Substantive and Procedural Rationality in Decisions under Uncertainty*

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Abstract

We report a laboratory experiment that enables us to study systematically the substantive and procedural rationality of decision making under uncertainty. By using novel graphical representations of budget sets over bundles of state-contingent commodities, we generate a very rich data set well-suited to studying behavior at the level of the

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individual subject. We test the data for consistency with the maximization hypothesis, and we recover underlying preferences using both nonparametric and parametric methods. We find that individual behaviors are complex and highly heterogeneous. In spite of this heterogeneity, we identify 'prototypical' heuristics that inform subjects' decision rules. To account for these heuristics, we propose a type-mixture model based on Expected Utility Theory employing only combinations of three heuristics which correspond to the behavior of individuals who are infinitely risk averse, risk neutral, and expected utility maximizers with intermediate risk aversion. The decision rules of the type-mixture model accord well with the large-scale features of the data at the level of the individual subject.

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1 Introduction

Because uncertainty is endemic in every aspect of human activity, models of decision making under uncertainty play a key role in every field of economics. The standard model of decisions under uncertainty is based on von Neumann and Morgenstern Expected Utility Theory (EUT), so it is natural that experimentalists should want to test the empirical validity of the Savage axioms on which EUT is based. Empirical violations of EUT raise intriguing questions about the rationality of individual behavior and, at the same time, raise criticisms about the status of the Savage axioms as the touchstone of rationality. These criticisms have generated the development of various theoretical alternatives to EUT, and the investigation of these theories has led to new empirical regularities.

For the most part, these laboratory experiments use several pairwise choices, à la Allais, to test EUT and its various generalizations, such as weighted utility (Chew (1983)), implicit expected utility (Dekel (1986)), and prospect theory (Kahneman and Tversky (1979)), among others.[1] Each of these theories gives rise to indifference curves with distinctive shapes in some part of the Marschak (1950) and Machina (1982, 1987) probability triangle, so each theory can be tested against the others by choosing alternatives that the various theories rank differently. In these studies, the criterion

[1] Camerer (1995) provides a comprehensive discussion of the experimental and theoretical work, and Starmer (2000) provides a more recent review that focuses on evaluating non-EUT theories. Kahneman and Tversky (2000) collect many theoretical and empirical papers that emerged from their pioneering work on prospect theory.

used to evaluate a theory is the fraction of choices it predicts correctly.[2] A few studies have also estimated parametric utility functions for individual subjects.[3] In general, existing experimental work has, on the one hand, collected only a few decisions from each subject and, on the other, presented subjects with an *extreme* binary choice designed to discover violations of specific theories.

Although this practice is understandable given the purposes for which the experiments were designed, it limits the usefulness of the data for other purposes. Most importantly, designing choices to reveal violations does not necessarily tell us very much about how choices are made in economic settings that are more often encountered in practice. Hence, while these experiments reveal that violations exist, they give us little sense of how important they are or how frequently they occur. Finally, the small data sets generated for each subject force experimenters to pool data, thus ignoring individual heterogeneity and precluding the possibility of statistical modeling at the level of the individual subject.

In this paper, we study individual choice under uncertainty in economically important settings. The study is motivated by five types of questions (Varian 1982): (*i*) *Consistency.* Is behavior under conditions of uncertainty consistent with a model of utility maximization? (*ii*) *Structure.* Is the observed data consistent with a utility function with some special structural properties? (*iii*) *Recoverability.* Can underlying preferences be recovered from observed choices? (*iv*) *Heterogeneity.* To what degree do preferences over risky alternatives differ across subjects? (*v*) *Heuristics.* Can heuristic procedures be identified when they occur?

Our objective of producing a general account of choice under uncertainty has led us to develop an experimental design that is innovative in a couple of ways. First, we present subjects with a standard economic decision problem that can be interpreted either as a portfolio choice problem (the allocation of wealth between two risky assets) or a consumer decision problem (the selection of a bundle of contingent commodities from a standard budget set).[4] Secondly, the decision problems are presented on a user-friendly graphical interface that allows for the collection of a rich individual-level data set.[5]

[2] Camerer (1992) and Harless and Camerer (1994) summarize the experimental evidence of testing the various utility theories.

[3] See, for example, Currim and Sarin (1989, 1990), Daniels and Keller (1990), and Lattimore, Baker and Witte (1992), among others.

[4] In Loomes (1991) subjects also allocate wealth in a portfolio of risky assets. The focus of this paper is on providing tests of several choice theories, so the results are not directly comparable to those presented here.

[5] Fisman, Kariv, and Markovits (2005) employ a similar experimental methodology to

Our data set has several advantages over data sets from earlier experiments. First, the choice of a portfolio subject to a budget constraint provides more information than a binary choice. Second, because of the user-friendly interface, each subject faces a large number of decisions with widely varying budget sets. The large amount of data generated by this design allows us to apply statistical models to individual data, rather than pooling data or assuming homogeneity across subjects. This allows us to generate better individual-level estimates of risk aversion than has heretofore been possible. Third, these decision problems are representative, both in the statistical sense and in the economic sense, rather than being narrowly tailored to test specific axioms. Finally, the graphical representation does not emphasize any particular portfolio; in particular, it does not offer subjects discrete choices that might suggest prototypical preference types.

We begin our analysis of the experimental data by using revealed preference theory to determine whether the observed choices are consistent with utility maximization. The theory of revealed preference tells us that if the data generated by our experiment satisfy the Generalized Axiom of Revealed Preference (GARP), the observed choices can be rationalized by a well-behaved utility function (see Varian (1982) for a description of these techniques). Although individual behaviors are complex and heterogeneous, we find that most subjects' choices come close to satisfying GARP according to a number of standard measures. We conclude that, for most subjects, the violations are sufficiently minor that we can ignore them for the purposes of recovering preferences or constructing appropriate utility functions. We emphasize that while GARP implies rationality in the sense of a complete, transitive preference ordering, it does not imply the Savage axioms. There is no need to assume EUT to investigate rational behavior under uncertainty.

Whereas the analysis of consistency provides support for *substantive rationality*, it has nothing to say about *procedural rationality*, that is, how subjects come to make decisions that are consistent with an underlying preference ordering. Another advantage of our data set is that it allows us to distinguish systematic behavior from what appear to be mistakes and identify *heuristics* when they occur. We find that most subjects use one or more underlying 'prototypical' heuristics, which we call *types*. These heuristics correspond to the behavior of individuals who are infinitely risk averse, risk neutral, and expected utility maximizers with intermediate (constant)

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study social preferences. While the papers share a similar experimental methodology that allows for the collection of a rich individual-level data set, they address very different questions and produce very different behaviors.

relative risk aversion. Where several of these prototypical heuristics are used, consistent behavior requires subjects to choose among heuristics in a consistent manner as well as behaving consistently in applying a given heuristic.

Motivated by these patterns, we propose and estimate a *type-mixture model* (TMM) in which boundedly rational individuals use heuristics in their attempt to maximize an underlying preference ordering. In implementing this framework, we assume their preferences have an expected utility representation. Individuals are assumed to choose the heuristic that offers the highest payoff in a given decision, taking into account the possibility of making mistakes. Thus, the probability of choosing a particular heuristic is a function of the parameters of the budget set. We find that a TMM, employing only the three heuristics mentioned above, helps to explain the choice of heuristics and allows us to estimate measures of risk aversion that agree with estimates from other studies. Although the TMM allows for the possibility of errors and the use of heuristics, there remains an important role for EUT in analyzing choice under uncertainty, because the choice of heuristics is motivated by the underlying expected utility representation.

The rest of the paper is organized as follows. The next section describes the experimental design and procedures. Section 3 evaluates the consistency of the data with the maximization a preference ordering. Section 4 summarizes some important features of the individual-level data. Section 5 reports individual-level estimates of a constant relative risk aversion (CRRA) utility function. Section 6 describes the TMM analysis and Section 7 contains some concluding remarks. The experimental instructions are reproduced in Section 8.

2 Experimental design and procedures

2.1 Design

In the experimental task we study, individuals make decisions under conditions of uncertainty about the objective parameters of the environment. In our preferred interpretation, there are two *states of nature* denoted by $s = 1, 2$ and two associated *Arrow securities*, each of which promises a dollar payoff in one state and nothing in the other. We consider the problem of allocating an individual's wealth between the two Arrow securities. Let x_s denote the demand for the security that pays off in state s and let p_s denote its price. Without essential loss of generality, assume the individual's wealth is normalized to 1. The budget set is then $p_1 x_1 + p_2 x_2 = 1$ and the individual can choose any portfolio $(x_1, x_2) \geq 0$ that satisfies this constraint. This simple state-of-nature representation of uncertainty provides a test of rational decision making under uncertainty.

An example of a budget set defined in this way is the straight line AB drawn in Figure 1. The axes measure the future value of a possible portfolio in each of the two states. The point C, which lies on the 45 degree line, corresponds to a portfolio with a certain outcome. By contrast, point A (point B) represents a portfolio in which all wealth is invested in the security that pays off in state 1 (state 2). Notice that given the objective probabilities of each state, positions on AB do not represent *fair* bets (i.e. outcomes with the same expected value as point C). For example, if π is the probability of state 1 and the slope of the budget line $-p_1/p_2$ is steeper than $-\pi/(1-\pi)$, positions along AC have a higher payoff in state 1, a lower payoff in state 2, and a lower expected portfolio return than point C.

[Figure 1 here]

If individuals only care about the distribution of monetary payoffs and if their preferences over probability distributions of monetary payoffs (lotteries) satisfy the Savage axioms, then the preference ordering over portfolios (x_1, x_2) can be represented by a function of the form

$$\pi u(x_1) + (1-\pi)u(x_2),$$

where $u(\cdot)$ is the Bernoulli utility function defined on amounts of money. Such an individual will choose a portfolio (x_1^*, x_2^*) to maximize the expected value of utility subject to the budget constraint. Under standard conditions, the solution to this problem will satisfy the first-order condition

$$\frac{\pi}{1-\pi}\frac{u'(x_1^*)}{u'(x_2^*)} = \frac{p_1}{p_2}$$

and the budget constraint $p_1 x_1^* + p_2 x_2^* = 1$. Of course, there is no need to assume the Savage axioms in order to investigate rational behavior in general under uncertainty. Any consistent preference ordering over lotteries is admissible.

2.2 Procedures

The experiment was conducted at the Experimental Social Science Laboratory (X-Lab) at UC Berkeley under the X-Lab Master Human Subjects

Protocol. The 93 subjects in the experiment were recruited from all undergraduate classes and staff at UC Berkeley. After subjects read the instructions (reproduced in Section 8), the instructions were read aloud by an experimenter. At the end of the instructional period subjects were asked if they had any questions or difficulties understanding the experiment. No subject reported difficulty understanding the procedures or using the computer interface. Each experimental session lasted about one and a half hours. A \$5 participation fee and subsequent earnings, which averaged about \$19, were paid in private at the end of the session.

Each session consisted of 50 independent decision problems. In each decision problem, a subject was asked to allocate tokens between two accounts, labeled x and y. The x account corresponds to the x-axis and the y account corresponds to the y-axis in a two-dimensional graph. Each choice involved choosing a point on a budget line of possible token allocations. Each decision problem started by having the computer select a budget line randomly from the set of lines that intersect at least one axis at or above the 50 token level and intersect both axes at or below the 100 token level. The budget lines selected for each subject in his decision problems were independent of each other and of the budget lines selected for other subjects in their decision problems.

The x-axis and y-axis were scaled from 0 to 100 tokens. The resolution compatibility of the budget lines was 0.2 tokens. At the beginning of each decision round, the experimental program dialog window went blank and the entire setup reappeared. The appearance and behavior of the pointer were set to the Windows mouse default and the pointer was automatically repositioned randomly on the budget line at the beginning of each round. To choose an allocation, subjects used the mouse or the arrows on the keyboard to move the pointer on the computer screen to the desired allocation. Subjects could either left-click or press the Enter key to record their allocation. The computer program dialog window is shown in Section 8.

The payoff at each decision round was determined by the number of tokens in the x account and the number of tokens in the y account. At the end of the round, the computer randomly selected one of the accounts, x or y. Each subject received the number of tokens allocated to the account that was chosen. We studied a *symmetric* treatment (subjects ID 201-219 and 301-328), in which the two accounts were equally likely ($\pi = 1/2$) and two *asymmetric* treatments (subjects ID 401-417, 501-520 and 601-609) in which one of the accounts was always selected with probability 1/3 and the other account was selected with probability 2/3 ($\pi = 1/3$ or $\pi = 2/3$). The treatment was held constant throughout a given experimental session.

This procedure was repeated until all 50 rounds were completed. Subjects were not informed of the account that was actually selected at the end of each round. At the end of the experiment, the computer selected one decision round for each participant, where each round had an equal probability of being chosen, and the subject was paid the amount he had earned in that round. Payoffs were calculated in terms of tokens and then converted into dollars. Each token was worth \$0.5. Subjects received their payment privately as they left the experiment.

The experiments provide us with a data set consisting of $93 \times 50 = 4650$ individual decisions over a wide range of budget sets. This variation in budget sets (prices and incomes) is essential for a non-trivial test of consistency and also gives us an opportunity to recover preferences for individual subjects.

3 Testing Rationality

We first test whether choices can be utility-generated. Let $\left\{(p^i, x^i)\right\}_{i=1}^{50}$ be the data generated by some individual's choices, where p^i denotes the i-th observation of the price vector and x^i denotes the associated portfolio. A portfolio x^i is *directly revealed preferred* to a portfolio x^j, denoted $x^i R^D x^j$, if $p^i \cdot x^i \geq p^i \cdot x^j$. A portfolio x^i is *revealed preferred* to a portfolio x^j, denoted $x^i R x^j$, if there exists a sequence of portfolios $\left\{x^k\right\}_{k=1}^{K}$ with $x^1 = x^i$ and $x^K = x^j$, such that $x^k R^D x^{k+1}$ for every $k = 1, ..., K-1$.

We wish to examine whether the data observed in our experiment could have been generated by an individual maximizing a well-defined utility function. The crucial test for this is provided by the Generalized Axiom of Revealed Preference (GARP). In the notation introduced above, GARP (which is a generalization of various other revealed preference tests) requires that if $x^i R x^j$ then $p^j \cdot x^j \leq p^j \cdot x^i$ (i.e. if x^i is revealed preferred to x^j, then x^i must cost at least as much as x^j at the prices prevailing when x^j is chosen). It is clear that if the data are generated by a non-satiated utility function, then they must satisfy GARP. Conversely, the following result due to Afriat (1967) tells us that if a *finite* data set generated by an individual's choices satisfies GARP, then the data can be rationalized by a well-behaved utility function.

Afriat's Theorem If the data set $\left\{(p^i, x^i)\right\}$ satisfies GARP, then there exists a piecewise linear, continuous, increasing, concave utility function

$u(x)$ such that for each observation (p^i, x^i)

$$u(x) \leq u(x^i) \text{ for any } x \text{ such that } p^i \cdot x \leq p^i \cdot x^i.$$

Hence, in order to show that the data are consistent with utility-maximizing behavior we must check whether it satisfies GARP. While verifying GARP is conceptually straightforward, it can be difficult in practice. Even moderately large data sets require an efficient algorithm to compute the transitive closure R of the direct revealed preference relation R^D and check that GARP is satisfied for every pair of portfolios x^i and x^j satisfying $x^i R x^j$. In addition, since GARP offers an exact test (either the data satisfy GARP or they do not) and choice data almost always contain at least some violations, it is desirable to measure the *extent* of GARP violations.

The various indices that have been proposed for this purpose are all computationally intensive.[6] We report measures of GARP violations based on three indices: Afriat (1972), Varian (1991), and Houtman and Maks (1985). Afriat's (1972) *critical cost efficiency index* (CCEI) measures the amount by which each budget constraint must be adjusted in order to remove all violations of GARP. Figure 2 illustrates one such adjustment for a simple violation of GARP involving two portfolios, x^1 and x^2. It is clear that x^1 is revealed preferred to x^2 because $p^1 \cdot x^1 > p^1 \cdot x^2$, yet x^1 is cheaper than x^2 at the prices at which x^2 is purchased, $p^2 \cdot x^1 < p^2 \cdot x^2$. If we shifted the budget constraint through x^2 as shown $(A/B < C/D)$, the violation would be removed.[7]

[Figure 2 here]

This suggests the following approach. For any number $0 \leq e \leq 1$, define the direct revealed preference relation $R^D(e)$ as $x^i R^D(e) x^j$ if $ep^i \cdot x^i \geq p^i \cdot x^j$, and define $R(e)$ to be the transitive closure of $R^D(e)$. Let e^* be the largest value of e such that the relation $R(e)$ satisfies GARP. Afriat's CCEI is the value of e^* associated with the data set $\{(p^i, x^i)\}$. It is bounded between zero and one and can be interpreted as saying that the consumer is 'wasting' as much as $1 - e^*$ of his income by making inefficient choices. The closer the CCEI is to one, the smaller the perturbation of the budget constraints required to remove all violations and thus the closer the data are to satisfying GARP.

[6] The computer program and details of the algorithm are available from the authors upon request.

[7] Here we have a violation of the Weak Axiom of Revealed Preference (WARP) since $x^1 R^D x^2$ and $x^2 R^D x^1$.

Although the CCEI provides a summary statistic of the overall consistency of the data with GARP, it does not give any information about which of the observations (p^i, x^i) are causing the most severe violations. A single large violation may lead to a small value of the index while a large number of small violations may result in a much larger efficiency index. Varian (1991) refined Afriat's CCEI to provide a measure that reflects the minimum adjustment required to eliminate the violations of GARP associated with each observation (p^i, x^i). In particular, fix an observation (p^i, x^i) and let e^i be the largest value of e such that $R(e)$ has no violations of GARP within the set of portfolios x^j such that $x^i R(e) x^j$. The value e^i measures the efficiency of the choices when compared to the portfolio x^i.

Knowing the efficiencies $\{e^i\}$ for the entire set of observations $\{(p^i, x^i)\}$ allows us to say where the inefficiency is greatest or least. These numbers may still overstate the extent of inefficiency, however, because there may be several places in a cycle of observations where an adjustment of the budget constraint would remove a violation of GARP and the above procedure may not choose the 'least costly' adjustment. Varian (1991) provides an algorithm that will select the least costly method of removing all violations by changing each budget set by a different amount. When a single number is desired, as here, one can use $e^* = \min\{e^i\}$. Thus, Varian's (1991) index is a lower bound on the Afriat's CCEI.

The third test, proposed by Houtman and Maks (1985) (HM), finds the largest subset of choices that is consistent with GARP. This method has a couple of drawbacks. First, some observations may be discarded even if the associated GARP violations could be removed by small perturbations of the budget constraint. Further, since the algorithm is computationally very intensive, we were unable to compute the HM index for a small number of subjects (ID 211, 324, 325, 406, 504 and 608) with a large number of GARP violations. In those few cases we report upper bounds on the consistent set.

Table 1 lists, by subject, the number of violations of the Weak Axiom of Revealed Preference (WARP) and GARP, and also reports the values of the three indices. Subjects are ranked according to (descending) CCEI scores. We allow for small mistakes resulting from the imprecision of a subject's handling of the mouse. The results presented in Table 1 allow for a narrow confidence interval of one token (i.e. for any i and $j \neq i$, if $d(x^i, x^j) \leq 1$ then x^i and x^j are treated as the same portfolio). Turning now to GARP violations, out of the 93 subjects, 80 subjects (86.0 percent) had CCEI scores above 0.90 and of those, 75 subjects (80.6 percent) were above 0.95.

[Table 1 here]

While these scores look satisfactory, there is no natural threshold for determining whether subjects are close enough to satisfying GARP that they can considered utility maximizers. Varian (1991) suggests a threshold of 0.95 for the CCEI, but this is purely subjective. A more scientific approach was proposed by Bronars (1987). His method calibrates the various indices using the choices of a hypothetical subject whose choices are uniformly distributed on the budget line. We generated a random sample of 25,000 subjects and found that their scores on the Afriat and Varian efficiency indices averaged 0.60 and 0.25 respectively.[8] Furthermore, all 25,000 random subjects violated GARP at least once. If we choose the 0.9 efficiency level as our critical value, we find that only 12 of the random subjects' CCEI scores were above the threshold and none of the random subjects' Varian efficiency scores were above this threshold.

Figure 3A compares the distributions of the CCEI scores generated by the sample of hypothetical subjects and the distributions of the scores for the actual subjects. The horizontal axis shows the value of the index and the vertical axis measures the percentage of subjects corresponding to each interval. Similarly, Figure 3B compares the distributions of the Varian efficiency index. The histograms show that actual subject behavior has high consistency measures compared to the behavior of the hypothetical random subjects. The graph clearly shows that a significant majority of the subjects did much better than the randomly generated subjects and only a bit worse than an ideal (substantively rational) subject. Finally, Figure 3C shows the distribution of the HM index. Note that we cannot generate a distribution of this index for random subjects because of the computational load.

[Figure 3 here]

The power of Bronars' (1987) test is defined to be the probability that a random subject violates GARP. It has been applied to experimental data by Cox (1997), Sippel (1997), Mattei (2000) and Harbaugh, Krause and Berry (2001). Here all the random subjects had violations, implying the Bronar criterion attains its maximum value. Our experiment is sufficiently powerful to exclude the possibility that consistency is the accidental result of random behavior. Therefore, the consistency of our subjects' behavior under these conditions is not accidental. The power of the Bronars test in this case depends on two factors. The first is that the range of choice sets is generated so that budget lines cross frequently (see Andreoni and Harbaugh,

[8] Each of the 25,000 random subjects makes 50 choices from randomly generated budget sets, in the same way as the human subjects do.

2005). The second is that the number of decisions made by each subject is large. This is a crucial point, because in most experimental studies, the number of individual decisions is too small to provide a powerful test.

To illustrate this point, we simulated the choices of random subjects in two experiments which used the design of this paper except that in one subjects made 10 choices and in the other they made 25 choices. In each case, the simulation was based on 25,000 random subjects. In the simulated experiment with 25 choices, 4.3 percent of random subjects were perfectly consistent, 14.3 percent had CCEI scores above Varian's 0.95 threshold, and 28.9 percent had values above 0.90. In the simulated experiment with only 10 choices, the corresponding percentages were 20.2, 37.3, and 50.6. In other words, there is a very high probability that random behavior will pass the GARP test if the number of individual decisions is as low as it usually is in experiments. As a practical note, the consistency results presented above suggest that subjects did not have any difficulties in understanding the procedures or using the computer program.

4 Data Description

We next provide an overview of some basic features of the individual-level data. Naturally, subjects bring to any decision task a number of rules of thumb, or *heuristics*, that they have acquired previously and that help them solve the problem at hand.[9] Moreover, a subject's "success" or "failure" in the experiment results from the appropriateness of the heuristics he uses as much as the inherent difficulty of the decision-making. It is plausible, of course, that subjects following a heuristic might behave "as if" they were rational maximizing individuals, even though it would be quite implausible to expect them to be able to "solve" for the optimal choice. To the extent that a subject's behavior in an experiment approximates that of a rational maximizing individual, it is probably because the task is sufficiently transparent to allow him to apply heuristics that approximate the optimal decision rule.

The fact that choices are sufficiently consistent to be considered utility-generated is a striking result in its own right, but consistency is endogenous: in a complex decision problem subjects may be forced to adopt heuristics; and the decision-making problem may be simplified as a result. Neverthe-

[9] Heuristics have been proposed to explain how individuals choose when facing complex decison problems. These rules work well under most circumstances, but in certain cases lead to systematic cognitive biases.

less, beyond satisfying consistency, the individual-level data yield a number of surprising features which provide important insights into the heuristic procedures followed by our subjects and the structure of preferences. In our experimental design, the symmetric treatment, in which the two states have equal probabilities ($\pi = 1/2$), is particularly transparent so it is possible readily to identify, simply from the scatterplots of their choices, subjects whose choices correspond to *prototypical heuristics* or *types*. We will focus on examples that reveal some of the unexpected features of the data and illustrate the role of heuristics. This also helps us get a sense of the challenges of substantive rationality. One must remember, however, that for most subjects the data is much less regular.

4.1 Heuristics

We next describe particular allocations chosen by individual subjects. As a preview, Figure 4 shows the choices made by subjects who follow easily identifiable heuristics in making their decisions.[10] In each case, the heuristic may easily be related to some notion of risk aversion, but we defer this discussion to the next section, and focus here on procedural rationality via the identification of heuristics.

For each subject, the left panel of Figure 4 depicts the portfolio choices (x_1, x_2) as points in a scatterplot. This panel provides information about the particular portfolios chosen by an individual subject. The middle and right panels show, respectively, the relationship between $\log(p_1/p_2)$, on the one hand, and $x_1/(x_1 + x_2)$ and $p_1x_1/(p_1x_1 + p_2x_2)$, on the other. These panels examine the sensitivity of portfolio decisions to changes in relative prices in terms of token shares and expenditure shares, respectively. The scatterplots reveal striking regularities *within* and marked heterogeneity *across* subjects.

[Figure 4 here]

Figure 4A depicts the choices of a subject (ID 304) who always chose nearly equal portfolios $x_1 = x_2$. We refer to this heuristic as the *diagonal heuristic* (D). Additionally, we find many cases of subjects whose choices demonstrate "smooth" responsiveness of portfolio allocations to the prices, which we refer to below as the *smooth price responsiveness heuristic* (S). Among these subjects, we find considerable heterogeneity in price sensitivity. Figure 4B depicts the choices of a subject (ID 309) who decreases the

[10] The scatterplots for the full set of subjects are available for downloading at http://ist-socrates.berkeley.edu/~kariv/CFGK_A1.pdf.

fraction of his portfolio invested in asset x_1 (middle panel) and the fraction of expenditure on x_1 (right panel) as $\log(p_1/p_2)$ increases (i.e. positive price responsiveness). This subject is therefore concerned with increasing expected payoffs rather than reducing differences in payoffs. In contrast, Figure 4C depicts the choices of a subject (ID 306) who decreases the fraction of his portfolio invested in asset x_1 (middle panel) but increases the fraction of expenditure on x_1 (right panel) as $\log(p_1/p_2)$ increases (i.e. negative price responsiveness). This subject is thus concerned with reducing differences in payoffs rather than increasing expected payoffs.

Perhaps a more interesting kind of regularity is illustrated in Figure 4D, which depicts the decisions of subject (ID 205) who chooses some minimum level of consumption in each state, and allocates the residual to the less expensive security. We denote the minimum level of consumption in each state by $\omega \geq 0$, where $\omega = 0$ if the subject chose the boundary allocation and $\omega > 0$ if he demands a positive minimum payoff in each state. We refer to this guaranteed minimum payoff as a *secure level heuristic* ($B(\omega)$). Most interestingly, in the symmetric treatment, no subject followed heuristic $B(0)$, corresponding to boundary allocations, which is the natural limit of the secure level heuristic $B(\omega)$. Nevertheless, heuristic $B(0)$ was used in combination with one or both of the prototypical heuristics, D and S, by many subjects, as we will see below.

Most interestingly, many subjects combine two or more of the prototypical heuristics D, S and $B(\omega)$. Figure 4E depicts the decisions of the subject (ID 307) who combines heuristic D, for values of $\log(p_1/p_2)$ in a neighborhood of zero, with heuristic $B(0)$ for values of $\log(p_1/p_2)$ that give a steep or flat budget line. There is obviously something distinctly discontinuous in the behavior and it thus seems natural to think of this subject as switching between two distinct heuristics, D and $B(0)$, rather than following a single complicated heuristic. Further, some subjects combine heuristic S in their mixture of heuristics. For example, the subject (ID 216) whose choices are depicted in Figure 4F combines a heuristic $B(0)$ with heuristic S, and the subject (ID 318) whose choices are depicted in Figure 4G combines all three distinct prototypical heuristics, D, S and $B(\omega)$. Finally, note that there are yet more complex cases, such as the subject (ID 213) whose choices are depicted in Figure 4H.

We have obviously shown just a small subset of our full set of subjects, and have chosen them to illustrate the role of heuristics. These are of course special cases, where the regularities in the data are very clear. There are many subjects for whom the behavioral rule is much less clear and there is no taxonomy that allows us to classify all subjects unambiguously. But even in cases that are harder to classify, we can distinguish elements of the prototypical heuristics, D, S and $B(\omega)$, described above. Overall, a review of the full data set confirms the heterogeneity of individual behavior and the prevalence of identifiable heuristics that inform subjects' decision rules.

4.2 Preferences

The particular portfolios chosen by individual subjects tell us a lot about how subjects come to make decisions that are almost consistent with GARP. Now we turn to the problem of recovering underlying preferences using the revealed preference techniques developed by Varian (1982, 1983). This approach is purely non-parametric and uses only information about the revealed preference relations. In particular, it makes no assumptions about the form, parametric or otherwise, of the underlying utility function. Since we observe many choices over a wide range of budget sets, we can in many cases describe preferences with some precision.

Varian's algorithm provides the tightest possible bounds on indifference curves through a portfolio x^0, which has not been observed in the previous data (p^i, x^i) for $i = 1, ..., 50$. First, we consider the set of prices at which x^0 could be chosen, consistently with the observed data and the implied revealed preference relations. This set of prices is the solution to a system of linear inequalities constructed from the data and revealed preference relations. Call this set $S(x^0)$. Second, we use $S(x^0)$ to generate the set of portfolios, $RP(x^0)$, revealed preferred to x^0 and the set of portfolios, $RW(x^0)$, revealed worse than x^0. It is not difficult to show that $RP(x^0)$ is simply the convex monotonic hull of all observations revealed preferred to x^0. To understand the construction of $RW(x^0)$, note that if $x^0 R^D x^i$ for all prices $p^0 \in S(x^0)$, then $x^0 R x^j$ for any portfolio x^j such that $x^i R^D x^j$, and so on. Hence, the two sets $RP(x^0)$ and the complement of $RW(x^0)$ form the tightest inner and outer bounds on the set of allocations preferred to x^0. Similarly, $RW(x^0)$ and the complement of $RP(x^0)$ form the tightest inner and outer bounds on the set of allocations worse than x^0.

Figure 5 depicts the construction of the bounds described above through some portfolio x^0 for the same group of subjects that we examine in Figure 4. Since the data are clustered in very different areas of the graphs for different subjects, we look at indifference curves through the "average" choices of each subject. In addition to the $RW(x^0)$ and $RP(x^0)$ sets, Figure 5 also shows the subjects' choices $(x^1, ..., x^{50})$ as well as the budget sets used to construct $RW(x^0)$. Most importantly, note the tightness of some of the sets and the differences among subjects. Finally, we note that our computational experience with this technique reveals that if the data are not very close to satisfying GARP then $RP(x^0)$ and $RW(x^0)$ often overlap.

[Figure 5 here]

Interestingly, for the subject (ID 304) whose choices were governed by heuristic D and the subject (ID 306) whose choices were governed by heuristic S with negative price responsiveness, the bounds on the indifference curve suggest a near right angled indifference curve, implying a very high degree of risk aversion. Another interesting case is the subject (ID 205) whose choices were governed by heuristic $B(\omega)$ with positive minimum level of consumption in each state. In this case, the indifference curve bounds suggest a kink at the secure level. For the subject (ID 307) who combines heuristic D with heuristic $B(0)$, the bounds on the indifference curve imply a linear indifference curve with slope close to -1. The bounds in Figure 5 show a particularly close fit, but experience suggests that we can generally provide reasonably precise bounds for subjects with a high consistency index, as long as x^0 is chosen within the convex hull of the data $\{x_n^i\}$.

4.3 Theoretical and empirical implications

How can we explain the very distinct types of individual behavior revealed by the data? Since subjects' choices are close to being consistent, Afriat's theorem tells us that there exists a well-behaved utility function that rationalizes most of the data. So one approach would be to posit the "kinky" preference ordering implied by Afriat's theorem and go no further in attempting to rationalize the data. This approach has its attractions, but the "switching" behavior that is evident in the data leads us to prefer an alternative approach, one that emphasizes procedural rationality.

Suppose a subject does have an underlying preference ordering over portfolios that represents his true preferences, but that it is difficult for him to be sure he is making the correct choice in a particular decision problem. If his cognitive ability is low and the cost of computing an optimal decision is high, he may find it better to adopt a heuristic that only roughly approximates his optimal decision in some cases. For example, if the security prices are very different and the true optimum is likely to be close to the boundary, it is not worth calculating the true optimum. Instead, he chooses the boundary portfolio with the larger expected payoff. On the other hand, if the security prices are very similar and the true optimum is likely to be near the diagonal, it is not worth thinking hard about the true optimum. Instead, he chooses the diagonal portfolio.

For intermediate prices ratios, neither of these "short cuts" will be attractive. In this case, the subject may attempt to find the optimal tradeoff between risk and return. Note that mistakes are more likely to occur when a subject attempts to maximize than when he chooses the diagonal or the boundary portfolios and this is another "cost" of adopting more complex decision rules. This also suggests that consistency is endogenous: in a complex situation subjects may be forced by bounded rationality to adopt simple decision rules. As a result of this "simplification," their decision making is more likely to be consistent.

Finally, guaranteeing a minimum payoff as a secure level can be rationalized in a similar way. It is a crude approximation to maximizing behavior when the subject is too risk averse to choose the boundary, but not sufficiently risk averse to choose the diagonal, and finds an attempt at maximizing the risk-return tradeoff too costly.

The procedural rationality approach outlined above has several advantages compared to what we might call the substantive rationality approach based on Afriat's theorem. It is more informative, in the sense that it attempts to derive a general account of behavior from simple and familiar elements - constant relative risk aversion and discrete choice among heuristics - that can be applied in other contexts, whereas a non-parametric utility function of the kind guaranteed by Afriat's theorem is hard to interpret and impossible to apply to other settings. Further, since our approach assumes an underlying preference ordering, it provides a unified account of both procedural and substantive rationality, which is interesting in its own right. Finally, by allowing expected utility maximization to play the role of the underlying preference ordering, we can link our approach to the classical theory of decision making under uncertainty.

To implement our approach, we need to estimate a structural model that will simultaneously account for subjects' underlying preferences and their choice of decision rules. In the next section, we will describe a *type-mixture model* (TMM) in which a subject chooses among the fixed set of types or prototypical heuristics, D, S and $B(\omega)$, in order to approximate the behavior that is optimal for his true underlying preferences. Obviously, with a sufficient number of heuristics, any choice data may be explained. However, the description of individual-level data in this section suggests that we may be able to effectively characterize behaviors with a minimal number of heuristics.

5 Risk aversion

Before undertaking the structural estimation route, we first use the "low-tech" approach of estimating an individual-level parametric constant relative risk aversion (CRRA) utility function directly from the data. In addition to providing individual-level estimates of a simple expected-utility model, this exercise will also provide standard measures of risk aversion which we can compare to the estimates of risk aversion that come out of the TMM. We emphasize again that the graphical representation enables us to collect many more observations per subject than has heretofore been possible and therefore to generate better individual-level estimates.

A particular functional form commonly employed in the analysis of choice under uncertainty is the power utility function

$$u(x) = \frac{x^{1-\rho}}{(1-\rho)},$$

where ρ is the Arrow-Pratt measure of relative risk aversion. The power utility function approaches pure risk neutrality as $\rho \to 0$ and pure risk aversion as $\rho \to \infty$ (the aversion to risk increases as ρ increases). As ρ approaches 1, the power utility function approaches $\ln(x)$. Logarithmic preferences imply that the fraction of the portfolio invested in each security is independent of the price ratio. Further, if $\rho > 1$ (resp. $0 < \rho < 1$) a fall in the price of a security lowers (resp. raises) the fraction of the portfolio invested in that security. Thus, a risk parameter $\rho > 1$ indicates a preference weighted towards reducing differences in payoffs, whereas a risk aversion parameter $0 \leq \rho < 1$ indicates preferences weighted towards increasing total expected payoffs.

By straightforward calculation, the solution to the maximization problem (x_1^*, x_2^*) satisfies the first-order condition

$$\frac{\pi}{1-\pi}\left(\frac{x_2^*}{x_1^*}\right)^{\rho} = \frac{p_1}{p_2}$$

and the budget constraint $p \cdot x^* = 1$. This generates the following individual-level econometric specification for each subject n:

$$\log\left(\frac{x_{2n}^i}{x_{1n}^i}\right) = \alpha_n + \beta_n \log\left(\frac{p_{1n}^i}{p_{2n}^i}\right) + \epsilon_n^i$$

where ϵ_n^i is assumed to be distributed normally with mean zero and variance σ_n^2. We generate estimates of $\hat{\alpha}_n$ and $\hat{\beta}_n$ using ordinary least squares (OLS), and use this to infer the values of the underlying parameter $\hat{\rho}_n = 1/\hat{\beta}_n$ and the implied *subjective* probability $\hat{\pi}_n = 1/(1 + e^{\hat{\alpha}_n/\hat{\beta}_n})$. By contrast with studies that pool data from different subjects, our data set is large enough to allow the estimation of the parameters $\hat{\rho}_n$ and $\hat{\pi}_n$ for each subject n separately. In particular, it allows us to test for heterogeneity of risk preferences.

Before proceeding to the estimations, we omit the nine subjects with CCEI scores below 0.80 (ID 201, 211, 310, 321, 325, 328, 406, 504 and 603) as their choices are not sufficiently consistent to be considered utility-generated, three subjects (ID 205, 218 and 320) who almost always follow the secure level heuristic $B(10)$, and a single subject (ID 508) who almost always chose a boundary portfolio.[11] This leaves a total of 80 subjects (86.0 percent) for whom we need to recover the attitudes towards risk by estimating the power model. For these subjects, we discard the boundary observations using a narrow confidence interval of one token (i.e. if $x_1^i \leq 1$ or $x_2^i \leq 1$ then x^i is treated as a boundary portfolio), for which the power function is not well defined. The boundary observations will be directly incorporated into our TMM estimation below. Our taxonomy of heuristics suggests that the risk aversion parameter is best estimated using choices that correspond to the smooth price responsiveness heuristic S. We include diagonal observations as well, though their inclusion does not substantially affect the estimated coefficients.

Table 2 presents the results of the estimations $\hat{\alpha}_n$, $\hat{\beta}_n$, $\hat{\rho}_n$ and $\hat{\pi}_n$ sorted according to ascending values of $\hat{\rho}_n$ for the symmetric ($\pi = 1/2$) and asymmetric ($\pi = 1/3$ and $\pi = 2/3$) treatments separately, as well as the number of observations per subject. Notice again that we screen the data for boundary observations, which results in many fewer observations for a small number of subjects. Nevertheless, out of the 80 subjects listed in Table 3, 33 subjects (41.3 percent) have no boundary observations and this increases to a total of 60 subjects (75.0 percent) if we consider less than five boundary observations.

[Table 2 here]

Figure 6 presents the distribution of the estimated individual Arrow-Pratt measures $\hat{\rho}_n$ for all subjects listed in Table 2, split by symmetric (black) and asymmetric (gray) treatments (subjects ID 304, 307, 311 and

[11] This subject (ID 508) almost always chose $x_1 = 0$ if $p_1 > p_2$ and $x_2 = 0$ otherwise. However, he participated in the asymmetric treatment $\pi_1 = 2/3$ and thus his choices do not correspond to pure risk neutrality.

324 are excluded because they have extreme $\hat{\rho}$-values). One notable feature of the distributions in Figure 6, is that both the symmetric and asymmetric subsamples exhibit considerable heterogeneity in preferences, though the two distributions are quite similar. Most interestingly, a significant fraction of our subjects in both treatments exhibit moderate to high levels of risk preferences around $\hat{\rho} = 0.8$. For 41 of the 80 subjects listed in Table 2 (51.3 percent), $\hat{\rho}_n$ is in the $0.6 - 1.0$ range. This increases to a total of 51 subjects (63.8 percent) if we consider the bounds $0.4 - 1.2$.

[Figure 6 here]

There have been many attempts to recover risk preferences from subjects' decisions in a variety of laboratory experiments. Our levels of ρ are slightly higher than some recent estimates. For comparison, Chan and Plott (1998) and Goeree, Holt and Palfrey (2002) report, respectively, $\rho = 0.48$ and 0.52 for private-value auctions. Goeree, Holt and Palfrey (2003) estimate $\rho = 0.44$ for asymmetric matching pennies games, and Goeree and Holt (2004) report $\rho = 0.45$ for a variety of one-shot games. Holt and Laury (2002) estimate individual degrees of risk aversion from ten paired lottery-choices under both low- and high-money payoffs. Most of their subjects in both treatments exhibit risk preferences around the $0.3 - 0.5$ range. Note, however, that our estimates are possibly biased upward because we omitted boundary observations.

Finally, notice that, on average, the estimated subjective probabilities $\hat{\pi}_n$ are close to the true probabilities (averaging 0.49 in the symmetric treatment, and 0.38 and 0.62 in the asymmetric treatments $\pi = 1/3$ and $\pi = 2/3$, respectively), though in the asymmetric treatments, the averages are biased towards 1/2.

In summary, we find that a simple OLS regression based on the CRRA utility function gives plausible estimates of individual risk aversion in the laboratory. The distinct patterns observed in individual data, however, suggest that a more complex formulation, incorporating the use of one or more heuristics, is necessary to fully interpret the data. Introducing heuristics will also provide an explicit linkage between substantive and procedural rationality, as we see in the next section.

6 Type-Mixture Model (TMM)

The patterns observed in the individual-level data suggest very distinct patterns of individual behavior, typically governed by mixtures of a small number of heuristics. A coherent account of individual behavior requires us to explain the choice of heuristics as well as the behavior of the individual who is following one of these heuristics. For this purpose, the natural econometric specification is a TMM and in what follows we will estimate a TMM for each subject n using the data $\left\{\left(p_n^i, x_n^i\right)\right\}_{i=1}^n$.

Other things being equal, the more heuristics are included in the model the easier it is to fit the data and the less we learn from the exercise. For this reason we have chosen a very parsimonious specification in which we attempt to explain subjects' behavior using the three heuristics, D, S and $B(\omega)$, that are easily identified in the data. Before we can estimate a structural model of the choice of heuristics, we need a theoretical model of choice. Again, we have tried to do this in the simplest possible way in order to avoid introducing unnecessary parameters. The basic problem is to explain why heuristics D and $B(\omega)$ are not dominated by S. We could introduce ad hoc "psychic costs" of using the S heuristic, but since we do not observe such costs we prefer to take another route. By comparison with heuristics D and $B(\omega)$, which are very simple to implement, making smooth tradeoffs between risk and return by using heuristic S is difficult. Whereas most subjects can find the diagonal and the boundary without difficulty, it seems unlikely that an individual whose "true" underlying preferences are represented by a power utility function chooses the optimal point without error. This provides us with an intuitive and observable cost of using the S heuristic: subjects are likely to make mistakes, and this randomness together with risk aversion will lower expected utility below that achieved by a substantively rational individual. As a result, a subject may prefer under certain circumstances to choose heuristic $B(\omega)$ or D instead of the noisy version of heuristic S.

Another advantage of this approach, apart from its simplicity, is that we can estimate the probability of mistakes at the same time as we estimate the other parameters of the model. For any budget set defined by the price vector $p = (p_1, p_2)$ we can represent the optimal choice $\varphi(p)$ by the share of expenditure invested in x_1 and represent the errors by a disturbance term ε with zero mean and variance σ^2. For any value of the risk aversion parameter ρ and secure level ω, the optimal share is a function of $p = (p_1, p_2)$. The error ε is the difference between the predicted share and the observed share. It is the distribution of these errors that allows us to estimate the value of σ.

A subject with a secure level ω faced with a budget set defined by the price vector p will have three choices. He can choose heuristic $B(\omega)$ and receive a payoff (expected utility) that we denote by $U_B(p)$; he can choose heuristic S and receive a random payoff of $U_S(p)$; or he can choose heuristic

D and receive a payoff $U_D(p)$. The exact expressions for these payoffs are given below; the important point is that the individual is assumed to know the payoff associated with each heuristic and that the payoff specification for heuristic S incorporates the possibility of mistakes. Because individuals make mistakes, we allow for the possibility that subjects do not necessarily choose the highest payoff. Instead we assume that the probability of choosing a particular heuristic is a function of the relative payoff to that heuristic.

Specifically, we implement this by using a standard logistic discrete choice model, in which the probability of choosing heuristic $k = D, S, B$ is given by

$$\Pr(\text{heuristic } \tau \,|p, \beta, \rho, \sigma\,) = \frac{e^{\beta U_\tau}}{\sum\limits_{k=D,S,B} e^{\beta U_k}}.$$

The coefficient $\beta \in [0, \infty)$ reflects the sensitivity to differences in payoffs, where the choice of heuristic becomes purely random as $\beta \to 0$, whereas the heuristic with the highest payoff is chosen with certainty as $\beta \to \infty$. Further, the probability of choosing heuristic k is an increasing function of the payoff U_k for any $\beta > 0$.

Before proceeding to the estimation, we note the effect of ρ and σ on choice probabilities via the logistic function given above. For moderate values of σ and moderate to high levels of $\rho > 0$, heuristic D yields the highest payoff when the prices are quite similar. For any σ, increasing ρ increases the range of intermediate prices around the diagonal $p_1 = p_2$ on which heuristic D yields the highest payoff. On the other hand, for any $\rho > 0$, increasing σ decreases the payoff for heuristic S. Hence, the concavity of the power utility function (positive risk aversion) has a crucial role in determining the choice probabilities.

Our model of boundedly rational choice thus depends on three parameters: the logistic parameter β, the risk aversion parameter ρ, and the standard deviation σ. The secure level ω is directly observed by the econometrician, and is thus not estimated. In the remainder of this section we explain the estimation procedure and discuss the empirical results. The reader who is not interested in technicalities may wish to skip to the discussion of the results in Table 3 below.

6.1 Specification

A subject's type is determined both by his underlying preferences, which represent his (substantive) rationality, and by the limits of his cognitive ability. Specifically, the underlying preferences of each subject n are assumed to be represented by a power utility function

$$u_n(x) = \frac{x^{1-\rho_n}}{(1-\rho_n)}$$

as long as his consumption in each state meets the secure level ω_n.[12, 13] Let $\varphi(p)$ be the portfolio which gives the subject the maximum (expected) utility achievable at given prices p.

The subject might not have the cognitive ability necessary to discover his optimal portfolio $\varphi(p)$. Clearly, his ability to calculate is limited and he is thus likely to make "mistakes" if the decision problem is not simplified under some circumstances. This is the behavioral interpretation of the TMM specification. In order to account for subjects' propensity to choose a portfolio different from the one predicted by the basic model $\varphi(p)$, we assume that subjects anticipate making mistakes in their attempt to maximize expected utility. The possibility of mistakes results in a random portfolio $\tilde{\varphi}(p)$ such that $p \cdot \tilde{\varphi}(p) = 1$ for every p.

This generates a simple payoff specification for each heuristic D, S and B that we note, respectively, by $U_D(p)$, $U_S(p)$ and $U_B(p)$. More precisely, the *ex ante* expected payoff from attempting to maximize expected utility by employing the smooth price responsiveness heuristic S is given by

$$U_S(p) = \mathbb{E}[\pi u(\tilde{\varphi}_1(p)) + (1-\pi)u(\tilde{\varphi}_2(p))].$$

For parametric tractability, the optimal fraction of total expenditure going to asset 1 is implicitly defined by

$$p_1\varphi_1(p) = \frac{1}{1 + \left(\frac{1-\pi}{\pi}\right)^{1/\rho}\left(\frac{p_1}{p_2}\right)^{(1-\rho)/\rho}}.$$

The difference between the actual share of expenditure $p_1\tilde{\varphi}_1(p)$ and the optimal share of expenditure $p_1\varphi_1(p)$ is denoted by the random variable ε, whose density function $\phi(\cdot;\sigma)$, is normal with mean zero and standard

[12] In practice, the individual secure level ω_n can be inferred from the observed choices. Only three subjects (ID 205, 218 and 320) choose strictly positive secure levels, $\omega = 10$. For these three subjects, we assume utility is a function of the net demand, i.e., the utility of x tokens is $u_n(x - \omega)$.

[13] The utility function $u(x)$ is not well defined when $x = 0$ and $\rho > 1$. For computational purposes we substitute the perturbed function $u(0.1 + x)$, where x is the number of tokens observed.

deviation σ. We therefore rewrite the *ex ante* expected payoff from heuristic S as

$$U_S(p;\rho,\sigma)=\int_{-\infty}^{\infty}\left[\pi u\left(\frac{g\left(p_1\varphi_1(p)+\varepsilon\right)}{p_1}\right)+(1-\pi)\,u\left(\frac{\left(1-g\left(p_1\varphi_1(p)+\varepsilon\right)\right)}{p_2}\right)\right]\phi(\varepsilon,\sigma)\,d\varepsilon,$$

where the function $g:\mathbf{R}\rightarrow[0,1]$ is defined by

$$g(\xi)=\begin{cases}0 & \text{if}\quad \xi<0\\ \xi & \text{if}\quad 0\leq\xi\leq 1\\ 1 & \text{if}\quad \xi>1.\end{cases}$$

While it would be quite implausible to expect subjects to be able to solve for the optimal portfolio $\varphi(p)$, because of the problem's complexity, it is plausible to assume that when following heuristic D or B subjects' hands do not tremble. By direct calculation, we therefore write

$$U_D(p)=u\left(\frac{1}{p_1+p_2}\right)$$

and

$$U_B(p)=\begin{cases}\pi u(0)+(1-\pi)u(\frac{1}{p_2}) & \text{if}\quad \pi p_1\leq(1-\pi)\,p_2\\ \pi u\left(\frac{1}{p_1}\right)+(1-\pi)u(0) & \text{if}\quad \pi p_1>(1-\pi)\,p_2\end{cases}.$$

[14]

6.2 Estimation

Let $\{(p_n^i,x_n^i)\}_{i=1}^{50}$ be the observed individual data of subject n. Then the probability of an observation (p_n^i,x_n^i) is given by the likelihood function

$$L_n\left((p_n^i,x_n^i)\right)=\sum_{k=D,S,B}\Pr\left(\tau=k;p_n^i,\beta_n,\rho_n,\sigma_n\right)\Pr\left(x_n^i|\tau=k,p_n^i\right)$$

where the probability that (p_n^i,x_n^i) is the result of choosing heuristic B is given by

$$\Pr\left(x_n^i|\tau=B,p_n^i\right)=\begin{cases}1 & \text{if } x_{1n}^i\in[0,1) \text{ and } \pi p_{1n}^i\leq(1-\pi)p_{2n}^i\\ 1 & \text{if } x_{2n}^i\in[0,1) \text{ and } \pi p_{1n}^i>(1-\pi)p_{2n}^i\\ 0 & \text{otherwise;}\end{cases}$$

[14] Note the indifference assumption that $x_1=0$ when $\pi p_1=(1-\pi)p_2$. Since these are probability zero events, this does not affect the estimation.

the probability that (p_n^i, x_n^i) is the result of choosing heuristic D is given by

$$\Pr\left(x_n^i|\tau = D, p_n^i\right) = \begin{cases} 1 & x_{1n}^i \in \left(x_{2n}^i - 1, x_{2n}^i + 1\right) \\ 0 & \text{otherwise;} \end{cases}$$

and the probability that (p_n^i, x_n^i) is the result of choosing heuristic S is given by the two-sided Tobit model

$$\begin{aligned}\Pr\left(x_n^i|\tau = S, p_n^i\right) = \; & \Phi\left(-\frac{p_{1n}^i \varphi_1(p_n^i)}{\sigma_n}\right)^{1\left\{x_{1n}^i \in [0,1)\right\}} \left[1 - \Phi\left(\frac{1 - p_{1n}^i \varphi_1(p_n^i)}{\sigma_n}\right)\right]^{1\left\{x_{2n}^i \in [0,1)\right\}} \\ & \times \left[\left(\frac{1}{\sigma_n \sqrt{2\pi}}\right) \exp\left(-\frac{\left(p_{1n}^i x_{1n}^i - p_{1n}^i \varphi_1(p_n^i)\right)^2}{\sigma_n^2}\right)\right]^{1\left\{x_{1n}^i \notin [0,1), x_{2n}^i \notin [0,1)\right\}},\end{aligned}$$

where $1_{x_{1n}^i}$ is an indicator function which allows for a narrow confidence interval of one token and $\Phi(\cdot)$ is the cumulative standard normal distribution. Note again that we identify the boundary observations using a narrow confidence interval of one token (i.e. if $x_1^i \leq 1$ or $x_2^i \leq 1$ then x^i is treated as a boundary portfolio). Finally, the extended likelihood function has the form:

$$\mathcal{L}\left(\beta_n, \rho_n, \sigma_n; \{(p_n^i, x_n^i)\}_{i=1}^{50}\right) = \prod_{i=1}^{50} \left\{ \sum_{k=D,S,B} \Pr\left(\tau = k; p_n^i, \beta_n, \rho_n, \sigma_n\right) \Pr\left(x_n^i|\tau = k, p_n^i\right) \right\}.$$

We again omit the nine subjects with CCEI scores below 0.80 (ID 201, 211, 310, 321, 325, 328, 406, 504 and 603) as well the subject (ID 508) for whom the TMM is not well defined. This leaves the group of 80 subjects for whom we estimated parameters for the simple power formulation above, as well as the three subjects (ID 205, 218 and 320) who almost always follow the secure level heuristic $B(10)$. We impose the restriction $0 \leq \rho \leq 5$ on the estimation in order to avoid two identifications problems. We require $\rho \geq 0$ because it is impossible to distinguish pure risk neutrality ($\rho = 0$) from risk loving behavior ($\rho < 0$). We require $\rho \leq 5$ because very high degrees of risk aversion imply portfolio choices close to the diagonal, making it impossible to distinguish D from S. Table 3 presents parameter estimates, $\hat{\beta}_n$, $\hat{\rho}_n$ and $\hat{\sigma}_n$ sorted according to ascending values of $\hat{\rho}_n$. For comparative purposes, the additional columns list the values of $\hat{\rho}_n$ derived from the simple OLS estimation based on CRRA and the CCEI scores.

[Table 3 here]

Although there are some differences in the $\hat{\beta}_n$ estimates across subjects, for most of them these estimates are significantly positive, implying that the TMM has predictive power in interpreting the behavior of selecting heuristics at the level of the individual subject. For a few subjects who use only one heuristic, the values of $\hat{\beta}_n$ are insignificant because the likelihood function becomes flat in the neighborhood of $\hat{\beta}_n$. For the underlying risk parameter, ρ, there are many subjects with intermediate values of $\hat{\rho}_n$, as was the case in the OLS estimation: 29 subjects (34.9 percent) have $0.6 \leq \hat{\rho}_n \leq 1.0$ and this increases to a total of 55 subjects (66.3 percent) who have $0.4 \leq \hat{\rho}_n \leq 1.2$. More interestingly, there is a strong correlation between the estimated $\hat{\rho}_n$ parameters from the individual OLS and the TMM estimations. The advantage of the TMM estimation over the OLS estimation is that it appears to effectively capture subjects' attitudes towards risk while also explaining the selection of heuristics. Figure 7 presents the distribution of $\hat{\rho}_n$ for the TMM estimation, with the sample split by symmetric (black) and asymmetric (gray) treatments. Notice that the distribution shifts slightly to the left when calculated using the TMM estimation as compared to the distribution calculated using the analogous OLS estimator presented in Figure 6. The reason may be the downward bias in the OLS estimates due to the omission of boundary observations. Notice that we obtain once more very similar distributions for the symmetric and asymmetric subsamples.

[Figure 7 here]

6.3 Goodness-of-fit

The results of the TMM estimation show some power in predicting the highly heterogeneous individual behaviors observed in the laboratory. It is instructive to follow the subjects we have considered in the previous section to examine the ability of the parametric TMM to capture the choice of heuristics. To this end, we perform a series of graphical comparisons between the individual choice probabilities predicted by the TMM and empirical choice probabilities. The predicted TMM choice probabilities are calculated using the individual-level $(\hat{\beta}_n, \hat{\rho}_n, \hat{\sigma}_n)$ estimates, and the empirical choice probabilities are estimated using nonparametric regressions over the observed choices of the heuristics D and S. Additionally, we calculate the relationship between $\log(p_1/p_2)$ and the fraction of $x_1/(x_1+x_2)$ for each subject, as predicted by the TMM, and compare this with the analogous relationship derived from the nonparametric estimates.

To generate the nonparametric estimates, we employ the Nadaraya-Watson estimator with a Gaussian kernel function. The optimal bandwidth in the nonparametric kernel regression with a single independent variable is proportional to $(\#\text{obs})^{-1/5}$. For most of the subjects listed in Table 3, the bandwidths selected by trial and error provided properly smoothed kernel regression estimates.[15] To illustrate, Figure 8 shows this set of comparisons for the same group of subjects as in Figure 4 and Figure 5.[16] Notice that the logistic function is not homogeneous of degree zero in prices, whereas the rest of the theory is. In order to remove any "income effects" in the estimation of the TMM, we normalized prices so that $p_1 + p_2 = 1$ when calculating the payoffs from the three heuristics.

[Figure 8 here]

In each of the graphs in Figure 8, a solid blue line represents the nonparametric estimation and a dotted red line represents the analogous parametric TMM estimation. The selected bandwidth is reported in the legend of each panel. For each subject, the left and middle panels compare the nonparametric and parametric probabilities of D^* and $B^*(\omega)$ respectively. We define D^* to be the event that the subject's choice belongs to the diagonal. Given $\sigma > 0$, the probability that this is chosen by heuristic S is approximately zero and thus the event D^* is considered to be chosen by heuristic D. Similarly, the event $B^*(\omega)$ is defined as the event that the portfolio conforms to the secure level ω allocation. Note that the event $B^*(\omega)$ is either chosen by heuristic $B(\omega)$ or S. The non-parametric regression is based on a manual assignment of observations to one of the following three events: D^*, $B^*(\omega)$, and the joint complement of them. Hence, in order to make a meaningful comparison, we contrast the non-parametric regression with the probabilities of D^* and $B^*(\omega)$ generated by the TMM. The right panel compares the nonparametric and parametric relationships between $\log(p_1/p_2)$ and $\hat{x}_1/(\hat{x}_1 + \hat{x}_2)$.

These graphical comparisons provide some indication of goodness-of-fit for the several subjects we used for illustrative purposes throughout the paper. The fit is generally very good except when we compare the probabilities of choosing heuristic D (left panel) for subjects who often chose portfolios near the diagonal. The identification problem in these cases is caused by the difficulty in distinguishing between heuristic D and heuristic S when

[15] The literature on bandwidth selection in nonparametric regression indicates that automatic bandwidth-selection such as such as Generalized Cross Validation is not always preferable to graphical methods with a trial and error approach. See Pagan and Ullah (1999, p.120).

[16] The scatterplots for the full set of subjects are available for downloading at http://ist-socrates.berkeley.edu/~kariv/CFGK_A2.pdf.

the degree of risk aversion is high. This does not reduce the ability of the TMM to describing the relationship between prices and the associated portfolios (right panel). Overall, the empirical data are supportive of the TMM model. We note, however, that the model appears to fit the data best in the symmetric treatment ($\pi = 1/2$). In the asymmetric treatments ($\pi = 1/3$ and $\pi = 2/3$), where decision-making is more complex than in the symmetric treatment, the predictions of the TMM are not as close to relationships revealed in the data.

In conclusion, the TMM combines the distinctive types of behavior observed in the raw data in a coherent theory based on underlying preferences. It produces reasonable estimates of risk aversion, exhibits significant explanatory power in the choice of heuristics, and matches the behavior of subjects as represented by the relationships between the log-price ratio and expenditure shares and between the log-price ratio and token shares. The expected utility maximization underlying the TMM effectively unifies the substantive rationality of the classical theory with the procedural rationality approach.

7 Conclusion

In this paper, we attempt to provide a more general account of choice under uncertainty. Our experimental design contains a couple of fundamental innovations over existing work. We employ graphical representations of the portfolio choice problem, rather than extreme (binary) choices designed to reveal violations of specific axioms. This allows for the collection of a rich individual-level data set. We do not pool data or assume that subjects are homogeneous. Most importantly, our experiment employs a broad range of budget sets that provide a serious test of the ability of EUT and a structural TMM to interpret the data. In this way, we present a systematic experimental study of individual choice under uncertainty.

The basic regularities from our experiment may be summarized as follows: First, a significant majority of our subjects exhibit behavior that appears to be “almost optimizing” in the sense that their choices are close to satisfying GARP. Thus, choices satisfy the standard notion of rationality. Second, individual behaviors are complex and highly heterogeneous, despite the prevalence of a few heuristics that inform their decision rules. Third, a TMM based on EUT employing only few intuitive heuristics - infinite risk aversion, risk neutrality, and intermediate (constant) relative risk aversion - provides reasonable fit at the level of the individual subject and can account for the highly heterogeneous behaviors observed in the laboratory.

Many open questions remain about choice under uncertainty. The experimental techniques that we have developed provide some promising tools for future work, and our results also suggest a number of potential directions. One particularly promising possibility would be to employ a similar methodology incorporating three states and three associated securities, which be utilized to systematically examine the axioms of EUT using both nonparametric and parametric methods. This complexities presented by such a study may demand further methodological and theoretical innovations, which will hopefully spur yet more interesting answers and further questions about choice under uncertainty.

8 Experimental Instructions ($\pi = 2/3$)

Introduction

This is an experiment in decision-making. Research foundations have provided funds for conducting this research. Your payoffs will depend partly only on your decisions and partly on chance. It will not depend on the decisions of the other participants in the experiments. Please pay careful attention to the instructions as a considerable amount of money is at stake.

The entire experiment should be complete within an hour and a half. At the end of the experiment you will be paid privately. At this time, you will receive $5 as a participation fee (simply for showing up on time). Details of how you will make decisions and receive payments will be provided below.

During the experiment we will speak in terms of experimental tokens instead of dollars. Your payoffs will be calculated in terms of tokens and then translated at the end of the experiment into dollars at the following rate:

2 Tokens = 1 Dollar

A decision problem

In this experiment, you will participate in 50 independent decision problems that share a common form. This section describes in detail the process that will be repeated in all decision problems and the computer program that you will use to make your decisions.

In each decision problem you will be asked to allocate tokens between two accounts, labeled x and y. The x account corresponds to the x-axis and the y account corresponds to the y-axis in a two-dimensional graph. Each choice will involve choosing a point on a line representing possible token allocations. Examples of lines that you might face appear in Attachment 1.

[Attachment 1 here]

In each choice, you may choose any x and y pair that is on the line. For example, as illustrated in Attachment 2, choice A represents a decision to allocate q tokens in the x account and r tokens in the y account. Another possible allocation is B, in which you allocate w tokens in the x account and z tokens in the y account.

[Attachment 2 here]

Each decision problem will start by having the computer select such a line randomly from the set of lines that intersect with at least one of the axes at 50 or more tokens but with no intercept exceeding 100 tokens. The lines selected for you in different decision problems are independent of each other and independent of the lines selected for any of the other participants in their decision problems.

To choose an allocation, use the mouse to move the pointer on the computer screen to the allocation that you desire. When you are ready to make your decision, left-click to enter your chosen allocation. After that, confirm your decision by clicking on the Submit button. Note that you can choose only x and y combinations that are on the line. To move on to the next round, press the OK button. The computer program dialog window is shown in Attachment 3.

[Attachment 3 here]

Your payoff at each decision round is determined by the number of tokens in your x account and the number of tokens in your y account. At the end of the round, the computer will randomly select one of the accounts, x or y. For each participant, account y will be selected with 1/3 chance and account x will be selected with 2/3 chance. You will only receive the number of tokens you allocated to the account that was chosen.

Next, you will be asked to make an allocation in another independent decision. This process will be repeated until all 50 rounds are completed. At the end of the last round, you will be informed the experiment has ended.

Earnings

Your earnings in the experiment are determined as follows. At the end of the experiment, the computer will randomly select one decision round from each participant to carry out (that is, 1 out of 50). The round selected depends solely upon chance. For each participant, it is equally likely that any round will be chosen.

The round selected, your choice and your payment will be shown in the large window that appears at the center of the program dialog window. At the end of the experiment, the tokens will be converted into money. Each token will be worth 0.5 Dollars. Your final earnings in the experiment will be your earnings in the round selected plus the $5 show-up fee. You will receive your payment as you leave the experiment.

Rules

Your participation in the experiment and any information about your payoffs will be kept strictly confidential. Your payment-receipt and participant form are the only places in which your name and social security number are recorded.

You will never be asked to reveal your identity to anyone during the course of the experiment. Neither the experimenters nor the other participants will be able to link you to any of your decisions. In order to keep your decisions private, please do not reveal your choices to any other participant.

Please do not talk with anyone during the experiment. We ask everyone to remain silent until the end of the last round. If there are no further questions, you are ready to start. An instructor will approach your desk and activate your program.

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Table 1: WARP and GARP violations and the three indices by subject (treatment by treatment sorted according to descending CCEI)

Symmetric treatment (π=1/2)

ID	WARP	GARP	Afriat	Varian	HM
205	0	0	1.000	1.000	50
213	0	0	1.000	1.000	50
215	0	0	1.000	1.000	50
216	0	0	1.000	1.000	50
219	0	0	1.000	1.000	50
303	0	0	1.000	1.000	50
304	0	0	1.000	1.000	50
306	0	0	1.000	1.000	50
314	0	0	1.000	1.000	50
316	0	0	1.000	1.000	50
317	0	0	1.000	1.000	50
320	0	0	1.000	1.000	50
326	0	0	1.000	1.000	50
301	3	11	0.997	0.951	48
323	3	3	0.991	0.978	47
302	2	7	0.990	0.943	48
210	1	1	0.988	0.967	49
311	3	3	0.986	0.804	48
313	2	2	0.986	0.970	48
217	7	14	0.986	0.935	46
207	3	15	0.981	0.941	47
204	4	10	0.973	0.970	47
318	4	6	0.972	0.809	48
202	6	12	0.968	0.944	46
203	4	14	0.966	0.946	48
319	3	20	0.966	0.727	48
327	2	5	0.965	0.915	49
315	10	33	0.959	0.795	45
312	4	13	0.957	0.952	47
309	4	17	0.952	0.890	48
218	5	10	0.951	0.907	48
214	8	21	0.949	0.916	45
206	9	147	0.948	0.855	47
208	8	14	0.942	0.912	45
308	2	6	0.938	0.930	49
209	15	94	0.929	0.825	46
307	5	12	0.916	0.914	46
322	8	96	0.905	0.768	47
212	5	111	0.866	0.697	47
305	17	182	0.852	0.695	45
324	18	453	0.840	0.657	29
201	16	147	0.797	0.526	42
321	27	375	0.757	0.356	44
325	27	702	0.739	0.398	32
328	21	559	0.705	0.401	33
310	22	241	0.690	0.366	43
211	83	669	0.611	0.361	34

Table 1 cont.

Asymmetric treatments (π=1/3 and π=2/3)

ID	WARP	GARP	Afriat	Varian	HM
508	0	0	1.000	1.000	50
509	0	0	1.000	1.000	50
604	0	0	1.000	1.000	50
411	2	4	0.999	0.978	48
416	1	1	0.999	0.979	49
405	2	2	0.999	0.933	48
417	1	1	0.998	0.996	49
505	1	1	0.996	0.995	49
501	2	2	0.995	0.985	48
605	5	5	0.992	0.982	45
414	1	1	0.990	0.951	49
413	5	7	0.989	0.979	47
408	1	1	0.987	0.986	49
415	4	5	0.987	0.934	47
402	5	7	0.987	0.834	47
410	4	4	0.984	0.954	47
515	5	6	0.984	0.973	46
407	3	3	0.984	0.972	48
503	2	5	0.982	0.961	49
512	8	8	0.982	0.960	43
601	1	1	0.981	0.981	49
516	4	4	0.981	0.975	46
520	8	9	0.979	0.907	46
412	7	12	0.976	0.928	46
514	2	3	0.975	0.952	49
502	5	17	0.971	0.880	47
609	3	5	0.969	0.880	47
519	4	5	0.963	0.944	47
513	10	37	0.957	0.822	45
602	6	11	0.947	0.861	45
510	8	13	0.946	0.914	45
409	6	15	0.943	0.935	46
511	16	231	0.936	0.472	42
507	16	39	0.929	0.843	44
403	8	27	0.916	0.724	46
404	26	117	0.915	0.729	42
517	13	32	0.911	0.845	43
506	5	294	0.892	0.568	48
401	3	3	0.874	0.838	49
607	37	179	0.870	0.712	37
608	23	549	0.847	0.570	29
606	18	241	0.839	0.470	44
518	26	121	0.816	0.732	43
504	29	794	0.697	0.355	33
603	12	322	0.686	0.229	47
406	39	881	0.653	0.225	30

Table 2: Results of individual-level power utility function estimation
(sorted according to ascending order of the Arrow-Pratt measure of relative risk aversion)

Symmetric treatment (π=1/2)

ID	α	Std(α)	β	Std(β)	ρ	π	# obs.
311	-0.002	0.002	-0.005	0.008	-214.124	0.384	24
324	0.002	0.003	-0.008	0.009	-123.286	0.557	19
307	-0.011	0.011	-0.019	0.052	-52.248	0.361	18
302	0.081	0.086	2.701	0.243	0.370	0.492	40
207	0.091	0.118	2.492	0.246	0.401	0.491	38
216	0.005	0.058	2.045	0.101	0.489	0.499	40
318	-0.112	0.093	2.034	0.177	0.492	0.514	31
305	0.082	0.122	1.779	0.151	0.562	0.489	50
209	-0.019	0.110	1.523	0.192	0.656	0.503	49
309	-0.112	0.060	1.483	0.099	0.674	0.519	50
217	-0.018	0.088	1.429	0.110	0.700	0.503	49
208	0.035	0.084	1.398	0.131	0.715	0.494	48
319	0.089	0.108	1.348	0.170	0.742	0.483	34
322	0.121	0.146	1.322	0.167	0.756	0.477	50
315	0.004	0.121	1.258	0.153	0.795	0.499	50
313	-0.040	0.050	1.210	0.070	0.827	0.508	49
326	-0.050	0.061	1.118	0.127	0.895	0.511	48
301	0.036	0.057	1.107	0.094	0.904	0.492	49
327	0.178	0.097	1.083	0.126	0.924	0.459	50
312	0.111	0.098	1.065	0.178	0.939	0.474	45
303	0.004	0.003	0.989	0.005	1.011	0.499	46
215	-0.208	0.130	0.982	0.227	1.019	0.553	37
202	0.071	0.075	0.963	0.091	1.038	0.482	50
219	0.065	0.030	0.890	0.063	1.124	0.482	41
212	0.122	0.075	0.883	0.105	1.132	0.465	48
308	-0.087	0.061	0.812	0.083	1.231	0.527	49
317	-0.050	0.025	0.801	0.040	1.249	0.515	50
316	0.063	0.050	0.779	0.075	1.284	0.480	50
214	0.035	0.054	0.769	0.088	1.301	0.489	50
213	-0.058	0.099	0.653	0.184	1.533	0.522	45
323	0.066	0.059	0.630	0.074	1.588	0.474	50
306	0.076	0.035	0.301	0.046	3.318	0.437	50
206	0.062	0.040	0.278	0.064	3.596	0.444	50
203	-0.019	0.044	0.208	0.088	4.799	0.523	46
210	0.053	0.029	0.199	0.042	5.034	0.433	50
204	0.011	0.052	0.187	0.090	5.348	0.486	47
314	0.004	0.021	0.184	0.103	5.448	0.495	25
304	-0.001	0.001	0.008	0.002	132.742	0.525	50

Table 2 cont.

Asymmetric treatments (π=1/3 and π=2/3)

ID	α	Std(α)	β	Std(β)	ρ	π	# obs.
517	-0.149	0.246	2.897	0.401	0.345	0.513	47
609	-0.162	0.132	2.383	0.281	0.420	0.517	45
412	0.199	0.109	2.277	0.182	0.439	0.478	41
415	0.638	0.212	2.181	0.749	0.458	0.427	22
520	-0.611	0.113	2.098	0.171	0.477	0.572	35
506	0.042	0.149	2.043	0.206	0.489	0.495	41
417	0.191	0.061	2.029	0.101	0.493	0.477	49
505	-0.772	0.083	1.884	0.122	0.531	0.601	44
601	-0.877	0.111	1.846	0.482	0.542	0.617	17
403	0.362	0.076	1.726	0.130	0.579	0.448	40
605	-0.328	0.115	1.674	0.129	0.597	0.549	50
405	0.289	0.093	1.661	0.109	0.602	0.457	50
503	-0.411	0.068	1.614	0.231	0.620	0.563	35
411	1.193	0.042	1.579	0.134	0.633	0.320	32
510	-0.363	0.099	1.544	0.106	0.648	0.558	50
402	0.290	0.056	1.515	0.079	0.660	0.452	45
507	-0.121	0.097	1.465	0.156	0.683	0.521	46
401	0.180	0.113	1.417	0.158	0.706	0.468	50
414	0.331	0.077	1.365	0.113	0.732	0.440	49
514	-0.900	0.069	1.308	0.092	0.764	0.666	50
602	-0.010	0.091	1.227	0.118	0.815	0.502	49
501	-0.189	0.060	1.216	0.103	0.822	0.539	49
518	-0.504	0.113	1.207	0.172	0.829	0.603	48
409	0.682	0.079	1.143	0.099	0.875	0.355	50
512	-0.329	0.068	1.059	0.097	0.944	0.577	48
604	-0.730	0.025	1.009	0.034	0.991	0.673	50
416	0.600	0.040	0.983	0.057	1.018	0.352	50
513	-0.200	0.069	0.919	0.087	1.088	0.554	50
509	-0.084	0.047	0.918	0.068	1.089	0.523	50
511	-0.255	0.084	0.832	0.093	1.202	0.576	49
410	0.431	0.058	0.773	0.071	1.294	0.364	50
608	-0.473	0.148	0.701	0.202	1.426	0.662	46
519	-0.923	0.077	0.580	0.133	1.724	0.831	45
502	-0.060	0.059	0.528	0.072	1.896	0.528	50
407	0.461	0.042	0.403	0.053	2.479	0.242	50
408	0.634	0.054	0.393	0.060	2.544	0.166	50
404	0.059	0.093	0.384	0.145	2.601	0.461	50
606	-0.020	0.083	0.289	0.128	3.455	0.517	50
413	0.464	0.031	0.270	0.049	3.707	0.152	45
516	-0.391	0.022	0.214	0.031	4.672	0.861	50
515	-0.478	0.032	0.194	0.036	5.158	0.922	50
607	-0.394	0.100	0.180	0.128	5.569	0.900	50

Table 3: Results of individual-level type-mixture model (TMM) estimation
(sorted according to ascending order of the measure of relative risk aversion)

Symmetric treatment (π=1/2)

ID	β	Std(β)	ρ	Std(ρ)	σ	Std(σ)	F-value	OLS	CCEI
205	7229.917	599800	0.000	0.002	0.358	64.719	0.617	0.651	1.000
320	12.371	5.021	0.000	0.091	0.003	1.503	8.296	0.797	1.000
218	21.752	25.382	0.000	0.086	0.340	0.323	30.249	0.807	0.951
307	34.228	19.536	0.201	0.030	0.299	0.137	13.700	-52.248	0.916
324	5.454	4.407	0.205	0.197	0.545	0.841	28.229	-123.286	0.840
311	27.869	12.874	0.209	0.041	0.303	0.252	16.450	-214.124	0.986
314	11.777	5.775	0.302	0.037	0.133	0.049	14.976	5.448	1.000
302	70.274	36.924	0.311	0.014	0.105	0.015	-7.364	0.370	0.990
207	135.414	120.450	0.351	0.017	0.137	0.015	-6.498	0.401	0.981
318	22.547	8.948	0.357	0.023	0.136	0.025	11.694	0.492	0.972
216	115.920	56.802	0.423	0.009	0.077	0.009	-28.309	0.489	1.000
319	17.924	8.269	0.514	0.026	0.149	0.024	15.482	0.742	0.966
215	6.986	2.553	0.562	0.040	0.191	0.030	32.729	1.019	1.000
209	24.317	17.275	0.587	0.038	0.136	0.020	-2.676	0.656	0.929
305	44.890	29.671	0.588	0.036	0.132	0.013	-3.822	0.562	0.852
208	23.111	21.586	0.637	0.038	0.124	0.018	0.311	0.715	0.942
309	36.117	35.387	0.664	0.043	0.091	0.013	-20.954	0.674	0.952
217	11.357	3.219	0.673	0.034	0.105	0.029	-10.864	0.700	0.986
312	5.780	2.833	0.746	0.059	0.129	0.028	7.504	0.939	0.957
315	11.823	8.478	0.762	0.055	0.156	0.023	13.717	0.795	0.959
301	36.349	34.354	0.763	0.024	0.093	0.010	-19.405	0.904	0.997
322	14.135	10.476	0.771	0.056	0.155	0.019	10.068	0.756	0.905
313	12.233	3.324	0.798	0.027	0.078	0.015	-25.245	0.827	0.986
326	6.581	2.184	0.805	0.043	0.089	0.022	-14.736	0.895	1.000
327	11.435	7.483	0.849	0.073	0.137	0.023	2.990	0.924	0.965
202	2.456	17.367	1.000	0.082	0.111	0.018	-35.601	1.038	0.968
212	2.205	0.494	1.000	0.027	0.118	0.026	-23.491	1.132	0.866
308	2.788	0.809	1.000	0.056	0.103	0.047	-34.390	1.231	0.938
213	2.399	0.476	1.000	0.034	0.132	0.023	-2.571	1.533	1.000
316	5.081	4.269	1.000	0.084	0.087	0.065	-49.878	1.284	1.000
303	2.983	0.361	1.000	0.004	0.006	0.008	-147.839	1.011	1.000
214	14.368	39919	1.000	0.060	0.097	0.037	-45.492	1.301	0.949
219	1.193	0.503	1.117	0.045	0.047	0.013	-17.563	1.124	1.000
317	302.411	263.760	1.171	0.025	0.038	0.002	-69.654	1.249	1.000
323	36.860	26.641	1.303	0.086	0.085	0.007	-20.181	1.588	0.991
306	31.994	30.742	2.479	0.183	0.050	0.003	-43.842	3.318	1.000
206	0.397	26.849	2.879	0.812	0.065	0.015	-33.704	3.596	0.948
203	0.023	0.030	3.598	0.634	0.067	0.020	-16.765	4.799	0.966
210	0.056	49.545	4.080	0.914	0.049	0.026	-52.152	5.034	0.988
204	0.006	0.029	4.336	1.049	0.086	0.042	-9.822	5.348	0.973
304	87.862	274.020	5.000	0.922	0.020	0.011	-92.380	132.742	1.000

Table 3 cont.

Asymmetric treatments (π=1/3 and π=2/3)

ID	β	Std(β)	ρ	Std(ρ)	σ	Std(σ)	F-value	OLS	CCEI
601	8.955	9.588	0.050	0.121	0.300	0.150	30.327	0.542	0.981
415	3.283	2.765	0.378	0.110	0.277	0.077	61.804	0.458	0.987
520	2.300	1.550	0.552	0.081	0.191	0.040	42.503	0.477	0.979
411	4.464	1.458	0.582	0.026	0.042	0.021	-7.010	0.633	0.999
505	55.529	26.907	0.646	0.022	0.109	0.007	15.455	0.531	0.996
517	2.614	5.844	0.743	0.263	0.327	0.055	74.247	0.345	0.911
514	122.118	106.630	0.749	0.042	0.082	0.007	-38.897	0.764	0.975
412	4.680	1.932	0.783	0.122	0.239	0.034	69.475	0.439	0.976
403	5.239	1.391	0.838	0.083	0.174	0.021	54.382	0.579	0.916
506	2.600	1.537	0.855	0.151	0.253	0.048	70.771	0.489	0.892
604	811.519	53.457	0.991	0.005	0.037	0.002	-93.441	0.991	1.000
416	1348.199	17.717	0.997	0.001	0.066	0.002	-65.221	1.018	0.999
405	30.630	7.122	0.997	0.054	0.146	0.015	30.024	0.602	0.999
518	19.895	3.228	0.998	0.028	0.129	0.020	5.575	0.829	0.816
512	10.810	3.107	0.998	0.032	0.124	0.031	-3.148	0.944	0.982
510	9.788	3.195	0.999	0.063	0.170	0.029	26.120	0.648	0.946
401	13.457	55.267	1.001	0.109	0.168	0.022	28.774	0.706	0.874
402	4.628	0.916	1.001	0.056	0.146	0.033	24.270	0.660	0.987
609	9.018	1.764	1.002	0.050	0.173	0.019	44.424	0.420	0.969
414	55.914	218.010	1.003	0.048	0.120	0.017	-2.416	0.732	0.990
605	17.708	12.264	1.004	0.048	0.174	0.024	27.322	0.597	0.992
417	37.801	50.045	1.004	0.029	0.146	0.014	19.222	0.493	0.998
409	5452.187	N/A	1.006	N/A	0.084	N/A	-26.818	0.875	0.943
519	6.469	1.483	1.007	0.069	0.116	0.029	7.134	1.724	0.963
602	9.960	5.926	1.010	0.086	0.167	0.019	26.972	0.815	0.947
513	18.426	10.952	1.225	0.123	0.117	0.012	3.026	1.088	0.957
507	2.178	0.876	1.261	0.179	0.176	0.030	37.243	0.683	0.929
511	6.533	2.479	1.270	0.088	0.132	0.018	7.527	1.202	0.936
608	2.662	1.298	1.322	0.191	0.154	0.024	19.322	1.426	0.847
410	46.358	28.869	1.344	0.079	0.082	0.005	-28.973	1.294	0.984
501	3.183	1.319	1.390	0.224	0.133	0.024	12.136	0.822	0.995
408	47.652	29.347	1.518	0.063	0.077	0.004	-22.736	2.544	0.987
503	0.145	0.280	1.550	0.352	0.112	0.029	27.538	0.620	0.982
509	16.057	11.093	1.566	0.119	0.102	0.008	-4.083	1.089	1.000
407	99.665	57.894	1.735	0.098	0.055	0.002	-46.833	2.479	0.984
413	0.655	0.299	1.921	0.162	0.061	0.023	-15.354	3.707	0.989
516	149.692	91.172	2.226	0.119	0.042	0.001	-58.699	4.672	0.981
515	31.951	25.258	2.250	0.143	0.061	0.003	-34.884	5.158	0.984
502	0.817	4.718	2.539	0.359	0.102	0.019	-12.215	1.896	0.971
607	0.423	2.261	2.709	0.647	0.143	0.031	12.592	5.569	0.870
404	0.016	0.060	4.429	1.707	0.140	0.025	9.701	2.601	0.915
606	0.006	0.031	5.000	3.012	0.128	0.021	-9.416	3.455	0.839

Figure 1: An example of a budget set with two states and two assets

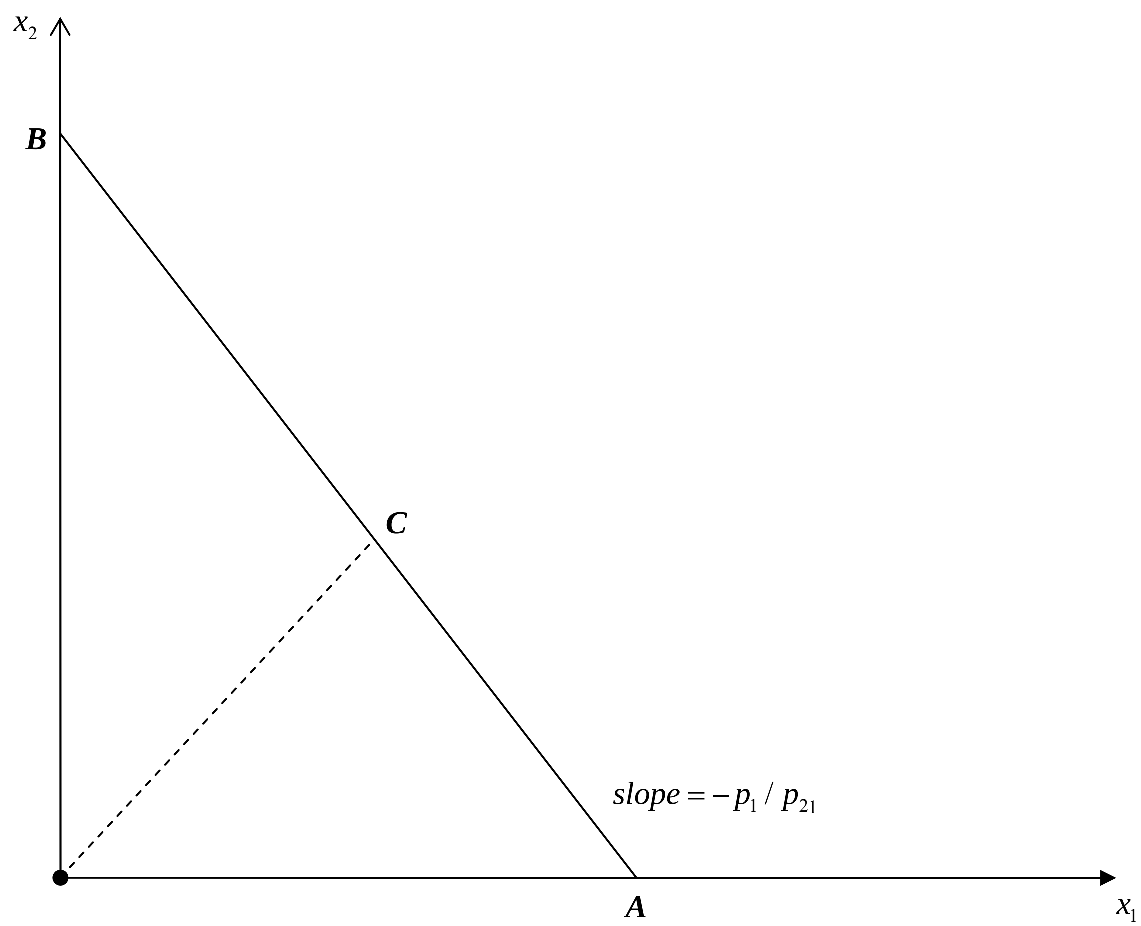

Figure 2: The construction of the CCEI for a simple violation of GARP

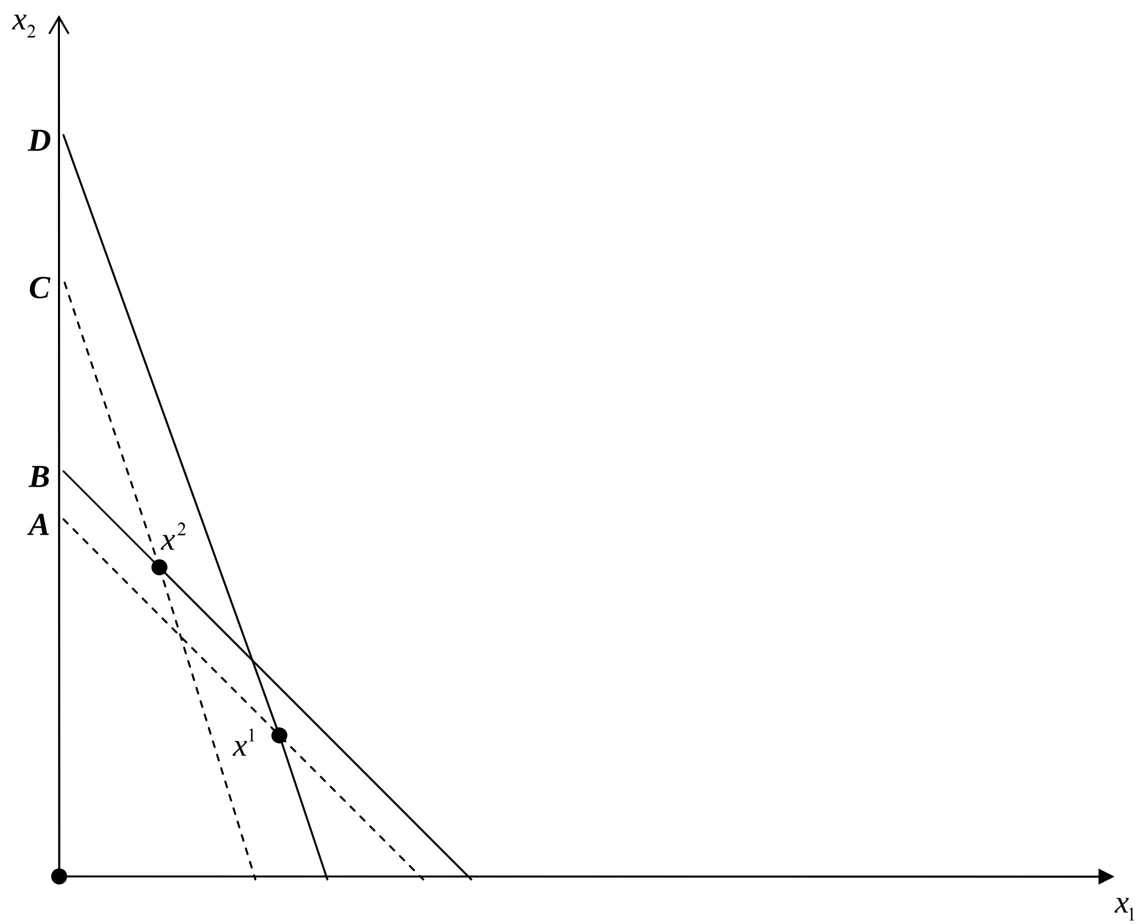

Figure 3A: The distributions of GARP violations
Afriat's (1972) efficiency index (CCEI)

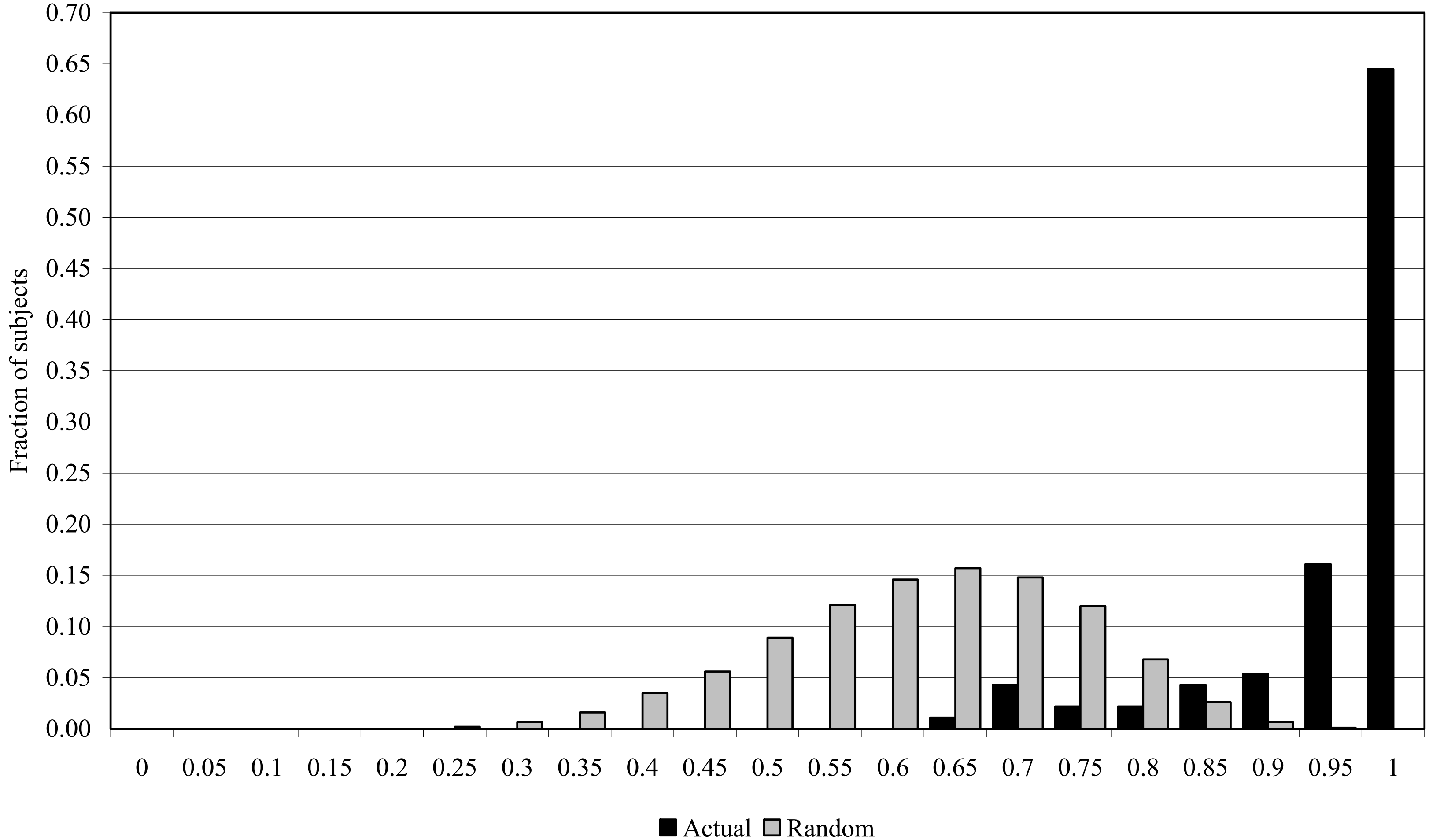

Figure 3B: The distributions of GARP violations
Varian (1991)

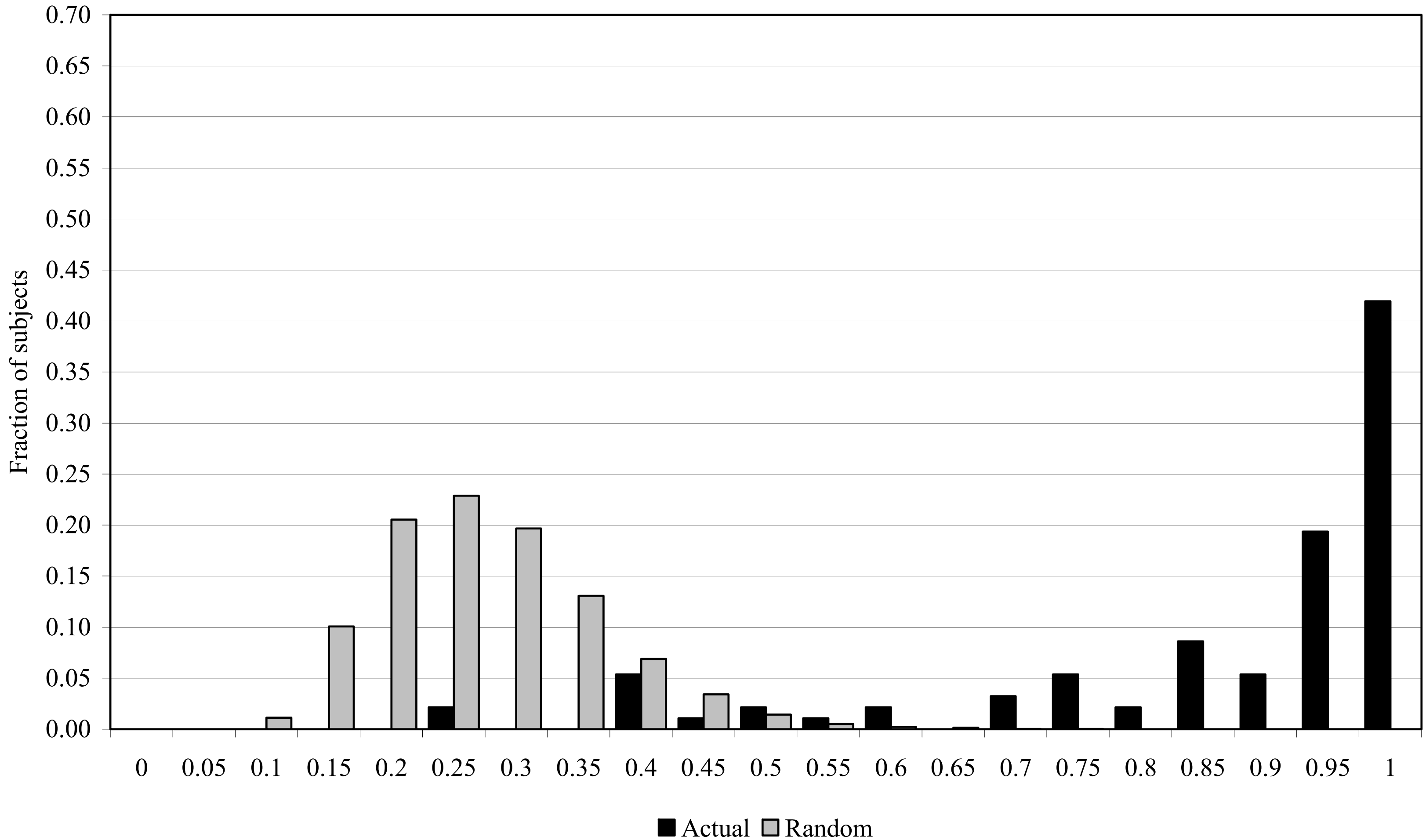

Figure 3C: The distributions of GARP violations
Houtman and Maks (1985)

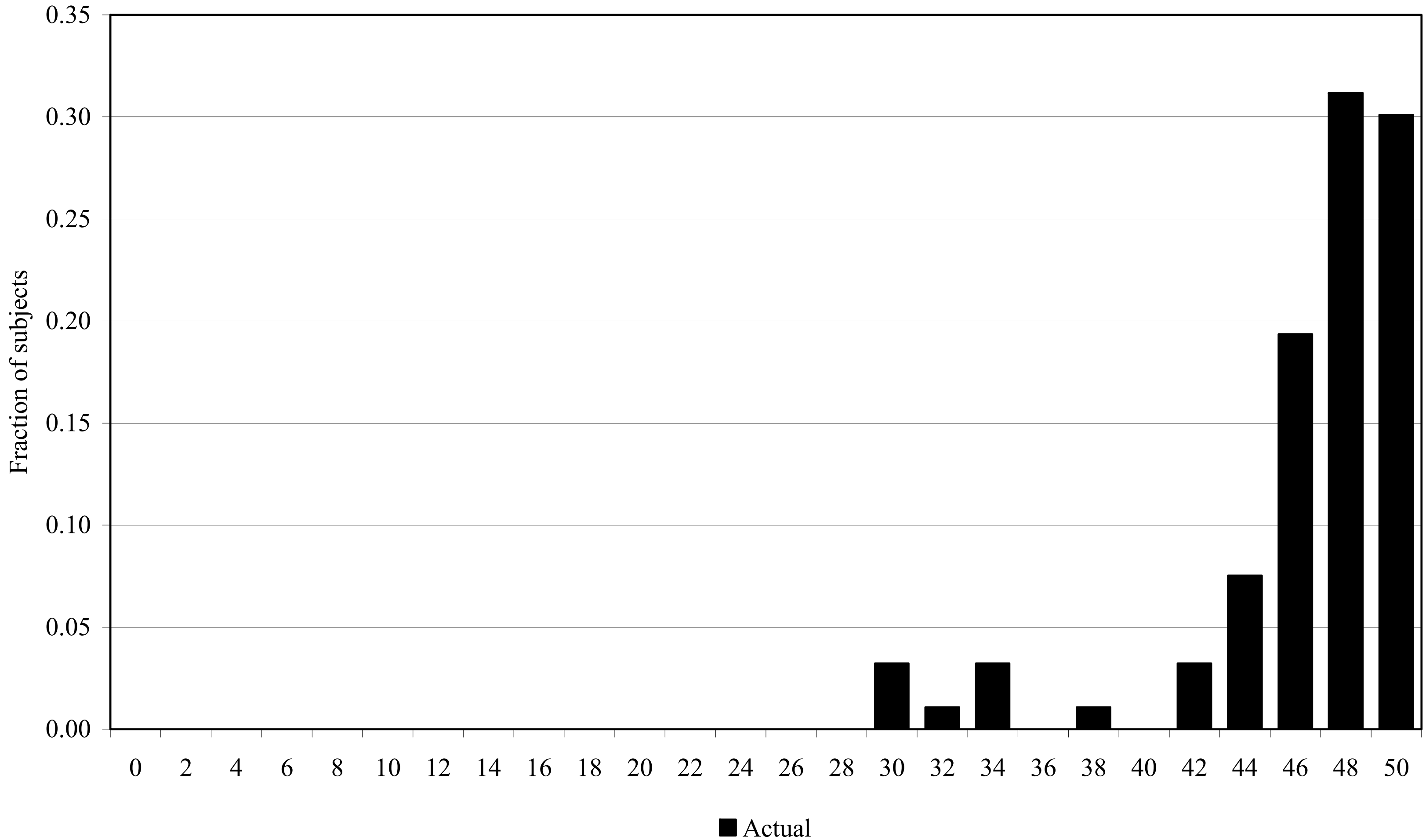

Figure 4: Individual-level data

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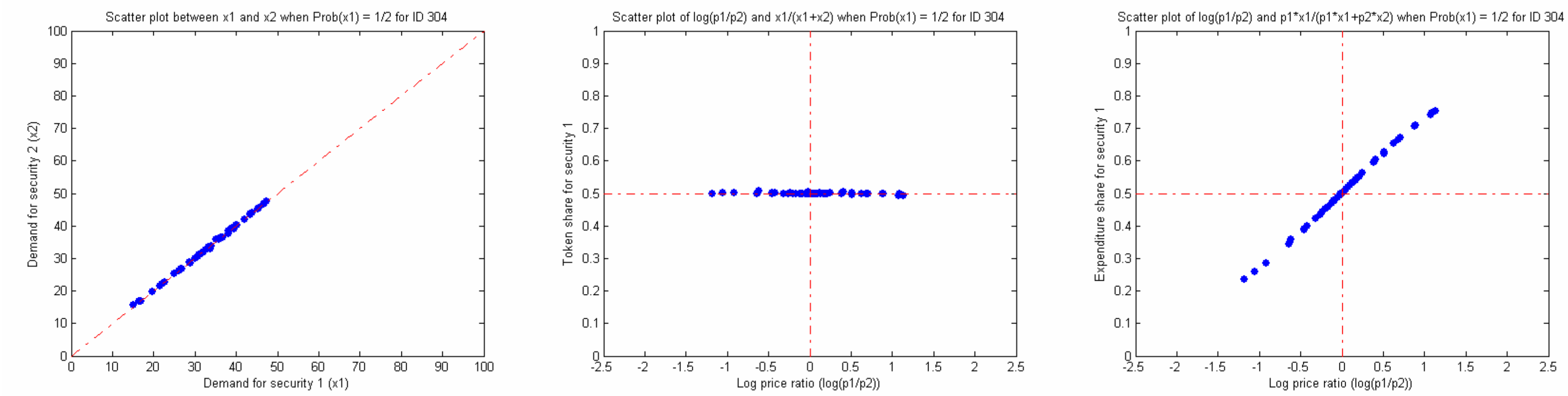

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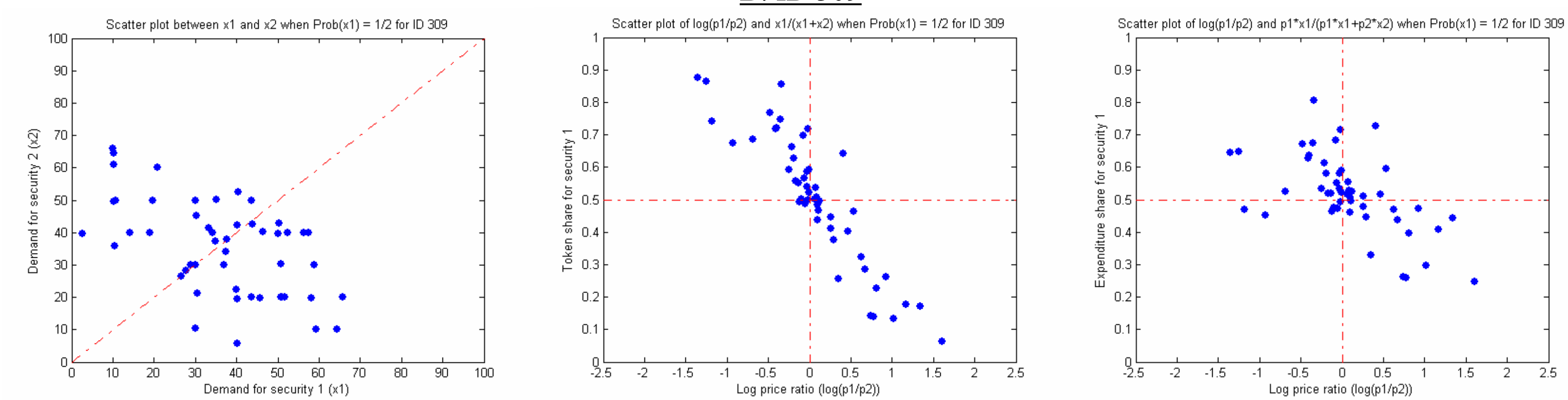

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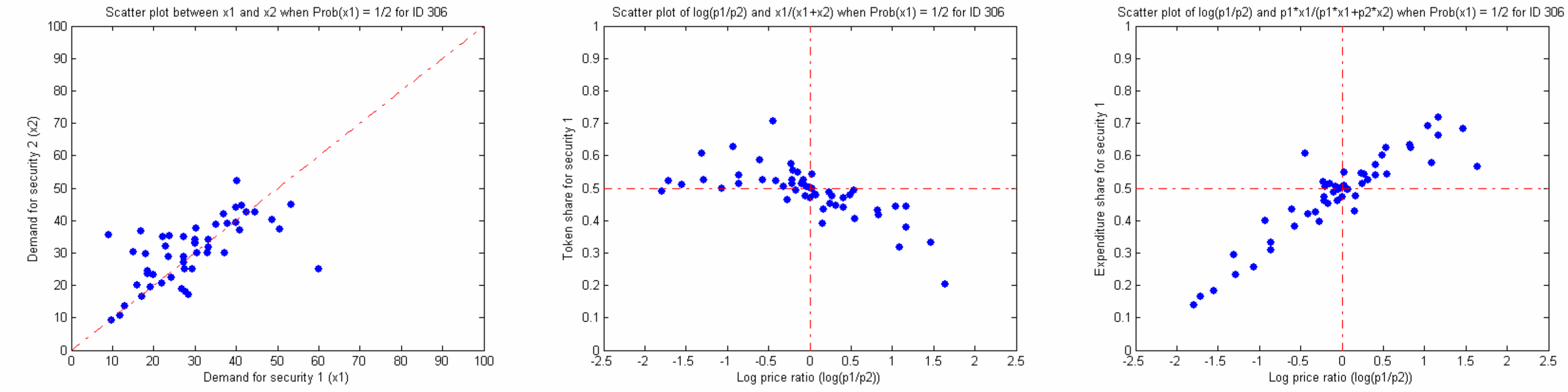

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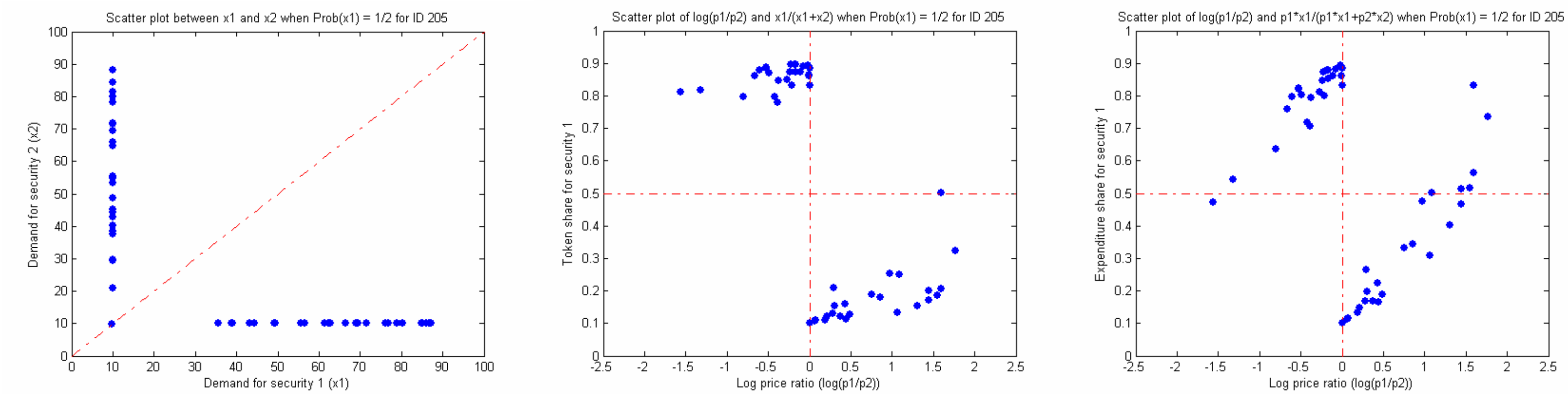

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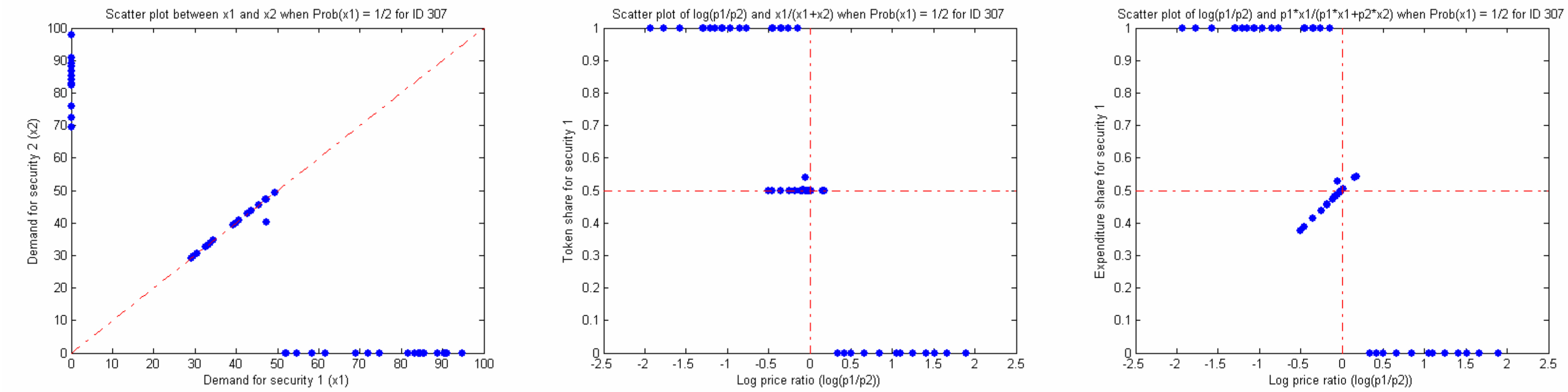

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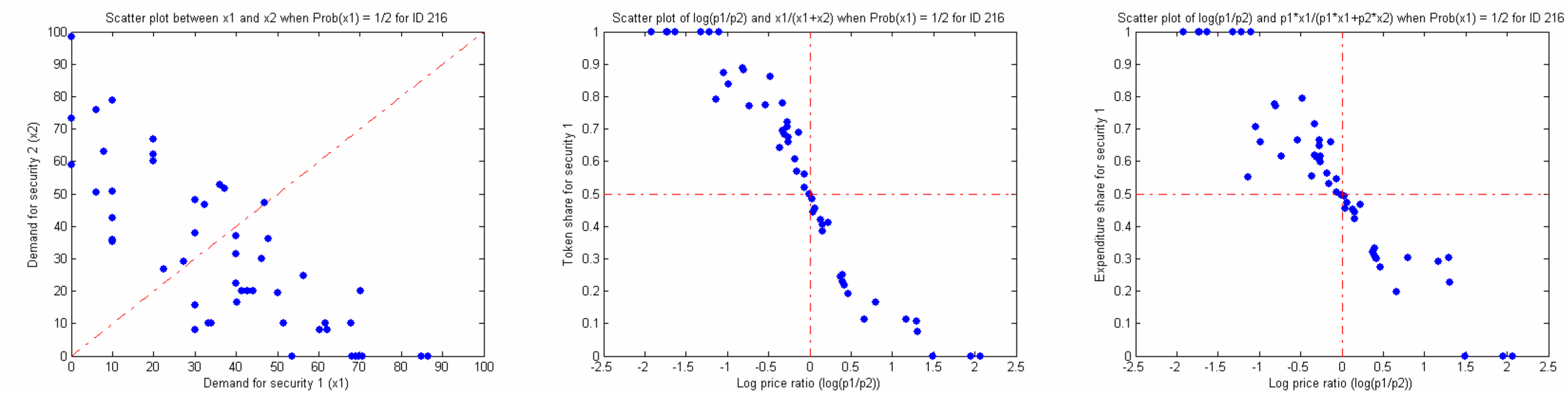
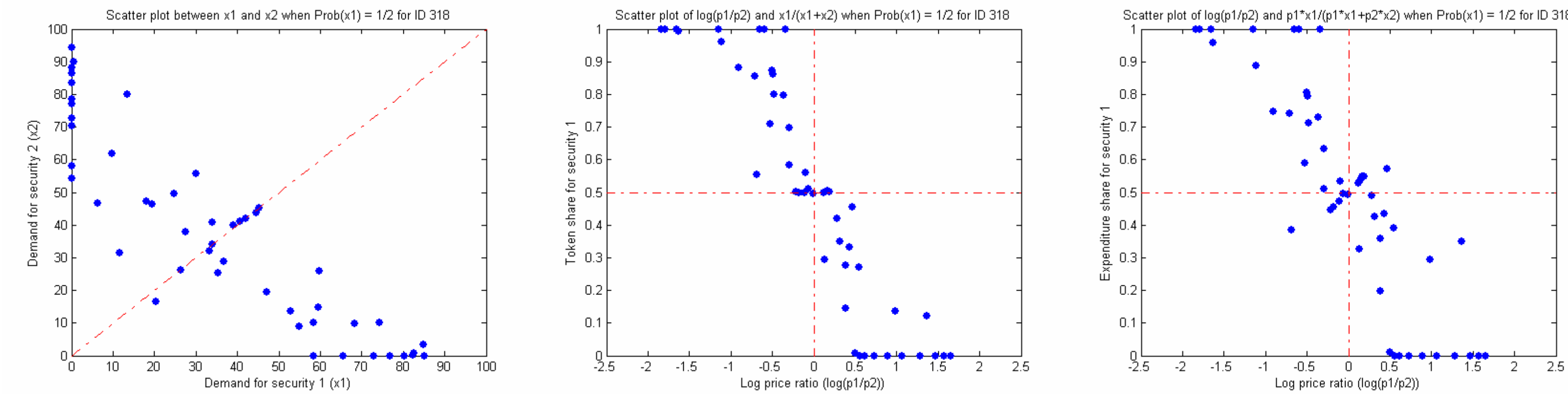


G: ID 318



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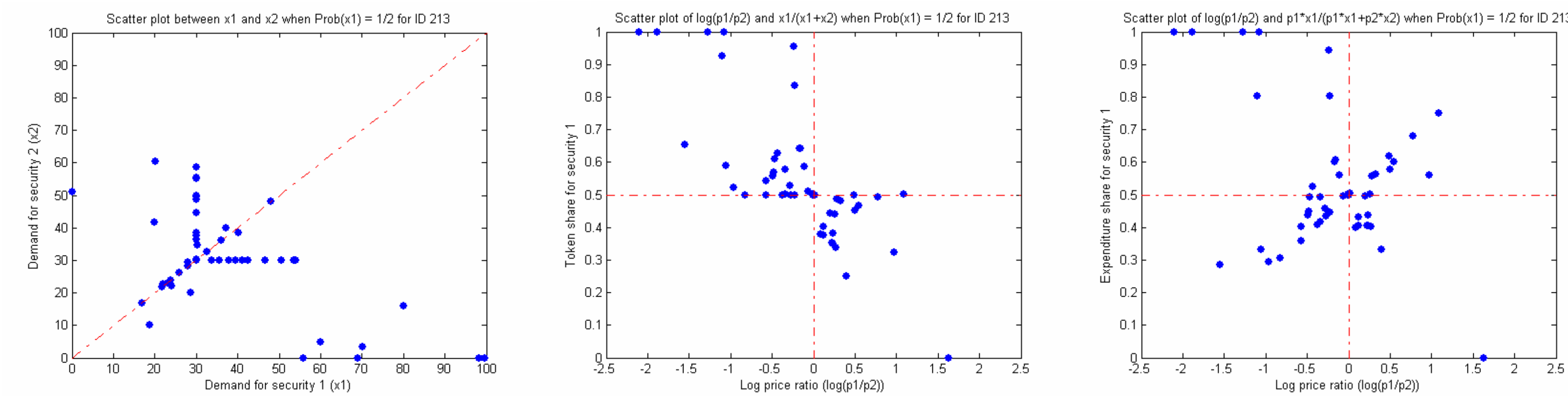
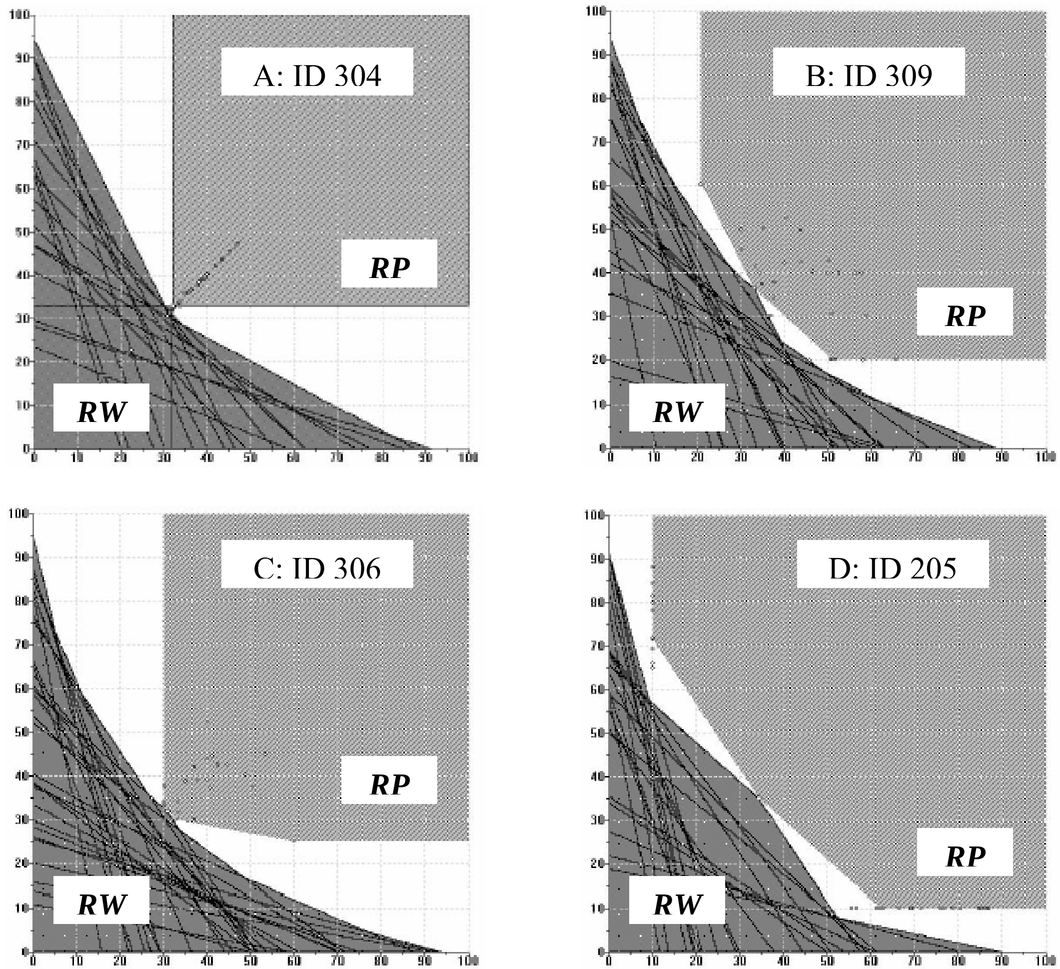
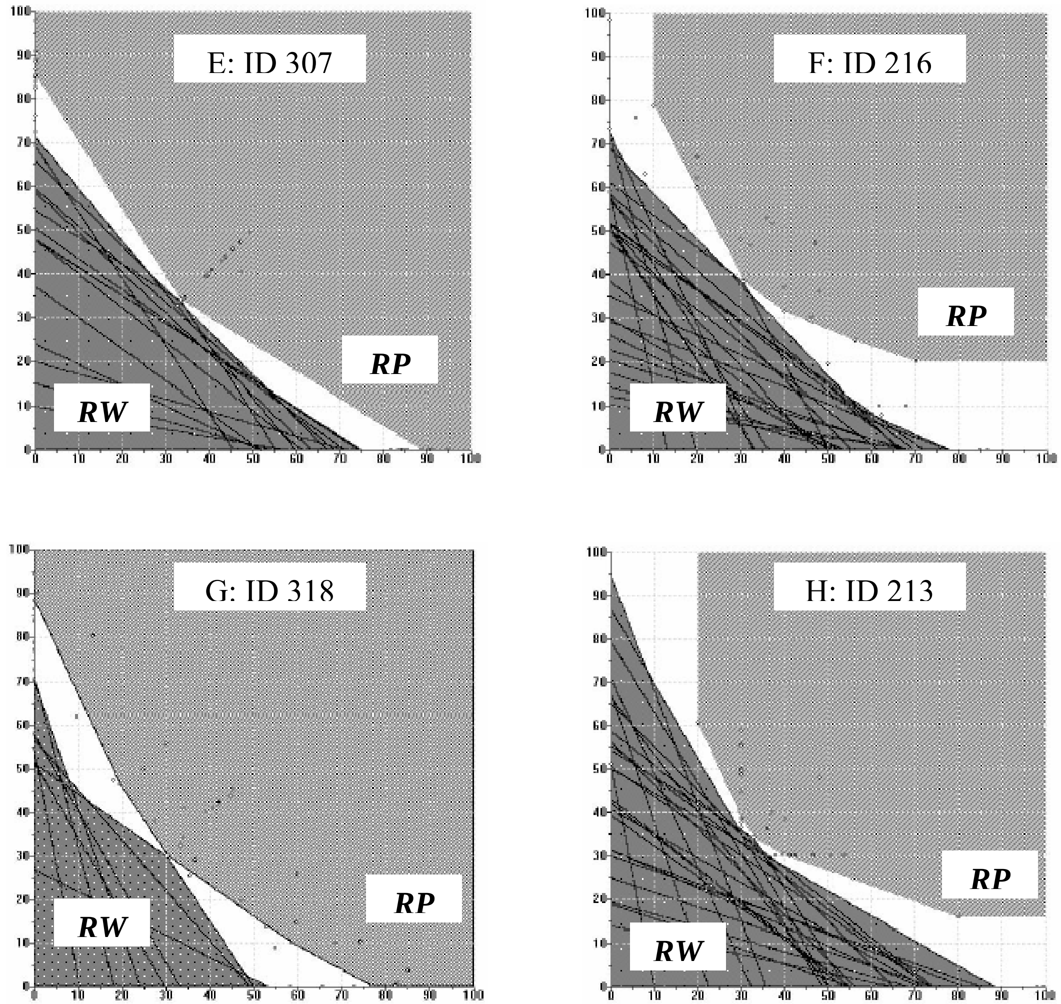


Figure 5: Illustration of recoverability for selected subjects





E: ID 307
RP
RW
F: ID 216
RP
RW
G: ID 318
RP
RW
H: ID 213
RP
RW

Figure 6: the distribution of the Arrow-Pratt measure of relative risk aversion
OLS estimation, treatment by treatment

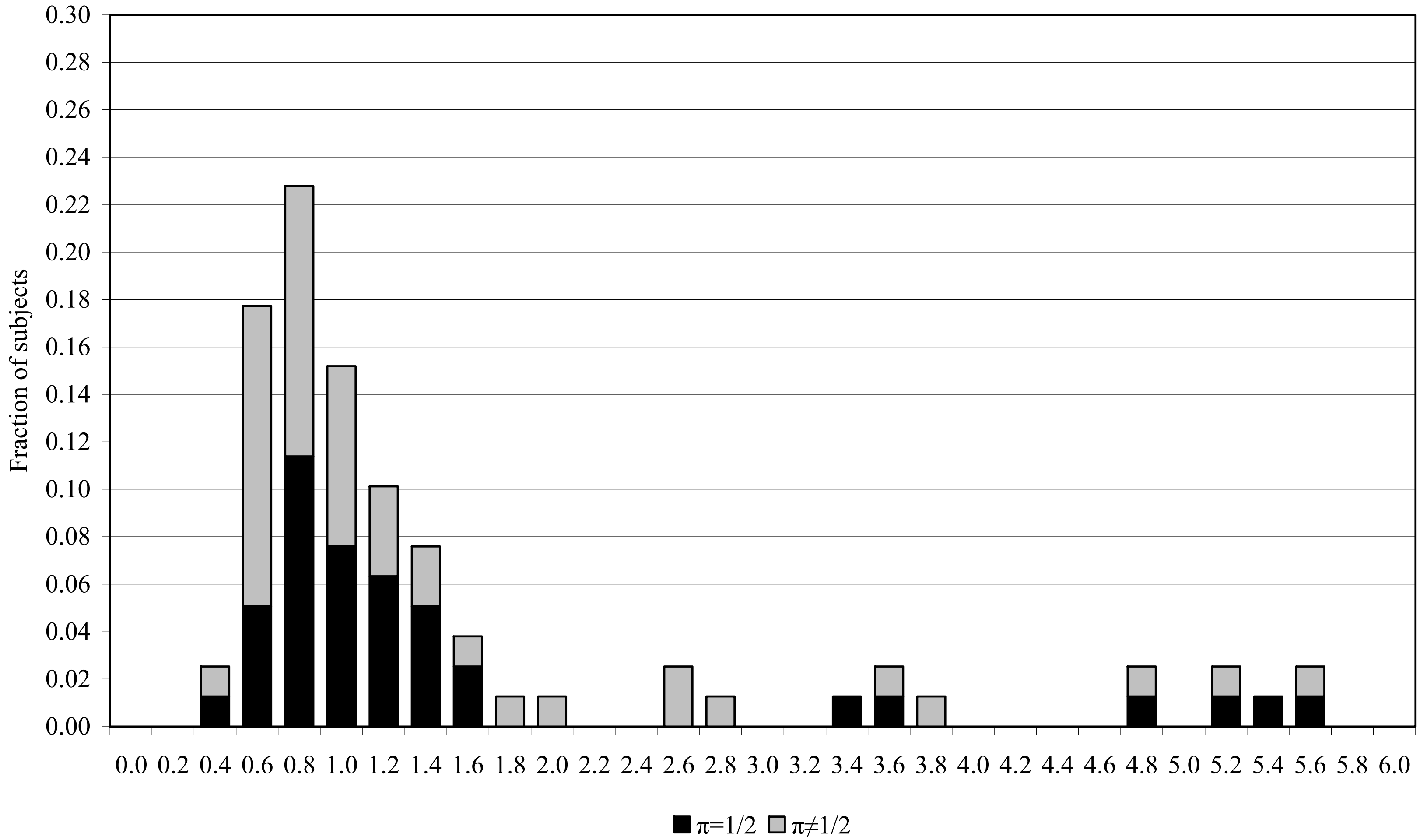


Figure 7: the distribution of the Arrow-Pratt measure of relative risk aversion
TMM estimation, treatment by treatment

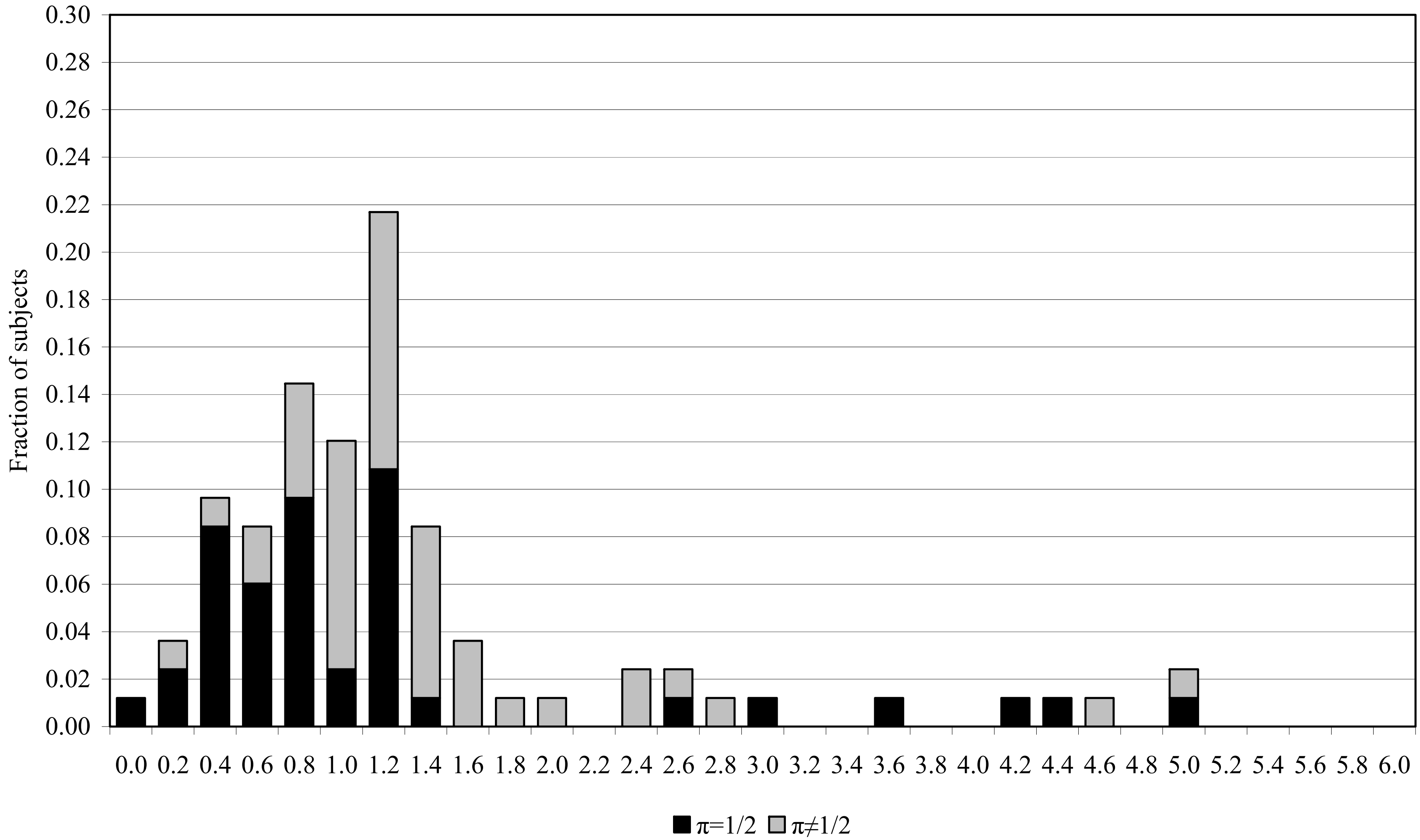


Figure 8: Individual-level comparisons of nonparametric and parametric (TMM) estimation

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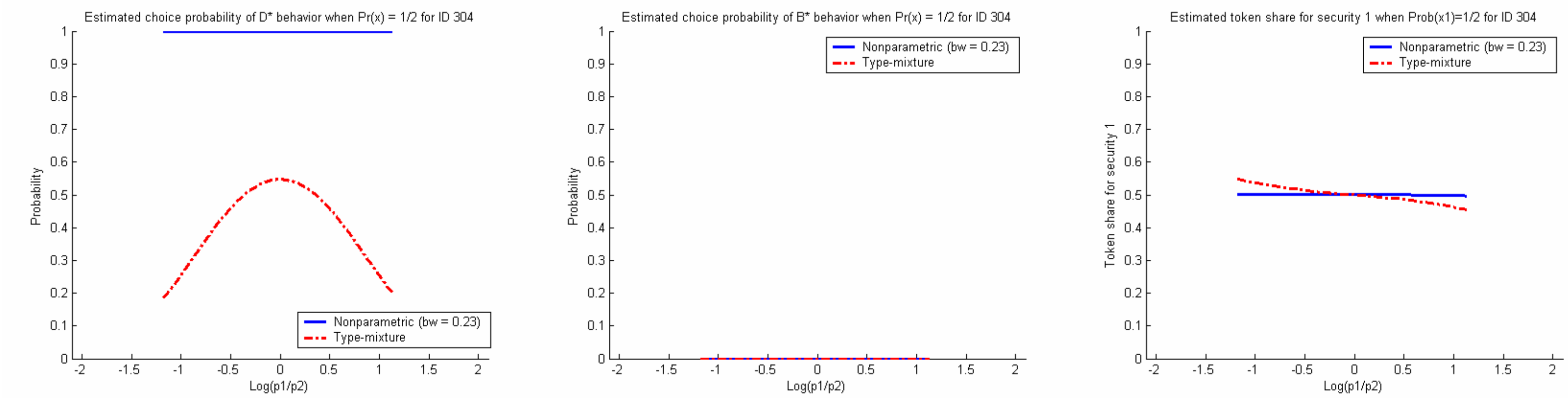

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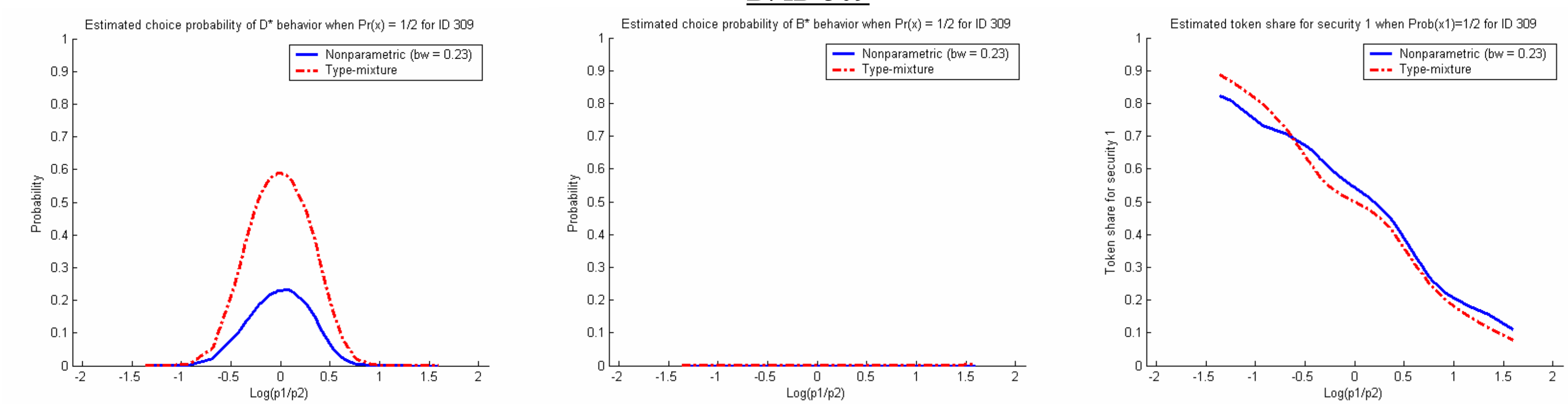

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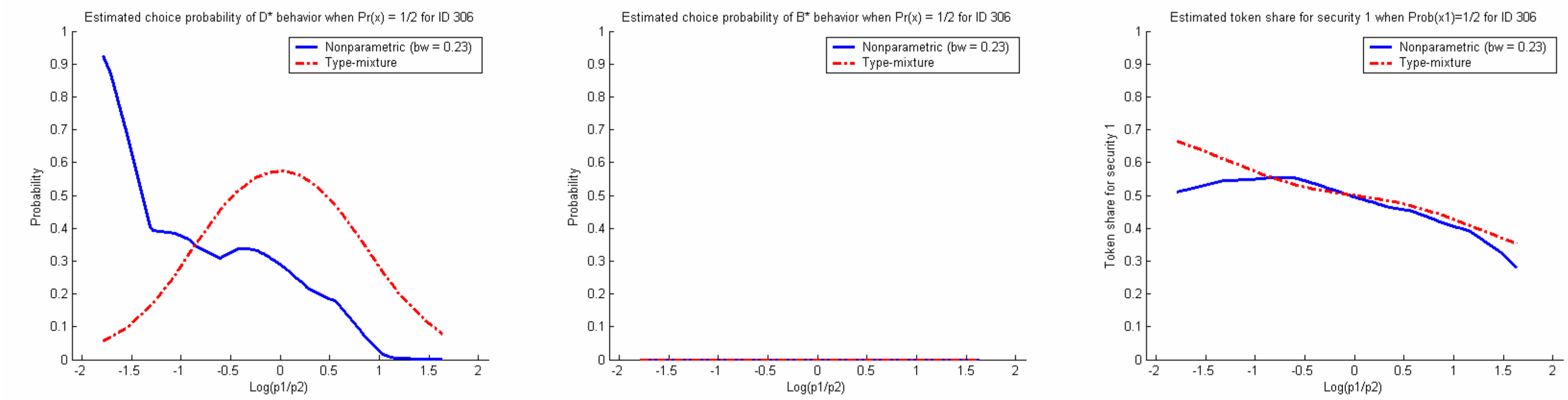

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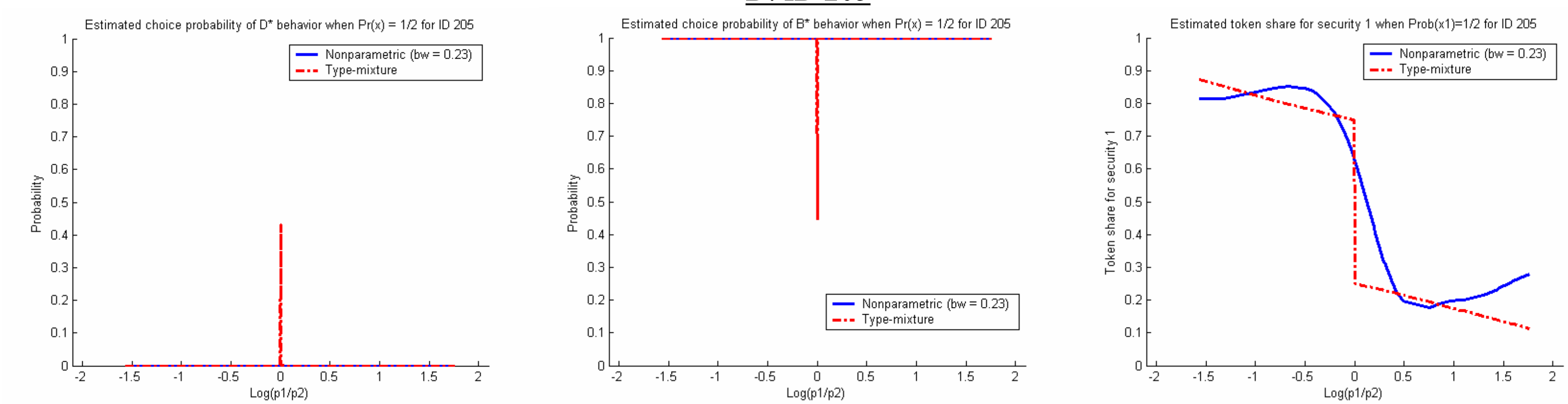

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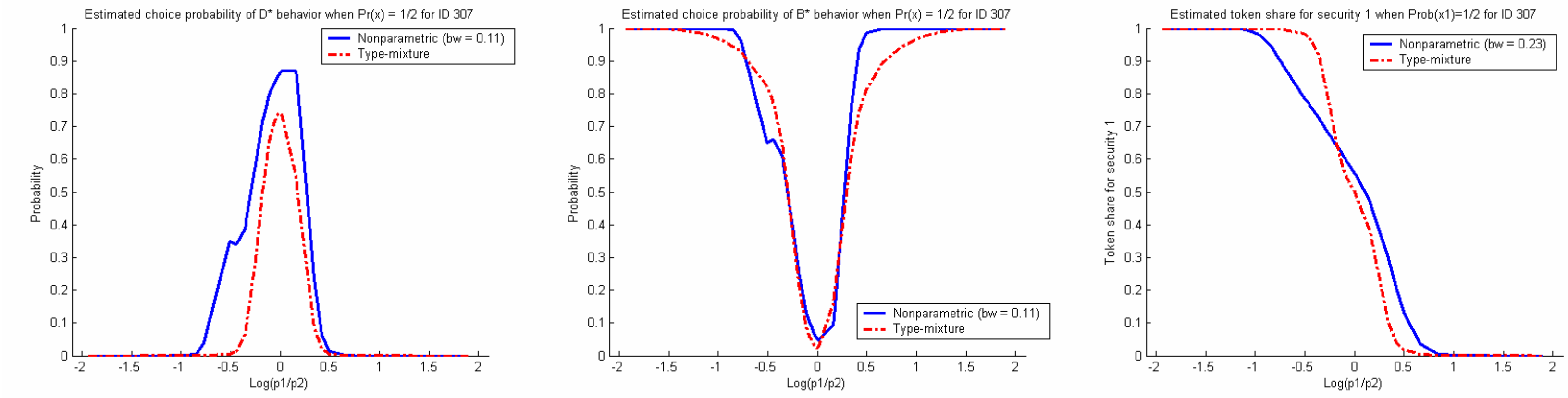

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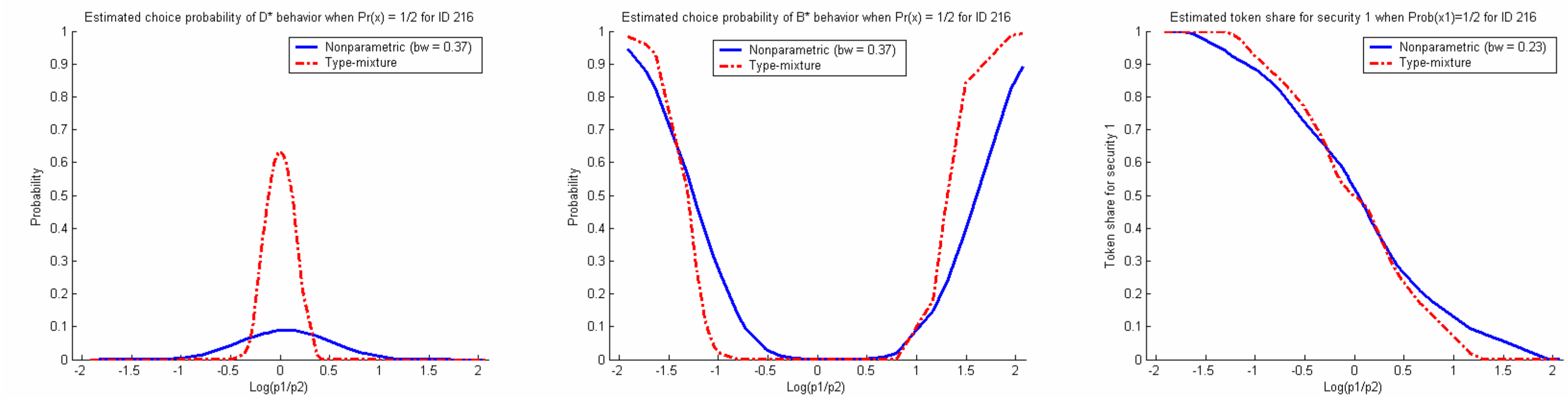
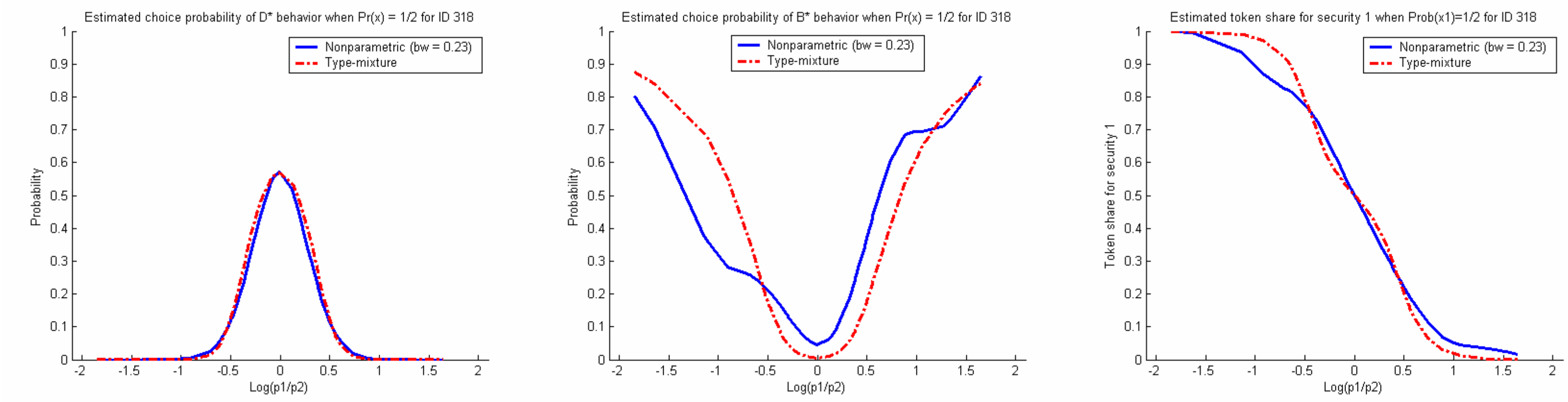


G: ID 318



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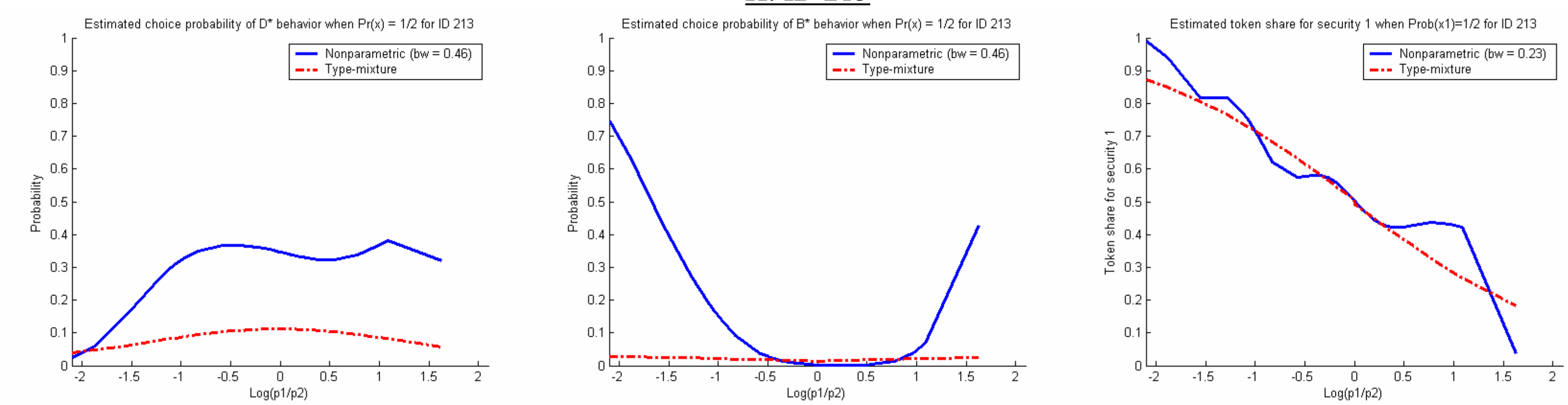
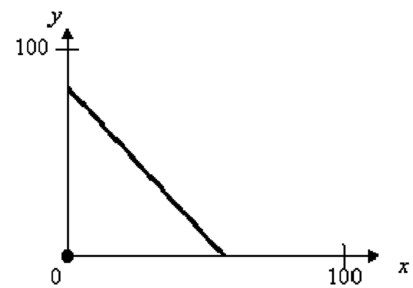
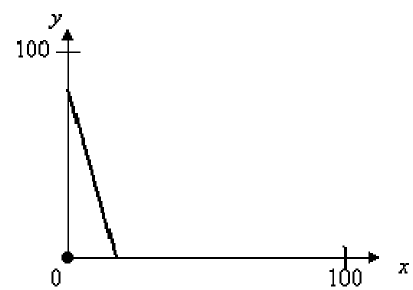
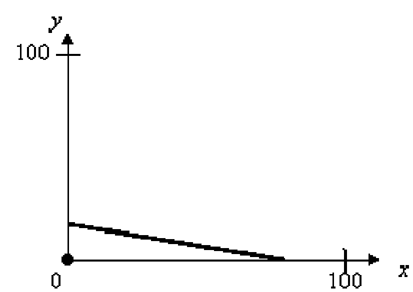


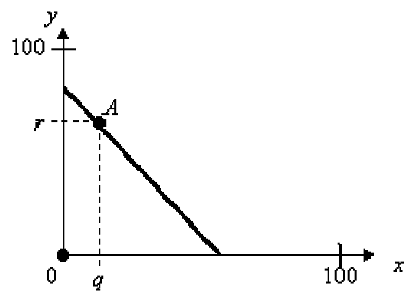
Attachment 1

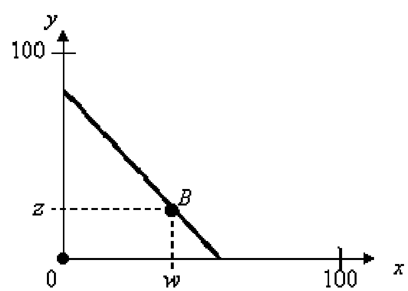






<u>Attachment 2</u>





Attachment 3