Economics 202A Lecture Outline, November 10-12, 2008 (version 1.1)

Maurice Obstfeld

The main point of the model we'll study today is to show how agency costs of investment can be mitigated by a larger decision-maker stake in projects. Thus, more plentiful internal funds can spur investment and, conversely, sharp reductions in decision-maker wealth can cause investment to collapse.

Idea of the Bernanke-Gertler (*QJE*, February 1990) model

To set the stage, start with a setting much more simple than that of Bernanke-Gertler (BG). A risk-neutral investor or entrepreneur with total real wealth w (observable by outsiders) faces a world capital market in which the *gross* interest rate on loans is (the given constant) r. There are two time periods; investment takes place in the first and consumption in the second.

A project requires the input of 1 unit of output on date 1 and has a date 2 payoff of R with probability p and of 0 with probability $1-p$. Importantly, p is the entrepreneur's private knowledge. An entrepreneur can undertake at most one project, and has the option of instead investing his or her wealth at the gross risk-free rate $r < R$. The cumulative distribution function for p within the population of entrepreneurs is $H(p)$.

Assume tentatively that an entrepreneur with wealth w can borrow $1-w$ at the world interest rate r. Lenders can observe the investment outcome and compel repayment up to the limit of the borrower's resources. For which values of p will entrepreneurs choose to invest in their risky projects? *If* there were no nonnegativity constraint on consumption, the cutoff value of p would be where the expected returns to risky and riskless investment coincide:

$$p\left[R - r(1-w)\right] - (1-p)r(1-w) = rw.$$

The solution is

$$p^*_{fb} = r/R,$$

which, you can confirm, gives an efficient amount of investment. But consumption *cannot* be negative. An entrepreneur whose investment goes sour can only repay 0 in period 2, so that the problem he or she solves in period 1 has a cutoff probability given by

$$p\left[R - r(1-w)\right] = rw,$$

with solution

$$p^* = \frac{rw}{R - r(1-w)} \leq \frac{r}{R}.$$

(The last inequality is strict if $w < 1$.) Notice that unless $w = 1$ (so the entrepreneur bears the entire risk of the project), $p^* < p^*_{fb}$. Too many projects will be undertaken relative to the efficient benchmark. There is a classic problem of adverse selection, because "bad" borrowers who know they have low p will borrow and invest. They have a small chance of a big win, but default at the lender's expense if the investment fails. Notice that the lower is w, the investor stake, the greater is the incentive to gamble on high-risk projects ($\mathrm{d}p^*/\mathrm{d}w > 0$).

Furthermore, rational lenders, anticipating the behavior above, would never lend at the interest rate r. Instead, they offer a loan contract designed in the expectation that the borrower will default if the project fails. The equilibrium loan contract is simple (and is equivalent to an equity contract in this simple setting). A borrower undertaking a risky project repays $R(1-w)$ if the project is successful and 0 otherwise (i.e., there is a default).[1] Following our earlier logic, we see that $p^* = p^*_{fb} = r/R$ and that the lender's expected return is $p^*_{fb}R = r$. The proposed contract entirely solves the adverse selection problem, delivering the first-best investment level while giving lenders their required expected return of r on loans. Thus, entrepreneurial stakes need not affect aggregate investment in this simple model.

To derive contrary results, BG introduce two additional assumptions. First entrepreneurs *must* pay a fixed charge e in order to invest, and paying that cost reveals to them their individual value of p. Second, lenders

[1] A borrower who undertakes the risk-free project repays $r(1-w)$ always. (Lenders can observe how borrowers use loan proceeds.)

cannot observe whether a borrower who claims to have paid e really has. The assumptions introduce a moral hazard problem, which an optimal loan contract must solve: to induce entrepreneurs to learn p and then to avoid investing if they turn out to have very low values of p. As we now see, the resulting contract generally does not attain the first-best, and its form makes investment sensitive to entrepreneurial wealth

The setup

The economy is closed. The population is a continuum indexed by $[0, 1]$. A fraction β consists of risk-neutral entrepreneurs (those with potential investment projects), a fraction $1 - \beta$ of risk-neutral nonentrepreneurs. A nonentrepreneur has wealth w^n, an entrepreneur wealth w^e, where $w^n \geq 1 \geq w^e$, and

$$w^{av} = \beta w^e + (1-\beta)w^n > \beta,$$

so that it is feasible (if not optimal in any sense) to fund all investment projects.[2] As in the earlier setting, there is a risk-free technology offering a gross rate of return r; call it storage. The last inequality implies that some storage will occur in equilibrium, so that we can again identify r with the real rate of interest between periods 1 and 2.

Let's look first at the first-best (socially efficient) allocation. To that end, define $H(p)$ (again) as the cumulative distribution function for p within the population of entrepreneurs. For any cutoff probability p^* define (see BG)

$$\hat{p} \equiv \mathrm{E}\left(p \mid p \geq p^*\right) = \frac{\int_{p^*}^{1} p dH}{1 - H(p^*)}.$$

(Keep in mind that $\hat{p}$ is a function of p^*—a fact that would only complicate the notation were we to continually make it explicit.) A first simplification: since all entrepreneurs are the same ex ante, it is socially optimal either for all or for none to evaluate and learn their projects' success probabilities.

[2]BG let entrepreneurs' wealth vary cross-sectionally, but this is inessential. The main consequence is that in their setup, constrained-optimal contracts tailored to different wealth levels coexist.

Assume first that it is socially optimal for all to pay the fixed investment charge e up front.[3] Having learned p, it is then socially optimal to invest in the risky technology if and only if the expected return is not below that on storage. So we get a cutoff probability given by

$$p_{fb}^{*}R = r \Leftrightarrow p_{fb}^{*} = r/R.$$

But when is it socially optimal to pay the up-front charge e? It is optimal to pay e only if the expected return on each project from paying e and then investing according to the preceding cutoff rule exceeds r. Formally the condition is[4]

$$\left[1 - H(p_{fb}^{*})\right]\left[\mathrm{E}\left(pR \mid p \geq p_{fb}^{*}\right) - r\right] - e = \left[1 - H(p_{fb}^{*})\right](\hat{p}_{fb}R - r) - e > 0.$$

If the last inequality fails to hold, it is optimal for society simply to invest all its resources in storage. In what follows we will assume this is not the case, so that the last two conditions define the first-best investment allocation.

The optimal incentive-compatible loan contract

Now assume that lenders cannot observe if an entrepreneur investing in storage has paid e or not.[5] Two distortions arise. Clearly the loan contract has to induce entrepreneurs to pay e up front; it must not provide incentives for entrepreneurs to pretend to have paid e when they have not. Furthermore, low-p entrepreneurs, who know they repay nothing if a project fails, must not confront such a large payoff to success that they are induced to gamble.

A state-contingent loan contract works as follows. Borrowers sign on date 1. They then have a chance to evaluate their projects, learn p, and decide

[3] As Bernanke and Gertler suggest, you can think of e as effort expended by the entrepreneur in the first period. Alternatively, you can think of it as a perishable endowment that could be consumed rather than spent to open up the investment opportunity, but that will disappear if not used in period 1. In either case, e will not enter directly into period 2 consumption and therefore does not figure in the consumption nonnegativity constraints discussed below.

[4] The law of large numbers implies that $1 - H(p_{fb}^{*})$ is the fraction of projects that will look viable after their success probabilities are ascertained.

[5] Lenders can observe whether an investment project is being undertaken.

whether to go ahead. A borrower who chooses not to proceed receives no loan on date 1 and pays Z_0 on date 2. A borrower who chooses to undertake a project receives $1-w^e$ in resources on date 1 and repays Z_s on date 2 if successful, Z_u if not. The repayment profile for the borrower can be summarized as:

Z_s : project is undertaken and succeeds
Z_u : project is undertaken and bombs
Z_0 : project is not undertaken.

Let p^* be the cutoff probability that the contract implies. For assumed values of p^* and of the preceding three Zs, I will write down the constrained maximization problem that the efficient contract solves and then find the optimal values of p^* and the Zs.

Competition among lenders ensures that the efficient contract will maximize entrepreneurs' expected utility subject to a break-even condition for the lenders, incentive-compatibilty constraints, and feasibility constraints.

Given that it is profitable to pay the evaluation cost e (otherwise the contract will not be signed), the contract gives an entrepreneur expected utility (expected date 2 consumption) equal to the sum of three terms:

$$[1-H(p^*)]\,[\hat{p}\,(R-Z_s)-(1-\hat{p})Z_u]+H(p^*)(rw^e-Z_0)-e. \tag{1}$$

The first term here is the probability of proceeding with a risky project times the project's expected net payoff (conditional on $p \geq p^*$). The second term is the probability of not proceeding times the net payoff to storage, given that the contract was signed. The third (negative) term is the evaluation cost.

The best incentive-compatible contract maximizes (1), but subject to a number of constraints. The constraint

$$Z_0 \geq 0 \tag{2}$$

ensures that no entrepreneur will wish to sign a contract and pretend they have evaluated a project; doing so would only result in a consumption of $rw^e - Z_0 \leq rw^e$ on date 2. We also have the nonnegativity constraints on consumption

$$rw^e - Z_0 \geq 0, \tag{3}$$

$$0 \geq Z_u. \tag{4}$$

Lenders must make nonnegative profits in equilibrium; thus the law of large numbers yields the profitability constraint[6]

$$[1 - H(p^*)]\,[\hat{p} Z_s + (1-\hat{p}) Z_u] + H(p^*) Z_0 \geq [1 - H(p^*)]\, r(1 - w^e). \tag{5}$$

Finally the constraints

$$p^*(R - Z_s) - (1 - p^*) Z_u = r w^e - Z_0, \tag{6}$$

$$R - Z_s \geq -Z_u, \tag{7}$$

ensure, respectively, that p^* is the optimal cutoff probability for entrepreneurs, given the contract, and that at probabilities *below* (rather than above) p^*, entrepreneurs will not proceed with risky projects.[7] We note that (4) and (7) imply that $R \geq Z_s$, so we needn't worry that successful entrepreneurs could have negative consumption.

Rather than going through a formal Kuhn-Tucker analysis (as in the appendix to BG), let's take an intuitive approach. Notice first that constraint (5) always holds as an equality, because higher expected profits for lenders imply lower expected utility for entrepreneurs.

Constraints (2) and (4) also always bind, that is, we always have $Z_0 = Z_u = 0$. Why? Values of Z_u below zero or of Z_0 above 0 tend to subsidize projects with low success probabilities, and thus worsen the moral hazard distortion. Forcing the two constraints to bind raises p^* closer to p^*_{fb}. Equation (6) implies that for $Z_u < 0$ and $Z_0 > 0$,

$$\frac{\mathrm{d}p^*}{\mathrm{d}Z_u} = \frac{1 - p^*}{R - Z_s + Z_u} > 0, \;\; \frac{\mathrm{d}p^*}{\mathrm{d}Z_0} = -\frac{1}{R - Z_s + Z_u} < 0,$$

[6] Remember that in a fraction $H(p^*)$ of cases, the borrower will not proceed and thus will not draw on the lender's resources. Thus, $[1 - H(p^*)]\,(1 - w^e)$ represents the amount a lender could place in storage instead of offering a contract with a loan amount of $1 - w^e$ (contingent in investment). (You can imagine that nonentrepreneurs take advantage of the law of large numbers by using competitive lending intermediaries as a risk pooling device.)

[7] Inequality (7) implies that the derivative of the left hand-side of (6) with respect to the success probability is positive.

so that raising Z_u (from negative to 0) or reducing Z_0 to 0 raises p^*.

If (2), (4), and (5) bind, however, (3) and (7) hold automatically, and we can characterize the optimal contract by simplified versions of (5) (in equality form) and (6):

$$\hat{p} Z_s = r(1 - w^e), \tag{8}$$

$$p^*(R - Z_s) = r w^e. \tag{9}$$

Since $\hat{p}$ is a function of p^*, with

$$\frac{\mathrm{d}\hat{p}}{\mathrm{d}p^*} = \frac{(\hat{p} - p^*)\, H'(p^*)}{1 - H(p^*)} > 0,$$

eqs. (8) and (9) yield a solution for Z_s, and thus for the optimal contract. Using (8) to solve for Z_s and substituting the result into (9) gives:

$$p^* \left[R - \frac{r(1 - w^e)}{\hat{p}} \right] - r w^e = 0.$$

(Notice that $p^* < r/R$ if $w^e < 1$.) Implicitly differentiating, we find that

$$\frac{\mathrm{d}p^*}{\mathrm{d}w^e} = \frac{r\left(1 - \dfrac{p^*}{\hat{p}}\right)}{R - Z_s\left(1 - \dfrac{p^*}{\hat{p}} \cdot \dfrac{\mathrm{d}\hat{p}}{\mathrm{d}p^*}\right)} > 0.$$

In addition, it is now clear from (8) that a rise in w^e reduces Z_s. Thus, a rise in entrepreneur wealth (even a rise that leaves *society's* wealth unchanged) reduces the agency cost, raises the quality of projects, and raises potential social welfare. A corollary [see BG, eq. (15)] is that for $w^e < 1$, the marginal expected value of nonborrowed or internal funds (an increment to w^e) exceeds that of borrowed or external funds, which is r.[8] Of course, if $w^e = 1$ agency costs are nil and we reach the first-best allocation.

Now comes a key result of the paper. For internal wealth levels below a critical positive value $\underline{w}^e$, it does not pay to evaluate and invest. We have so far assumed that the optimal contract makes it worthwhile to evaluate

[8] This conclusion assumes that the firm finds it worthwhile to borrow.

projects. Consider, however, the entrepreneurial wealth level $\underline{w}^e$ at which evaluation is no better than a break-even proposition. By (1), $\underline{w}^e$ is given by

$$\begin{aligned} & \Big[1 - H(\underline{p}^*)\Big] \, \hat{\underline{p}} \left[R - \frac{r(1 - \underline{w}^e)}{\hat{\underline{p}}} \right] + H(\underline{p}^*) r \underline{w}^e - e \\ = & \Big[1 - H(\underline{p}^*)\Big] \Big(\hat{\underline{p}} R - r \Big) + r \underline{w}^e - e = 0, \end{aligned} \tag{10}$$

where, by combining (8) and (9), $\underline{p}^*$ and $\hat{\underline{p}}$ are derived from

$$\underline{p}^* \left[R - \frac{r(1 - \underline{w}^e)}{\hat{\underline{p}}} \right] - r \underline{w}^e = 0.$$

If $w^e < \underline{w}^e$, evaluation is not profitable. Observe that $\underline{w}^e$ must be strictly greater than 0, for at $\underline{w}^e = 0$, the preceding equation implies $\hat{\underline{p}} R = r$. By (10), therefore, the return to evaluation is *strictly* negative at $\underline{w}^e = 0$.

This result raises the prospect of an investment collapse if internal real wealth levels fall too low (for example, due to an abrupt debt deflation). The most realistic setting in which to contemplate this result is one in which there are nondegenerate distributions of entrepreneur and nonentrepreneur wealth, as in BG (with contracts tailored to each borrower's wealth, and wealth, of course, observable). In that environment, a large unfavorable shift in the distribution of internal firm resources can reduce aggregate investment by pushing many firms into a position where high agency costs make it unprofitable to sign loan contracts. Alternatively, *gradually* falling wealth will lead to a rise in agency costs over time and a steady deterioration in the average quality of investments. Paradoxically, the rise in agency costs may initially lead investment to expand as entrepreneurs, facing less favorable incentives, progressively adopt riskier projects. The process can be cumulative, with wealth declining to the point where investment ultimately collapses altogether. This is the pattern seen in some east Asian countries during the 1997-98 financial crisis.

An example

To make this all more concrete, let's look at a specific example, in which $H(p)$ describes a uniform distribution over the unit interval [0,1], so that $H(p) = p$.

In this special case

$$\hat{p} = \frac{1 + p^*}{2}.$$

Combining eqs. (8) and (9), we see that p^* satisfies the quadratic equation

$$(p^*)^2 + \left[1 - p^*_{fb} - p^*_{fb}(1 - w^e)\right] p^* - p^*_{fb} w^e = 0.$$

Since $p^*_{fb} w^e > 0$, this equation has one positive root and one negative root, although only the positive one is economically meaningful:

$$p^* = \frac{-\left[1 - p^*_{fb} - p^*_{fb}(1 - w^e)\right] + \sqrt{\left[1 - p^*_{fb} - p^*_{fb}(1 - w^e)\right]^2 + 4p^*_{fb} w^e}}{2}.$$

Naturally, $p^* \to p^*_{fb}$ as $w^e \to 1$. Equation (8) or (9) can be used to solve for Z_s, which equals 0 if $w^e = 1$.