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The international diversification puzzle is not as bad as you think[1]

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Abstract

In simple one-good international macro models, the presence of non-diversifiable labor income risk means that country portfolios should be heavily biased toward foreign assets. The fact that the opposite pattern of diversification is observed empirically constitutes the international diversification puzzle. We embed a portfolio choice decision in a frictionless two-country, two-good version of the stochastic growth model. In this environment, which is a workhorse for international business cycle research, we derive a closed-form expression for the equilibrium shares of domestic and foreign assets in a country's portfolio. Equilibrium portfolios are biased towards domestic assets, as in the data. We show that home bias arises because endogenous international relative price fluctuations make domestic stocks a good hedge against non-diversifiable labor income risk. We then use the expression for equilibrium portfolios to link openness to trade to the level of diversification, and find that the theory offers a quantitatively compelling account for the patterns of international diversification observed across developed economies in recent years.

KEYWORDS: Home bias, international diversification

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1 Introduction

Although there has been rapid growth in international portfolio diversification in recent years, portfolios remain heavily biased towards domestic assets. For example, foreign assets accounted, on average, for only around 25% of the total value of the assets owned by U.S. residents over the period 1990-2004. There is a large theoretical literature that explores whether observed low diversification should be interpreted as evidence of incomplete insurance against country-specific risk (see, for example, Baxter and Jermann, 1997, and Lewis, 1999). These papers share a common conclusion: relative to the prediction of frictionless models, too little diversification is observed in the data. In response, recent theoretical work on diversification has focused on introducing frictions that can rationalize observed portfolios. The set of candidate frictions is long and includes proportional or fixed costs on foreign equity holdings (Lewis, 1996; Amadi and Bergin, 2006; Coeurdacier and Guibaud, 2006), costs in goods trade (Uppal, 1993; Obstfeld and Rogoff, 2000; Couerdacier, 2006), liquidity or short sales constraints (Michaelides, 2003; Julliard, 2004), price stickiness in product markets (Engel and Matsumoto, 2006), weak investor rights concentrating ownership among insiders (Kho et. al., 2006), non-tradability of nontraded-good equities (Tesar, 1993; Pesenti and van Wincoop, 2002; Hnatkovska, 2005) and asymmetric information in financial markets (Gehrig, 1993; Jeske, 2001; Hatchondo, 2005; and van Nieuwerburgh and Veldkamp, 2007).
In this paper, we take a different approach. We develop a frictionless model in which perfect risk sharing is in fact wholly consistent with relatively low levels of international diversification. We argue that previous theoretical benchmarks delivered the wrong answers because the models were too simple to capture the key diversification motives associated with country-specific business cycle risks. Our environment is the two-country extension of the stochastic growth model developed by Backus, Kehoe and Kydland (1992 and 1995, henceforth BKK), which is a workhorse model for quantitative international macroeconomics. While BKK allow for a complete set of Arrow securities to be traded between countries, we instead follow the tradition in the international diversification literature and assume that households only trade shares in domestic and foreign firms. BKK and others have shown that the international stochastic growth model is broadly consistent with a large set of international business cycle facts. We show that the same model rationalizes observed levels of international diversification. Since our model is frictionless, this finding casts doubt on the quantitative role of frictions in understanding observed portfolios.

One contribution of our paper is to show that given particular assumptions on preferences and technologies, equilibrium portfolio choices can be characterized analytically.[2] In this case, the equilibrium portfolio choice depends on only two parameters: (i) the relative preference in consumption for domestically-produced versus imported goods, and (ii) capital's share in production. When these parameters are set to the values used by BKK, our expression implies portfolios comprising 80% domestic stocks and 20% foreign stocks. Moreover, this portfolio perfectly insures consumers against country-specific productivity shocks. We conclude that observed low levels of diversification should not be interpreted as indicating a low degree of international risk sharing.

To better understand the predictions of our model for portfolio choice we compare and contrast our economy to those considered by Lucas (1982), Baxter and Jermann (1997), and Cole and Obstfeld (1991). Lucas (1982) points out that in a symmetric one-good two-country model, perfect risk pooling involves agents of each country owning half the claims to the home endowment and half the claims to the foreign endowment. Baxter and Jermann (1997) extend Lucas' model in one direction by introducing production while retaining the single-good assumption. They show that if returns to capital and labor are highly correlated within a country, then agents can compensate for non-diversifiable labor income risk by aggressively diversifying asset holdings. In their examples, fully diversified portfolios typically involve substantial short positions in domestic assets.

Cole and Obstfeld (1991) extend Lucas' analysis in a different direction. They retain the focus on an endowment economy, but assume that the two countries receive endowments of different goods that are imperfect substitutes. These goods are then traded, and agents consume bundles comprising both goods. They show that changes in relative endowments induce off-setting changes in the terms of trade. When preferences are log-separable between the two goods, the terms of trade responds one-for-one to changes in relative income, effectively delivering perfect risk-sharing. Thus, in sharp contrast to the results of Lucas or Baxter and Jermann, any level of diversification is consistent with complete risk-pooling, including portfolio autarky.[3]

One important difference in our analysis relative to Baxter and Jermann (1997) is that we allow for imperfect substitutability between domestic and foreign-produced traded goods, following Cole and Obstfeld (1991). Thus, in our model, changes in international relative prices provide some insurance against country-specific shocks and, in the flavor of the Cole and Obstfeld indeterminacy

[2]The assumptions required to derive an analytical expression for the portfolio choice are (i) preferences are separable between consumption and leisure and logarithmic in consumption, and (ii) all production technologies are Cobb-Douglas, which implies a unitary elasticity of substitution between traded goods.

[3]Kollmann (2006) considers a two-good endowment economy with more general preferences. He finds that equilibrium diversification is sensitive to both the intra-temporal elasticity of substitution between traded goods, and the inter-temporal elasticity of substitution for the aggregate consumption bundle.

result, portfolio choice does not have to do all the heavy-lifting when it comes to delivering perfect risk-sharing. In contrast to Cole and Obstfeld, however, the presence of production and particularly investment in our model means that returns to domestic and foreign stocks are not automatically equated, and thus agents face an interesting portfolio choice problem. Home bias arises because relative returns to domestic stocks move inversely with relative labor income in response to productivity shocks. This pattern can be traced to the role of international relative price movements, and accounts for the difference between the portfolio predictions of our model relative to those of one-good models explored in previous work.

We conduct a sensitivity analysis in which we consider the implications for diversification of varying two key parameters: the elasticity of substitution between domestic and foreign-produced goods, and the inter-temporal elasticity of substitution for the composite consumption good. We show that home bias is a robust prediction of the model for all plausible values for these parameters. We also extend the model to introduce preference shocks as a second source of risk, and one that induces very different relative price dynamics in response to a shock. Our low equilibrium diversification result also survives here, while the model with both productivity and preference shocks delivers a realistically low unconditional equilibrium correlation between relative consumption and the real exchange rate.

The closed-form expression we derive for equilibrium portfolios makes it straightforward to test the model by considering the extent to which differences in trade shares in a cross-section of countries predict differences in levels of diversification. We find that the theoretical relationship is both qualitatively and quantitatively consistent with the empirical pattern for relatively high-income economies. In particular, the ratio of trade to GDP is a powerful predictor of observed international diversification, and can explain around a third of the cross-country variation in portfolio diversification. The theory is also consistent with changes in diversification over time: countries that have expanded trade relatively rapidly have tended to see the largest increases in diversification.

In the next section we describe the model and state our main result. Section 3 offers some intuition for equilibrium portfolios. Section 4 discusses some extensions of the basic model. Section 5 contains the empirical analysis. Section 6 concludes. Proofs, details about numerical methods, and a description of the data are in the Appendix.

2 The Model

The modeling framework is the one developed by Backus, Kehoe and Kydland (1995). There are two countries, each of which is populated by the same measure of identical, infinitely-lived households. Firms in each country use country-specific capital and labor to produce an intermediate good. The intermediate good produced in the domestic country is labeled a, while the good produced in the foreign country is labeled b. These are the only traded goods in the world economy. Intermediate-goods-producing firms are subject to country-specific productivity shocks. Within each country the intermediate goods a and b are combined to produce country-specific final consumption and investment goods. The final goods production technologies are asymmetric across countries, in that they are biased towards using a larger fraction of the locally-produced intermediate good. This bias allows the model to replicate empirical measures for the volume of trade relative to GDP.

We assume that the assets that are traded internationally are shares in the domestic and foreign representative intermediate-goods-producing firms. These firms make investment and employment decisions, and distribute any non-reinvested earnings to shareholders.

2.1 Preferences and technologies

In each period t the economy experiences one event $s_t \in S$. We denote by $s^t = (s_0, s_1, ..., s_t) \in S^t$ the history of events from date 0 to date t. The probability at date 0 of any particular history s^t is given by $\pi(s^t)$.

Period utility for a household in the domestic country after history s^t is given by[4]

$$(1) \quad U\left(c(s^t), n(s^t)\right) = \ln c(s^t) - V\left(n(s^t)\right)$$

where $c(s^t)$ denotes consumption at date t given history s^t, and $n(s^t)$ denotes labor supply. Disutility from labor is given by the positive, increasing and convex function $V(.)$. The assumption that utility is log-separable in consumption will play a role in deriving a closed-form expression for equilibrium portfolios in our baseline calibration of the model. In contrast, the equilibrium portfolio in this case will not depend on the particular functional form for $V(.)$.

Households supply labor to domestically located perfectly-competitive intermediate-goods-producing firms. Intermediate goods firms in the domestic country produce good a, while those in the foreign

[4] The equations describing the foreign country are largely identical to those for the domestic country. We use star superscripts to denote foreign variables.

country produce good b. These firms hold the capital in the economy and operate a Cobb-Douglas production technology:

$$(2) \quad F\left(z(s^t), k(s^{t-1}), n(s^t)\right) = e^{z(s^t)} k(s^{t-1})^{\theta} n(s^t)^{1-\theta},$$

where $z(s^t)$ is an exogenous productivity shock. The vector of shocks $[z(s^t), z^*(s^t)]$ evolves stochastically. For now, the only assumption we make about this process is that it is symmetric. In the baseline version of the model, productivity shocks are the only source of uncertainty.

Each period, households receive dividends from their stock holdings in the domestic and foreign intermediate-goods firms, and buy and sell shares to adjust their portfolios. After completing asset trade, households sell their holdings of intermediate goods to domestically located final-goods-producing firms. These firms are perfectly competitive and produce final goods using intermediate goods a and b as inputs to a Cobb-Douglas technology:

$$(3) \quad G\left(a(s^t), b(s^t)\right) = a(s^t)^{\omega} b(s^t)^{(1-\omega)}, \qquad G^*\left(a^*(s^t), b^*(s^t)\right) = a^*(s^t)^{(1-\omega)} b^*(s^t)^{\omega},$$

where $\omega > 0.5$ determines the size of the local input bias in the composition of domestically produced final goods.

Note that the Cobb-Douglas assumption implies a unitary elasticity of substitution between domestically-produced goods and imports. The Cobb-Douglas assumption, in conjunction with the assumption that utility is logarithmic in consumption, will allow us to derive a closed-form expression for equilibrium portfolios. Note, however, that a unitary elasticity is within the range of existing estimates: BKK (1995) set this elasticity to 1.5 in their benchmark calibration, while Heathcote and Perri (2002) estimate the elasticity to be 0.9. In a sensitivity analysis we will explore numerically the implications of deviating from the logarithmic utility, unitary elasticity baseline.

We now define two relative prices that will be useful in the subsequent analysis. Let $t(s^t)$ denote the terms of trade, defined as the price of good b relative to good a. Because the law of one price applies to traded intermediate goods, this relative price is the same in both countries:

$$(4) \quad t(s^t) = \frac{q_b(s^t)}{q_a(s^t)} = \frac{q_b^*(s^t)}{q_a^*(s^t)}$$

Let $e(s^t)$ denote the real exchange rate, defined as the price of foreign relative to domestic consumption. By the law of one price, $e(s^t)$ can be expressed as the foreign price of good a (or good b)

relative to foreign consumption divided by the domestic price of good a (or b) relative to domestic consumption:

$$(5) \quad e(s^t) = \frac{q_a(s^t)}{q_a^*(s^t)} = \frac{q_b(s^t)}{q_b^*(s^t)}$$

2.2 Households' problem

The budget constraint for the domestic household is given by

$$(6) \quad \begin{aligned} & c(s^t) + P(s^t)\left(\lambda_H(s^t) - \lambda_H(s^{t-1})\right) + e(s^t)P^*(s^t)\left(\lambda_F(s^t) - \lambda_F(s^{t-1})\right) \\ = \;& q_a(s^t)w(s^t)n(s^t) + \lambda_H(s^{t-1})d(s^t) + \lambda_F(s^{t-1})e(s^t)d^*(s^t) \qquad \forall t \geq 0, s^t \end{aligned}$$

Here $P(s^t)$ is the price at s^t of (ex dividend) shares in the domestic firm in units of domestic consumption, $P^*(s^t)$ is the price of shares in the foreign firm in units of foreign consumption, $\lambda_H(s^t)$ ($\lambda_H^*(s^t)$) denotes the fraction of the domestic firm purchased by the domestic (foreign) agent, $\lambda_F(s^t)$ ($\lambda_F^*(s^t)$) denotes the fraction of the foreign firm bought by the domestic (foreign) agent, $d(s^t)$ and $d^*(s^t)$ denote domestic and foreign dividend payments per share, and $w(s^t)$ denotes the domestic wage in units of the domestically-produced intermediate good. The budget constraint for the foreign household is

$$(7) \quad \begin{aligned} & c^*(s^t) + P^*(s^t)\left(\lambda_F^*(s^t) - \lambda_F^*(s^{t-1})\right) + (1/e(s^t))P(s^t)\left(\lambda_H^*(s^t) - \lambda_H^*(s^{t-1})\right) \\ = \;& q_b^*(s^t)w^*(s^t)n^*(s^t) + \lambda_F^*(s^{t-1})d^*(s^t) + \lambda_H^*(s^{t-1})(1/e(s^t))d(s^t) \qquad \forall t \geq 0, s^t \end{aligned}$$

We assume that at the start of period 0, the domestic (foreign) household owns the entire domestic (foreign) firm: thus $\lambda_H(s^{-1}) = 1$, $\lambda_F(s^{-1}) = 0$, $\lambda_F^*(s^{-1}) = 1$ and $\lambda_H^*(s^{-1}) = 0$.

At date 0, domestic households choose $\lambda_H(s^t)$, $\lambda_F(s^t)$, $c(s^t) \geq 0$ and $n(s^t) \in [0, 1]$ for all s^t and for all $t \geq 0$ to maximize

$$(8) \quad \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t)\beta^t U\left(c(s^t), n(s^t)\right)$$

subject to (6).

The domestic households' first-order condition for domestic and foreign stock purchases are, respectively,

$$
\begin{aligned}
(9) \qquad U_c(s^t)P(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)U_c(s^t,s_{t+1})\left[d(s^t,s_{t+1})+P(s^t,s_{t+1})\right] \\
U_c(s^t)e(s^t)P^*(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)U_c(s^t,s_{t+1})e(s^t,s_{t+1})\left[d^*(s^t,s_{t+1})+P^*(s^t,s_{t+1})\right]
\end{aligned}
$$

where we use $U_c(s^t)$ for $\frac{\partial U(c(s^t),n(s^t))}{\partial c(s^t)}$ and (s^t, s_{t+1}) denotes the $t+1$ length history s^t followed by s_{t+1}

The domestic household's first-order condition for hours is

$$
\begin{aligned}
(10) \quad U_c(s^t)q_a(s^t)w(s^t)+U_n(s^t) &\geq 0 \\
&= \text{ if } n(s^t)>0
\end{aligned}
$$

Analogously, the foreign households' first-order condition for domestic and foreign stock purchases and hours are, respectively,

$$
\begin{aligned}
(11) \quad U_c^*(s^t)\frac{P(s^t)}{e(s^t)} &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)U_c^*(s^t,s_{t+1})\left[\frac{d\left(s^t,s_{t+1}\right)+P(s^t,s_{t+1})}{e\left(s^t,s_{t+1}\right)}\right] \\
U_c^*(s^t)P^*(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)U_c^*(s^t,s_{t+1})\left[d^*(s^t,s_{t+1})+P^*(s^t,s_{t+1})\right]
\end{aligned}
$$

and

$$
\begin{aligned}
(12) \quad U_c^*(s^t)q_b^*(s^t)w^*(s^t)+U_n^*(s^t) &\geq 0 \\
&= \text{ if } n^*(s^t)>0.
\end{aligned}
$$

2.3 Intermediate firms' problem

The domestic intermediate-goods firm's maximization problem is

$$
\max_{k(s^t),n(s^t)} \sum_{t=0}^{\infty}\sum_{s^t} Q(s^t)d(s^t)
$$

subject to $k(s^t)$, $n(s^t) \geq 0$, taking as given $k(s^{-1})$, where $Q(s^t)$ is the price the firm uses to value dividends at s^t relative to consumption at date 0, and dividends (in units of the final good) are given by

$$(13) \quad d(s^t) = q_a(s^t) \left[F\left(z(s^t), k(s^{t-1}), n(s^t)\right) - w(s^t)n(s^t)\right] - \left[k(s^t) - (1-\delta)k(s^{t-1})\right].$$

In this expression δ is the depreciation rate for capital. Analogously, foreign firms use prices $Q^*(s^t)$ to price dividends in state s^t, where foreign dividends are given by

$$(14) \quad d^*(s^t) = q_b^*(s^t) \left[F\left(z^*(s^t), k^*(s^{t-1}), n^*(s^t)\right) - w^*(s^t)n^*(s^t)\right] - \left[k^*(s^t) - (1-\delta)k^*(s^{t-1})\right].$$

The domestic and foreign firms' first order conditions for $n(s^t)$ and $n^*(s^t)$ are

$$(15) \quad w(s^t) = (1-\theta)F\left(z(s^t), k(s^{t-1}), n(s^t)\right)/n(s^t)$$

$$(16) \quad w^*(s^t) = (1-\theta)F\left(z^*(s^t), k^*(s^{t-1}), n^*(s^t)\right)/n^*(s^t).$$

The corresponding first order conditions for $k(s^t)$ and $k^*(s^t)$ are

$$(17) \quad Q(s^t) = \sum_{s_{t+1} \in S} Q(s^t, s_{t+1}) \left[q_a(s^t, s_{t+1})\theta F\left(z(s^t, s_{t+1}), k(s^t), n(s^t, s_{t+1})\right)/k(s^t) + (1-\delta)\right]$$

$$(18) \quad Q^*(s^t) = \sum_{s_{t+1} \in S} Q^*(s^t, s_{t+1}) \left[q_b^*(s^t, s_{t+1})\theta F\left(z^*(s^t, s_{t+1}), k^*(s^t), n^*(s^t, s_{t+1})\right)/k^*(s^t) + (1-\delta)\right]$$

The state-contingent consumption prices $Q(s^t)$ and $Q^*(s^t)$ obviously play a role in intermediate–goods firms' state-contingent decisions regarding how to divide earnings between investment and dividend payments. We assume that domestic firms use the discount factor of the representative domestic household to price the marginal cost of foregoing current dividends in favor of extra investment.[5] Thus

$$(19) \quad Q(s^t) = \frac{\pi(s^t)\beta^t U_c(s^t)}{U_c(s^0)}, \qquad Q^*(s^t) = \frac{\pi(s^t)\beta^t U_c^*(s^t)}{U_c^*(s^0)}.$$

[5]Note that each agent takes $Q(s^t)$ as given, understanding that their individual atomistic portfolio choices will not affect aggregate investment decisions. In Heathcote and Perri (2004) we experiment with alternative discount factors.

2.4 Final goods firms' problem

The final goods firm's static maximization problem in the domestic country after history s^t is

$$\max_{a(s^t),b(s^t)} \left\{G(a(s^t), b(s^t)) - q_a(s^t)a(s^t) - q_b(s^t)b(s^t)\right\}$$

subject to $a(s^t)$, $b(s^t) \geq 0$.

The first order conditions for domestic and foreign firms may be written as

$$\begin{array}{lcc} (20) & q_a(s^t) = \omega G(a(s^t), b(s^t))/a(s^t), & q_b(s^t) = (1-\omega)G(a(s^t), b(s^t))/b(s^t), \\ & q_b^*(s^t) = \omega G^*\left(a^*(s^t), b^*(s^t)\right)/b^*(s^t), & q_a^*(s^t) = (1-\omega)G^*\left(a^*(s^t), b^*(s^t)\right)/a^*(s^t). \end{array}$$

2.5 Definition of equilibrium

An equilibrium is a set of quantities $c(s^t)$, $c^*(s^t)$, $k(s^t)$, $k^*(s^t)$, $n(s^t)$, $n^*(s^t)$, $a(s^t)$, $a^*(s^t)$, $b(s^t)$, $b^*(s^t)$, $\lambda_H(s^t)$, $\lambda_H^*(s^t)$, $\lambda_F(s^t)$, $\lambda_F^*(s^t)$, prices $P(s^t)$, $P^*(s^t)$, $r(s^t)$, $r^*(s^t)$, $w(s^t)$, $w^*(s^t)$, $Q(s^t)$, $Q^*(s^t)$, $q_a(s^t)$, $q_a^*(s^t)$, $q_b(s^t)$, $q_b^*(s^t)$, productivity shocks $z(s^t)$, $z^*(s^t)$ and probabilities $\pi(s^t)$ for all s^t and for all $t \geq 0$ which satisfy the following conditions:

1. The first order conditions for intermediate-goods purchases by final-goods firms (equation 20)
2. The first-order conditions for labor demand by intermediate-goods firms (equations 15 and 16)
3. The first-order conditions for labor supply by households (equations 10 and 12)
4. The first-order conditions for capital accumulation (equations 17 and 18),
5. The market clearing conditions for intermediate goods a and b :

$$\begin{array}{lrcl} (21) & a(s^t) + a^*(s^t) & = & F\left(z(s^t), k(s^{t-1}), n(s^t)\right) \\ & b(s^t) + b^*(s^t) & = & F\left(z^*(s^t), k^*(s^{t-1}), n^*(s^t)\right). \end{array}$$

6. The market-clearing conditions for final goods:

$$\begin{array}{lrcl} (22) & c(s^t) + k(s^t) - (1-\delta)k(s^{t-1}) & = & G\left(a(s^t), b(s^t)\right) \\ & c^*(s^t) + k^*(s^t) - (1-\delta)k^*(s^{t-1}) & = & G^*\left(a^*(s^t), b^*(s^t)\right). \end{array}$$

7. The market-clearing condition for stocks:

$$(23) \quad \lambda_H(s^t) + \lambda_H^*(s^t) = 1 \qquad \lambda_F(s^t) + \lambda_F^*(s^t) = 1.$$

8. The households' budget constraints (equations 6 and 7)

9. The households' first-order conditions for stock purchases (equations 9 and 11).

10. The probabilities $\pi(s^t)$ are consistent with the stochastic processes for $\left[z(s^t), z^*(s^t)\right]$

2.6 Equilibrium portfolios

PROPOSITION 1: Suppose that at time zero, productivity is equal to its unconditional mean value in both countries ($z(s^0) = z^*(s^0) = 0$) and that initial capital is equalized across countries, $k(s^{-1}) = k^*(s^{-1}) > 0$. Then there is an equilibrium in this economy with the property that portfolios in both countries exhibit a constant level of diversification given by

$$(24) \quad 1 - \lambda = \lambda_F(s^t) = \lambda_H^*(s^t) = 1 - \lambda_H(s^t) = 1 - \lambda_F^*(s^t) = \frac{1-\omega}{1+\theta-2\omega\theta} \qquad \forall t, s^t$$

Moreover, in this equilibrium stock prices are given by

$$(25) \quad P(s^t) = k(s^t),\ P^*(s^t) = k^*(s^t) \qquad \forall t, s^t.$$

PROOF: See the appendix

We prove this result by showing that these portfolios decentralize the solution to an equal-weighted planner's problem in the same environment. In particular, we consider the problem of a planner who seeks to maximize the equally-weighted expected utilities of the domestic and foreign agents, subject only to resource constraints of the form (21) and (22). We then describe a set of candidate prices such that if the conditions that define a solution to the planner's problem are satisfied, then the conditions that define a competitive equilibrium in the stock trade economy are also satisfied when portfolios are given by equation (24).

3 Intuition for the result

What explains the finding that two stocks are sufficient to effectively complete markets in this economy, and how should we understand the particular expression for the portfolios that deliver perfect risk sharing in equation (24)? We now build intuition for these results from two different perspectives. First, we take a macroeconomic general equilibrium perspective, and combine a set of equilibrium conditions that link differences between domestic and foreign aggregate demand and aggregate supply in this economy. These equations shed light on how changes in relative prices coupled with modest levels of international portfolio diversification allow agents to achieve perfect risk-sharing. We then take a more micro agent-based perspective, and explore how, from a price-taking individual's point of view, returns to labor and to domestic and foreign stocks covary in such a way that agents prefer to bias portfolios towards domestic assets.

3.1 Macroeconomic Intuition

We now develop three key equations that are helpful for understanding the macroeconomics of how the equilibrium portfolio choices, defined in equation (24), deliver perfect risk-sharing.

The first equation is the hallmark condition for complete international risk-sharing, relating relative marginal utilities from consumption to the international relative price of consumption. Since the utility function is log-separable in consumption, this condition is simply

$$c(s^t) = e(s^t)c^*(s^t) \ \forall s^t, \tag{26}$$

which we can write more compactly as $\Delta c(s^t) = 0$, where $\Delta c(s^t)$ denotes the difference between domestic and foreign consumption in units of the domestic final good.

The second key equation uses budget constraints to express the difference between foreign and domestic consumption as a function of relative investment and relative GDP. Assuming constant portfolios, where λ denotes the fraction of the domestic (foreign) firm owned by domestic (foreign) households, domestic consumption is given by

$$\begin{aligned} c(s^t) &= q_a(s^t)w(s^t)n(s^t) + \lambda d(s^t) + (1-\lambda)e(s^t)d^*(s^t) \\ &= (1-\theta)y(s^t) + \lambda\left(\theta y(s^t) - x(s^t)\right) + (1-\lambda)e(s^t)\left(\theta y^*(s^t) - x^*(s^t)\right) \end{aligned} \tag{27}$$

where the second line follows from the definitions for dividends, and the assumption that the intermediate-goods production technology is Cobb-Douglas in capital and labor. Given a similar expression for foreign consumption, the difference between the value of consumption across countries is given by

$$(28) \quad \Delta c(s^t) = (1 - 2(1-\lambda)\theta)\Delta y(s^t) + (1 - 2\lambda)\Delta x(s^t)$$

Note that in the case of complete home bias ($\lambda = 1$), the relative value of consumption across countries would simply be the difference between relative output and relative investment. For $\lambda < 1$, financial flows mean that some fraction of changes in relative output and investment are financed by foreigners.

Equations (26) and (28) do not depend on the elasticity of substitution between traded goods, and can therefore be applied unchanged to the one-good models that have been the focus of much of the previous work on portfolio diversification (in a one-good model $e(s^t) = 1$). It is useful to briefly revisit some important results in this existing literature, prior to explaining why the portfolio predictions from the two-good model that is the focus of this paper differ so sharply.

Lucas considers a one-good endowment economy, which we can reinterpret in the context of (28) by setting $\theta = 1$ and $\Delta x(s^t) = 0$ for all s^t. In this case it is immediate that perfect risk pooling is achieved when agents hold 50 percent of both domestic and foreign shares in each period, *i.e.* $\lambda = 0.5$.[6]

Baxter and Jermann (1997) study a one-good economy with production. They argue that since the Cobb-Douglas technology implies correlated returns to capital and labor, agents can effectively diversify non-diversifiable country-specific labor income risk by aggressively diversifying claims to capital. Assuming firms in both countries target a constant capital stock, in which case $\Delta x = 0$, achieving perfect risk-sharing ($\Delta c = 0$) in the context of equation (28) means picking a value for λ such that the coefficient on ΔGDP is zero. The implied value for diversification is $1 - \lambda = 1/(2\theta)$, which is exactly the portfolio described by equation (2) in Baxter and Jermann. If capital's share θ is set to a third, the value for λ that delivers equal consumption in the two countries is -0.5. Thus, as Baxter and Jermann emphasize, a diversified portfolio involves a negative position in domestic assets.[7]

[6] Cantor and Mark extend Lucas' analysis to a simple environment with production. However, they make several assumptions that ensure that their economy inherits the properties of Lucas'. In particular, (i) domestic and foreign agents have the same log-separable preferences over consumption and leisure, (ii) productivity shocks are assumed to be iid through time, (iii) firms must purchase capital and rent labor one period before production takes place, and (iv) there is 100% depreciation. When their two economies are the same size, assumptions (ii) and (iii) ensure that in an efficient allocation capital and labor are always equalized across countries. Thus to deliver perfect risk-sharing, the optimal portfolio choice simply has to ensure an equal division of next period output, which is ensured with Lucas' 50-50 portfolio split.

[7] Note that equation 28 suggests that there will always exist a portfolio that delivers perfect risk sharing as

Our model enriches the Baxter and Jermann analysis along two dimensions. First, we explicitly endogenize investment. With stochastic investment, equation (28) indicates that, in general, no constant value for λ will deliver $\Delta c(s^t) = 0$, the perfect risk-sharing condition. Thus, in a one-good model, perfect risk-sharing is not achievable with constant portfolios. However, our second extension relative to Baxter and Jermann is to assume that the two countries produce different traded goods that are imperfect substitutes when it comes to producing the final consumption-investment good. As we now explain, the Cobb-Douglas technology we assume for combining these traded goods implies an additional equilibrium linear relationship between $\Delta y(s^t)$, $\Delta(c^t)$ and $\Delta x(s^t)$ - our third key equation - such that perfect risk-sharing can be resurrected given appropriate constant portfolios.

From equations (5), (20) and (21), domestic GDP (in units of the final good) is given by

$$\begin{aligned}(29)\quad y(s^t) &= q_a(s^t)\left(a(s^t) + a^*(s^t)\right) = q_a(s^t)a(s^t) + e(s^t)q_a^*(s^t)a^*(s^t) \\ &= \omega G(s^t) + e(s^t)(1-\omega)G^*(s^t)\end{aligned}$$

Similarly, foreign GDP is given by

$$(30)\quad y^*(s^t) = (1/e(s^t))(1-\omega)G(s^t) + \omega G^*(s^t)$$

Combining the two expressions above, $\Delta y(s^t)$, the difference between the value of domestic and foreign GDP, is linearly related to the difference between domestic and foreign absorption:

$$\begin{aligned}(31)\quad \Delta y(s^t) &= (2\omega - 1)\left(G(s^t) - e(s^t)G^*(s^t)\right) \\ &= (2\omega - 1)\left(\Delta c(s^t) + \Delta x(s^t)\right)\end{aligned}$$

This equation indicates that changes to relative domestic versus foreign demand for consumption or investment automatically change the relative value of intermediate output. The fact that countries devote a constant fraction of total final expenditure to each of the two intermediate goods

long as Δx is strictly proportional to ΔGDP. Thus, as an alternative to assuming $\Delta x = 0$, we could assume, for example, that firms invest a fixed fraction of output, so that $x(s^t) = \kappa GDP(x^t)$. In this case, in a one-good world, $\Delta x = \kappa \Delta GDP$. Now consumption equalization requires that $\Delta c = [(1 - 2(1-\lambda)\theta) + (1 - 2\lambda)\kappa]\,\Delta GDP = 0$ which implies $\lambda = \frac{2\theta - 1}{2(\theta - \kappa)}$.

As an example, if the investment rate κ is equal to 0.2 and capital's share is 1/3, the value for λ that delivers consumption equalization is -1.25, implying an even larger short position in domestic assets than the one predicted by Baxter and Jermann. The intuition is simply that foreign stocks are now a less effective hedge, since following an increase in foreign output, foreign investment rises, reducing income from foreign dividends.

means that the size of the effect is proportional to the change in demand, where the constant of proportionality is $(2\omega - 1)$. When the technologies for producing domestic and foreign final goods are the same ($\omega = 0.5$), changes to relative demand do not impact the relative value of the outputs of goods a and b. When final goods are produced only with good a ($\omega = 1$), an increase in domestic demand translates into an equal-sized increase in the relative price of good a (assuming no supply response). For intermediate values for ω, the stronger the preference for home-produced goods, the larger the impact on the relative value of domestic output.

Note that this equation is independent of preferences and the asset market structure, and follows solely from our Cobb-Douglas assumption, implying a unitary elasticity of substitution between the two traded goods.

We can now combine our three key equations, (26), (28) and (31) to explore the relationship between portfolio choice, relative price movements, and international risk-sharing. We start by substituting (31) into (28) to express the difference in consumption as a function solely of the difference in investment:[8]

$$(32) \quad \Delta c(s^t) = (1 - 2(1-\lambda)\theta)\,(2\omega - 1)(\Delta c(s^t) + \Delta c(s^t)) + (1 - 2\lambda)\Delta x(s^t)$$

which implies that

$$(33) \quad \mu \Delta c(s^t) = \underbrace{(1 - 2\lambda)}_{direct\ foreign\ financing} \Delta x(s^t) + \underbrace{(2\omega - 1)\,(1 - 2(1-\lambda)\theta)}_{indirect\ foreign\ financing} \Delta x(s^t)$$

where μ is a constant.

There is a unique value for λ such that the right hand side of (33) is always equal to zero. In particular, simple algebra confirms that this value is defined in (24).

As a first step towards understanding the implications of equation (33) for portfolio choice, we first revisit a result due to Cole and Obstfeld (1991), who consider a two-country endowment economy. They show that when domestic and foreign agents share the same log-separable preferences for consuming the two goods, then a regime of portfolio autarky (100 percent home bias or $\lambda = 1$) delivers the same allocations as a world with a complete set of internationally-traded assets. In the context of our model, considering an endowment economy effectively implies $\Delta x = 0$, in which case equations (28) and (31) become two independent equations in two unknowns, Δc and Δy. The only

[8] Alternatively, one could substitute out investment to derive an equation linking $\Delta y(s^t)$ to $\Delta c(s^t)$.

possible solution is $\Delta c = \Delta y = 0$. Thus for any choice for λ, including the portfolio autarky value $\lambda = 1$ emphasized by Cole and Obstfeld, perfect risk-pooling is achieved. The reason is simply that differences in relative quantities of output are automatically offset one-for-one by differences in the real exchange rate, so $y = ey^*$. Thus movements in the terms of trade provide automatic and perfect insurance against fluctuations in the relative quantities of intermediate goods supplied.[9]

In contrast to the Cole and Obstfeld result, only one portfolio delivers perfect risk-pooling in our economy. Furthermore, portfolio autarky is only efficient in the case when there is complete specialization in tastes, so that $\omega = 1$. The reason for these differences relative to their results is that with partial depreciation and persistent productivity shocks, efficient investment will not be either constant or a constant fraction of output; rather, as in a standard growth model, positive persistent productivity shocks will be associated with a surge in investment. Thus dividends are not automatically equated across domestic and foreign stocks, and asset income is sensitive to portfolio choice. Moreover, these investment responses mediate relative price movements, so that relative earnings also fluctuate in response to productivity shocks. Nonetheless, the Cole and Obstfeld result is useful in that it reminds us that absent changes in relative investment, automatic insurance delivered through changes in the terms of trade would automatically deliver perfect risk-pooling. Thus one way to think about the role of portfolio diversification is to ensure that the cost of funding changes in investment is efficiently split between domestic and foreign residents.

We can use equation (33) to understand the effect of an investment shock $\Delta x(s^t)$ on relative consumption, $\Delta c(s^t)$. Absent any diversification, an increase in $\Delta x(s^t)$ would reduce $\Delta c(s^t)$ proportionately. For $\lambda < 1$ some of the cost of additional domestic investment is paid for by foreign shareholders directly (the first term on the right hand side) or indirectly through changes in relative prices (the second term). The direct foreign financing effect depends on the difference between the fraction of domestic stock held by foreigners relative to domestic agents $((1 - \lambda) - \lambda)$. The indirect effect works as follows: an increase in relative domestic investment increases the relative value of domestic output in proportion to the factor $(2\omega - 1)$ (see eq. 31). This captures the fact that an increase in relative demand for domestic final goods has a positive effect on the terms of trade for

[9] Cole and Obstfeld also consider a version of the model with production. In this version the two goods may be consumed or used as capital inputs to produce in the next period. Like Cantor and Mark (1988) they assume 100 percent capital depreciation. When production technologies are Cobb-Douglas in the quantities of the two goods allocated for investment, portfolio autarky once again delivers perfect risk-sharing. The reason is that the assumptions of log separable preferences and full depreciation imply that consumption, investment and dividends are all fixed fractions of output, so that $\Delta x = \kappa \Delta GDP$. Given this relationship, equations 28 and 31 reduce to two independent equations in two unknowns, Δc and ΔGDP. Thus total dividend income in any given period is again independent of the portfolio split.

the domestic economy. The fraction of this additional output that accrues as income to domestic shareholders is given by the term $(1-2(1-\lambda)\theta)$, which in turn amounts to labor's share of income $(1-\theta)$ plus the difference between domestic and foreign shareholder's claims to domestic capital income $(\lambda\theta - (1-\lambda)\theta)$. The equilibrium value for λ is the one for which the direct effect and the indirect effects exactly offset, so that changes in relative investment have no effect on relative consumption.

Why do portfolios exhibit home bias? If the lion's share of income goes to labor ($\theta < 0.5$) and, and if preferences are biased towards domestically-produced goods ($\omega > 0.5$), then the indirect effect of an increase in relative domestic investment on relative consumption is positive (the second term in (33) is positive). It is positive because the change in the terms of trade triggered by an increase in domestic demand favors domestic agents. Because the relative values of domestic earnings increases, domestic residents can afford to finance (by holding most of domestic equity) the bulk of an increase domestic investment while still equalizing consumption across countries.

3.2 Microeconomic intuition

The key to understanding optimal portfolio choice from the perspective of an individual agent is to understand how the returns to domestic and foreign stocks co-vary with non-diversifiable labor income. If returns to domestic stocks co-vary negatively with labor earnings, then domestic stocks will offer a good hedge against labor income risk, and agents will prefer a portfolio biased towards domestic firms. In Section 2.6 we described an equilibrium in which perfect risk sharing is achieved, and in which home bias is in fact observed. This suggests that domestic stock returns do in fact co-vary negatively with labor income. At first sight, this might seem a rather puzzling result, given that the production technology is Cobb-Douglas, suggesting a constant division of output between factors. We now explain how two key features of the BKK environment, durable capital and relative price dynamics, interact to give rise to this negative covariance.

First, recall that perfect risk sharing means equalizing the value of consumption across countries, state by state: $c(s^t) = e(s^t)c^*(s^t)$.

The difference between the value of domestic and foreign earnings (in units of the domestic final good) is

$$(34) \quad q_a(s^t)w(s^t)n(s^t) - e(s^t)q_b^*(s^t)w^*(s^t)n^*(s^t) = q_a(s^t)(1-\theta)\left(F(s^t) - t(s^t)F^*(s^t)\right)$$

Thus the relative value of domestic earnings rises in response to an increase in $z(s^t)$ relative to $z^*(s^t)$ if and only if the increase in the relative production of good a relative to good b exceeds the increase in the terms of trade (ie the price of good b relative to good a). In our economy this condition is satisfied: thus a positive domestic productivity shock is good news for domestic workers.

Now to rationalize the finding that agents prefer to bias their portfolios towards domestic stocks we need to show that in response to a positive domestic productivity shock, the return to domestic stocks declines relative to the return to foreign stocks, and thus that domestic stocks offer a good hedge against non-diversifiable labor income risk.

Period t returns on domestic and foreign stocks (in units of the domestic final good) are given by

$$(35)\quad r(s^t) = \frac{d(s^t) + P(s^t)}{P(s^{t-1})}, \qquad r^*(s^t) = \frac{e(s^t)}{e(s^{t-1})} \frac{d^*(s^t) + P^*(s^t)}{P^*(s^{t-1})}$$

Using the expressions for equilibrium stock prices - $P(s^t) = k(s^t)$ and $P^*(s^t) = k^*(s^t)$ - along with the definitions for dividends, these returns can alternatively be expressed as

$$(36)\quad r(s^t) = \frac{\theta q_a(s^t) F(s^t)}{k(s^{t-1})} + 1 - \delta, \qquad r^*(s^t) = \frac{e(s^t)}{e(s^{t-1})} \left(\frac{\theta q_b^*(s^t) F^*(s^t)}{k^*(s^{t-1})} + 1 - \delta \right)$$

The difference between the aggregate returns to domestic versus foreign stocks is then

(37)

$$r(s^t)P(s^{t-1}) - r^*(s^t)e(s^{t-1})P^*(s^{t-1}) = \theta q_a(s^t) \left[F(s^t) - t(s^t)F^*(s^t) \right] + (1-\delta) \left[k(s^{t-1}) - e(s^t)k^*(s^{t-1}) \right]$$

The first term in this expression captures the change in relative income from capital, and it has exactly the same flavor as the change in relative earnings: through this term, a positive domestic productivity shock will increase the relative return on domestic stocks as long as the terms of trade does not respond too strongly. However, there is also a second term in the expression for relative returns, as long as depreciation is only partial. This captures the fact that part of the return to buying a stock is the change in its price. A positive domestic productivity shock drives up the real exchange rate $e(s^t)$ and thus drives down the relative value of undepreciated domestic capital (since final consumption and investment are perfectly substitutable in production, the relative price of capital is equal to the relative price of consumption). Whether relative returns to domestic stocks rise or fall in response to a positive productivity shock depends on whether the first or second term dominates. In the model described above, the second term dominates, meaning that when faced with a positive shock, owners of domestic stocks lose more from the ensuing devaluation of domestic capital than they gain from a higher rental rate.

We are not the first to relate portfolio choice to the pattern of comovement between labor income and domestic and foreign stock returns. Cole (1988), Brainard and Tobin (1992), and Baxter and Jermann (1997) argued that in models driven entirely by productivity shocks, one should expect labor income to co-move more strongly with domestic rather than foreign stock returns, thereby indicating strong incentives to aggressively diversify. Bottazzi, Pesenti and van Wincoop (1996) argued that this prediction could be over-turned by extending models to incorporate additional sources of risk that redistribute income between capital and labor, and thereby lower the correlation between returns on human and physical capital. They suggested terms of trade shocks as a possible candidate. We have shown that in fact it is not necessary to introduce a second source of risk: the endogenous response of the terms of trade to productivity shocks is all that is required to generate realistic levels of home bias. The existing empirical evidence on correlations between returns to labor and domestic versus foreign stocks is, for the most part, qualitatively consistent with the pattern required to generate home bias. Important papers on this topic are Bottazzi et. al. (1996), Palacios-Huerta (2001), and Julliard (2002).

3.2.1 Impulse responses

To further our understanding of how perfect risk sharing is achieved with time-invariant and home-biased portfolios, it is helpful to examine the response of macro variables to a productivity shock in this economy. In order to do so, we must first fully parameterize the model. We discuss our calibration in detail in the next section, and report parameter values in Table 1. Figure 1 plots impulse responses to a persistent (but mean reverting) positive productivity shock in the domestic country. The path for productivity in the two countries is depicted in panel (a). Stock returns, labor earnings, financial wealth and stock prices are all plotted in units of the domestic final consumption good.

In the period of the shock, the relative return to domestic labor increases, and the gap between relative earnings persists through time (see panel c). The differential can persist because labor is immobile internationally. In the period of the shock, realized returns to foreign stocks exceed returns to domestic stocks, reflecting a decline in the relative value of domestic capital (panel b). After the first period, however, returns to domestic and foreign stocks are equalized. The reason

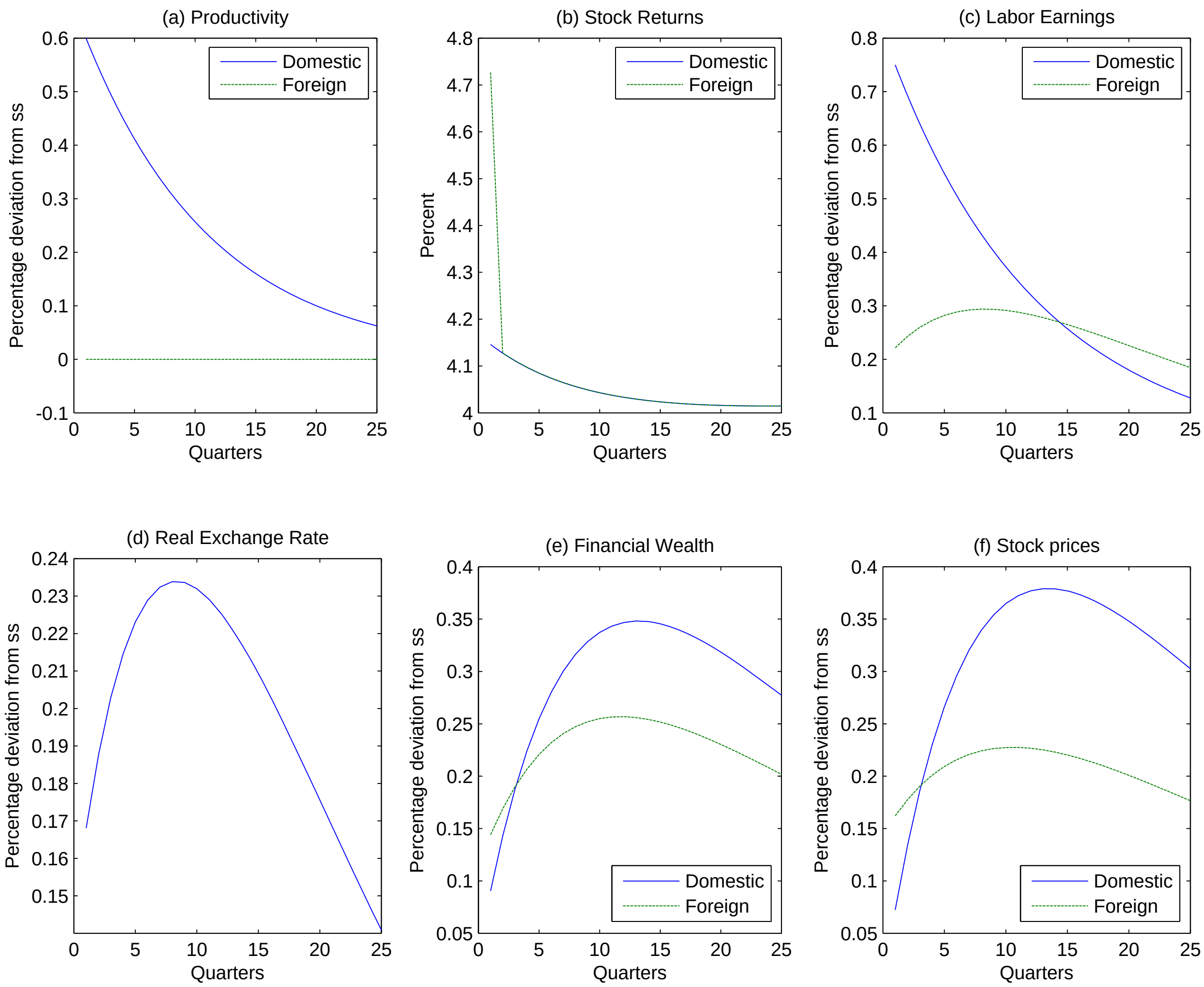

Figure 1: Impulse responses to a domestic productivity shock

for this result is simply that stocks are freely traded and thus equilibrium stock prices must adjust to equalize expected returns, up to a first-order approximation.[10]

Because agents do not adjust their portfolios in response to the shock, the decline in the relative value of domestic stocks on impact means that financial wealth for home-biased domestic agents declines relative to the wealth of foreigners (panel e). This means that in the periods immediately following the shock, even though returns are equalized, the total asset income accruing to foreign agents is larger, because they hold more financial wealth in total. This additional asset income exactly offsets foreigners' lower labor income, and the relative value of consumption is equalized.

Over time, the domestic productivity shocks decays, while the real exchange rate remains above its steady state level. As a consequence, foreign labor income eventually rises above domestic labor income. But notice that now, because of capital accumulation in country 1 (panel f), domestic wealth now exceeds foreign wealth, and this compensates domestic residents for the fact that they expect relatively low earnings during the remainder of the transition back to steady state.

To summarize, from the point of view of an individual worker / investor, optimal portfolio choice can be interpreted in the usual way as depending on the covariances between non-diversifiable labor income and the returns on domestic and foreign stocks. The key feature of this environment, however, is that these covariances are endogenous and depend critically on the dynamics of investment and relative prices. An important message from the preceding analysis is that the model makes clear predictions about the signs of these covariances, and, perhaps surprisingly, returns to domestic labor and capital tend to co-move negatively, even though the model is frictionless and the only shocks are Hicks-neutral innovations to TFP.

3.3 Diversification and the trade share

When $\omega = 0.5$, so that changes in demand fall equally on domestic and foreign intermediate goods, relative output and earnings are automatically equated across countries ($\Delta y(s^t) = 0$ in equation 31). This reflects the fact that changes in relative quantities are exactly canceled out by offsetting changes in the terms of trade, as in Cole and Obstfeld (1991). In this case, perfect risk sharing implies a constant real exchange rate ($e(s^t) = 1$), so that relative stock returns are also equated across countries (the second term in equation 37 drops out). Thus, as in Cole and Obstfeld's

[10] More formally, comparing the first order conditions for the domestic agent for domestic and foreign stocks we get

$$E_{s^t}\left[\frac{r(s^t, s_{t+1})}{c(s^t, s_{t+1})}\right] = E_{s^t}\left[\frac{r^*(s^t, s_{t+1})}{c(s^t, s_{t+1})}\right].$$

endowment economy, any portfolio automatically delivers perfect insurance against country-specific risk, and the equilibrium value for λ is indeterminate.

For $\omega \neq 0.5$, there is a unique equilibrium portfolio defined by equation (24). A lower trade share (a larger value for ω) implies a lower value for diversification, $(1-\lambda)$. The intuition is as follows. For $\omega > 0.5$, reducing the trade share implies that in response to a positive domestic productivity shock, the associated increase in domestic investment is increasingly targeted towards domestic intermediate goods. This attenuates the increase in the relative price of the (relatively scarce) foreign intermediate good, and magnifies the increase in relative domestic earnings. Thus, as the import share is reduced, non-diversifiable labor income becomes a more important risk that agents want to hedge in financial markets. This pushes agents towards more asymmetric portfolios, which continue to favor the asset (domestic stocks) whose return comoves negatively with earnings.

3.4 Diversification and labor's share

Equation (24) indicates that the larger is labor's share the stronger is home bias. This is the opposite of the Baxter and Jermann (1997) result, who found that introducing labor supply made observed home bias even more puzzling from a theoretical standpoint. Both results are easy to rationalize. The larger is labor's share, the larger is the increase in relative domestic earnings following a positive productivity shock, and thus the greater is the demand for asset's whose return covaries negatively with domestic output. In our economy, that asset is the foreign stock. In the Baxter and Jermann one-good world, it is the domestic stock.

Van Wincoop and Warnock (2006) emphasize a different force that can also deliver home bias in two-good models: negative covariance between the real exchange rate and the return differential between domestic and foreign stocks. If domestic stocks pay a relatively high return in states of the world in which domestic goods are expensive (i.e. the real exchange rate is low) then, since domestic residents mostly consume domestic goods, they may prefer to mostly hold domestic stocks. Note that this effect is not the driver of home bias in our basic set-up. In fact Van Wincoop and Warnock (2006) show that this mechanism generates home bias only when the coefficient of relative risk aversion exceeds one. By contrast, our model generates substantial home bias even with risk aversion equal to one.[11] The most important difference between our environment and theirs is that they abstract from labor income. In the presence of nondiversifiable labor income, portfolio choice is driven primarily by the covariance between relative excess stock returns and labor income (rather

[11]We experiment with alternative values for risk aversion in Section 4.2.

than exchange rates). We conclude that abstracting either from imperfect substitutability between traded goods (as in Baxter and Jermann) or from labor supply (as in van Wincoop and Warnock) leads to an incomplete account of the theoretical determinants of portfolio choice.

4 Sensitivity Analysis

Three key assumptions are required to deliver our closed-form expression for portfolio choice: first, that the elasticity of substitution between traded intermediate goods is unity (so that the G functions are Cobb-Douglas); second, that utility is logarithmic in consumption; and third, that there is only one type of shock. We now experiment with relaxing these assumptions. The main finding from these experiments is that a strong bias toward domestic assets is a robust feature of this model: the only case in which home bias disappears is when the elasticity of substitution between domestic and foreign goods is very high, in which case portfolios resemble those in the one-good model.

In order to compute equilibrium country portfolios in a general set-up we must fully specify the remaining parameters of the model, including a stochastic process for productivity shocks. Most parameters are straightforward to calibrate, since variations on this model have been widely studied. Here we mostly follow Heathcote and Perri (2004), which show that a similar model economy can successfully replicate a set of key international business cycle statistics for the U.S. v/s an aggregate of industrial countries over the period 1986-2001. Table 1 below reports the values.

Table 1. Parameter values

Preferences	
Discount factor	$\beta = 0.99$
Disutility from labor	$V(.) = v\frac{n^{1+\phi}}{1+\phi}$
	$v = 9.7, \quad \phi = 1$
Technology	
Capital's share	$\theta = 0.34$
Depreciation rate	$\delta = 0.025$
Import share	$1 - \omega = 0.15$
Productivity Process	

$$\begin{bmatrix} z(s^t) \\ z^*(s^t) \end{bmatrix} = \begin{pmatrix} 0.91 & 0.00 \\ 0.00 & 0.91 \end{pmatrix} \begin{bmatrix} z(s^{t-1}) \\ z^*(s^{t-1}) \end{bmatrix} + \begin{bmatrix} \varepsilon(s^t) \\ \varepsilon^*(s^t) \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon(s^t) \\ \varepsilon^*(s^t) \end{bmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.006^2 & 0.00 \\ 0.00 & 0.006^2 \end{pmatrix} \right)$$

We then solve the model numerically, and compute average values for diversification in simulations.[12] Solving for equilibria numerically requires a non-standard numerical method, since standard linearization techniques cannot handle the consumers' portfolio problem. The numerical technique we employ is described in detail in Appendix B.

4.1 Elasticity of substitution and risk aversion

The Cobb-Douglas aggregator for producing final goods implies a unitary elasticity of substitution between the traded goods a and b. This elasticity is towards the low end of estimates used in the business cycle literature. Panel (a) of figure 2 shows how the average equilibrium level of diversification changes as the elasticity of substitution, σ, is varied from 0.8 to 2.5, given a CES aggregator of the form $G(a,b) = (\omega a^{\frac{\sigma-1}{\sigma}} + (1-\omega)b^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$. The main message of the picture is that for commonly-used elasticities, theory predicts strong (even too strong) home bias. Notice also that increasing substitutability strengthens home bias within this range of values for σ. The logic for this result is that the more substitutable are a and b, the less relative prices change in response to shocks. This means that following a positive domestic shock, the increase in the relative value

[12] Note that, in general, the share of foreign assets in wealth need not to be constant.

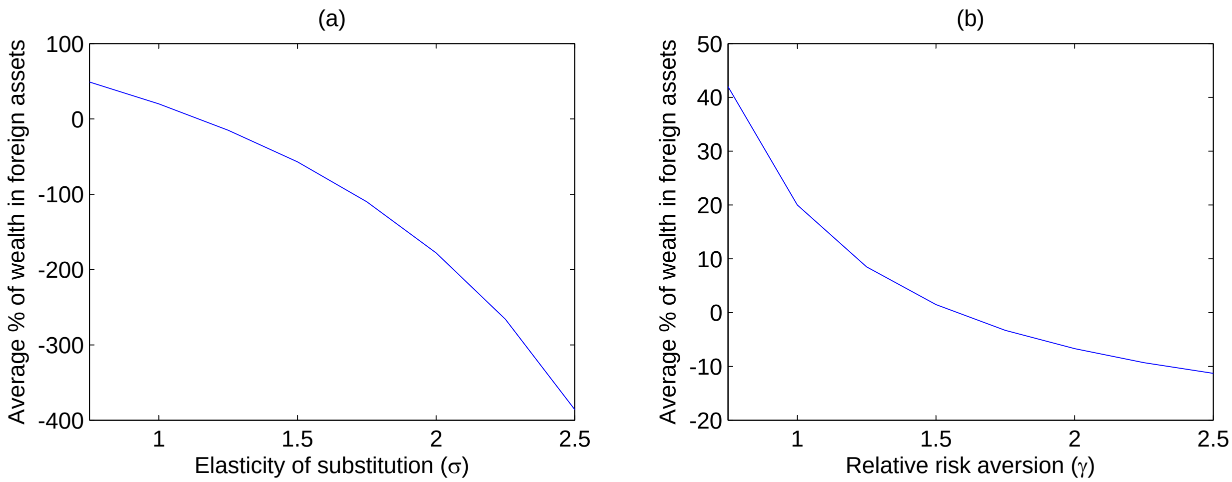

Figure 2: International diversification and elasticity of substitution

of domestic labor earnings becomes larger and, at the same time, the decline in relative domestic stock returns becomes smaller. Thus agents must overweight domestic stocks to an even greater extent in order to hedge such risks.

For very high elasticities (values for σ exceeding 4), price movements become so small that, following a positive domestic shock, returns to domestic stocks exceed returns to foreign stocks, and the correlation between relative labor income and relative domestic stock returns turns positive. For such high elasticities, the two-good model is sufficiently close to the one-good model that its portfolio implications are similar. In particular it is optimal for the individual to hedge against shocks to relative labor income by shorting domestic assets. Thus the average portfolio displays a very strong - and counter-factual - foreign bias.

Panel (b) of figure 2 shows how diversification changes as we change the coefficient of relative risk aversion, γ. Notice that higher risk aversion leads to higher home-bias. Changing the risk aversion coefficient does not impact the two equilibrium relationships (equations 28 and 31) developed in Section 3.1. Changing γ does, however, change the pattern of co-movement between domestic and foreign consumption consistent with perfect risk-sharing. In particular, since higher risk aversion corresponds to a lower inter-temporal elasticity of substitution for consumption, γ^{-1}, desired consumption becomes less sensitive to changes in relative prices. Thus in choosing portfolios, agents want to ensure that their total income does not decline too much in periods when domestic productivity falls and the relative price of domestic consumption increases ($e(s^t)$ declines). This pushes agents further towards domestic stocks, whose relative return rises in periods when domestic productivity and earnings decline.

4.2 Preference shocks and the Backus Smith evidence

When productivity shocks are the only source of risk, perfect risk-sharing implies that the real exchange rate is perfectly correlated with the ratio between real domestic consumption and real foreign consumption. This follows immediately from equation (26). As Backus and Smith (1993) first noted, however, for most countries this correlation is either close to zero or negative. This failure of the prototypical international business cycle model is well-known, but it raises the question of whether the model delivers realistic portfolios only at the cost of counter-factual comovement between international relative prices and relative quantities. In this section we argue that this is not the case. In particular, we modify the basic model to make it consistent with the Backus-Smith evidence, and then show that the strong home bias motive remains (in fact it is strengthened).

There is strong empirical evidence that in response to supply side shocks, relative quantities and relative prices co-move positively, as in the productivity-shock driven version of the model (see Acemoglu and Ventura, 2002; Debaere and Lee, 2004; and Pavlova and Rigobon, 2007). This suggests that the unconditional correlation is close to zero because demand side risks induce a negative conditional correlation between relative consumption and the real exchange rate. We now experiment with introducing taste shocks, following Stockman and Tesar (1995), as a simple reduced-form way to model these demand-side shocks.

We modify the representative agent's utility functions as follows:

$$
\begin{aligned}
(38) \qquad U\left(c(s^t), n(s^t), \zeta(s^t)\right) &= e^{\zeta(s^t)} \ln c(s^t) - v\frac{n(s^t)^{1+\phi}}{1+\phi} \\
U\left(c^*(s^t), n^*(s^t), \zeta^*(s^t)\right) &= e^{\zeta^*(s^t)} \ln c^*(s^t) - v\frac{n^*(s^t)^{1+\phi}}{1+\phi}
\end{aligned}
$$

where the vector of taste shocks $[\zeta(s^t)\ \zeta^*(s^t)]$ evolves exogenously according to a similar process to the one we assumed for productivity shocks. In particular, we assume that innovations to taste shocks are uncorrelated across countries, uncorrelated with innovations to productivity shocks, and that taste shocks and productivity shocks are equally persistent.

To understand why taste shocks lower the correlation between the real exchange rate and relative consumption, consider the effect of a positive taste shock in, say, country 1. In response to the shock, consumers in country 1 will want to increase current consumption and this, because of the home preference bias in consumption, will raise the world demand for good a. Since productivity in country 1 (the producer of good a) is unchanged, the price of good a relative to good b will tend to rise, inducing more labor input in country 1. Thus, in equilibrium, relative consumption and relative output will increase, while the terms of trade and the real exchange rate will fall, inducing a negative conditional correlation between relative consumption and the real exchange rate. Since productivity shocks induce a positive conditional correlation, the equilibrium unconditional correlation between relative consumption and the real exchange rate will depend on the relative volatility of the two types of risk.

Panel (a) of Figure 3 shows how the correlation changes as the volatility of taste shocks is increased from 0 to 1.2 times the volatility of productivity shocks. Notice that when the volatility of taste shocks is similar to the volatility of productivity shocks, the correlation between the real exchange rate and relative consumption is close to 0, and thus consistent with the Backus-Smith evidence.[13] Panel (b) of the figure shows the crucial part of this experiment: how the equilibrium average share of foreign assets changes as we increase the size of taste shocks. The panel shows that the larger are taste shocks, the stronger is the bias toward domestic assets. This indicates that domestic stocks are a good hedge against taste shocks. To understand why, recall that when domestic demand increases ($\zeta(s^t)$ is high), domestically-produced goods become relatively expensive, and the real exchange rate appreciates. This change in international relative prices raises the relative return on domestic stocks, which is good news for domestic high-marginal-utility investors. Thus the same relative price movement that resolves the Backus-Smith puzzle also makes local stocks even more attractive to investors.

5 Explaining international diversification across countries and across time

The main analytical result of this paper, summarized in Proposition 1, offers a prediction for the levels of international diversification we should observe across countries, and establishes a link between international diversification and the trade share. In this section, we take these predictions

[13] It is also easy to assess how taste shocks affect standard business cycle statistics produced by the model. Broadly speaking taste shocks mostly affect statistics related to consumption; in particular, relative to model without taste shocks, they tend to increase volatility of consumption relative to output, to reduce the correlation between domestic consumption and domestic output and to lower the international correlation of consumption relative to the one of output.

Most importantly even for volatile taste shocks (volatility equal 1.5 times the volatility of productivity shocks) the statstics generated by the model are well within the range of data statistics measured for various OECD countries.

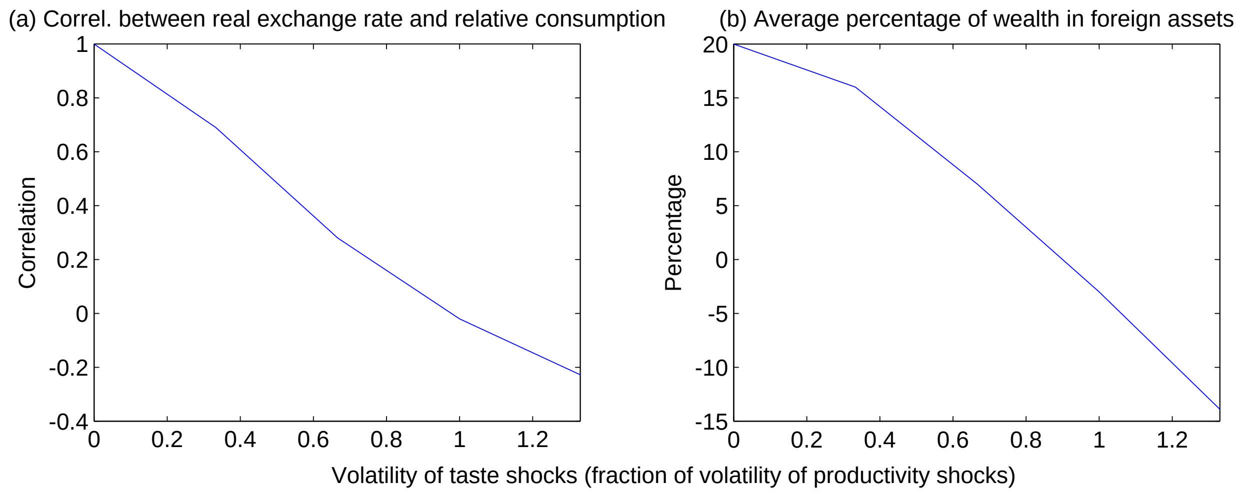

Figure 3: The role of taste shocks

to the data in order to assess the extent to which our model can shed light on the patterns of international diversification that we see across countries and over time. The first issue we need to confront is that our model focuses on a world with two symmetric countries, while international diversification data are drawn from countries which are heterogenous in many dimensions, including size, level of development, and the extent of financial liberalization. One possible way to deal with this issue would be to enrich our basic model to include many heterogenous countries and to then bring such a model to the data; we view that as an interesting project, but one that is beyond the scope of this paper.[14]

Here we address the issue in two ways. First we restrict most of our empirical analysis to a relatively homogenous and financially liberalized group of countries: high income (as classified by the World bank) economies after 1990. Second, within that group, we assess whether factors omitted in the model, such as size or level of development, are important empirical factors in explaining diversification patterns.

[14]We did experiment with one dimension of heterogeneity. In particular we considered an extension of our main model in which the two countries differ in terms of population. We then solve this version of the model numerically, given the parameter values described in Table 1, and compare the average equilibrium level of diversification to the level predicted by equation 24. We find that, for the smaller economy, the equilibrium level of diversification exceeds that which would be observed in the corresponding symmetric-size economy, while for the larger economy, the equilibrium level of diversification is below that which would be prediction by (24), given the country's import share. However, these differences are generally small (less than 1%), unless the smaller country is both very open and very small.

5.1 Data

Taking the reciprocal of the relation in Proposition 1 we obtain

$$(39)\quad \frac{1}{1-\lambda} = 2\theta + (1-\theta)\frac{1}{1-\omega}$$

which is a linear relationship between the reciprocal of diversification $1/(1-\lambda)$ and the reciprocal of the trade share, $1/(1-\omega)$. Our measure of international diversification is $1-\lambda$, which in the model is both the ratio of gross foreign assets to wealth and the ratio of gross foreign liabilities to wealth. In order to construct these ratios we need data on gross foreign assets, gross foreign liabilities and total country wealth. We obtain data on total gross foreign assets (FA)and total gross foreign liabilities (FL) from the exhaustive dataset collected by Lane and Milesi Ferretti (2006). In particular both FA and FL include portfolio equity investment, foreign direct investment, debt (including loans or trade credit), financial derivatives and reserve assets (excluding gold). We then construct total financial wealth as $K+FA-FL$, where K is the capital stock. One important issue regarding the capital stock is whether it is measured at book value (i.e. by cumulating investment) or at market value (i.e. measuring the market value of the assets as reflected, for example, by the stock market). Ideally one would like to construct a measure of capital that is consistent with the measure of foreign assets and foreign liabilities. Unfortunately, some asset categories (for example foreign direct investment) are constructed using book values, while others (for example portfolio equity investment) are constructed using market values, so it is not immediately clear what is the right measure of capital to use. In order to partly deal with issue we construct two series for capital stock. Our baseline measure is computed simply by starting from initial capital stock figures from Dhareshwar and Nehru (1993), updated by cumulating investments from the Penn World Tables 6.2 (as for example in Kraay and al. 2006). We label this measure K_B. We also compute another measure of capital stock, which we label K_M which uses information on the value of the stock market and the stock market growth in the different countries to revalue part of the capital stock.[15] We then measure international diversification for country i in period t as

$$(1-\lambda)_{it} = average\left(\frac{FA_{it}}{K_{it}+FA_{it}-FL_{it}}, \frac{FL_{it}}{K_{it}+FA_{it}-FL_{it}}\right)$$

[15] The construction of both measures of K is discussed in detail in appendix C

We measure trade share for country i in period t, using national income data from the the Penn World Table 6.2, as

$$(1-\omega)_{it} = \left(\frac{\text{Imports}_{it} + \text{Exports}_{it}}{2GDP_{it}}\right)$$

The final of piece of evidence we need regards the share of labor income $1-\theta$. Consistently with evidence reported in Gollin (2002) we will assume $1-\theta$ to be constant over time and across countries at a value of 0.66.

5.2 Diversification across countries

In this section we abstract from time variation in diversification and focus on explaining average diversification across countries. Figure 4 summarizes our main findings. The circles in the figure represent the time averages (over the period 1990-2004) for the reciprocal of diversification $1/(1-\lambda)_{it}$ (computed using K_B) and the reciprocal of trade shares $1/(1-\omega)_{it}$ for each country in the group of high income economies for which we have data. Notice how there is a great deal of heterogeneity both in the trade share and in the diversification shares, with both shares ranging from around 10% to over 100%. The solid line is the relation between them obtained estimating equation (39) using OLS, the shaded area represent the 95% confidence band (obtained using heteroscedasticity corrected standard errors) around the OLS relation while the dashed line is the relation between them implied by the model assuming that $\theta = 0.34$. The figure suggests that trade share is an important factor in explaining the patterns of international diversification in different countries[16] and that the quantitative predictions from our model regarding both the level of diversification and the relation between diversification and trade share lie within two standard deviation of the data estimates. In table 2 we report results of the the regression depicted in the picture (column 1), along with which various other robustness checks.

In all regressions the dependent variable is the reciprocal of the average diversification. Regressions 1,3,5 only include a constant and the reciprocal of trade share as independent variables, while regressions 2,4,6 also include, as controls, the log of average GDP (PPP adjusted) per capita and the log of average population. Regressions 1 through 4 use diversification measures computed our benchmark measure of capital stock (constructed cumulating investment) while regressions 5 and 6 use capital stock constructed using also stock market information. Finally regressions 3 and 4

[16] Portes and Rey (2003) and Collard et al. (2007) also highlight a strong empirical relation between trade in assets and trade in goods.

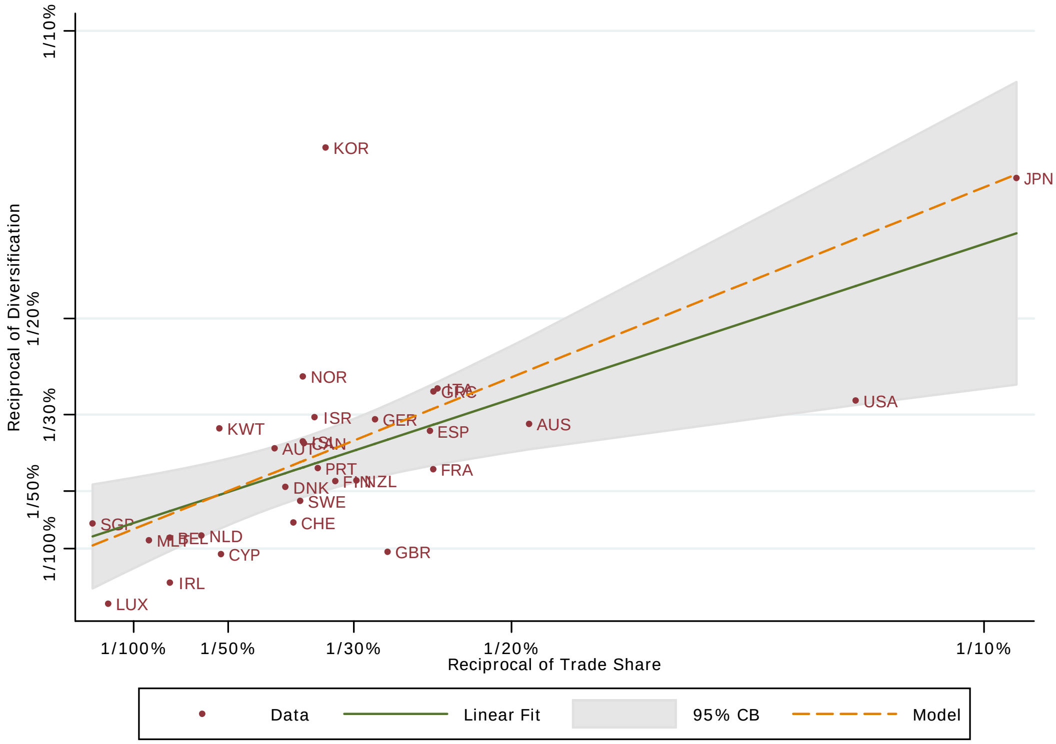

Figure 4: Diversification and trade shares: data and model

(LAD) compute the coefficients by minimizing absolute deviations, since these coefficients are less sensitive to the presence of outliers. The first row reports the coefficient of the reciprocal of trade share, as estimated in the data (the first 6 columns) and as predicted by the model (the last column). Notice that in all cases the coefficient is significantly different from 0 (at the 1% level) confirming the strong link between trade and financial diversification. Quantitatively the coefficients estimated in the data are not far from those predicted by the model: in all specifications except (5) one cannot reject at the 5% significance the hypothesis that the estimated coefficient in the data is equal to the one predicted by the model. The second and third row assess the effect of GDP per capita and size on international diversification. Our symmetric model is silent about the effects of those variables; nevertheless it is interesting to assess i) whether these variables are indeed statistically correlated with diversification, and ii) whether the relation between trade and diversification is affected by the inclusion of these variables. In particular, since it is well known that small countries and rich countries tend to trade more, one might wonder whether the trade only matters for diversification only through size and GDP per capita. The numbers in the table suggest instead that GDP per capita and size are not statistically related to diversification, once trade is included[17] and that the relation between diversification and trade is largely unaffected by their inclusion. The row labeled "Predicted median diversification" reports the predicted diversification for a country which has the median value of the independent variables, in the data and in the model. The numbers in the row suggest that the model's prediction for the level of diversification are statistically close to the average, conditional on a given trade share, diversification observed in the data (for specification 1 this result could have been anticipated simply by looking at figure 4 and noticing that the model line lies within the shaded area). Finally the R^2 figures in the last row suggest that differences in openess to trade alone can explain between 30% and 40% of the cross sectional variation in portfolios.

[17] This result crucially hinges on the fact that we have selected a group of fairly homogenous countries. We have also repeated the analysis for a larger group of countries including developing economies and found that, although the strong link between trade and diversification persist, income per capita becomes an important determinant of diversification, with richer countries being more diversified.

Table 2. Cross sectional regressions
Dependent variable is reciprocal diversification, $1/(1-\lambda)_i$

	OLS, K_B		LAD, K_B		OLS, K_M		Model $\theta = 0.34$
	(1)	(2)	(3)	(4)	(5)	(6)	
Reciprocal Openess	**0.54**†	**0.48**†	**0.69**†	**0.59**†	**0.45**	**0.41**†	0.66
	(0.16)	(0.17)	(0.11)	(0.24)	(0.08)	(0.13)	
Log GDP per capita		-2.05		-0.72		-1.49	
		(1.48)		(1.02)		(1.02)	
Log population		0.09		0.05		0.06	
		(0.26)		(0.27)		(0.23)	
Predicted median diversif.	**0.41**†	**0.43**†	**0.45**†	**0.50**†	**0.41**†	**0.42**†	0.40
	(0.04)	(0.04)	(0.06)	(0.08)	(0.04)	(0.04)	
Obs	29	29	29	29	29	29	
R^2	0.38	0.44	0.30	0.33	0.39	0.43	

Note: The number in parentheses are heteroscedasticity corrected (for the OLS specifications) standard errors. Bold statistics are significantly different from 0 at the 1% level. A data statistic with † indicates that the corresponding model statistic lies within the two standard deviation band around the data

5.3 Diversification across time

In this section we explore the panel dimension of our dataset to assess whether our framework can also be used to understand the evolution of diversification over time. Literally in equilibria of our baseline model diversification does not change over time; it is easy to see though that, in the model economy, a permanent and unanticipated change in trade openness, *ceteris paribus*, would affect equilibrium diversification. In particular if, say, between period t and period $t+k$ there is an unexpected and permanent change in trade openess then equation (39) would hold in both periods and taking differences we obtain

$$\left(\frac{1}{1-\lambda}\right)_{i,t+k} - \left(\frac{1}{1-\lambda}\right)_{i,t} = (1-\theta)\left[\left(\frac{1}{1-\omega}\right)_{i,t+k} - \left(\frac{1}{1-\omega}\right)_{i,t}\right]$$

suggesting that changes over time in the reciprocal of diversification of a country should be linearly related to changes over time in the reciprocal of its trade share. In order to identify permanent and unanticipated changes in the trade data we consider changes in the reciprocal trade shares over 5 years intervals. In Table 3 we explore how these changes are related to diversification.[18] Regressions (1) through (3) use diversification constructed using our benchmark capital measure K_B, while regressions (3) through (6) use our alternative capital measure K_M.Columns (1) and (4) shows that time differences in reciprocal openess are linearly related to time differences in reciprocal diversification and that the estimated regression coefficient estimated in the data are not statistically different from the one predicted by the model. Notice though that the relation estimated in the data shows a significant negative constant term, suggesting an additional increase in diversification which is not explained by the growth in trade, and thus which is is not captured by our model.[19] Columns 2, 3, 5 and 6 introduce controls in the basic regressions. We find remarkable that even after controlling for country and period dummies (specifications 3 and 6) and thus allowing for a variety of factors to affect changes in diversification, the link between trade and diversification is still strongly significant and quantitatively similar to the link predicted by the model.

[18] Specifically we look at changes in reciprocal diversification and in reciprocal trade over the periods 1990-1994,1995-2000 and 2000-2004 for all 29 countries for which we have data.

[19] The size of the additional increase in diversification does not depend only on the constant but on the specific values for the increase in trade shares and the initial diversification. We find that it is small for most observations in our sample. For example if a country with 20% diversification experiences an increase of its trade share from 20% to 25% the model suggest its diversification should increase to around 23% while the data a constant equal to -0.16 implies an increase in diversification to around 24%.

Table 3. Changes regression
Dependent variable is change in reciprocal diversification

	OLS, K_B			OLS, K_M			Model $\theta = 0.34$
	(1)	(2)	(3)	(4)	(5)	(6)	
Change in recipr. openess	**1.00**†	**1.07**†	**0.95**†	**0.57**†	**0.64**†	**0.51**†	0.66
	(0.38)	(0.40)	(0.15)	(0.22)	(0.22)	(0.18)	
Constant	**-.16**	-0.09†	**-0.44**	**-0.18**	-0.11†	**0.38**	0
	(0.02)	(0.06)	(0.12)	(0.02)	(0.05)	(0.1)	
Log GDP per cap. growth		-1.48	-0.83		-1.53	-0.99	
		(1.29)	(1.44)		(1.59)	(1.51)	
Log population growth		-0.23	-2.11		0.17	-2.34	
		(3.48)	(3.77)		(4.37)	(5.18)	
Country & period dummies	No	No	Yes	No	No	Yes	
Obs	85	85	85	85	85	85	
R^2	0.25	0.26	0.74	0.11	0.13	0.64	

Note: The number in parentheses are heteroscedasticity corrected (for the OLS specifications) standard errors. Bold statistics are significantly different from 0 at the 1% level. A data statistic with † indicates that the corresponding model statistic lies within the two standard deviation band around the data

The overall message of the data analysis is that the predictions of the model regarding the level of diversification and the relation between diversification can help us understand qualitatively and quantitatively the cross section of country portfolios in developed economies. Obviously there still a significant fraction of heterogeneity in diversification which is not explained by our model; for example the fact that the United Kingdom and Luxembourg are significantly more diversified relative to what the model predicts (see figure 4) is probably related to the fact that those countries are international financial centers and our model does not capture this type of heterogeneity. Still our results suggest that using our model in conjunction with trade openness data can help us understand an important fraction of the variation in international diversification across developed economies and across time.

6 Conclusion

In this paper we have shown that standard macroeconomic theory predicts patterns for international portfolio diversification that are broadly consistent with those observed empirically in recent years.

The economic model we used to generate theoretical predictions for portfolio choice was a standard two-country two-good version of the stochastic growth model that has been widely used in business cycle research. We conclude that, from the perspective of standard macroeconomic theory, the observed bias towards domestic assets is not a puzzle. We have explored the economics underlying this result, and argued that the dynamics of investment and international relative price movements, elements which have been largely overlooked in the existing literature on portfolio choice, are central to understanding portfolio choice.

Important questions remain. In our analysis we have focussed primarily on the predictions of our model for portfolio diversification. However, it is well known that it is difficult to reconcile many features of asset prices with the predictions of stochastic general equilibrium production economies. For example, relative to the predictions of our model, actual stock prices and real exchange rates appear excessively volatile. A general equilibrium theoretical resolution of these pricing puzzles is required to bridge the gap between the macroeconomic theory and empirical finance literatures on portfolio choice. A somewhat less ambitious task for future work is to build a multi-country version of the model which allows for multi-lateral trade in goods, and multi-lateral diversification in assets. Such a model would lead to a better understanding of the separate roles of size and openness in understanding portfolio diversification. It would also generate richer predictions linking trading patterns to distributions of foreign assets and liabilities by country.

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Appendix A. Proof of proposition 1

Let $G(s^t)$ and $F(s^t)$ be compact notations for $G(a(s^t), b(s^t))$ and $F\left(z(s^t), k(s^{t-1}), n(s^t)\right)$.
The equations that characterize a solution to the planner's problem are:

1. First order conditions for hours:

$$
\begin{aligned}
U_c(s^t)\frac{\omega G(s^t)}{a(s^t)}\frac{(1-\theta)F(s^t)}{n(s^t)} + U_n(s^t) &\geq 0 \\
U_c^*(s^t)\frac{\omega G^*(s^t)}{b^*(s^t)}\frac{(1-\theta)F^*(s^t)}{n^*(s^t)} + U_n^*(s^t) &\geq 0
\end{aligned}
$$

2. First order conditions for allocating intermediate goods across countries:

$$
\begin{aligned}
U_c(s^t)\omega G(s^t)/a(s^t) &= U_c^*(s^t)(1-\omega)G^*(s^t)/a^*(s^t) \\
U_c(s^t)(1-\omega)G(s^t)/b(s^t) &= U_c^*(s^t)\omega G^*(s^t)/b^*(s^t)
\end{aligned}
$$

3. First order conditions for investment:

$$
\begin{aligned}
\widetilde{Q}(s^t) &= \sum_{s_{t+1}\in S} \widetilde{Q}(s^t, s_{t+1})\left[\frac{\omega G(s^t, s_{t+1})}{a(s^t, s_{t+1})}\frac{\theta F(s^t, s_{t+1})}{k(s^t)} + (1-\delta)\right] \\
\widetilde{Q}^*(s^t) &= \sum_{s_{t+1}\in S} \widetilde{Q}^*(s^t, s_{t+1})\left[\frac{\omega G^*(s^t, s_{t+1})}{b^*(s^t, s_{t+1})}\frac{\theta F^*(s^t, s_{t+1})}{k^*(s^t)} + (1-\delta)\right]
\end{aligned}
$$

   where

$$
\begin{aligned}
\widetilde{Q}(s^t) &= \frac{1}{2}\pi(s^t)\beta^t U_c(s^t) + \frac{1}{2}\pi(s^t)\beta^t U_c^*(s^t)\frac{(1-\omega)}{\omega}\frac{G^*(s^t)}{G(s^t)}\frac{a(s^t)}{a^*(s^t)} \\
\widetilde{Q}^*(s^t) &= \frac{1}{2}\pi(s^t)\beta^t U_c^*(s^t) + \frac{1}{2}\pi(s^t)\beta^t U_c(s^t)\frac{\omega}{(1-\omega)}\frac{G(s^t)}{G^*(s^t)}\frac{a^*(s^t)}{a(s^t)}
\end{aligned}
$$

4. Resource constraints of the form (21) and (22).

Consider the set of allocations that satisfies this set of equations, *i.e.* the solution to the planner's problem. We now show there exists a set of prices at which these same allocations also satisfy the set of equations defining equilibrium in the stock trade economy (see Section 2.5), given the portfolios described in equation (24). In other words, we can decentralize the complete markets allocations with asset trade limited to two stocks and constant portfolios.

Let intermediate-goods prices be given by equations (20). Then condition (1) for the stock trade economy is satisfied. Let wages be given by equations (15) and (16). Then condition (2) for the stock trade economy is satisfied. Substituting these prices into condition (1) from the planner's problem gives condition (3) for the stock trade economy. Let the real exchange rate by given by equation (5). Then combining conditions (2) and (3) from the planner's problem gives condition (4) for the stock trade economy. Condition (4) from the planner's problem translates directly into conditions (5) and (6) for the stock trade economy. Condition (7) - stock market clearing - follows immediately from the symmetry of the candidate stock purchase rules.

Condition (8) is that households' budget constraints are satisfied. Given constant portfolios, the domestic household's budget constraint simplifies to

$$c(s^t) = q_a(s^t)w(s^t)n(s^t) + \lambda d(s^t) + (1-\lambda)e(s^t)d^*(s^t)$$

Substituting in the candidate function for $w(s^t)$, the resource constraint for intermediate goods, and the definitions for dividends (and suppressing the state-contingent notation) gives

$$c = q_a(1-\theta)(a+a^*) + \lambda\left(q_a\theta(a+a^*) - x\right) + (1-\lambda)e\left(q_b^*\theta(b+b^*) - x^*\right)$$

Using the candidate expression for the real exchange rate gives

$$c = (1-\theta+\lambda\theta)\left(q_a a + e q_a^* a^*\right) - \lambda x + (1-\lambda)\theta\left(q_b b + e q_b^* b^*\right) - (1-\lambda)e x^*$$

Now using the candidate expressions for intermediate goods prices and collecting terms gives

$$c = \left[\omega + (1-\lambda)(\theta - 2\omega\theta)\right]G + e\left[(1-\omega) - (1-\lambda)\left(\theta - 2\omega\theta\right)\right]G^* - \lambda x - (1-\lambda)e x^*$$

Using the resource constraint for final goods firms gives

$$\begin{aligned} G &= \left[\omega + (1-\lambda)(\theta - 2\omega\theta)\right]G + e\left[(1-\omega) - (1-\lambda)\left(\theta - 2\omega\theta\right)\right]G^* \\ &\quad + (1-\lambda)(G-c) - (1-\lambda)e(G^* - c^*) \end{aligned}$$

Given the candidate expression for the real exchange rate, and exploiting the assumption that utility is logarithmic in consumption, condition (2) for the planners problem implies

$$c = ec^*.$$

Thus the budget constraint can be rewritten as

$$G = \left[\omega + (1-\lambda)(1+\theta - 2\omega\theta)\right]G + e\left[(1-\omega) - (1-\lambda)\left(1+\theta - 2\omega\theta\right)\right]G^*$$

Finally substituting in the candidate expression for λ confirms that the domestic consumer's budget constraint is satisfied. The foreign consumer's budget constraint is satisfied by Walras' Law.

Condition (9) is the households' inter-temporal first order conditions for stock purchases. Substituting condition (2) from the planner's problem into condition (3), the planner's first order conditions for investment may be rewritten as

$$\begin{aligned} U_c(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t) U_c(s^t, s_{t+1}) \left[\frac{\omega G(s^t, s_{t+1})}{a(s^t, s_{t+1})} \frac{\theta F(s^t, s_{t+1})}{k(s^t)} + (1-\delta)\right] \\ U_c^*(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t) U_c^*(s^t, s_{t+1}) \left[\frac{\omega G^*(s^t, s_{t+1})}{b^*(s^t, s_{t+1})} \frac{\theta F^*(s^t, s_{t+1})}{k^*(s^t)} + (1-\delta)\right] \end{aligned}$$

Multiplying both sides of the first (second) of these two equations by $k(s^t)$ $(k^*(s^t))$ gives

$$\begin{aligned} U_c(s^t)k(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)U_c(s^t,s_{t+1})\left[\frac{\omega G(s^t,s_{t+1})}{a(s^t,s_{t+1})}\theta F(s^t,s_{t+1}) + (1-\delta)k(s^t)\right] \\ U_c^*(s^t)k^*(s^t) &= \beta \sum_{s_{t+1}\in S} \pi(s_{t+1}|s^t)U_c^*(s^t,s_{t+1})\left[\frac{\omega G^*(s^t,s_{t+1})}{b^*(s^t,s_{t+1})}\theta F^*(s^t,s_{t+1}) + (1-\delta)k^*(s^t)\right] \end{aligned}$$

Let stock prices by given by

(40) $P(s^t) = k(s^t),\ P^*(s^t) = k^*(s^t) \qquad \forall t, s^t.$

Substituting these candidate prices for stocks, the prices for intermediate goods, the wage, and the expressions for dividends into the planner's first order conditions for investment gives the domestic household's first order condition for domestic stock purchases, and the foreign household's first order condition for foreign stock purchases. The remaining two first-order conditions for stock purchases follow immediately by substituting condition (2) from the planner's problem into these two conditions.

Appendix B. Computational algorithm

Here we describe the algorithm that allows us to solve for equilibrium portfolio holdings in the generalized version of the model described in Section 4. By generalized, we mean parameterizations for which Proposition 1 does not apply, and for which portfolios must be characterized numerically. Our algorithm can be used to solve for equilibria in more general international macro models with portfolio choice, and thus it complements the recent work of Devereux and Sutherland (2006), Tille and van Wincoop (2007), and Evans and Hnatkovska (2007). Matlab programs that implement this algorithm are available at `http://www.fperri.net/research_data.htm`. We now outline the steps of the algorithm

Step 1. Pick a non-stochastic symmetric steady state equilibrium (i.e. an equilibrium in which agents know that productivities $z(s^t), z^*(s^t)$ are constant and equal to 0). We denote such a steady state with the vector $[\lambda_H, \lambda_F^*, X, Y]$, where $\lambda_H, \lambda_F^* \in \mathbb{R}$ are the fractions of local stocks held by home and foreign residents, respectively, $X \in \mathbb{R}^n$ is the vector of non portfolio state variables (i.e. productivities and capital stock.).while $Y \in \mathbb{R}^m$ is the vector of non portfolio control variables (i.e. consumption, investment, terms of trade etc.). Notice that first order conditions plus symmetry uniquely pin down X and Y, while any value $\lambda_0 = \lambda_H = \lambda_F^*$ is a non-stochastic steady state symmetric equilibrium.

Step 2. Compute decision rules $\lambda_{H,t+1} = g_1(\lambda_{H,t}, \lambda_{F,t}^*, X_t)$, $\lambda_{H^*,t+1} = g_2(\lambda_{H,t}, \lambda_{F,t}^*, X_t)$, $X_{t+1} = g_3(\lambda_{H,t}, \lambda_{F,t}^*, X_t, \varepsilon_{t+1})$, $Y_t = g_4(\lambda_{H,t}, \lambda_{F,t}^*, X_t)$ that characterize the solution to a second-order approximation of the stochastic economy around the steady state. The functions g_1, g_2 , g_3, and g_4 are quadratic forms in their arguments and can be computed using the methods described by Schmitt-Grohe and Uribe (2004) or Gomme and Klein (2006) among others. Note that in order to apply those method here it is necessary to slightly modify the model by adding a small adjustment cost of changing the portfolio from its steady state value. This step yields decision rules for all variables (including porfolio decisions) that are correct up to a second-order approximation, in a neighborhood of the steady state around which the economy is linearized. However, we do not yet know whether the steady state portfolio λ_0 we started with is equal to the average equilibrium portfolio in the true stochastic economy.

Step 3: Starting from our guess for the steady state, we simulate the model for a large number of periods using the decision rules from Step 2, and compute the average share of wealth held by domestic agents along the simulation. If this average share is different from the initial steady state share, we set the new guess for the steady state portfolio, λ_1, equal to the average simulated share and return to step 1. If the simulated average is equal (up to a small tolerance error) to the initial steady state λ_0, then λ_0 constitutes a good approximation of the long run portfolio holdings and we take it as the solution to our portfolio problem.

As a test, we apply this method to our benchmark parameterization and to the one-good model of Baxter and Jermann (for both these cases we know the true portfolio solution). In both cases our algorithm converges very rapidly to the true solution, regardless of the initial guess. We also find that the portfolio adjustment costs can be set to an arbitrarily small (but positive) number, such that changing the size of these costs locally does not affect the solution.

Appendix C. Data

The countries we use in our analysis in Section 6 are the high income economies (as classified by the World Bank) for which have foreign asset position and capital stock data. This group includes the following 29 countries (The codes used in figure 5 are in parentheses) Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Cyprus (CYP), Denmark (DNK), Finland (FIN), France (FRA), Germany (GER), Greece (GRC), Iceland (ISL), Ireland (IRL), Israel (ISR), Italy (ITA), Japan (JPN), Korea (KOR), Kuwait (KWT), Luxembourg (LUX), Malta (MLT), Netherlands (NLD), New Zealand (NZL), Norway (NOR), Portugal (PRT), Singapore (SGP), Spain (ESP), Sweden (SWE), Switzerland (CHE), United Kingdom (GBR), United States (USA). The data on gross international diversification positions (total foreign assets and foreign liabilities) are in US dollars and are from Lane and Milesi Ferretti (2006).

We denote by $K_{B,i,t}$ our baseline measure (in US Dollars) for the capital stock in country i in period t. We construct $K_{B,i,t}$ by multiplying GDP in US dollars (as reported by Lane and Milesi Ferretti, 2006) by the capital-output ratio. The capital-output ratio is computed as follows: in 1989 we take it directly from Dhareshwar and Nehru (1993), who report both physical capital stock and GDP figures. After 1989 we construct the capital-output ratio in period $t+1$ for country i, $\left(\frac{k}{y}\right)_{i,t+1}$ using the following recursion:

$$\left(\frac{k}{y}\right)_{i,t+1}\left(\frac{y_{i,t+1}}{y_{i,t}}\right) = (1-\delta)\left(\frac{k}{y}\right)_{i,t} + \left(\frac{x}{y}\right)_{i,t}$$

where $\left(\frac{y_{i,t+1}}{y_{i,t}}\right)$ is the growth rate of GDP for country i (PPP adjusted, in constant prices (chain method) from the Penn World Tables (PWT) 6.2), $\left(\frac{x}{y}\right)_{i,t}$ is the ratio between investment and GDP (both PPP adjusted, in current prices, from PWT 6.2), δ is the depreciation rate which, in absence of better information, we set equal to 6% (this value is also used by Kraay et al. 2005) for all countries and for all years.

Given $K_{B,i,t}$, we can derive a panel for our alternative measure of capital, $K_{M,i,t}$ The idea behind this alternative measure is that part of the capital stock of a country is comprised of assets of firms quoted on the stock market, and thus we can measure the growth of the value of this capital simply by measuring the growth of stock prices. For the remaining (non-publicly-traded) portion of the capital stock we simply assume the same growth rate as for our baseline measure $K_{B,i,t}$ More specifically, we assume that in 1989 $K_{M,i,t} = K_{B,i,t}$, while for the subsequent years we use the following recursion:

$$K_{M,i,t+1} = (K_{M,i,t} - S_{i,t})g(K_{B,i,t}) + S_{i,t}g(P_{i,t})$$

where $S_{i,t}$ is the value of the stock market in country i at year t (from Beck et. al., 2000), $g(K_{B,i,t})$ is the growth rate of the baseline capital stock in country i, and $g(P_{i,t})$ is the growth rate for stock prices in country i (computed using the growth rate of the Morgan Stanley Capital International (MSCI) index for country i). For five countries in our sample (Cyprus, Iceland, Kuwait, Malta and Luxembourg) we do not have the MSCI country index, and so we simply replace $g(P_{i,t})$ with the growth rate of the total stock market value, $g(S_{i,t})$.
Our complete dataset is available online at `http://www.fperri.net/research_data.htm`