Economics 202A Final Exam

December 15, 2007

Instructions: You have 1 hour and 35 minutes. Answer all 3 questions, each of which carries an equal weight. Notice that within question 2, you have a choice between two options. There is no extra credit for picking both of those options – you may only pick one. Take the first 5 minutes to look over all 4 pages of the exam before you start. That way you can better pace yourself. If you get stuck, go on to something you can answer more easily and return to the difficult bits later on. Good luck, and happy holidays!

1. A representative consumer maximizes

$$\int_0^\infty u\left[c(t)\right] \mathrm{e}^{-\delta t} \mathrm{d}t$$

subject to

$$\dot{a} = y + ra - c, \quad a(0) \text{ given,}$$

where y and r are constant through time, y is perishable output, and a represents a stock of interest-bearing real financial assets. We do not necessarily impose that the subjective discount rate δ is equal to the market real interest rate r. A no-Ponzi condition also is imposed on the problem.

(a) Use the Maximum Principle to write down the necessary conditions for this maximization problem, based on the Hamiltonian.

(b) For the isoelastic function

$$u(c) = \frac{c^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}},$$

derive from the Maximum Principle first-order conditions the Euler equation relating the proportional growth rate of consumption c to the parameter σ and the difference $r - \delta$.

(c) The solution for the linear differential equation describing the consumption path is

$$c(t) = c(0)e^{\sigma(r-\delta)t}.$$

On the other hand, integrating the $\dot{a}$ equation above and imposing the no-Ponzi condition preventing unlimited debt, we find the intertemporal budget constraint,

$$a(0) = \int_0^\infty [c(t) - y]\, e^{-rt} dt.$$

Substituting the equation for $c(t)$ into this last budget constraint, calculate the optimal value of $c(0)$ by calculating the value of the integral

$$\int_0^\infty e^{[(\sigma-1)r-\sigma\delta]t} dt.$$

You should assume that $(\sigma - 1)\, r - \sigma\delta < 0$.

(d) Explain the three distinct channels through which a change in the interest rate affects consumption in this model.

(e) (Extra credit) In part (c) above it was assumed that $(\sigma - 1)\, r - \sigma\delta < 0$. What happens if $(\sigma - 1)\, r - \sigma\delta > 0$?

2. Pick one *and only one* of the following two questions to answer:

(a) State and prove the Modigiliani-Miller theorem for a complete-markets economy. Be sure to show the implications for the firm's investment policy.

(b) Generally speaking, it is unexpected rather than expected inflation that gives a government the most net policy benefit (either in terms of higher output or higher government revenue). Using the Barro-Gordon model of dynamic inconsistency in monetary policy, explain how a government's inability to commit to a monetary-policy rule may land it in an equilibrium where *expected* inflation is high relative to the government's preferred rate.

3. Consider the Lucas "tree" model, but with only two periods and one tree. There is a representative competitive agent who can buy or sell shares in the tree. In period 1, people trade shares. In period 2, the tree pays off a random amount of fruit, people consume the fruit, and the world ends. I also assume

that in period 1, the representative agent has a perishable endowment of y_1 to consume — we won't ask where it came from; perhaps it is the fruit of earlier investments. Output of a tree in period 2 is the random variable y_2.

The representative individual maximizes

$$U = u(c_1) + \beta \mathrm{E}\left\{u(c_2)\right\}$$

where $0 < \beta < 1$ and the usual concavity assumptions are made.

(a) Using a general utility function $u(c)$,write down the intertemporal Euler equation for the tree if its period 1 price is q.

(b) Substituting equilibrium consumption for consumption, derive the equilibrium price of the tree.

(c) Why does higher y_1 imply a higher price of a tree in equilibrium? Explain intuitively.

(d) Assume a quadratic utility function, $u(c) = ac - \frac{b}{2}c^2$. (We assume the parameters of the problem are such that marginal utility of c is always strictly positive.) Use this to solve explicitly for q in terms of the mean of y_2, μ_y, and the variance of y_2, σ_y^2.(Hint: You'll have to express the second moment $\mathrm{E}\left\{(y_2)^2\right\}$ in terms of μ_y and σ_y^2. If you don't remember the formula, refresh your memory by using the definition of the variance, $\sigma_y^2 = \mathrm{E}\left\{(y - \mu_y)^2\right\}$.) How does a rise in the variance of second-period tree yield y_2 affect the first-period price? Is this intuitive?

(e) Now we assume there are two countries in the world, each with a representative inhabitant and a tree. The representative inhabitants both have the same quadratic utility functions as above. Foreign consumption and yields are indicated by stars. The two countries are otherwise absolutely symmetric as well, implying that $y_1 = y_1^*$, $\mu_y = \mu_y^*$, and $\sigma_y^2 = \sigma_y^{*2}$. However, we allow the correlation coefficient ρ between national yields,

$$\rho = \frac{\mathrm{Cov}\left(y_2, y_2^*\right)}{\sigma_y^2},$$

to be any number between -1 and 1. You may take it as given that, in an equilibrium, $c_1 = y_1 = y_1^* = c_1^*$ and

$$c_2 = c_2^* = \frac{y_2 + y_2^*}{2}.$$

The last equation comes from an assumption that, in period 1, residents of the two countries diverse their tree portfolios internationally, each country swapping half its claims to fruit with the other country. Given these assumptions, what is the price of a tree in equilibrium, in terms of μ_y, σ_y^2, and ρ? Use the equilibrium Euler equation to answer this question. (Hint: You will have to express the expected product $\mathrm{E}\left\{y_2 y_2^*\right\}$ in terms of μ_y, σ_y^2, and ρ. If you don't remember the formula, use the definition of a covariance to figure it out.)

(f) If two previously separated countries integrate their equity markets, what happens to the price of trees when $\rho < 1$? What happens when $\rho = 1$? Please explain intuitively.