

Problem Set 2 – Solutions

(please address any questions regarding the grading of this problem set and these solutions to Matt Levy)

1. (a) Neither of these facts are consistent with a model of exponential discounting. Choosing to get the EITC payment as a single payment at the end of the tax year is strictly worse from a (purely) financial standpoint than getting the payments over the course of the year. Getting it as a single large payment is essentially like saving it in a zero interest bearing account. One could do strictly better by taking the monthly payments and putting them in a (positive interest bearing) bank account. At the end of the year, one would have the EITC payment, plus a non-trivial amount of accrued interest.

The “refund anticipation” loan is also not consistent with a model of dynamic consistency. If the family foresaw needing the EITC payment a few weeks before their income tax refund would arrive, they should have chosen the monthly payments. It is highly unlikely that so many families would have unexpected shocks right at tax season, and even less likely that EITC recipients are so borrowing-constrained that paying over 100% interest on a \$2,000 loan makes sense (remember, one has to be substantially employed to receive the EITC).

- (b) A *sophisticated* hyperbolic discounter could make these decisions. Knowing that his future selves have a taste for immediate gratification, such a taxpayer correctly predicts that he would squander the small monthly payments if he chose the first option. Even though it is financially inferior to the monthly payments, he chooses the lump-sum as a self-control device against his future selves’ impatience. When he gets the large tax refund, he can spend the EITC payment on some larger purchase that his current self (may) find more beneficial.

Note that choosing the lump sum payment is not consistent with a *naïve* hyperbolic discounter. A naïf believes that he will not be impatient in the future, so a self-control device is unwarranted. A naïf would behave much like an exponential discounter and choose the monthly payments.

The sort of extreme impatience demonstrated by the “refund anticipation loan” is exactly what is predicted by hyperbolic discounting. Because the future is heavily discounted relative to the present, a short-term, high-interest loan is appealing to someone with a sufficiently small β . (This doesn’t mean they’d take any bad loan – consider how he might feel about a loan with a similarly high interest rate, but over a much longer time frame)

- (c) Part of the puzzle here is why someone would accrue debt (on which she has to pay interest) and wait to receive the EITC payment (on which she earns no interest), when she could just opt for the monthly payments and pay off the debt as she goes along.

We’ve already established in part 1b that only sophisticated hyperbolic discounters would choose the lump-sum payment over the monthly payments. Since sophisticates correctly predict their future behavior, they know they’ll be running up this debt. One possible interpretation is that sophisticates know they’ll run up this debt, but that they’d spend even more if they had the monthly payments. That is, their future consumption is only constrained by their credit card borrowing capacity. If they keep their cards maxed out, their future selves are constrained to consume less. (Whereas any amount used over the

year to pay off the debt would get spent again right away). When they get the EITC lump sum, they use it to pay down this very expensive self-control device.

- (d) This is much easier to explain using hyperbolic discounting. Durable goods, which give a person utility over a long period of time, are exactly the sort of good that can be troublesome for hyperbolic discounters. Long-run patience makes them very attractive, but short-run impatience means people would be tempted to spend money now rather than save for the durable good. Thus, even though someone could do strictly better by putting the monthly payments in an interest-bearing account, a sophisticated hyperbolic discounter chooses the lump sum as a “forced savings” program. (This is similar to “Christmas Clubs”, where people put money into a zero-interest account but cannot withdraw it until Christmas)

2. (a) The payoffs from the four possible consumption paths is given in the following table:

Consumption Path	Self 0	Self 1
Hit, Hit	$4 + \beta(-9 + 4 - 9)$ $= 4 - 7 = -3$	$-9 + 4 + \beta(-9)$ $= -9.5$
Hit, Refrain	-0.5	-9
Refrain, Hit	-2	0.5
Refrain, Refrain	0	0

- Naïve: From Self 0’s perspective, the optimal consumption path is (Refrain, Refrain). She therefore refrains in period 0. Self 1 prefers (Refrain, Refrain) over (Refrain, Hit), so she also refrains in period 1.
- Sophisticate: Regardless of what Self 0 does, Self 1 will choose to refrain. Knowing this, Self 0 will also choose to refrain.

- (b) With the higher payoffs from hitting, the discounted utility from the four consumption paths is now:

Consumption Path	Self 0	Self 1
Hit, Hit	$4 + \beta(-9 + 5 - 9)$ $= -2.5$	$-9 + 5 + \beta(-9)$ $= -8.5$
Hit, Refrain	0.5	-9
Refrain, Hit	-2	0.5
Refrain, Refrain	0	0

- Naïve: From Self 0’s perspective, the optimal consumption path is (Hit, Refrain). She therefore hits in period 0 (and expects to refrain in period 1). Self 1 prefers (Hit, Hit) over (Hit, Refrain), so she also winds up hitting in period 1.
- Sophisticate: Regardless of what Self 0 does, Self 1 will choose to hit. Knowing this, Self 0 will also choose to hit.

- (c) This bizarre result comes from the fact that hyperbolic discounting leads people to overconsume harmful goods. When the instantaneous utility is only 4, the current period self is not tempted to hit. When the instantaneous utility is 5 – just on the other side of the threshold – the current period self *is* tempted to hit. But even with the higher instantaneous utility, it is a mistake to hit. Both naïfs and sophisticates would prefer

never to hit than to hit in both periods. They can achieve this when the instantaneous utility is low enough, but not when it rises. The naïf doesn't realize she'll consume in the future, and the sophisticate knows she can't commit not to consume.

- (d) Now that her hitting in period 0 affects her period-1 payoffs from hitting, the discounting utilities from her four consumption paths are:

Consumption Path	Self 0	Self 1
Hit, Hit	$5 + \beta(-9 + 5 - 9)$ $= -1.5$	$-9 + 5 + \beta(-9)$ $= -8.5$
Hit, Refrain	0.5	-9
Refrain, Hit	-2.5	-0.5
Refrain, Refrain	0	0

- Naïve: From Self 0's perspective, the optimal consumption path is (Hit, Refrain). She therefore hits in period 0 (and expects to refrain in period 1). Self 1 prefers (Hit, Hit) over (Hit, Refrain), so she also winds up hitting in period 1.
- Sophisticate: She correctly predicts that Self 1 will hit if Self 0 hit, and will refrain if Self 0 refrained. Because Self 0 prefers (Refrain, Refrain) over (Hit, Hit), she refrains in period 0 (and hence also in period 1).

The naïf is overly optimistic about her future actions – she thinks she can refrain in period 1 even if she hits currently. She therefore hits now, expecting not to hit in the future. When period 1 rolls around, she finds herself impatient again and winds up hitting. The sophisticate understands that she will hit in the future *if and only* if she hits now (i.e. if she gets addicted). This *incentive effect* causes her to refrain today in order to control herself in the future.

- (e) Once again we make the table of her payoffs from each of the four consumption paths:

Consumption Path	Self 0	Self 1
Hit, Hit	$4 + \beta(-9 + 7 - 9)$ $= -1.5$	$-9 + 7 + \beta(-9)$ $= -6.5$
Hit, Refrain	-0.5	-9
Refrain, Hit	-2	0.5
Refrain, Refrain	0	0

- Naïve: From Self 0's perspective, the optimal consumption path is (Refrain, Refrain). She therefore refrains in period 0 (and expects to refrain in period 1). Self 1 prefers (Refrain, Hit) over (Refrain, Refrain), so she winds up hitting in period 1.
- Sophisticate: She correctly predicts that Self 1 will hit if Self 0 hit, and will hit if Self 0 refrained. Because Self 0 prefers (Hit, Hit) over (Refrain, Refrain), she hits in period 0 (and hence also in period 1).

As usual, the naïf is overly optimistic about her future behavior. In particular, she thinks that she can get her future self to refrain if she refrains from hitting today. The sophisticate recognizes that her future self will hit no matter what she does. The sophisticate views hitting as inevitable, and because of this *pessimism effect* she hits in both periods.

3. (a) If she goes to the doctor, then she can always pursue the right course of action (i.e. get treatment if he tells he she has cancer, and not get treatment if he tells her that she's cancer-free). If she doesn't go to the doctor, then she will choose not to get treated (since she has only a 1/10 chance of needing treatment).

The person's Expected Utility is:

$$\begin{aligned} \text{If she goes :} \quad EU &= 0.1 [v \cdot (1 - |1 - 1|) + w \cdot (0)] + 0.9 [v \cdot (1 - |0 - 0|) + w \cdot (1)] \\ &= v + 0.9w \end{aligned}$$

$$\begin{aligned} \text{If she doesn't go :} \quad EU &= 0.1 [v \cdot (1 - |0 - 1|)] + 0.9 [v \cdot (1 - |0 - 0|)] + w \cdot (0.9) \\ &= 0.9v + 0.9w \end{aligned}$$

Because $v > 0$, she will choose to go to the doctor. (Note the importance of the linearity of her "anxiety". If it were concave like the example in lecture, she might have chosen not to see the doctor)

- (b) Although her probability of having cancer has risen to $\frac{1}{2}$, she is still just as well off by not getting treated if she doesn't go to the doctor as she would be by getting treated if she doesn't see the doctor (either way, she's wrong half the time).

Now her expected utilities are:

$$\begin{aligned} \text{If she goes :} \quad EU &= 0.5 [v \cdot (1 - |1 - 1|) + w \cdot (0)] + 0.5 [v \cdot (1 - |0 - 0|) + w \cdot (1)] \\ &= v + 0.5w \end{aligned}$$

$$\begin{aligned} \text{If she doesn't go :} \quad EU &= 0.5 [v \cdot (1 - |0 - 1|)] + 0.5 [v \cdot (1 - |0 - 0|)] + w \cdot (0.5) \\ &= 0.5v + 0.5w \end{aligned}$$

Just as in part (a), the linearity of her anxiety means that information acquisition has no effect on her anticipatory utility. She therefore goes to the doctor again, in order to get the correct treatment.

- (c) This question is subtle. Importantly, we asked when going to the doctor yields higher utility than not going to the doctor – not when she would go to the doctor. This means that we should use the *actual* probabilities of events when calculating her expected utility, and her *subjective* beliefs when calculating her anxiety.

Thus her expected utilities are:

$$\begin{aligned} \text{If she goes :} \quad EU &= 0.5 [v \cdot (1 - |1 - 1|) + w \cdot (0)] + 0.5 [v \cdot (1 - |0 - 0|) + w \cdot (1)] \\ &= v + 0.5w \end{aligned}$$

$$\begin{aligned} \text{If she doesn't go :} \quad EU &= 0.5 [v \cdot (1 - |0 - 1|)] + 0.5 [v \cdot (1 - |0 - 0|)] + w \cdot (0.9) \\ &= 0.5v + 0.9w \end{aligned}$$

Thus going to the doctor yields higher utility exactly when:

$$\begin{aligned}v + 0.5w &\geq 0.5v + 0.9w \\10v + 5w &\geq 5v + 9w \\5v &\geq 4w \\v &\geq \frac{4}{5}w\end{aligned}$$

Intuitively, going to the doctor will let her get the correct treatment but also increases the expected amount of anxiety she'll suffer. As long as getting the right treatment is sufficiently important relative to avoiding anxiety, she will see the doctor.