# 4

## Balance of Payments and the Foreign Exchange Market: A Dynamic Partial Equilibrium Model

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The modern theory of flexible exchange rates appears to have no connection with actual market processes. There is no explicit treatment of the sources of supply and demand in the foreign exchange market and no explicit analysis of how supply and demand interact in that market to determine the exchange rate. In part this is because such details are hidden in the background of general equilibrium models; but in part it is because analysis has been reduced to terms that bear no connection to supply and demand in the foreign exchange market. An example of the latter is the monetarist theory of exchange rate determination according to which "the exchange rate is a relative price of two monies and as such is determined by the relative supplies of these monies on the one hand and the relative demands for these monies on the other" (see Frankel, 1976; Mussa, 1976; and Bilson, 1978). This cannot be a statement about supply and demand in the foreign exchange market because, as mentioned, they derive from demands for and supplies of all kinds of assets and goods and services. In fact holdings of foreign money are so small that the direct effect of changes in money demand or supply on the exchange rate must be insignificant. Rather, such shifts affect the exchange rate through their effect, *ex ante*, on capital movements and other components of the balance of payments.

In many of the recent models, including Dornbusch (1976), it is assumed that all other assets but monies are perfect substitutes. In such models

balance of payments pressures have no effect on the exchange rate, which can deviate from its purchasing power parity, or long-run equilibrium, value only to the extent that monetary conditions permit differences in interest rates, as is shown in Kouri and de Macedo (1978). These models cannot, however, explain observed movements in exchange rates in recent years because these movements have been far in excess of differences in inflation rates even if allowance is made for anticipated differences in future inflation rates as reflected in interest rate differentials. The models of perfect substitutability simply assume away market pressures that could account for the observed behavior. As one example, they cannot explain the tendency of the currencies of surplus countries to appreciate and those of deficit countries to depreciate continuously in real terms. This stylized fact of recent years cannot be explained either by the "textbook" partial equilibrium model, based on Bickerdike (1920), Robinson (1937), and Machlup (1939, 1940). In that model the capital flow account is independent of the exchange rate—in fact, it is treated as a transfer payment. This implies that *ex ante* shifts in the current account or in the capital flow account give rise to once-and-for-all adjustments in the exchange rate. Furthermore, shifts in the trade balance *ex ante* have no effect on the current account *ex post* because the *ex post* current account is determined independently of the exchange rate.

The purpose of this chapter is to develop a dynamic supply-demand model of the foreign exchange market, consistent with the modern general equilibrium approach—given the assumption that prices or quantities in other markets are exogenous. The burden of balance of payments adjustment is thus assumed to fall on the exchange rate alone.

It is assumed that the foreign exchange market clears continuously, as indeed it almost does. Therefore, at a given time only stock demands for and supplies of foreign exchange derived from stock transactions on the capital account matter in determining the short-run equilibrium value of the exchange rate. This is similar to the liquidity preference theory of interest rate determination, or the theory of the determination of the demand price of capital assets in terms of stocks rather than flows. It is also consistent with the general equilibrium "asset market approach" to international monetary economics that determines all of these prices simultaneously by conditions of stock equilibrium.

Given the short-run equilibrium value, the exchange rate must change *per unit of time* in such a way as to equilibrate flow demands for and supplies of foreign exchange derived from capital flows on the one hand and current account transactions on the other. Capital flows are functions of the rate of change of the exchange rate because the *stock* demand for

foreign assets is a function of the level of the exchange rate. Therefore in order for the foreign exchange market to stay in equilibrium, domestic currency must depreciate whenever the current account is in deficit (in excess of "normal deficit") and appreciate whenever the current account is in surplus (in excess of "normal surplus"). This *acceleration hypothesis* accords well with the behavior of the major currencies in recent years.

In long-run equilibrium the real exchange rate is constant and is determined by the condition that the current account is at its normal level, which is taken to be zero in this analysis.

## 4.1 Capital Account and the Exchange Rate: Short-Run Equilibrium in the Foreign Exchange Market

The foreign exchange market is but one of many interconnected financial markets. Its special nature is that intermediation and arbitrage between financial markets in different currency areas must go through it: To invest abroad in excess of their current holdings of foreign assets domestic residents must first acquire foreign currency in the amount of their planned investment; to add to their asset holdings in the domestic economy foreign residents must similarly first acquire domestic currency in the foreign exchange market. Demand for and supply of foreign currency, or what comes to the same, demand for and supply of domestic currency *at a given time* can thus be derived from domestic demand for foreign assets in excess of already existing holdings on the one hand and foreign demand for domestic assets in excess of existing holdings on the other. This description must be qualified when domestic residents lend to foreign residents in domestic currency or when foreign residents lend to domestic residents in foreign currency: This case is analyzed separately in appendix 3.

Consider now how domestic demand for foreign assets and foreign demand for domestic assets depend on the exchange rate. For this purpose assume that prices and rates of return of all assets in the domestic economy are exogenously given in domestic currency, and that prices and rates of return of foreign assets are exogenously given in foreign currency. Thus all domestic assets can be aggregated into a single "domestic asset" whose price for a foreign investor is the foreign currency price of domestic currency; and similarly all foreign assets can be aggregated into a single foreign asset whose price for a domestic investor is the domestic currency price of foreign currency, *ceteris paribus*.

Portfolio theory suggests that, given total domestic marketable wealth,

domestic residents want to hold some fraction of it in foreign assets, this fraction depending on expected returns and risk characteristics of domestic and foreign assets.[1] Therefore, taking prices of foreign assets as exogenous in foreign currency and the stock of domestic marketable wealth as exogenous in domestic currency, it follows that domestic demand for foreign assets is a declining function of the price of foreign currency with elasticity of minus one.[2] By the same reasoning, demand for domestic assets by foreign residents is a declining function of the price of domestic currency in terms of foreign currency with elasticity of minus one. This simple result linking stock transactions on the capital account to the level of the exchange rate has not, surprisingly, been noted in the literature. Thus in a recent paper on foreign exchange market intervention John Williamson (1976) states that "in striking contrast [to the current account] there are rather few reasons for expecting the capital account to depend on the *level* of the exchange rate, but compelling reasons for expecting it to depend on the expected change in the rate."[3] Similarly in his well-known paper on flexible rates Dornbusch (1976) assumes away the dependence of capital account transactions on the level of the exchange rate with the assumption of perfect substitutability between domestic and foreign assets, except for money. Therefore the exchange rate is indeterminate in his model in a system of dual exchange rates, when the capital account is isolated from the trade account.[4]

### 4.1.1 The Formal Model of Short-Run Equilibrium

To develop the model formally let $F_0$ be the stock of foreign assets held by domestic residents at time 0 valued in foreign currency, and let $G_0$ be the stock of domestic assets held by foreign residents at time 0 valued in domestic currency. $F_0$ and $G_0$ are both given at the initial moment. In contrast, the *desired holdings* of foreign assets by domestic residents $F^d$ and of domestic assets by foreign residents $G^d$ are subject to choice. Let $s$ be the domestic currency price of foreign exchange. Then $F^d - F_0$ is the stock demand for foreign exchange derived from domestic excess demand for foreign assets (in excess of initial holdings) and $(G^d - G_0)/s$ the stock supply of foreign exchange derived from foreign excess demand for domestic assets. Momentary equilibrium in the foreign exchange market is then defined by

$$(F^d - F_0) + (I^d - I_0) = (G^d - G_0)/s, \tag{1}$$

where

$I_0$ = initial stock of central bank's foreign exchange reserves,

$I^d$ = desired stock of foreign exchange reserves,

$I^d - I_0$ = central bank's purchase of foreign exchange at time 0.

With fixed exchange rates change in reserves, $I^d - I_0$ is the residual that equilibrates the foreign exchange market; with clean floating $I^d - I_0$ is zero and the exchange rate adjusts to equilibrate private demand and supply. Between these two extremes there are other types of regimes which can be specified by appropriate characterization of intervention behavior. In the present paper we restrict the analysis to floating with no systematic intervention.

Next we need to specify the asset demand functions. Let $V$ be the value of domestic marketable wealth in domestic currency and $V^*$ the value of foreign marketable wealth in foreign currency. Then, with reference to earlier discussion,

$$F^d \cdot s = f(R, R^* + \pi, z)V, \qquad G^d/s = g(R - \pi, R^*, z)V^*, \qquad (2)$$

where

  $R$ = nominal rate of return on domestic assets in domestic currency,

  $R^*$ = nominal rate of return on foreign assets in foreign currency,

  $\pi$ = expected rate of change in the domestic currency price of foreign currency,

  $z$ = vector of other determinants of international investment.

Substituting these demand functions in equation (1) above and setting $I^d - I_0$ equal to zero, we get the condition of short-run equilibrium in the foreign exchange market with no intervention:

$$f(R, R^* + \pi, z)V/s - F_0 = g(R - \pi, R^*, z)V^* - G_0/s. \qquad (3)$$

The left-hand side gives the stock demand for foreign exchange at time 0 derived from desired domestic purchases of foreign assets at time 0. It is illustrated by the downward sloping $DD$ schedule in figure 4.1. The right-hand side gives the stock supply of foreign exchange derived from desired foreign purchases of domestic assets at time 0. It is illustrated by the upward sloping $SS$ schedule in figure 4.1. Short-run equilibrium in the foreign exchange market, which clearly is unique, obtains at the intersection of these demand and supply schedules at $A_0$. As the schedules are drawn no transactions take place at the equilibrium exchange rate $s_0$ because initial asset holdings just happen to equal desired asset holdings. This does not mean, however, that the exchange rate is indeterminate, for a small increase in the price of foreign currency would bring forward an excess supply of foreign exchange; and a small decrease, an excess demand
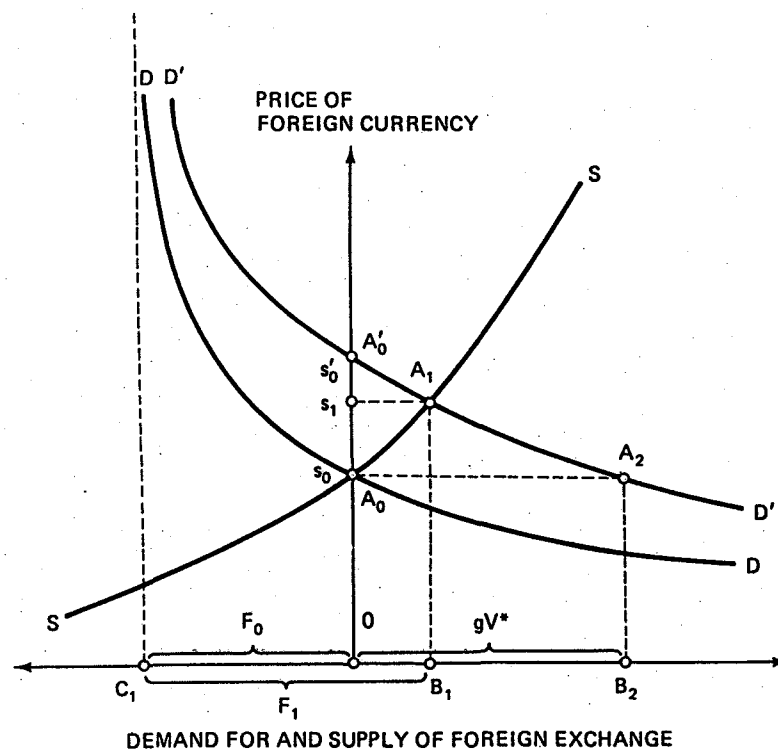
DEMAND FOR AND SUPPLY OF FOREIGN EXCHANGE

Figure 4.1
Short-run equilibrium in the foreign exchange market.

for foreign exchange. Thus, although no transactions take place, potential transactions keep the exchange rate at $s_0$. It is clear then that short-run equilibrium is not only unique but also stable. It will be assumed from now on that the foreign exchange market is always in short-run equilibrium.

Suppose now that domestic residents want to increase their holdings of foreign assets for some reason—for example, because of an increase in foreign interest rates. In figure 4.1 the demand schedule shifts to the right, to $D'D'$, by $OB_2$ at the initial exchange rate $s_0$. If the central bank pegged the exchange rate at $s_0$, it would, of course, lose reserves in that amount. With no intervention, domestic currency instead depreciates to $s_1$. This enables domestic residents to increase their foreign asset holdings by $OB_1$—which is equal in value to the additional purchases of the now cheaper domestic assets by foreign residents (measured in

foreign currency). Once the new equilibrium position has been reached, and by assumption it is reached instantaneously, the exchange rate stays at $s_1$ and again no transactions take place in the market (ignoring for now "marginal" supplies and demands coming from flow transactions). Technically, the origin shifts to $B_1$ in figure 4.1: The initial stock of foreign assets is now $C_1 B_1$ $(F_1)$ rather than $C_1 O$ $(F_0)$.

It is not necessary for transactions actually to take place in order for the exchange rate to change, as is the case above. For example, suppose that foreign residents do not hold domestic assets, a case typically assumed in recent literature. Then an attempt by domestic residents as a group to increase their holdings of foreign assets would only succeed, in the short run, in raising the price of foreign assets, or the exchange rate, to a level at which the existing stock would be willingly held and thus to a level at which no transactions would actually take place in the market.

To develop the above analysis formally, let $f$ and $g$ be the initial portfolio proportions, and $F_0$ and $G_0$ the initial asset holdings. From equation (3), equilibrium exchange rate, $s_0$ in figure 4.1, is given by

$$s_0 = \frac{fV + G_0}{gV^* + F_0}. \tag{4}$$

Let $f_1$ and $g_1$ be the new portfolio properties at time 0 (unlike in the figure, the supply schedule is allowed to shift at well). The new equilibrium value of the exchange rate, $s_1$ in figure 4.1, is then

$$s_1 = \frac{f_1 V + G_0}{g_1 V^* + F_0}. \tag{5}$$

At this new exchange rate, asset holdings are

$$F_1 = f_1 V / s_1, \qquad G_1 = g_1 V^* s_1. \tag{6}$$

$F_1$ and $G_1$ become the new initial conditions. If nothing changes after the shift from $(f, g)$ to $(f_1, g_1)$ at the initial moment, the exchange rate will stay at $s_1$ with no transactions taking place thereafter. To check this apply formula (4) again to the new situation:

$$s_1' = \frac{f_1 V + G_1}{g_1 V^* + F_1} = \frac{f_1 V + g_1 V^* s_1}{g_1 V^* + f_1 V / s_1} = s_1, \tag{7}$$

which confirms that the exchange rate will indeed stay at $s_1$.

Figure 4.2 illustrates the effect of a once-and-for-all sale of foreign exchange by the central bank. The market is initially in equilibrium at $A_0$ with exchange rate $s_0$. The central bank sells foreign exchange in the
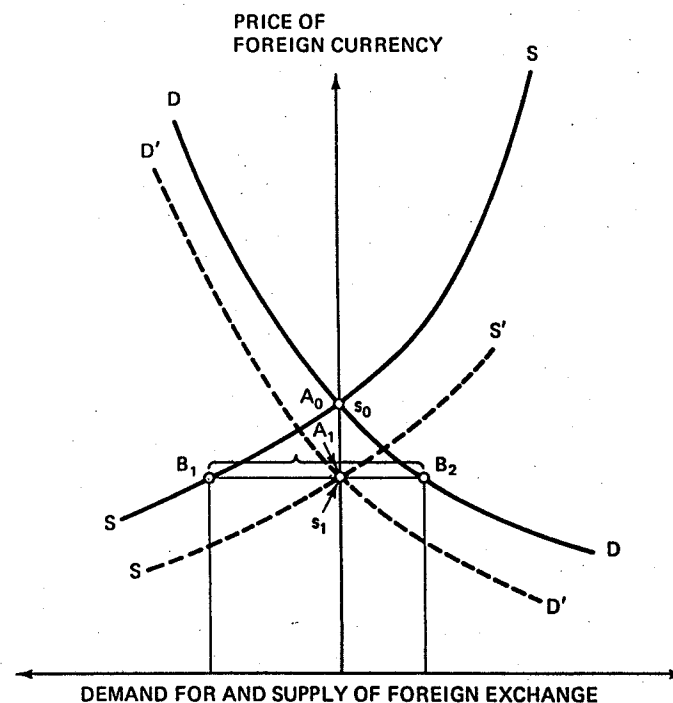
**Figure 4.2**
Effect of a once-and-for-all sale of foreign exchange by the central bank.

amount $B_1 B_2$ ($= \Delta I_0$), thus forcing the price of foreign currency down to $s_1$. At this new exchange rate, $B_1 A_1$ of the decline in central bank reserves finances an increase in domestic holdings of foreign assets, while the rest, $A_1 B_2$ in the figure, finances a reduction in foreign holdings of domestic assets. The exchange rate will not return to $s_0$ immediately as the central bank leaves the market; instead it stays at $s_1$ because the initial conditions change, thus shifting the demand and supply schedules so that they intersect at $s_1$. From equation (1) the new equilibrium exchange rate $s_1$ is

$$s_1 = \frac{fV + G_0}{gV^* + F_0 + \Delta I_0}. \tag{8}$$

This formula is useful because it shows the limited possibilities that central banks have to influence the exchange rate when official reserves are only a small fraction of private asset holdings.

### 4.1.2 Comparison with the Bickerdike-Robinson-Machlup Model

The supply-demand model developed above resembles the "textbook" model of the foreign exchange market based on the work of Bickerdike (1920), Robinson (1937), and Machlup (1939, 1940) in that the supply and demand schedules in the foreign exchange market are derived from economic transactions between the two countries or currency areas. The difference is that the textbook model derives these schedules from flow transactions on the trade account, while the present model derives them from stock transactions on the capital account. In the textbook model, depreciation of domestic currency increases net supply of foreign exchange *per unit of time* by inducing a reduction in domestic purchases of foreign goods and services, and an increase in foreign purchases of domestic goods and services, provided that the "elasticities condition" holds, while capital movements are treated as "transfer payments" that do not depend on the level of the exchange rate. In contrast, in the present model depreciation of domestic currency brings forward an excess supply of foreign currency *at a point in time* by inducing a reduction in domestic holdings of foreign assets and an increase in foreign holdings of domestic assets. Furthermore, net supply of foreign exchange is everywhere an increasing function of the price of foreign currency; thus equilibrium is both unique and globally stable. The elasticities condition is met because the elasticities of domestic demand for foreign assets, and of foreign demand for domestic assets with respect to the exchange rate are both equal to one in absolute value. (See, however, the discussion in appendix 1.)

If there was a dual exchange rate system with capital account transactions going through one market, market $A$, and current account transactions, including interest payments going through another market— market $B$—the model developed above would be a complete partial equilibrium model of market $A$, while the Bickerdike-Robinson-Machlup model would be a complete partial equilibrium model of market $B$. Market $B$ would be linked to market $A$ because the net inflow or outflow of interest income would have to be effected through exchange rate adjustments in market $B$. In a unified floating rate system there is, as it were, also a feedback from market $B$ to market $A$ because the initial conditions on which equilibrium in market $A$ depends change whenever the current account is different from zero. We now turn to study this dynamic process.

### 4.2 Interaction between the Current Account and the Capital Account: The Dynamics of the Exchange Rate

To study the dynamics of the foreign exchange market we make two simplifying assumptions initially: first, there is no inflation or real growth in either country; and second, all interest earnings on foreign assets are spent on imports. The first assumption implies that current account must be zero in long-run equilibrium, while the second assumption implies that the long-run equilibrium value of the exchange rate, associated with zero current account, does not depend on the level or direction of international investment. The reason for these assumptions is to abstract from the complex problems of intertemporal transfer whose satisfactory treatment requires general equilibrium analysis.

#### 4.2.1 A Special Case

We begin the analysis with the special case, typically assumed in recent literature, in which foreign residents do not hold domestic assets. Short-run equilibrium condition (3) then becomes

$$F \cdot s = f(R, R^* + \pi, z)V, \tag{9}$$

where $\pi$ is still set equal to zero. This equilibrium condition is illustrated by the $FF$ schedule in figure 4.3b. With initial stock of foreign assets equal to $F_0$, initial equilibrium obtains at $A_0$ with exchange rate $s_0$.

The stock of foreign assets changes whenever the current account is different from zero, or

$$\dot{F} = B(s; y), \tag{10}$$

where $B$ is current account surplus in foreign currency and $y$ is a vector of determinants of the current account, including domestic and foreign prices and activity levels. Equations (9) and (10) together with the initial condition $F = F_0$ define the complete dynamic partial equilibrium model of the foreign exchange market.

Assume first that the current account is everywhere an increasing function of the price of foreign currency, as illustrated by the $BB$ schedule in figure 4.3a. At the initial exchange rate $s_0$ the current account is in surplus by $B_0$. Thus, thereafter the stock of foreign assets increases and domestic currency appreciates as the market converges to long-run equilibrium at $A^*$ with exchange rate $s^*$ and stock of foreign assets $F^*$. If, instead, the stock of foreign assets was initially above its long-run level, the price of foreign currency would be below its long-run equilibrium
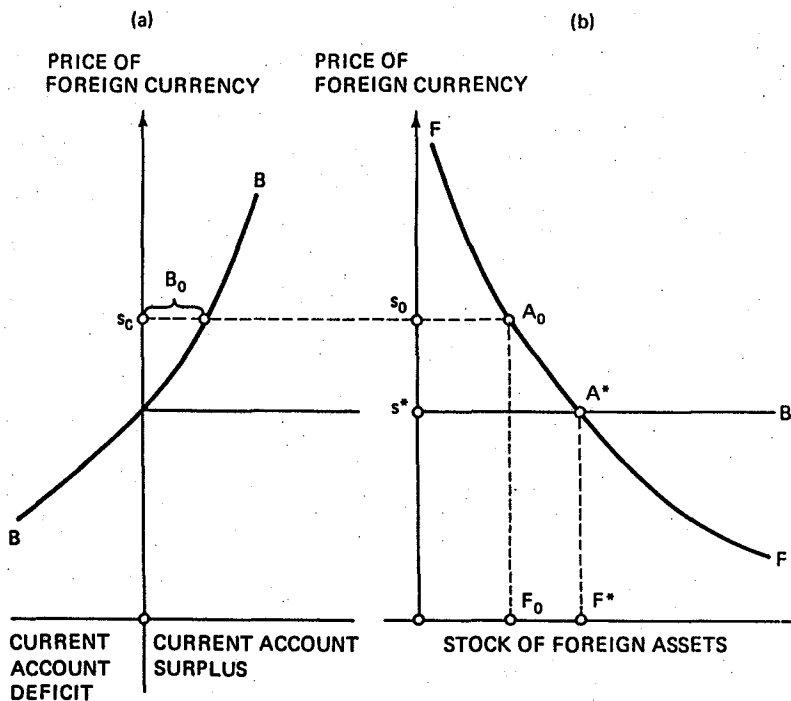
(a)                                                                          (b)



**Figure 4.3**
Interaction between current and capital accounts 1: a special case.

value, and the market would converge to equilibrium at $A^*$ along $FF$ from the right-hand side with domestic currency depreciating and the current account in deficit. It is thus evident that, given the monotonicity of the $BB$ schedule, long-run equilibrium is both unique and globally stable.

The dynamics of the foreign exchange market, illustrated in figure 4.3, is consistent with observed exchange rate behavior in recent years with the currencies of surplus countries appreciating and those of deficit countries depreciating in excess of differences in inflation rates. It is useful to restate this implication of the model as the *acceleration hypothesis*: Domestic currency appreciates whenever the current account is in surplus and depreciates whenever it is deficit.

## 4.2.2 Multiple Equilibria and Dynamic Instability
Consider next the possibility that long-run equilibrium is not unique because the current account is not a monotonic function of the exchange
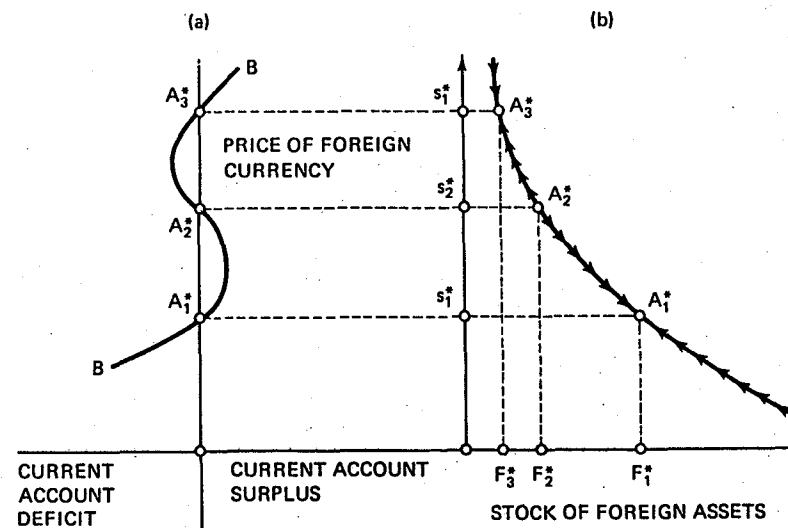
(a)                                                                          (b)



**Figure 4.4**
Interaction between current and capital accounts 2: multiple equilibria.

rate. Figure 4.4 gives an example in which there are three long-run equilibria, namely, $A_1^*$, $A_2^*$, and $A_3^*$. Of these, only $A_1^*$ and $A_2^*$ are stable. Which one of these the market will converge to depends on where it starts. As is indicated in figure 4.4a, it converges to $A_1^*$ if it is initially on the right-hand side of $A_2^*$, and to $A_3^*$ if it is initially on the left-hand side of $A_2^*$; and it stays at $A_2^*$ if it happens to be there. With multiple equilibria, the foreign exchange market is thus dynamically unstable in that perturbations may move it from one equilibrium to another, unlike the case above, in which it always ended in the same long-run equilibrium.

Before considering the general case it is illuminating to derive the results of the above analysis in a slightly different way. In order for the foreign exchange market to stay in equilibrium after the initial moment, the exchange rate has to change per unit of time in such a way as to equilibrate the net flow supply of foreign exchange from current account transactions with the net flow demand for foreign exchange from marginal additions to domestic holdings of foreign assets, or capital outflow for short. But capital outflow is a function of *the rate of change* of the exchange rate; that is,

$$\dot{F}^d = -f\frac{V}{s}\frac{\dot{s}}{s}, \tag{11}$$

where expectations and other determinants of $f$ are held constant. Along the adjustment path the exchange rate must then satisfy a "dynamical balance of payments equilibrium condition":

$$\dot{F}^d = -f\frac{V}{s}\frac{\dot{s}}{s} = B(s; x). \tag{12}$$

This can be rewritten

$$\frac{\dot{s}}{s} = -\frac{sB(s; x)}{fV} = -\frac{B}{F}, \tag{13}$$

which is an algebraic statement of the acceleration hypothesis: Domestic currency appreciates whenever the current account is in surplus, and depreciates when it is in deficit. Furthermore, the rate of appreciation (depreciation) is equal to the ratio of the current account surplus (deficit) to the stock of foreign assets.[5]

### 4.2.3 The General Case
To build on the previous section it is useful to rearrange short-run equilibrium condition (3) in the form

$$f(R, R^* + \pi, z)V/s - g(R - \pi, R^*; z)V^* \equiv NFA^d = NFA^s$$

$$\equiv F_0 - G_0/s. \tag{14}$$

The right-hand side can be interpreted as net supply of foreign assets and the left-hand side as net demand for foreign assets.[6] These two schedules are illustrated by the $FF$ and $SS$ schedules in figure 4.5. The $FF$ schedule corresponds to the $FF$ schedule in figures 4.3 and 4.4; they are the same when foreign residents do not hold domestic assets. Unlike in the special case, however, net stock of foreign assets is not predetermined at a given time—its value in foreign currency increases as domestic currency depreciates. For any exchange rate excess demand for foreign exchange at a given time can be read as the horizontal distance between the $FF$ and $DD$ schedules (see figure 4.1). Excess demand is zero at $A_0$, which is the same short-run equilibrium position as $A_0$ in figure 4.1.

Given its initial value $s_0$, the exchange rate must change in such a way as to equilibrate the current account with "marginal" net outflow of capital. Net outflow of capital is a function of the rate of change of the exchange rate; that is,

$$CF = \frac{\dot{G}^d}{s} - \dot{F}^d = \left(gV^* + f\frac{V}{s}\right)\frac{\dot{s}}{s} = \left(F + \frac{G}{s}\frac{\dot{s}}{s}\right) \tag{15}$$
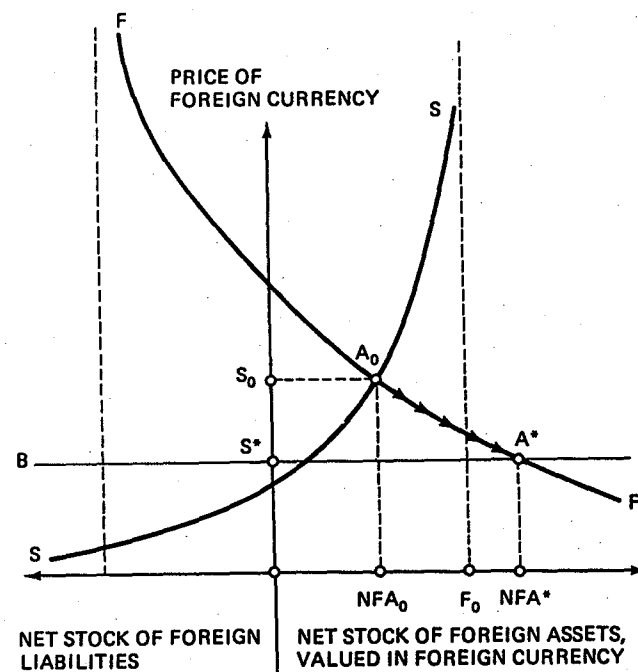
Figure 4.5
Interaction 3: the general case.

where $\dot{G}^d/s = gV^*(\dot{s}/s) =$ inflow of capital and $\dot{F}^d = -f(V/s)(\dot{s}/s) =$ outflow of capital.[7]

For the foreign exchange market to stay in flow equilibrium, it is then necessary that the exchange rate satisfies the dynamical balance of payments equilibrium condition:

$$CF = \left(gV^* + f\frac{V}{s}\right)\frac{\dot{s}}{s} = B(s; x), \tag{16}$$

where $B$ is the current account surplus in foreign currency as before. This can be written

$$\frac{\dot{s}}{s} = -B(s; x)/\left(f\frac{V}{s} + gV^*\right), \tag{17}$$

which is simply the "acceleration equation." Thus the rate of appreciation (depreciation) of domestic currency is equal to the ratio of the current account to the level of international investment as measured by the sum

of domestic holdings of foreign assets on the one hand and foreign holdings of domestic assets on the other.

Change in the net stock of foreign assets, valued on foreign currency, is governed by the dynamical equation

$$NF\dot{A} = -f\frac{V}{s}\frac{\dot{s}}{s} = \frac{f(V/s)}{f(V/s) + gV^*}B(s; x),\tag{18}$$

which is similar to equation (11). In particular, net stock of foreign assets is constant whenever the current account is in zero, increases when it is in surplus, and decreases when it is in deficit.

The complete dynamic partial equilibrium model of the foreign exchange market consists of equations (14) and (18). It is illustrated in figure 4.6, where the horizontal BB line is the schedule of long-run equilibrium in the balance of payments (zero current account). It is assumed to be unique.[8] Short-run equilibrium obtains at the intersection of the FF and SS schedules at $A_0$; long-run equilibrium obtains at the intersection of the FF and BB schedules at $A^*$; and adjustment from short-run to long-run equilibrium takes place along the FF schedule as indicated by the arrows. Given that long-run equilibrium is unique, it is clearly also globally stable; Wherever the market starts, it always ends there.

### 4.2.4 Comparative Dynamics

We now apply the model to study the dynamic response of the foreign exchange market to (i) a decline in foreign demand for domestic assets; (ii) a once-and-for-all purchase of foreign exchange by the central bank; and (iii) an increase in domestic demand for imported foreign goods and services. Except for the analysis in figure 4.7 it is assumed that long-run equilibrium is unique.

Figure 4.6 illustrates the dynamic response of the foreign exchange market to a decline in foreign demand for domestic assets from an initial situation of long run equilibrium at $A_0$. The FF schedule shifts to the right to $F'F'$: At the initial exchange rate there is excess supply of domestic currency and thus excess demand for foreign currency in the amount $A_0A^*$. Short-run equilibrium is restored as domestic currency depreciates to $s_0'$. Foreign residents are able to reduce their holdings of domestic assets immediately to the extent that domestic residents reduce their holdings of the now more expensive foreign assets. They are able to do so over time by importing more from and exporting less to the domestic economy, in effect by gradually exchanging their assets for goods and
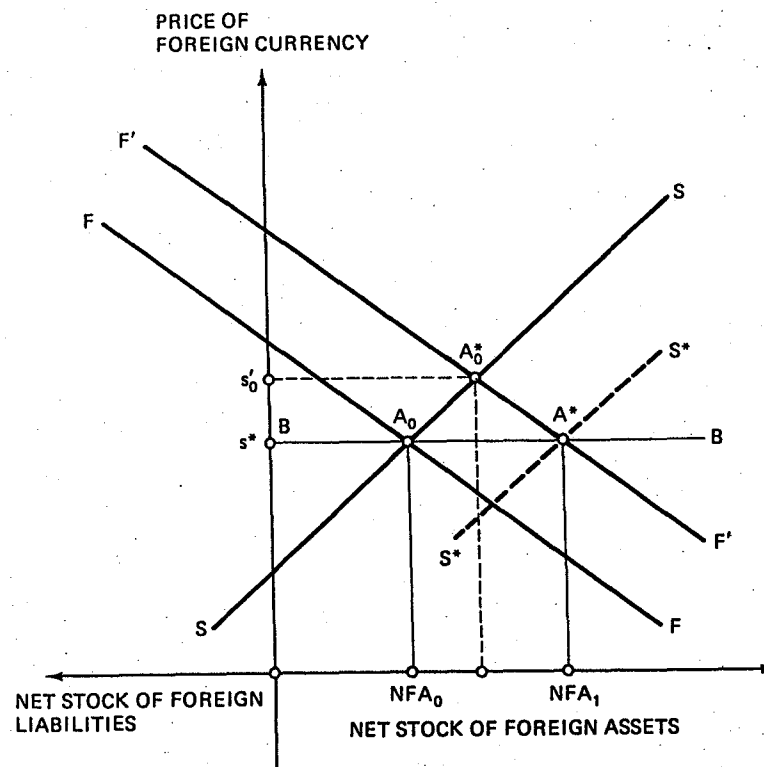
**Figure 4.6**
Dynamic response 1: effect of a decline in foreign demand for domestic assets.

services. As is shown in figure 4.6 the exchange rate eventually returns back to its "normal" level $s^*$.

The cumulative current account surplus in the course of the adjustment process is approximately equal to $A_0A^*$: The initial excess demand for foreign exchange in the amount $A_0A^*$ translates over time into an equal cumulative current account surplus.[9]

One can interpret the capital account disturbance as a once-and-for-all financial transfer payment that is effected over a period of time through surpluses in the current account brought about by transitory depreciation of the domestic currency. The time it takes the foreign exchange market to effect such capital transfers depends on the level of trade in relation to the level of asset holdings on the one hand and the exchange rate

PRICE OF
FOREIGN CURRENCY



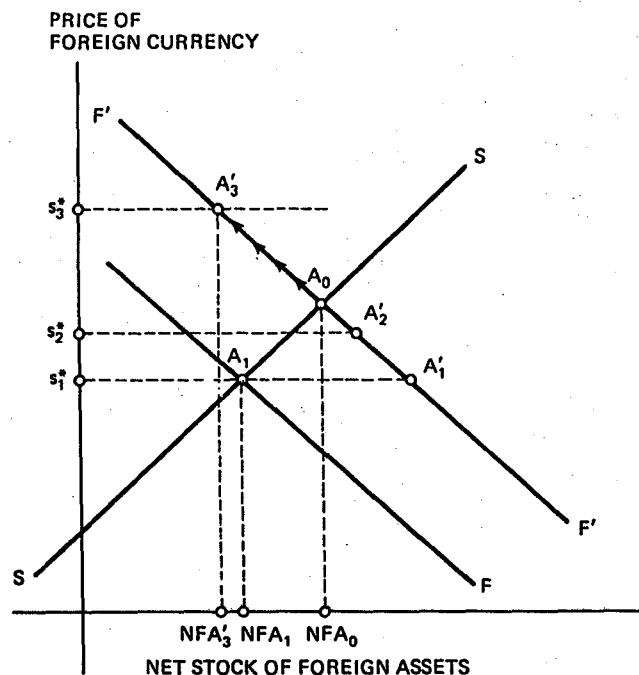NET STOCK OF FOREIGN ASSETS

**Figure 4.7**
Dynamic response with multiple equilibria.

elasticity of the current account on the other (see appendix 1 and in particular table 4.1).

When long-run equilibrium is unique, as above, the exchange rate effect of a capital account disturbance is only transitory. With multiple equilibria, however, the effect may be permanent because the market may move from one equilibrium to another in response to a disturbance. Figure 4.7 illustrates such a possibility. There are three possible long-run equilibrium values of the exchange rate, namely, $s_1^*$, $s_2^*$, and $s_3^*$, of which only $s_1^*$ and $s_3^*$ are stable. The market is initially at $A_1$ with exchange rate $s_1^*$ and net stock of foreign assets $NFA_1$. Reduction in foreign demand for domestic assets shifts the $FF$ schedule to the right to $F'F'$. To restore equilibrium, domestic currency depreciates to $s_0$, which is above the exchange rate associated with the unstable equilibrium, namely, $s_2^*$. Therefore the exchange rate will not come back to $s_1^*$, but instead increases to $s_3^*$. Thus there is a permanent decline in the value of domestic currency and a period of *deficits* in the current account which may in fact cause the
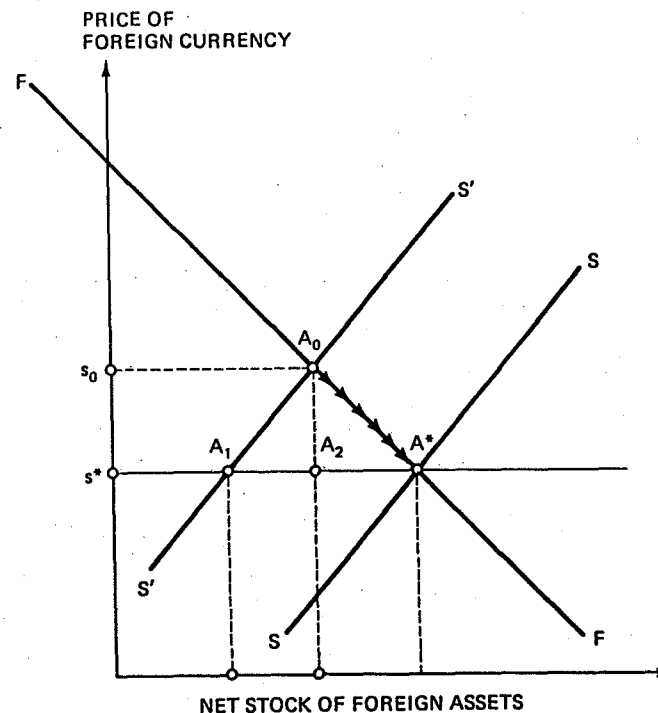
PRICE OF
FOREIGN CURRENCY



NET STOCK OF FOREIGN ASSETS

**Figure 4.8**
Dynamic response 2: effect of a once-and-for-all purchase of foreign exchange by the central bank.

net stock of foreign assets to decline in the long run. Thus with multiple equilibria, it is difficult to infer whether observed currency depreciation is caused, *ex ante*, by "capital outflow" or an adverse shift in the trade balance, because, as the example shows, *ex ante* "capital outflow" may turn into *ex post* "capital inflow."

Consider next the response of the market to a once-and-for-all purchase of foreign exchange by the central bank. This is shown in figure 4.8 as a leftward shift of the $SS$ schedule by $A_1A^*$ at the initial exchange rate $s^*$, assumed to be the long-run equilibrium exchange rate. The *ex ante* excess demand of $A_1A^*$ is eliminated by depreciation of the domestic currency to $s_0$. At this exchange rate, $A_2A^*$ of the increase in the central bank's foreign exchange reserves comes from domestic residents who reduce their holdings of foreign assets by this amount, and the rest from foreign residents who supply foreign exchange in the amount $A_1A_2$ in order to increase their holdings of domestic assets. Over time, however,

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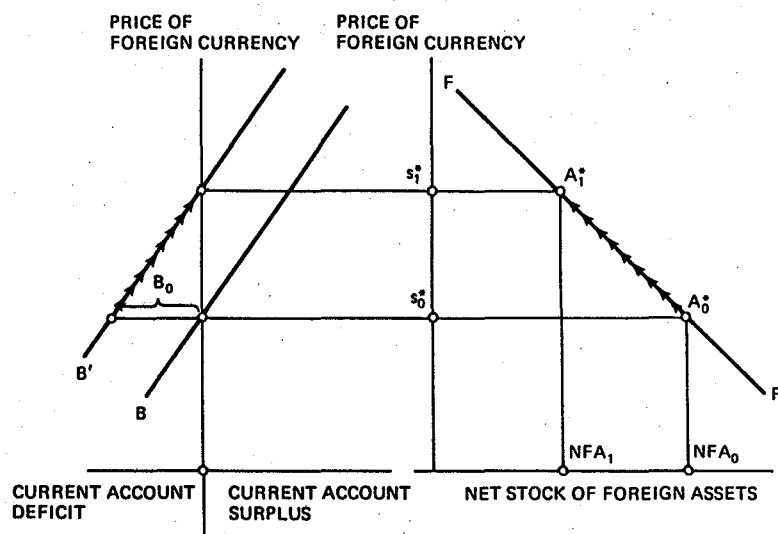CURRENT ACCOUNT DEFICIT | CURRENT ACCOUNT SURPLUS | NET STOCK OF FOREIGN ASSETS

**Figure 4.9**
Dynamic response 3: effect of a permanent increase in the domestic demand for foreign goods.

the purchase of foreign exchange translates to an exactly equal cumulative surplus in the current account, and after the initial depreciation domestic currency appreciates back to the same long-run equilibrium level. The initial impact of intervention on the exchange rate depends on the size of the central bank's purchase of foreign exchange in relation to total international investment, while the duration of the impact depends on the strength of the induced current account response, as is clear from earlier analysis.

To conclude this section we show in figure 4.9 the response of the foreign exchange market to a permanent increase in domestic demand for imported goods and services. In the textbook model such shift would cause an immediate depreciation of domestic currency with no change in the current account balance which is determined *ex post* by the capital flow account independently of the exchange rate. In contrast, in figure 4.9 the increase in import demand causes an equal increase in the current account deficit at the initial moment ($B_0$). Domestic currency depreciates to its new long-run equilibrium value $s_1^*$ only gradually as successive, although diminishing, current account deficits are financed by reductions in domestic holdings of foreign assets on the one hand and increases in

foreign holdings of domestic assets on the other. The total cumulative current account deficit resulting from the increase in import demand is approximately equal to $NFA_1 - NFA_0$.

### 4.2.5 Balance of Payments Equilibrium with Inflation

We have assumed so far that there is no inflation in either country. Suppose now that prices in the domestic economy are increasing at a constant rate $\pi_p$ and prices in the foreign country at constant rate $\pi_p^*$. The rate of change of the price of foreign currency in long-run equilibrium is then $\pi_p - \pi_p^*$. Even in the absence of real growth the current account is not zero in long-run equilibrium: In order to maintain a constant real stock of foreign assets domestic residents must purchase new assets at the rate of $\pi_p^*(F/p^*)$ in real terms, where $(F/p^*)$ is the real value of domestic holdings of foreign assets in long-run equilibrium. Similarly, foreign residents must purchase new domestic assets at the rate of $\pi_p(G/p)$ in order to keep the real value of their holdings of domestic assets constant, where $G/p$ is the real value of foreign holdings of domestic assets in long-run equilibrium. The current account surplus is thus equal to $\pi_p^*(F/sp^*) - \pi_p(G/p)$ in the units of domestic output. If the balance of payments accounts were properly measured this notional surplus of the current account would instead be counted as outflow of interest income: in real terms, net inflow of interest income would be $(R^* - \pi_p^*)(F/sp^*) - (R - \pi_p)(G/p)$. If we continue to assume that real interest earnings are spent on imports, the above analysis still applies to the real exchange rate $s(p^*/p)$ and the real, or inflation adjusted, balance of payments. A satisfactory treatment of inflation as well as of real growth requires, however, an analysis of its own. For this reason we continue to assume in the following that there is no inflation or real growth.

### 4.3 Implications of "Rational" Speculation in the Foreign Exchanges

In this section we study the implications of "rational" speculative behavior for the dynamics of the foreign exchange market. Such behavior is technically interpreted as perfect foresight, that is, $\pi = \dot{s}/s$, in contrast to the assumption of stationary expectations maintained so far. The analysis is limited to the special case in which foreigners do not hold domestic assets because space does not permit satisfactory treatment of the general case. For the same reason, long-run equilibrium is assumed to be unique.

The dynamic partial equilibrium model of the foreign exchange market is now defined by the following two differential equations:
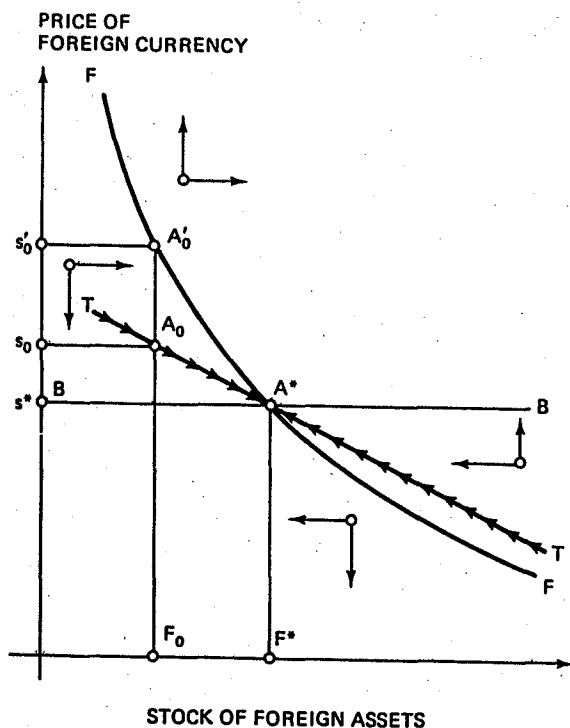
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**Figure 4.10**
Rational expectations equilibrium.

$$f\left(R, R^* + \frac{\dot{s}}{s}, z\right)\frac{V}{s} = F \qquad \text{[see equation (9)],} \qquad (19)$$

$$\dot{F} = B(s; x). \qquad (20)$$

The dynamics of this system can be studied with the aid of figure 4.10. The FF schedule, defined by setting $\dot{s}/s$ equal to zero in equation (18), gives for each stock of foreign assets the exchange rate that is consistent with no expected appreciation or depreciation. It is the same as the FF schedule in figure 4.3. As the arrows indicate, domestic currency must be expected to appreciate at any point above the FF schedule and depreciate at any point below it for such a point to be an equilibrium point. The dynamics of the stock of foreign assets can be determined with the aid of the BB schedule, defined by setting $\dot{F}$ equal to zero in equation (20) [see equation (23) in appendix 1]. As indicated by the arrows, the stock

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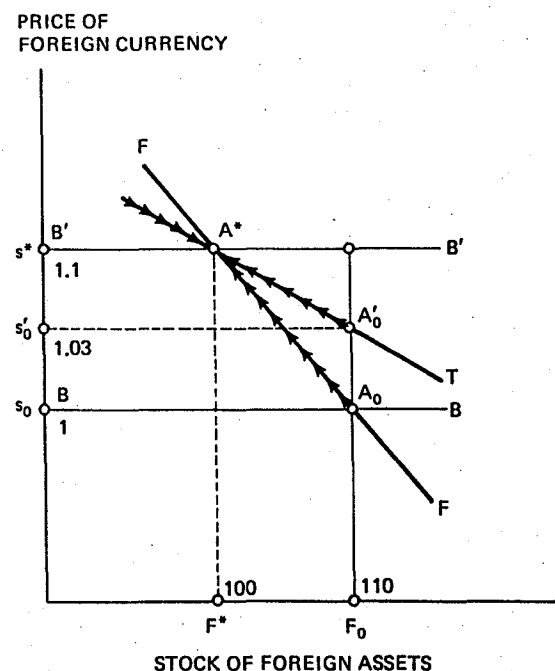
**Figure 4.11**
Effect of an unanticipated increase in domestic demand for foreign assets under rational expectations 1.

of foreign assets increases above the BB schedule and decreases below it. Inspection of the direction of movement in each of the four regions separated by the FF and BB schedules reveals that there is only one path along which expectations are continuously realized such that it takes the market to long-equilibrium at $A^*$. This is the "rational expectations" path and it is illustrated by the TT schedule in the figure. With initial stock of foreign assets equal to $F_0$ the equilibrium value of the exchange rate consistent with "rational expectation" of subsequent appreciation is $s_0$, in comparison with $s_0'$ that would obtain with stationary expectations [see figure 4.3]. How the market could reach $s_0$ and stay on TT, which is the only path that takes it to long-run equilibrium, is an open question and an answer will not be attempted here.

### 4.3.1 Comparative Dynamics
The response of the foreign exchange market to *unanticipated permanent disturbances* with rational expectations differs from the response pattern
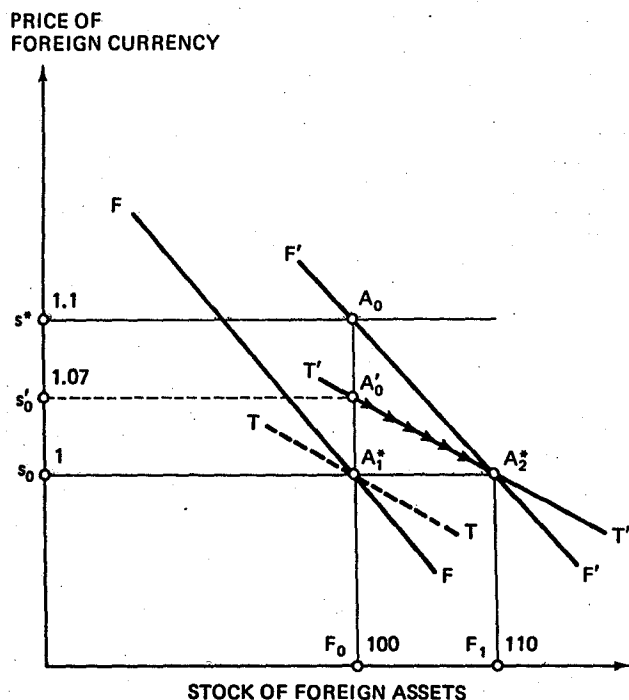
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**Figure 4.12**
Effect of a permanent unanticipated increase in domestic demand 2.

with stationary expectations only in that the short-run effect of capital account disturbances on the exchange rate is weakened while the short-run effect of trade account disturbances on the exchange rate is strengthened.

Figure 4.11 illustrates response to an increase in domestic demand for foreign assets on an initial situation of long-run equilibrium with exchange rate $s_0$ and stock of foreign assets $F_0$. The exchange rate jumps to $s_0'$ from $s_0$, which is consistent with equilibrium under stationary expectations. Thereafter, the market converges back to equilibrium at $A^*$ along the $TT$ schedule. The analysis of appendix 1 and appendix 2 permits a quantitative illustration. The numbers reported in the figure give an example with $\alpha = 0.50$ and $\beta = 1$. The initial impact of the trade account disturbance on the exchange rate is to increase it from 1 to 1.03. The average exchange rate for the first year is 1.04, compared to 1.02 under stationary expectations. However, the market reaches long-run equilibrium faster under stationary expectations than it does under rational expectations—from

table 4.4 the market reaches 1.09 in 4.6 years under stationary expectations and only in 6 years under rational expectations. The cumulative current account deficit along the adjustment path from $A_0$ to $A^*$ is 10 units. From table 4.6 the deficit is 3.1 in the first year, compared to 3.9 under stationary expectations. This example shows that when the level of international investment is high in relation to trade flows, permanent shifts in the trade account give rise to long periods of deficits or surpluses in the current account associated with continuous currency depreciation or appreciation, *ceteris paribus*.

Figure 4.12 illustrates the response of market to a permanent increase in domestic demand for foreign assets in an initial situation of long-run equilibrium at $A_1^*$ with exchange rate $s_0$ and stock of foreign assets $F_0$ ($s_0 = 1$, $F_0 = 100$). The $FF$ schedule shifts to $F'F'$ and the $TT$ schedule to $T'T'$. Rational expectations equilibrium obtains initially at $s_0'$ and thereafter the market converges to long-run equilibrium at $A_2^*$ along the $TT$ schedule. Using the same parameter values as in the above example ($\alpha = 0.5$, $\beta = 1$), $s_0'$ is equal to 1.07. Average exchange rate in the first year is, from table 4.7, 1.06, compared to 1.08 under stationary expectations. However, the exchange rate reaches 1.09 only in 6 years under rational expectations, compared to 4.6 years under stationary expectations (table 4.4). This is also the time it takes the market to effect 90 percent of the total transfer of $F_1 - F_0$.

### 4.3.2 Market Response to Anticipated Disturbances

As an efficient speculative market, the foreign exchange market responds to disturbances when they are anticipated rather than when they occur: Disturbances that have been correctly discounted in advance have no effect when they actually occur. Using another terminology, the market responds only to new information. In this section we show the effects of permanent capital account and trade account disturbances when they are anticipated in advance.

Figure 4.13 shows market response to an anticipated central bank purchase of foreign exchange at some future date $T$. The market is initially in equilibrium at $A_0$ with exchange rate $s^*$ and stock of foreign assets $F_0$. If the central bank purchase of foreign exchange occurred immediately at time 0, domestic currency would depreciate to $s_3'$. Thereafter, the market would return back to equilibrium along the $TT$ schedule. If, in contrast, the intervention is expected to occur at some future date $T$, domestic currency depreciates immediately to $s_0'$. Thereafter it continues to depreciate while the current account is in surplus. When the interven-
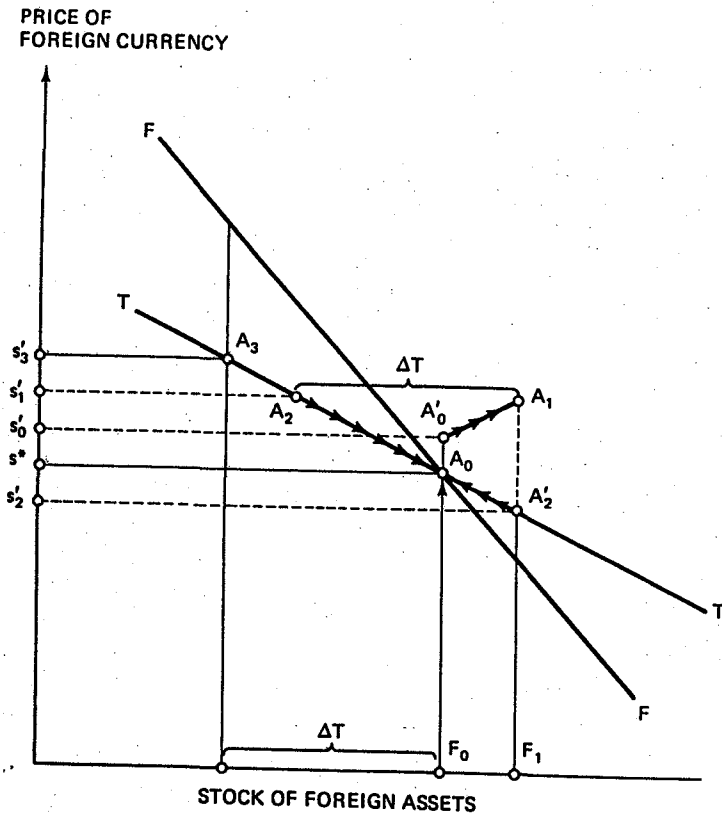
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**Figure 4.13**
Effect of an anticipated cental bank purchase of foreign exchange.

tion actually occurs at time $T$, there is no effect on the exchange rate, and after the intervention domestic currency gradually appreciates back to its normal value $s^*$. If the intervention, although anticipated, does not occur, domestic currency appreciates discretely to $s'_2$ and thereafter depreciates gradually back to equilibrium at $A_0$. In summary, only new information, whether in the form of current unanticipated events, anticipated future events, or mistakes in past expectations, gives rise to discrete and noticeable movements in the exchange rate.

To conclude the analysis, figure 4.14 illustrates market response to an anticipated permanent increase in import demand at some future date $T$ from an initial situation of equilibrium at $A_0$. Domestic currency depreciates immediately to $s'_0$ and thereafter continues to depreciate while the

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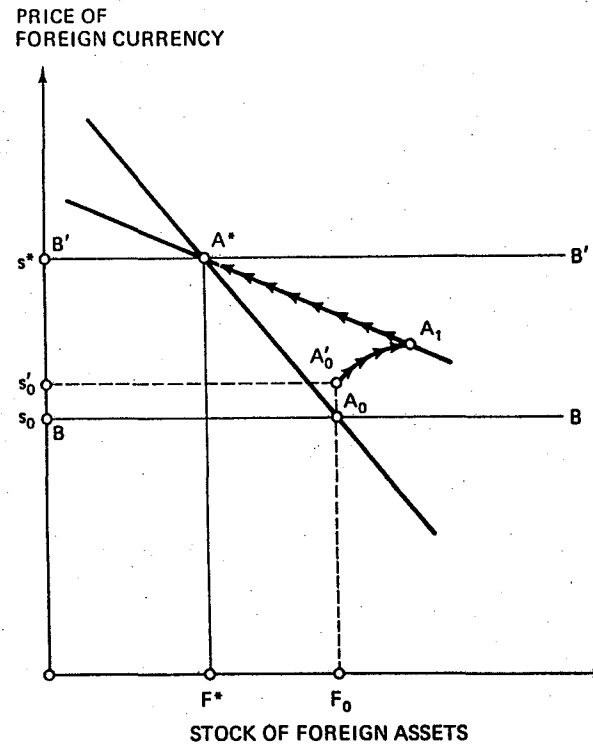
**Figure 4.14**
Effect of an anticipated increase in import demand.

current account is in surplus in reflection of the expectations induced outflow of capital—an obvious modification to the acceleration hypothesis. When the turnaround in the trade balance occurs at time $T$ there is again no noticeable jump in the exchange, although there is a discrete change in the balance of payments. Adjustment to equilibrium at $A_2$ now takes a longer time than in the case when the disturbance is not anticipated in advance because the market has to eliminate the "overhang" of speculative holdings of foreign assets.

**4.4 Conclusion**

This chapter has synthesized the modern theory of exchange rate determination with the older balance of payments approach in the framework

of a dynamic partial equilibrium model that is consistent with the nature of the foreign exchange market as an intermediary between the markets for goods and services and assets in two separate currency areas. Explicit analysis of the sources of supply and demand—stocks as well as flows—makes the model a useful tool in empirical analysis of exchange rate behavior. As one example, the model explains the observed tendency of the currencies of countries in current account surplus to appreciate and those of countries in current account deficit to depreciate. This phenomena cannot be explained by the monetarist theory of exchange rate determination, nor can it be explained by the "textbook" supply-demand model.

The partial equilibrium nature of the model is an obvious qualification to the analysis of this chapter, although simple general equilibrium models (Kouri, 1976, 1980; Kouri and de Macedo, 1978; Branson, 1978; Calvo and Rodriguez, 1977; Dornbusch and Fischer, 1980; Niehans, 1978; and Rodriguez, 1980) yield similar response patterns for the exchange rate. In the partial equilibrium framework, however, there are a number of problems that still need to be analyzed. One problem is the observed slow, and initially perverse, response of the trade balance to changes in the exchange rate, known as the J-curve problem. Another is the distinction between the currency denomination of an asset and its country of origin. This brings in the forward currency market that enables investors to decide separately about currency composition of assets on the one hand and the "nationality" of assets on the other. A third problem, connected with the J-curve problem is the problem of leads and lags in trade payments. In the present chapter, trade credit is treated as any other assets. Finally, models of the foreign exchanges should clearly recognize the existence of more than two currencies and of more than one exchange rate.

## Appendix 1    Linear Approximation of the Model[9]

The diagram developed in section 4.2 enables us to study qualitatively the dynamic response of the foreign exchange market to capital account as well as current account disturbances. To get some idea of the quantitative effects of such disturbances, or rather to identify the crucial parameters, we need to consider the approximate solution of the nonlinear model. For this purpose we write the current account equation more explicitly as

$$B(s; x) = E(s; x)/s - M(s; x),$$ \hfill (21)

where $E$ is the value of exports in domestic currency and $M$ is the value of imports in foreign currency.[10] Substituting this in equation (17) in section 4.2 and linearizing around $s^*$, we obtain

$$\dot{s} = -\frac{M^*}{f(V/s^*) + gV^*}(\eta_E + \eta_M - 1)(s - s^*) \equiv -\alpha(s - s^*),$$ \hfill (22)

where

$\eta_E$ = elasticity of export earnings in domestic currency with respect to the exchange rate,

$\eta_M$ = elasticity of imports in foreign currency with respect to the exchange rate,

$s^*$ = long-run equilibrium value of the exchange rate.

Long-run equilibrium value of the exchange rate is determined by

$$B(s^*; x) = 0.$$ \hfill (23)

A necessary and sufficient condition for long-run equilibrium $s^*$ to be locally stable is that $\alpha > 0$, or that the "elasticities condition" $\eta_E + \eta_M - 1 > 0$ holds. The solution of the linear differential equation defined by equation (22) is

$$s_t = s^* + (s_0 - s^*)e^{-\alpha t},$$ \hfill (24)

where $s_0$ is the initial equilibrium value of the exchange rate as determined by equation (4) [or (14)], repeated below for convenience:

$$s_0 = \frac{fV + G_0}{gV^* + F_0}.$$ \hfill (25)

Equations (23)–(25) enable us to study the effects of capital account as well as trade account disturbances in the short run as well as in the long run, starting from an initial situation of equilibrium. Equations (23) and (24) seem to suggest that "only capital account disturbances matter in the short run," while "only trade account disturbances matter in the long run." This asymmetry disappears, however, when allowance is made for speculative expectations, as will be shown, and when the analysis applies to any *period* of time. To show this consider the approximate discrete solution of the above model in terms of annual averages. For this purpose we adopt the convention of measuring all flow variables and flow parameters "at annual rates." By integration of equation (24) we can write the average annual exchange rate for the $i$th year as

$$s_i = (1 - \theta_i)s^* + \theta_i s_0,$$ \hfill (26)

where

$$\theta_i = \frac{1}{\alpha}e^{-\alpha(i-1)}(1 - e^{-\alpha}), \qquad i = 1, 2, \dots.$$

The average exchange rate for any year after time 0 is thus a weighted average of the long-run equilibrium exchange rate $s^*$ on the one hand and the short run equilibrium value of the exchange rate $s_0$ at time 0 on the other. The weight of the long-run equilibrium exchange rate increases with time. It also increases with an increase in the exchange rate elasticity of the current account $\eta_E + \eta_M - 1$ and with an increase in the level of trade in relation to the level of asset holdings

$$(M^*/[f(V/s^*) + gV^*]), \qquad \text{for } \alpha = (M^*/[f(V/s^*) + gV^*])$$

$$(\eta_E + \eta_M - 1).$$

The cumulative current account surplus associated with the adjustment path is approximately

$$S = \int_0^\infty B_t\,dt = \left(f^*\frac{V}{s^*} - g^*V^*\right)\frac{s_0 - s^*}{s^*}. \tag{27}$$

If foreign residents do not hold domestic assets, the cumulative current account surplus is exactly equal to $F^* - F_0$. Current account surplus in the $i$th year after the initial moment is a fraction of the cumulative surplus, this fraction decreasing over time:

$$B_i = \psi_i S, \tag{28}$$

where

$$\psi_i = e^{-\alpha(i-1)}(1 - e^{-\alpha}).$$

Finally, there are two possible measures of the speed of adjustment. One is the time it takes the market to eliminate $x$ percent of the initial exchange rate discrepancy $s_0 - s^*$, or $x$ percent of the cumulative current account surplus $S$; that is,

$$T_1(x) = -\frac{1}{\alpha}\ln(1 - x), \tag{29}$$

where $s_T - s_0 = x(s^* - s_0)$ and $S_T = xS$. This measure is independent of the size of the disturbance. An alternative measure is the time it takes the exchange rate to get to within $x$ percent of its long-run equilibrium value; that is,

Table 4.1
Speeds of adjustment for different values of $\alpha$

| $\alpha$ | 0 | 0.10 | 0.25 | 0.50 | 0.75 | 1.0 | 2.0 | 2.5 | 5.0 | 10.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| $e^{-\alpha}$ | 1 | 0.90 | 0.78 | 0.61 | 0.47 | 0.37 | 0.14 | 0.08 | 0.01 | 0.00 |
| $\psi_1$ | 0 | 0.10 | 0.22 | 0.39 | 0.53 | 0.63 | 0.86 | 0.92 | 0.99 | 1.00 |
| $\psi_2$ | 0 | 0.09 | 0.17 | 0.24 | 0.25 | 0.23 | 0.12 | 0.08 | 0.01 | 0.00 |
| $1 - \theta_1$ | 0 | 0.00 | 0.12 | 0.21 | 0.30 | 0.37 | 0.57 | 0.63 | 0.80 | 0.90 |
| $1 - \theta_2$ | 0 | 0.10 | 0.31 | 0.52 | 0.67 | 0.77 | 0.94 | 0.97 | 1.00 | 1.00 |
| $T_1(0.90)$ (years) | $\infty$ | 23.0 | 9.2 | 4.6 | 3.1 | 2.3 | 1.2 | 0.9 | 0.5 | 0.2 |

$$T_2(x) = -\frac{1}{\alpha}\ln\left(\frac{x_0}{x}\right), \tag{30}$$

where $x_0 = |(s_0 - s^*)/s^*|$ is the initial deviation of the exchange rate from its long-run equilibrium value. See table 4.1.

## Appendix 2   Linear Approximation of the Model

To get an idea of the quantitative effects of disturbances we again consider a linear approximation of the dynamic model defined by equations (19) and (20) in section 4.3. The linear differential equation corresponding to equation (22) in appendix 1 is

$$\frac{\dot{s}}{s^*} = \frac{1}{\beta}\left(\frac{s - s^*}{s^*}\right) + \frac{1}{\beta}\left(\frac{F - F^*}{F^*}\right), \tag{31}$$

where

$\beta = (\partial f/\partial \pi)/f =$ the elasticity of demand for foreign assets with respect to the expected rate of depreciation of domestic currency, evaluated at long-run equilibrium.

The linear approximation of differential equation (30) around long-run equilibrium is

$$\frac{\dot{F}}{F^*} = \alpha\frac{s - s^*}{s^*}, \tag{32}$$

where

$\alpha = (M^*/F^*)(\eta_E + \eta_M - 1)$ as before, except for the assumption that $gV^* = 0$.

The characteristic roots of this system of two linear differential equations are $\frac{1}{2}(1/\beta) \pm \sqrt{(1/\beta)^2 + 4\alpha/\beta}$. If the "elasticities condition" holds, $\alpha$ is positive and therefore the characteristic roots are real and of opposite

sign, which means that the rest point of the system is a saddlepoint.[11] There is therefore only one initial value of the free variable, in this case the exchange rate, such that starting from that value the system converges to equilibrium. To determine the stable solution of the above system consider the solution

$$\frac{s_t - s^*}{s^*} = \mu_1 A_1 e^{-\lambda_1 t} + \mu_2 A_2 e^{\lambda_2 t}, \tag{33}$$

$$\frac{F_t - F^*}{F^*} = A_1 e^{-\lambda_1 t} + A_2 e^{\lambda_2 t}, \tag{34}$$

where $\lambda_1$ and $\lambda_2$ are the absolute values of the negative and positive characteristic roots, respectively; $(\mu_1, 1)$ and $(\mu_2, 1)$ are the associated characteristic vectors; and $A_1$ and $A_2$ are constants to be determined by initial and terminal conditions. The terminal condition is that the system converges to equilibrium, and it restricts $A_2$ to equal zero. The initial condition on the stock of foreign assets restricts $A_1$ to equal $(F_0 - F^*)/F^*$. From equation (33) the "rational expectations" equilibrium value of the exchange rate at the initial moment, denoted by $s_0'$, must then equal $\mu_1(F_0 - F)^*)/F^*$. From this we get the linear approximation of the rational expectations equilibrium schedule:

$$\frac{s - s^*}{s^*} = \mu_1 \frac{F - F^*}{F^*}. \tag{35}$$

This is illustrated by the $TT$ schedule in figure 4.15. The linear approximation of the stationary expectations equilibrium schedule, $FF$ in figure 4.15, is simply

$$\frac{s - s^*}{s^*} = -\frac{F - F^*}{F^*}. \tag{36}$$

The $BB$ schedule, also shown in figure 4.15, is defined by

$$s = s^*. \tag{37}$$

The rational expectations equilibrium value of the exchange rate $s_0'$ at time 0 is thus related to the stationary expectations value $s_0$ by

$$s_0' - s^* = -\mu_1(s_0 - s^*) \quad \text{(see figure 4.10)}. \tag{38}$$

The value of parameter $\mu_1$ is given by

$$\mu_1 = \tfrac{1}{2}\gamma - \tfrac{1}{2}\sqrt{\gamma^2 + 4\gamma}. \tag{39}$$

where $\gamma = 1/\alpha\beta$.
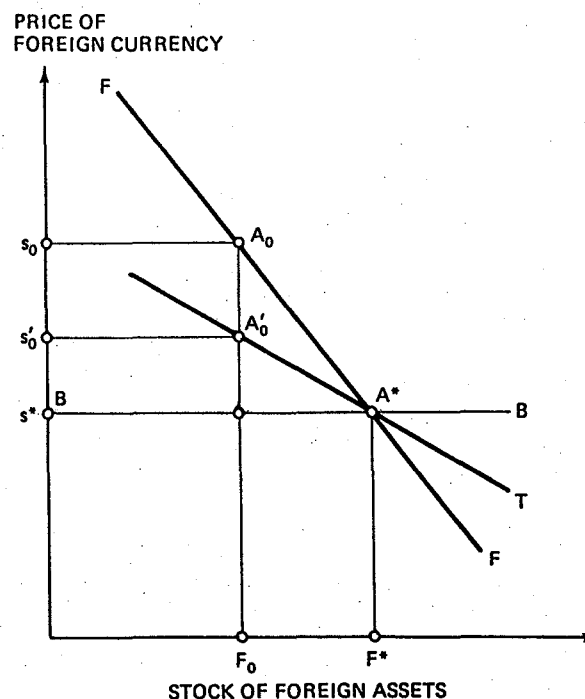
Figure 4.15
Linear approximations to the rational expectations equilibrium.

Table 4.2
Different values of $|\mu_1|$[a]

| $\alpha$ | $\beta$ 0 | 0.1 | 0.25 | 0.5 | 1 | 2 | 2.5 | 5 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 10 | 1 | 0.62 | 0.44 | 0.36 | 0.27 | 0.25 | 0.18 | 0.13 | 0.10 |
| 5 | 1 | 0.73 | 0.58 | 0.46 | 0.36 | 0.27 | 0.25 | 0.18 | 0.13 |
| 2.5 | 1 | 0.83 | 0.61 | 0.58 | 0.46 | 0.36 | 0.33 | 0.25 | 0.18 |
| 2 | 1 | 0.85 | 0.73 | 0.62 | 0.50 | 0.37 | 0.36 | 0.27 | 0.25 |
| 1 | 1 | 0.92 | 0.83 | 0.73 | 0.62 | 0.50 | 0.44 | 0.36 | 0.27 |
| 0.5 | 1 | 0.95 | 0.90 | 0.83 | 0.72 | 0.62 | 0.58 | 0.46 | 0.36 |
| 0.25 | 1 | 0.98 | 0.95 | 0.90 | 0.83 | 0.73 | 0.61 | 0.58 | 0.44 |
| 0.1 | 1 | 0.99 | 0.98 | 0.95 | 0.92 | 0.85 | 0.83 | 0.73 | 0.62 |

a. $\mu_1 = \tfrac{1}{2}\gamma - \tfrac{1}{2}\sqrt{\gamma^2 + 4\gamma}$, where $\gamma = 1/\alpha\beta$.

The value of $\mu_1$ goes from zero to minus one as $\gamma$ increases from zero to infinity. The higher are the interest elasticity of the capital account on the one hand and the exchange rate elasticity of the current account on the other, the closer is the $TT$ schedule to the horizontal $BB$ schedule. Also, an increase in the level of trade in relation to the level of asset holdings brings the $TT$ schedule closer to the $BB$ schedule. Table 4.2 shows different values of $|\mu_1|$ corresponding to different values of $\alpha$ and $\beta$.

Given $s'_0$ and $F_0$, the time path of the market under rational expectations is characterized by

$$s'_t - s^* = (s'_0 - s^*)e^{\lambda_1 t}, \tag{40}$$

$$F_t - F^* = (F_0 - F^*)e^{\lambda_1 t}, \tag{41}$$

where

$$\lambda_1 = \tfrac{1}{2}(1/\beta) - \tfrac{1}{2}\sqrt{(1/\beta)^2 + 4(\alpha/\beta)} < 0.$$

Table 4.3 shows different values of $\lambda_1$ corresponding to the same values of $\alpha$ and $\beta$ as above. The speed of adjustment of the stock of foreign assets decreases as the interest rate elasticity of the capital account ($\beta$) increases. This is illustrated in table 4.4, which reports different values of $T_1(0.10)$ corresponding to different values of $\alpha$ and $\beta$, where $T_1(x)$ is defined by

$$T_1(x) = -\frac{1}{\lambda_1}\ln(1 - x). \tag{42}$$

The column with $\beta$ equal to zero is the same as the last row of table 4.1.

$T_1(x)$ does not, however, correctly measure the speed of adjustment of the exchange rate because the exchange rate jumps discretely at the initial moment. An appropriate measure is instead the time it takes to eliminate $x$ percent of deviation $s_0 - s^*$ of which $s_0 - s'_0$ is eliminated immediately. This time is given by

$$T_3(x) = -\frac{1}{\lambda_1}\ln\frac{1 - x}{|\mu_1|}, \tag{43}$$

which is less than $T_1(x)$. Table 4.5 reports different values of $T_3(x)$ for the same range of values of $\alpha$ and $\beta$ as used in table 4.2.

Finally, the discrete solution of the model is given by

$$s'_t = (1 - |\mu_1|\theta'_t)s^* + |\mu_1|\theta'_t s_0, \tag{44}$$

**Table 4.3**
Different values of $\lambda_1$[a]

| α | β | | | | | | | | |
|---|---|------|------|------|------|------|------|------|------|
| | 0 | 0.1 | 0.25 | 0.5 | 1 | 2 | 2.5 | 5 | 10 |
| 10 | 10 | 6.20 | 4.40 | 3.60 | 2.70 | 2.50 | 1.80 | 1.30 | 1 |
| 5 | 5 | 3.65 | 2.90 | 2.30 | 1.80 | 1.35 | 1.25 | 0.90 | 0.65 |
| 2.5 | 2.5 | 2.08 | 1.53 | 1.45 | 1.15 | 0.90 | 0.83 | 0.63 | 0.45 |
| 2 | 2 | 1.70 | 1.46 | 1.24 | 1.00 | 0.78 | 0.72 | 0.54 | 0.50 |
| 1 | 1 | 0.92 | 0.83 | 0.73 | 0.62 | 0.50 | 0.44 | 0.36 | 0.27 |
| 0.5 | 0.5 | 0.48 | 0.45 | 0.42 | 0.37 | 0.31 | 0.29 | 0.23 | 0.18 |
| 0.25 | 0.25 | 0.25 | 0.24 | 0.23 | 0.21 | 0.18 | 0.15 | 0.15 | 0.11 |
| 0.1 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.09 | 0.08 | 0.08 | 0.06 |

a. $\lambda_1 = \tfrac{1}{2}(1/\beta) - \tfrac{1}{2}\sqrt{(1/\beta)^2 + 4(\alpha/\beta)} = \alpha\mu_1$.

**Table 4.4**
Different values of $T_1$ (years)[a]

| α | β | | | | | | | | |
|---|---|------|------|------|------|------|------|------|------|
| | 0 | 0.1 | 0.25 | 0.5 | 1 | 2 | 2.5 | 5 | 10 |
| 10 | 0.23 | 0.37 | 0.52 | 0.64 | 0.85 | 0.92 | 1.28 | 1.77 | 2.30 |
| 5 | 0.46 | 0.63 | 0.79 | 1.28 | 1.70 | 1.84 | 2.56 | 3.54 | 4.60 |
| 2.5 | 0.83 | 1.11 | 1.50 | 1.59 | 2.00 | 2.56 | 2.77 | 3.65 | 5.11 |
| 2 | 0.87 | 1.35 | 1.58 | 1.85 | 2.30 | 2.95 | 3.19 | 4.26 | 4.60 |
| 1 | 2.30 | 2.50 | 2.77 | 3.15 | 3.71 | 4.60 | 5.23 | 6.39 | 8.52 |
| 0.5 | 4.60 | 4.80 | 5.12 | 5.48 | 6.22 | 7.42 | 7.93 | 10.01 | 12.79 |
| 0.25 | 9.20 | 9.20 | 9.58 | 10.00 | 10.25 | 12.78 | 15.33 | 15.33 | 20.91 |
| 0.1 | 23.0 | 23.0 | 23.0 | 23.0 | 25.6 | 28.8 | 28.8 | 32.9 | 38.3 |

a. $T_1(x) = -(1/\lambda_1)\ln(1 - x), x = 0.90$.

**Table 4.5**
Different values of $T_3$[a]

| α | β | | | | | | | | |
|---|---|------|------|------|------|------|------|------|------|
| | 0 | 0.1 | 0.25 | 0.5 | 1 | 2 | 2.5 | 5 | 10 |
| 10 | 0.23 | 0.29 | 0.34 | 0.36 | 0.37 | 0.37 | 0.33 | 0.20 | 0 |
| 5 | 0.46 | 0.55 | 0.61 | 0.94 | 1.13 | 0.87 | 1.45 | 1.64 | 1.46 |
| 2.5 | 0.83 | 1.02 | 1.18 | 1.22 | 1.32 | 1.43 | 1.43 | 1.44 | 1.31 |
| 2 | 0.87 | 1.26 | 1.37 | 1.46 | 1.61 | 1.75 | 1.77 | 1.80 | 1.82 |
| 1 | 2.30 | 2.41 | 2.55 | 2.72 | 2.94 | 3.22 | 3.37 | 3.56 | 3.68 |
| 0.5 | 4.60 | 4.70 | 4.90 | 4.69 | 5.38 | 5.87 | 6.07 | 6.62 | 7.12 |
| 0.25 | 9.20 | 9.12 | 9.37 | 9.57 | 10.05 | 11.06 | 12.06 | 11.73 | 13.46 |
| 0.1 | 23.0 | 22.90 | 22.80 | 22.50 | 24.71 | 26.80 | 26.43 | 28.47 | 30.47 |

a. $T_3(x) = -\dfrac{1}{\lambda_1}\ln\dfrac{1 - x}{|\mu_1|} = T_1(x) + \dfrac{1}{\lambda_1}\ln|\mu_1| \leqslant T_1(x)$.

**Table 4.6**
Different values of $\psi_i'^a$

| $\alpha$ | $\beta$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0 | 0.1 | 0.25 | 0.5 | 1 | 2 | 2.5 | 5 | 10 |
| 10 | 1.00 | 1.00 | 0.99 | 0.97 | 0.93 | 0.92 | 0.83 | 0.73 | 0.63 |
| 5 | 0.99 | 0.97 | 0.94 | 0.90 | 0.83 | 0.74 | 0.71 | 0.59 | 0.48 |
| 2.5 | 0.92 | 0.87 | 0.78 | 0.76 | 0.68 | 0.59 | 0.56 | 0.47 | 0.36 |
| 2 | 0.86 | 0.82 | 0.77 | 0.71 | 0.63 | 0.54 | 0.51 | 0.42 | 0.39 |
| 1 | 0.63 | 0.60 | 0.56 | 0.52 | 0.46 | 0.39 | 0.35 | 0.30 | 0.24 |
| 0.5 | 0.39 | 0.38 | 0.36 | 0.34 | 0.31 | 0.27 | 0.25 | 0.21 | 0.16 |
| 0.25 | 0.22 | 0.22 | 0.21 | 0.20 | 0.19 | 0.16 | 0.14 | 0.14 | 0.10 |
| 0.1 | 0.09 | 0.09 | 0.09 | 0.09 | 0.09 | 0.08 | 0.08 | 0.07 | 0.06 |

a. $\psi_i' = 1 - e^{-\lambda_i}$.

**Table 4.7**
Different values of $1 - |\mu_1|\theta_i'^a$

| $\alpha$ | $\beta$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0 | 0.1 | 0.25 | 0.5 | 1 | 2 | 2.5 | 5 | 10 |
| 10 | 0.90 | 0.90 | 0.90 | 0.90 | 0.91 | 0.91 | 0.92 | 0.93 | 0.94 |
| 5 | 0.80 | 0.81 | 0.81 | 0.82 | 0.83 | 0.85 | 0.86 | 0.88 | 0.90 |
| 2.5 | 0.63 | 0.65 | 0.69 | 0.70 | 0.73 | 0.76 | 0.78 | 0.81 | 0.86 |
| 2 | 0.57 | 0.59 | 0.61 | 0.64 | 0.68 | 0.73 | 0.74 | 0.79 | 0.80 |
| 1 | 0.37 | 0.40 | 0.44 | 0.48 | 0.54 | 0.61 | 0.65 | 0.70 | 0.76 |
| 0.5 | 0.21 | 0.25 | 0.28 | 0.33 | 0.39 | 0.46 | 0.50 | 0.58 | 0.68 |
| 0.25 | 0.12 | 0.14 | 0.17 | 0.22 | 0.25 | 0.35 | 0.43 | 0.46 | 0.60 |
| 0.1 | 0.10 | 0.11 | 0.12 | 0.14 | 0.02 | 0.15 | 0.17 | 0.27 | 0.38 |

a. $\theta_i' = (1/\lambda_1)(1 - e^{-\lambda_i})$.

where

$\theta_i' = (1/\lambda_1)e^{-\lambda_1(i-1)}(1 - e^{-\lambda_i})$, $i = 1, 2, \ldots$,

$s_i' =$ average exchange rate for the $i$th year,

$s^* =$ long-run equilibrium value of the exchange rate,

$s_0 =$ initial exchange rate as determined by (36).

This equation is of the same form as equation (26).

Current account in the $i$th year is a fraction of the cumulative surplus $F^* - F_0$:

$$B_i = \psi_i' S, \tag{45}$$

where

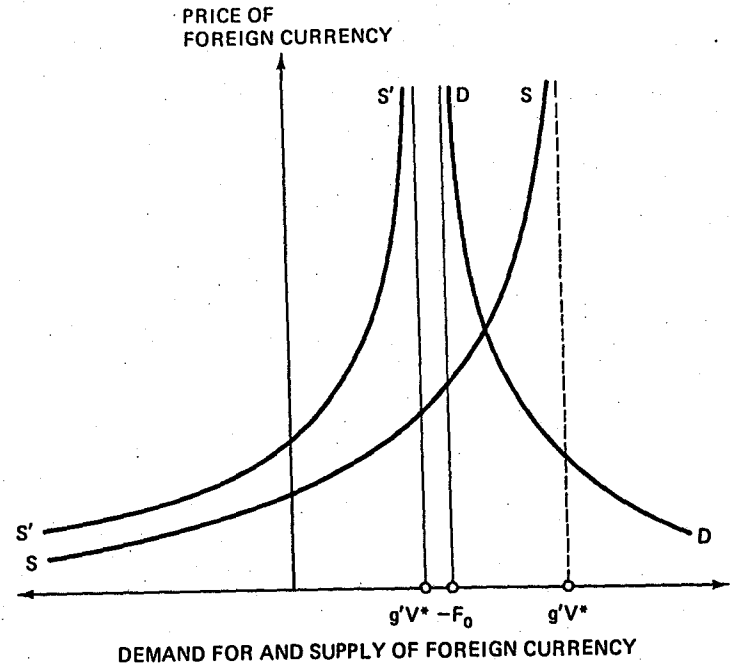$$S = F^* - F_0 \quad \text{and} \quad \psi_i' = e^{-\lambda_1(i-1)}(1 - e^{-\lambda_i}).$$

**Figure 4.16**
The foreign exchange market with foreign currency denominated debt (1).

Because $\lambda_1$ is smaller than $\alpha$, $\psi_i'$ is smaller than $\psi_1$, while $\theta_i'$ is greater than $\theta_i$ for sufficiently high values of $i$. This is, of course, another indication of the fact that, under rational expectations, adjustment to long-run equilibrium is slower, ceteris paribus. Tables 4.6 and 4.7 report different values of $\psi_1$ and $1 - |\mu_1|\theta_1'$ for a range of values of $\alpha$ and $\beta$. The columns with $\beta$ equal to zero are the same as the rows for $\psi_1$ and $1 - \theta_1$ in table 4.1.

## Appendix 3   Implications of Borrowing in Foreign Currency

Residents of most small countries cannot borrow in their own currencies in the international capital market, but instead have to borrow in one of the major currencies. Many of the small countries, furthermore, are net debtors. This introduces problems in the foreign exchange market. To analyze these problems, consider first the case when domestic residents currently are net debtors in foreign currency ($F_0 < 0$) but would like to hold a positive stock of foreign assets ($fV > 0$). If foreign demand
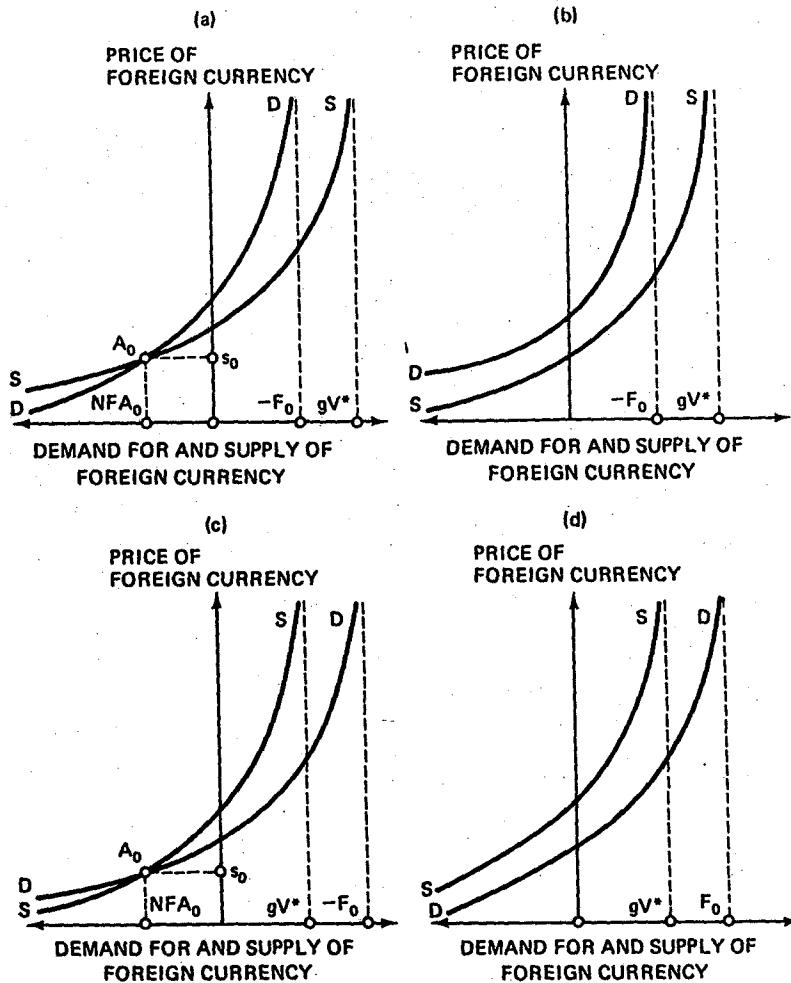
(a)

PRICE OF
FOREIGN CURRENCY



DEMAND FOR AND SUPPLY OF
FOREIGN CURRENCY

(b)

PRICE OF
FOREIGN CURRENCY



DEMAND FOR AND SUPPLY OF
FOREIGN CURRENCY

(c)

PRICE OF
FOREIGN CURRENCY



DEMAND FOR AND SUPPLY OF
FOREIGN CURRENCY

(d)

PRICE OF
FOREIGN CURRENCY



DEMAND FOR AND SUPPLY OF
FOREIGN CURRENCY

**Figure 4.17**
The foreign exchange market with foreign currency denominated debt (2):
(a) $G_0 > fV$, $-F_0 < gV^*$; (b) $G_0 < fV$, $-F_0 < gV^*$; (c) $G_0 < fV$, $-F_0 > gV^*$;
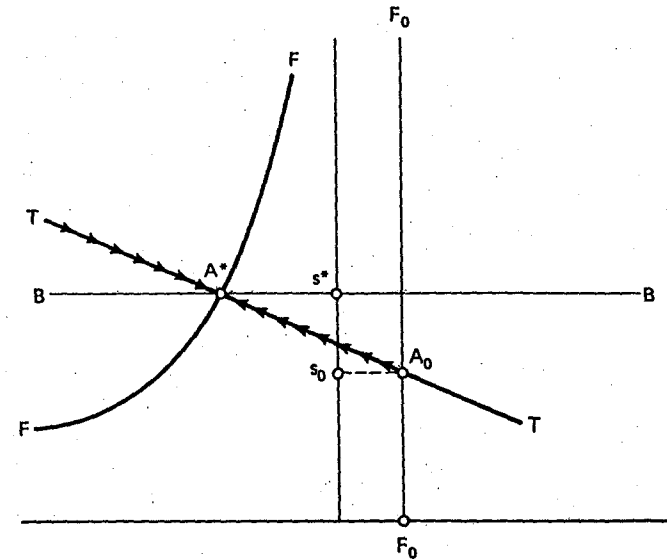(d) $G_0 > fV$, $-F_0 > gV^*$.

**Figure 4.18**
The foreign exchange market with foreign currency denominated debt (3).

for other types of domestic assets but bonds is larger (measured in foreign currency) than the foreign debt of domestic residents ($gV^* > F_0$), equilibrium still exists, and is unique and stable as is shown in figure 4.16, where the market reaches equilibrium at $A_0$ with exchange rate $s_0$. Suppose, however, that in this situation foreign demand for domestic assets declines so much as to shift the supply schedule to $S'S'$ ($g'V^* < F_0$). Then clearly there does not exist any equilibrium with stationary expectations: Domestic currency would tend to depreciate without limit.

Consider next the case when the desired stock of foreign assets is negative ($fV < 0$). The equilibrium condition is now

$$-f(R, R^* + \pi, z)V/s - F_0 = g(R - \pi, R^*, z)V^* - G_0/s, \qquad (46)$$

where $f$ is replaced by $-f$ to indicate that domestic residents want to borrow foreign currency. $f(R, R^* + \pi, z)V/s$ is the supply schedule of foreign currency denominated domestic loans. It is assumed that foreign demand for these loans is infinitely elastic at interest rate $R^*$. The equilibrium value of the exchange rate implied by equation (1) is

$$s_0 = \frac{G_0 - fV}{F_0 + gV^*}. \qquad (47)$$

There are four possible constellations in terms of initial asset holdings $(F_0, G_0)$ and desired asset holdings $(fV, gV^*)$, as illustrated in figure 4.17.

Given stationary expectations short run equilibrium exists if and only if $G_0 > fV$ and $-F_0 < gV^*$ or $G_0 < fV$ and $-F_0 > gV^*$ [(a) and (c) in figure 4.17]. However, equilibrium is stable if and only if $G_0 > fV$ and $-F_0 < gV^*$; loosely speaking, it is stable only if actual and desired foreign holdings of domestic currency denominated assets are sufficiently large.

### Rational Expectations Equilibrium

Figure 4.18 shows that when expectations are rational the problem of nonexistence of short-run equilibrium does not arise provided that long-run equilibrium is unique.[12] It is assumed in the figure that foreign residents do not hold domestic currency denominated assets. Domestic residents have initially a positive stock of foreign assets, equal to $F_0$ in the figure while they would like to be net debtors—in other words, they would like to borrow in foreign currency to invest in domestic assets. Clearly, there is no equilibrium under stationary expectations—the $FF$ and the $F_0F_0$ schedules do not intersect. However, there is a rational expectations equilibrium at $A_0$ where the $TT$ schedule intersects the vertical $F_0F_0$ schedule.

### Notes

1. For a theory of portfolio diversification between assets of different currency denomination, see Kouri and de Macedo (1978, pp. 118–30).

2. Strictly speaking, domestic marketable wealth cannot be taken as exogenous even in partial equilibrium analysis because of capital gains and losses resulting from exchange rate changes. This is not a real problem, however, because we could equally take the domestic component of wealth as exogenous. This is not done for expositional convenience.

3. J. Williamson (1976, p. 331).

4. Dornbusch (1976, footnote on p. 271).

5. The dimensions of $\dot{s}/s$ and $B/F$ are the same because both $\dot{s}$ and $B$ are flow variables while $s$ and $F$ are stock variables.

6. It is not necessary that the net stock of foreign assets be positive. The analysis applies equally if domestic residents are net debtors abroad. Problems arise only if domestic residents are net debtors in foreign currency, that is, either $fV/s$, $F_0$, or both are negative (see appendix 1).

7. Note that along the equilibrium path, after portfolio adjustment at the initial moment, $F^d = F$ and $G^d = G$.

8. From equation (7) in appendix 1 the cumulative current account surplus $S$ is

$$S = \left(f^*\frac{V^*}{s^*} + g^*V^*\right)\frac{s_0 - s^*}{s^*},\tag{i}$$

where $f^*$ and $g^*$ are the desired portfolio proportions corresponding to the $F'F'$ schedule. From equation (5) in appendix 1, $s_0$ is given by

$$s_0 = \frac{f^*V + G_0}{g^*V^* + F_0} = \frac{f^*V^* + gV^*s^*}{g^*V^* + f(V/s^*)},\tag{ii}$$

where $f$ and $g$ correspond to the $FF$ schedule. Substituting this in equation (i) gives

$$S = \frac{f^*V - fV}{s^*} - (g^*V^* - gV) = NFA^* - NFA_0 = A_0A^*.\tag{iii}$$

When foreign residents do not hold domestic assets $(g^*V^* = gV = 0)$ this approximation is exact: The cumulative surplus equals $F^* - F_0$.

9. The case of multiple long run equilibria is discussed in the next section.

10. Because of the assumption that interest earnings are spent on imports in both countries, the interest service account and the implied net imports cancel out.

11. If the elasticities condition is not met and thus $\alpha$ is negative, there are two possibilities. First, if $4|\alpha| \leqslant 0.25(1/\beta)$, where $|\alpha|$ is the absolute value of $\alpha$, both characteristic roots are real and positive; second, if $4|\alpha| > 0.25(1/\beta)$, the roots are complex conjugates with a positive real part $(1/\beta)$. In both cases there *does not* therefore *exist* a rational expectations equilibrium.

12. This analysis is not meant to be rigorous, only suggestive.

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