \frac{4(\lambda_e+1)\sigma_e^2}{(\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho)\sigma_u^2}\xi_t \\
\mathsf{E}\left[\nabla u_t | \xi_t\right] &= \frac{4(\lambda_u+1)(1-\rho)\sigma_u^2}{(\lambda_e+1)^2\sigma_e^2 + (\lambda_u+1)^2(1-\rho)\sigma_u^2}\xi_t
\end{aligned}$$

Finally, note that $\mathsf{E}_t^3\left(\Delta s_{t+i}^3 - \nabla r_{t+i}^k\right) = \mathsf{E}_t^3\left(\mathsf{E}_{t+i-1}^3 \Delta s_{t+i}^3 - \nabla r_{t+i}^k\right)$ by iterated expectations, so the result above establishes that $\mathsf{E}_t^3\left(\Delta s_{t+i}^3 - \nabla r_{t+i}^k\right) = 0$ for $i > 0$.