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Government Debt and Taxes

As of September 2008, government plans to underwrite the U.S. financial system (if not the entire world financial system) look likely to increase the U.S. government's debt significantly. Leaving aside the fascinating questions raised by the financial crisis itself, how do macroeconomists think about government debt and its effects? Should government debt matter at all – after all, leaving aside the possibility of borrowing from foreigners, we owe any public debt to ourselves! Because one logical possibility is that government debt somehow affects capital accumulation and growth, it is natural to consider the question in the context of our growth models.

The leading breakthrough on the subject is Diamond's (*AER* 1965) adaptation of Samuelson's overlapping generations model to incorporate capital, growth, and public debt. We shall study that model soon, but first take a look at the debt question within the Ramsey-Cass-Koopmans (RCK) setup. There the answers are less interesting (and perhaps less intuitive), yet they provide an essential benchmark case for understanding the Diamond model's different predictions.

Within the RCK framework we now wish to distinguish between the private sector and the government, two sectors that add up to be the total economy, of course. As we are now therefore dropping the idea that a government "planner" makes allocation decisions, we need to observe (following basic welfare economics) that the RCK allocation can be decentralized if private agents face the time path of real interest rates corresponding to that optimal allocation,

$$r_t = f'(k_t)$$

and earn real wages per unit labor given by the marginal product of labor,

$$w_t = f(k_t) - f'(k_t)k_t.$$

[Following Diamond 1965, I assume that the depreciation rate δ of capital is 0; otherwise the real interest rate would be $r = f'(k) - \delta$.] A key step in showing this is to contemplate the government and private sectors' budget constraints separately.

With respect to the private sector, household assets at the start of period t are the sum of capital K_t and debt issued by the government, D_t. If we redefine these stocks in per capita terms as k_t and d_t, and also assume that the household pays per capita lump-sum taxes τ_t to the government each period, then we may write the private asset-accumulation equation in terms of real per capital wealth $a \equiv k + d$ as

$$a_{t+1} = \frac{1}{1+n}\left[(1+r_t)a_t + w_t - \tau_t - c_t\right].$$

Above, r_t is the interest paid during period t on assets accumulated over $t-1$. It is now easy to see that if consumers invest at the real interest rate r_{t+1} between dates t and $t+1$, then the relevant Euler equation of optimality would be

$$u'(c_t) = \beta(1+r_{t+1})u'(c_{t+1}). \tag{1}$$

At the same time the government's debt evolves according to the equation

$$d_{t+1} = \frac{1}{1+n}\left[(1+r_t)d_t - \tau_t + g_t\right],$$

where g is per capita consumption of goods by the government. Since debt represents negative assets, simply subtract the second of these from the first to get

$$\begin{aligned} k_{t+1} &= \frac{1}{1+n}\left[(1+r_t)k_t + w_t - c_t - g_t\right] \\ &= \frac{1}{1+n}\left[f(k_t) + k_t - c_t - g_t\right], \end{aligned}$$

the aggregate relationship from the RCK model (where $g \equiv 0$).

The private and public asset-stock *flow* relationships above imply infinite-horizon intertemporal budget constraints for the two sectors. For the private sector, for example, we may write (for $t = 0$),

$$\begin{aligned}
a_0 &= \frac{c_0 - (w_0 - \tau_0)}{1 + r_0} + \frac{1+n}{1+r_0} a_1 \\
&= \frac{c_0 - (w_0 - \tau_0)}{1 + r_0} + \left(\frac{1+n}{1+r_0}\right) \frac{[c_1 - (w_1 - \tau_1)]}{1 + r_1} + \left(\frac{1+n}{1+r_0}\right)\left(\frac{1+n}{1+r_1}\right) a_2 \\
&= \frac{1}{1+r_0} \left(\sum_{t=0}^{\infty} \left\{ \prod_{s=1}^{t} \left(\frac{1+n}{1+r_s}\right) [c_t - (w_t - \tau_t)] \right\} \right. \\
&\quad \left. + (1+n) \lim_{t\to\infty} \prod_{s=1}^{t} \left(\frac{1+n}{1+r_s}\right) a_{t+1} \right).
\end{aligned}$$

Consider the reasonableness of imposing on households the condition that

$$\lim_{t\to\infty} \prod_{s=1}^{t} \left(\frac{1+n}{1+r_s}\right) a_{t+1} \geq 0.$$

In the Ramsey economy we can never have a negative capital stock. But in the decentralized economy, where households borrow subject to a real interest rate, we can imagine someone borrowing to consume and then always borrowing more to repay the previous loans, thereby never repaying at all. The preceding inequality constraint rules out such a Ponzi scheme and thus is called the "no-Ponzi-game" constraint.[1] Imposing it, we obtain the intertemporal constraint

$$(1+r_0)a_0 \geq \sum_{t=0}^{\infty} \left\{ \prod_{s=1}^{t} \left(\frac{1+n}{1+r_s}\right) [c_t - (w_t - \tau_t)] \right\}. \tag{2}$$

This restrictions says that household initial assets (along with their payout) must cover any discounted excess of consumption over after-tax wage income. [Can you see, using (1), why the transversality condition will normally ensure that in equilibrium, this condition holds as an equality?]

The government faces an analogous constraint: the excess of its tax receipts net of public spending, discounted to the present, must cover at least

[1] If we do not impose such a constraint, then anyone can consume infinite resources and there would be excess demand for output.

its initial debt to the private sector. Because government assets are equal to $-d$, we may write the public-sector constraint as

$$-(1+r_0)d_0 \geq \sum_{t=0}^{\infty} \left[\prod_{s=1}^{t} \left(\frac{1+n}{1+r_s} \right) (g_t - \tau_t) \right]. \tag{3}$$

Putting these together leads to

$$(1+r_0)k_0 \geq \sum_{t=0}^{\infty} \left[\prod_{s=1}^{t} \left(\frac{1+n}{1+r_s} \right) (c_t - w_t) \right]$$

for the economy as a whole.

The proposition I now wish to explore is the neutrality of public debt in this economy with lump-sum taxes and a single representative family. The proposition is known as the *Ricardian equivalence* of debt and future taxes. Suppose the government increases its own initial debt by showering a gift Δd of government bonds on people at the start of period 0. (Think of the recent U.S. fiscal stimulus package.) To finance the payments on this debt, the government raises taxes intertemporally (perhaps far in the future) by the amount

$$\Delta d = \sum_{t=0}^{\infty} \left[\prod_{s=1}^{t} \left(\frac{1+n}{1+r_s} \right) \Delta\tau_t \right]$$

[recall that τ denotes per capita taxes in (3)]. Notice that this experiment changes the left-hand and right-hand sides of the household constraint (2) by *equal* amounts: there is no change in intertemporal household consumption possibilities. Accordingly, private consumption behavior also is unchanged. In other words, the gift of government debt does not represent *net* wealth for households, because it arrives with the certainty of offsetting future tax payments to the government. (Of course, the private sector is likely to raise its *saving* so as to build a fund that can be used to pay the anticipated future taxes. Private saving is defined as total household income, including interest earned on government bonds, less consumption.)

That is the prediction of models featuring Ricardian equivalence. Here indeed, public debt does not matter because the same people who own the debt pay the taxes – indeed, we "owe it to ourselves." Diamond's overlapping generations model is not in this category.

The Diamond Overlapping-Generations Model: Basic Setup

The basic structure assumes that every individual lives for two periods, but that generations are born in a staggered fashion.

Thus, on a generic date t, a new cohort of agents is born, who live during period t (when they are young) and period $t+1$ (when they are old). However, the next generation is born already on date $t+1$, so that the young born on date $t+1$ and the date-$(t+1)$ old, who were born on date t, coexist (or *overlap*) during period $t+1$.

Only the young are able to work. Thus, if you are born in t, you work during t and enjoy retirement during $t+1$. Because you wish to consume on both dates, however, you will attempt to save during your youth. People cannot leave bequests to members of future generations (and have no motive to do so), nor are they born with any inherited wealth or with any endowment other than the labor power they have to sell. Otherwise, Ricardian equivalence could return, as in Barro's famous 1974 *JPE* paper.

The constant-returns production function is $Y_t = F(K_t, N_t)$, where N_t is the number of young workers on date t. (They supply their labor inelastically.) The labor force grows according to

$$N_{t+1} = (1+n)N_t.$$

A young worker will put his/her savings into capital, reap the marginal product of capital when old, and then also sell the capital to the contemporaneous young. Capital income and capital sales finance consumption in old age. (As noted above, capital does not depreciate.)

As usual $k \equiv K/N$. The young worker of date t receives a wage of

$$w_t = f(k_t) - f'(k_t)k_t,$$

while the date-t old receive a per capita income from their investment equal to

$$f'(k_t)\frac{K_t}{N_{t-1}} = (1+n)f'(k_t)k_t.$$

A young worker on date t pays taxes τ_t^y to the government, while an old worker pays taxes τ_t^o. (It could be that $\tau^o < 0$, for example, if the young pay social security taxes of τ_t^y and then receive $-\tau_{t+1}^o$ in pension payments in their old age. We will come back to social security later.) Suppose that a worker born on date t maximizes

$$U_t = u\left(c_t^y\right) + \beta u\left(c_{t+1}^o\right)$$

subject to the intertemporal constraint

$$c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} = w_t - \tau_t^y - \frac{\tau_{t+1}^o}{1+r_{t+1}}. \tag{4}$$

Then optimal consumption is determined by combining the budget constraint with the Euler equation

$$u'\left(c_t^y\right) = \beta\left(1+r_{t+1}\right)u'\left(c_{t+1}^o\right).$$

Let

$$s_t^y = w_t - \tau_t^y - c_t^y \tag{5}$$

denote per capita saving by the young of date t. In old age they will have a per capita saving rate of

$$s_{t+1}^o = r_{t+1}s_t^y - \tau_{t+1}^o - c_{t+1}^o \tag{6}$$

(because saving is income minus consumption). From the budget constraint and (5), however,

$$c_t^o = \left(1+r_t\right)\left(w_{t-1} - \tau_{t-1}^y - c_{t-1}^y\right) - \tau_t^o = \left(1+r_t\right)s_{t-1}^y - \tau_t^o,$$

so by (6), rewritten to apply to period t,

$$s_t^o = r_t s_{t-1}^y - \tau_t^o - c_t^o = r_t s_{t-1}^y - \left(1+r_t\right)s_{t-1}^y = -s_{t-1}^y :$$

what you save when young you simply consume (dissave) while old. As a result, the capital stock on any date equals the amount saved by the previously young:

$$K_t = N_{t-1}s_{t-1}^y \Leftrightarrow k_t = \frac{s_{t-1}^y}{1+n}.$$

Those who are old on date t eat this capital completely during t, leaving the contemporaneous young to put aside the next period's capital stock K_{t+1} through their own savings.

Without losing too much generality, let's compute the equilibrium explicitly for a specific example. Assume that $u(c) = \ln(c)$ and let $F(K,N) = AK^\alpha N^{1-\alpha}$. Then the Euler equation can be written as

$$c_{t+1}^o = \beta\left(1+r_{t+1}\right)c_t^y,$$

which, together with (4), leads to the solutions

$$\begin{aligned} c_t^y &= \frac{1}{1+\beta}\left(w_t - \tau_t^y - \frac{\tau_{t+1}^o}{1+r_{t+1}}\right), \\ c_{t+1}^o &= \frac{\beta\left(1+r_{t+1}\right)}{1+\beta}\left(w_t - \tau_t^y - \frac{\tau_{t+1}^o}{1+r_{t+1}}\right). \end{aligned}$$

Accordingly,

$$\begin{aligned} s_t^y &= w_t - \tau_t^y - c_t^y \\ &= \frac{\beta}{1+\beta}\left(w_t - \tau_t^y\right) + \frac{1}{1+\beta}\frac{\tau_{t+1}^o}{\left(1+r_{t+1}\right)}. \end{aligned} \tag{7}$$

We now can represent the equilibrium as a difference equation in k. Because $k_{t+1} = s_t^y/(1+n)$, $w_t = f(k_t) - k_t f'(k_t) = (1-\alpha)Ak_t^\alpha$, and $r_{t+1} = f'(k_{t+1}) = \alpha A k_{t+1}^{\alpha-1}$, the last equation can be written as:

$$k_{t+1} - \frac{1}{(1+n)\,(1+\beta)}\frac{\tau_{t+1}^o}{\left(1+\alpha k_{t+1}^{\alpha-1}\right)} = \frac{\beta}{(1+n)\,(1+\beta)}\left[(1-\alpha)Ak_t^\alpha - \tau_t^y\right]. \tag{8}$$

The Diamond Model: No Fiscal Policy

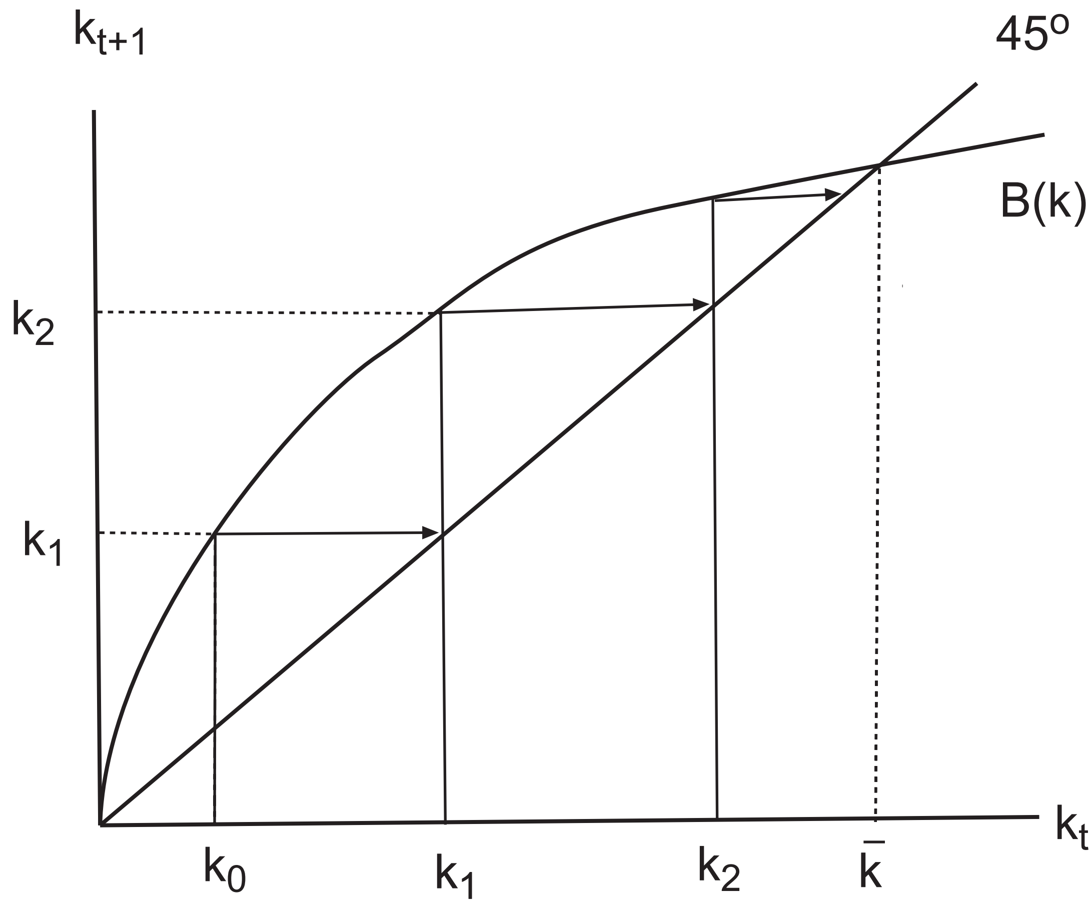
Equation (8) is a very general depiction of the economy's dynamics (which is why it looks so complex) and I will show how to analyze it in some fiscally relevant cases later. To make some initial points, however, it is useful to take the special case in which fiscal policy is absent, so that $\tau^y = \tau^o = 0$ on all dates. In that case, eq. (8) can be written in the much simpler form

$$k_{t+1} = \frac{\beta(1-\alpha)A}{(1+n)(1+\beta)}k_t^\alpha \equiv B(k_t).$$

A simple diagram (next page) allows us to analyze this difference equation. We use it as follows. Starting at any k_0 on the x-axis, the curved locus $B(k)$ indicates the value of k_1. Project that value horizontally to the 45° line, then down vertically to find the location of k_1 on the x-axis. Then repeat the process using k_1 as the new starting value, from which k_2 is derived.

The picture makes obvious that the economy will converge in a stable, monotonic fashion to a steady state capital/labor ratio $\bar{k}$ given by

$$\bar{k} = \left[\frac{(1-\alpha)\beta A}{(1+n)(1+\beta)}\right]^{\frac{1}{1-\alpha}} \tag{9}$$



Diamond Model with no Taxation

Steady state capital per worker will be higher if β is closer to 1 (people are more patient) and if n is lower. The steady state is a balanced growth path with constant capital per worker. In the steady state, a young worker consumes

$$\bar{c}^y = \frac{1}{1+\beta}\bar{w} = \frac{1-\alpha}{1+\beta}A\bar{k}^\alpha,$$

while an old retiree consumes

$$\bar{c}^o = (1+n)(\bar{k} + \alpha A\bar{k}^\alpha).$$

With labor-augmenting technical change at rate g, there would be a balanced growth path with consumptions per capita and capital growing at rate g.

Let us now consider the question of the Golden Rule in this economy; the situation is different from that in the RCK economy, where we saw that $f'\left(\bar{k}\right) > n$ always. A central planner might like to maximize total steady-state *lifetime* utility of a typical individual

$$\bar{U} = u\left(\bar{c}^y\right) + \beta u\left(\bar{c}^o\right)$$

subject to the constraint that $\bar{k}$ is constant over time

$$f(\bar{k}) = n\bar{k} + \bar{c}^y + \frac{\bar{c}^o}{1+n}.$$

If you form the Lagrangian for this problem, you will see that the first-order conditions for consumption boil down to

$$u'\left(\bar{c}^y\right) = \beta(1+n)u'\left(\bar{c}^o\right).$$

But compare this to the individual's Euler equation, eq. (1): the preceding condition will hold in the steady state – that is, the utility of a typical generation will be maximized – only if $\bar{k} = k^*$, where $f'\left(k^*\right) = r^* = n$. Thus, the Golden Rule prescription is unchanged from its usual form. *However*, unlike in the RCK model, it is perfectly possible that $\bar{k} > k^*$ in Diamond's model.

Why? The Golden Rule capital-stock in our specific (log, Cobb-Douglas) example is

$$\left(\frac{\alpha A}{n}\right)^{\frac{1}{1-\alpha}} = k^*.$$

Using (9), you can see that the Golden Rule will be violated if

$$\left[\frac{(1-\alpha)\beta A}{(1+n)(1+\beta)}\right]^{\frac{1}{1-\alpha}} > \left(\frac{\alpha A}{n}\right)^{\frac{1}{1-\alpha}},$$

that is, if

$$\left(\frac{n}{1+n}\right)\left(\frac{\beta}{1+\beta}\right)\left(\frac{1-\alpha}{\alpha}\right) > 1.$$

That this inequality holds is certainly possible (if not highly plausible).

If $f'\left(\bar{k}\right) < n$, we are in a dynamically inefficient situation in which everyone in the economy could enjoy higher consumption on all dates if some capital were permanently consumed. In this model, however, the decentralized market is not capable of accomplishing this this. An all-powerful economic planner could transfer income from young to old however needed to maximize the utility of a typical generation, as in the last optimization problem. But in the market economy, the old can consume only if they save when young.

Let's look at the problem more closely. Normally – that is, in models where resource allocation is efficient – agents trade in order to eliminate unexploited opportunities for mutual gain. Consider a dynamically inefficient steady-state equilibrium of the Diamond model with $f'\left(\bar{k}\right) < n$, however. Start at time 0, and imagine that members of the young generation of period $t = 0$ could strike the following deal with the young of $t = 1, 2, 3$, etc. (who, of course, have not yet been born): we will each pay an amount $\tau/(1+n)$ to the old of period $t = 0$ if, in turn, every future young generation member promises likewise to pay $\tau/(1+n)$ to its contemporaneous old folks. Let us further set τ so that saving by the young results in a capital-labor ratio of k^*. Since $k = s^y/(1+n)$, we need τ to satisfy the equation

$$k^* = \frac{1}{1+n}\left\{\frac{\beta\left[f\left(k^*\right) - nk^*\right]}{1+\beta} - \frac{\tau}{(1+n)}\right\}.$$

[Recall (7), and substitute in $w = f\left(k^*\right) - nk^*$, $\tau^y = \tau/(1+n)$, $\tau^o = -\tau$, and $r = n$.] In this equilibrium, a person pays to the old $\tau/(1+n)$ when young, but receives τ when old (because there are $1+n$ more young people next period); and because the interest rate is also equal to n, an individual's budget constraint in this steady state is:

$$c^{y*} + \frac{c^{o*}}{1+n} = f(k^*) - nk^*.$$

Observe that if agents can carry out these agreements, they fully replicate the (optimal) Golden Rule solution to the planning problem. The only obstacle to this clever scheme is that a generation cannot, in reality, contract with generations yet to be born! And so the private marketplace cannot bring about an exit from dynamic inefficiency.

The Role of Fiscal Policy

Unless we introduce some sort of redistributive fiscal policy, there is no avenue for government to transfer resources to the old so that they will save less. Fiscal policy is a way for the government to mimic the voluntary transfers described above, and it works when the (infinitely-lived) government can make binding commitments on behalf of generations that are yet to be born.[2] In that scenario, the government simply taxes the young to subsidize the old: the young pay $\tau/(1+n)$ per capita and the old receive τ per capita; the budget is balanced date by date.

The alert reader will ask the following: suppose we are at a $\bar{k}$ that is *below* the Golden Rule level k^*. By doing the above scheme in reverse, could we not move to the steady-state-consumption-maximizing Golden Rule? The answer is yes, but we would have to *tax* the initial old to pay the initial young, so the first generation of old is worse off even though everyone else may be better off. (They die after being taxed; so, unlike the young, they do not recoup their losses as a subsidy later on.) Thus, it is only in the case of dynamic inefficiency that there is scope for a *Pareto* improvement. In moving from $\bar{k} > k^*$ to k^*, we were able to make *everyone* better off (including the $t = 0$ old, who received a positive payment).

Public Debt

We stick with the log utility/Cobb-Douglas example. Let the government maintain a public debt of $D_t/N_t = d$ forever. To do so, assume that the young *only* are taxed; $\tau^o \equiv 0$. If τ_t^y is the per capita (lump-sum) tax that is levied on a young person, the flow government budget constraint is

$$D_{t+1} = (1 + r_t)D_t - N_t\tau_t^y.$$

[2]But can it? A young generation, outnumbering the old, could simply vote to change the law and thereby default on their payment to the old. In reality, the sustainability of an efficiency-enhancing fiscal scheme is therefore a question in political economy. Such matters are fascinating but beyond the scope of this course.

In order that the public debt per young person remain constant over time, we need

$$\begin{aligned}
d &= \frac{D_{t+1}}{N_{t+1}} = \frac{(1+r_t)D_t - N_t\tau_t^y}{N_{t+1}} \\
&= \frac{1+r_t}{1+n}d - \frac{\tau_t^y}{1+n} \\
&\Rightarrow \tau_t^y = (r_t - n)\,d \\
&= \left[f'(k_t) - n\right]d.
\end{aligned}$$

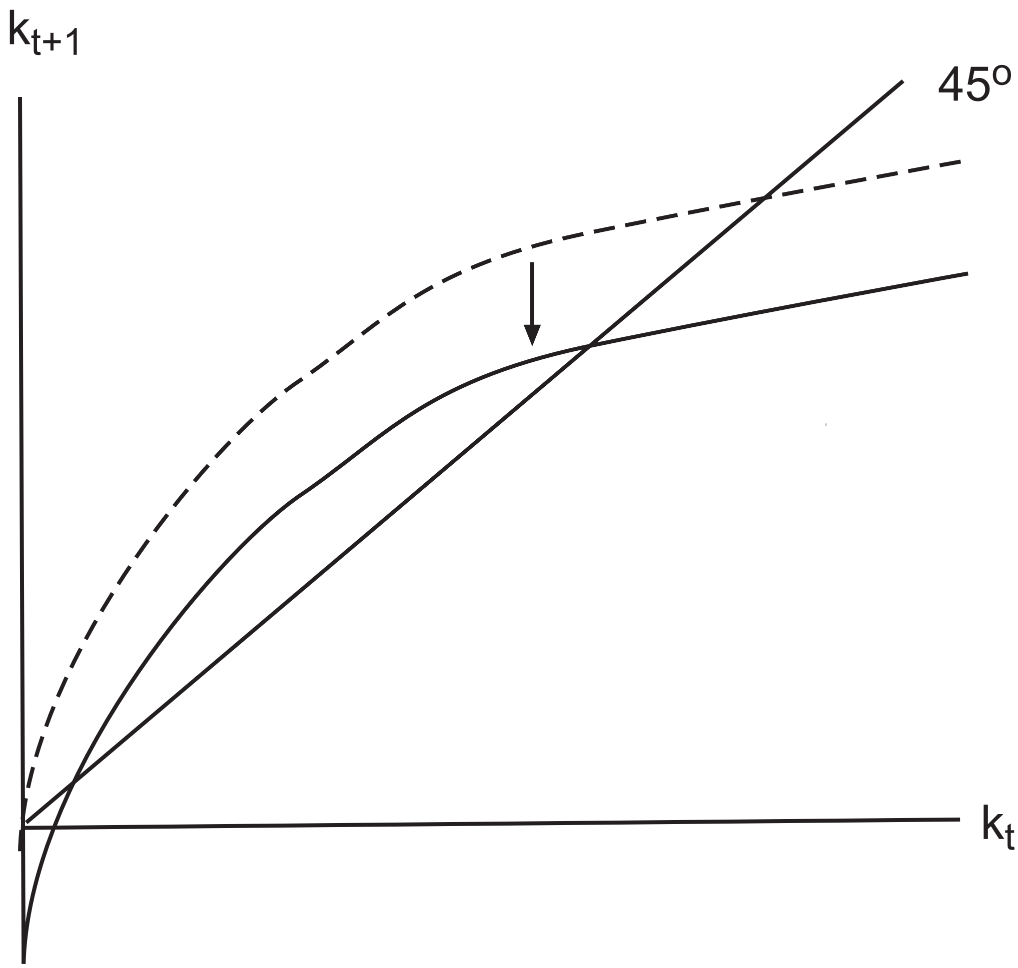
Imagine that the government endows the initial old with d and levies the indicated tax on the young at the same time. In the first period the old have very high consumption, and the young must buy the debt from them. In the second period the capital stock still reflects the impact of the very high period 1 consumption of the old. By period 3 the economy has settled down to the relation implied by eq. (8), modified for the fact that the young must now purchase the debt as part of their savings in addition to any capital they accumulate:

$$d + k_{t+1} = \frac{\beta}{(1+n)(1+\beta)}\left\{(1-\alpha)Ak_t^{\alpha} - \left[\alpha Ak_t^{\alpha-1} - n\right]d\right\}.$$

The effect is to shift downward the curved $B(k)$ locus in the Diamond diagram, as shown on the next page. There is a unique *stable* steady state, with a *lower* long-run capital stock per worker. (There is also a second steady state with a nonzero capital level, but it is *unstable*.)

What are the welfare effects? (Please verify what follows!) If initially the economy is dynamically efficient ($\bar{k} \leq k^*$), then the initial old who receive the gift of debt are better off, and all subsequent generations are worse off. There is a capital *crowding out effect* because people put their savings into unproductive public debt rather than productive capital; and because we are "to the left" of the Golden Rule, more capital is better. In this sense, accumulating public debt today impoverishes future generations, even though society owes the debt to itself. (Perhaps surprisingly, matters are even worse in the closed economy than if the debt is owed to foreigners! See Diamond 1965.)

If, however, the economy initially is dynamically *in*efficient ($\bar{k} > k^*$), public debt paradoxically makes all generations better off by crowding out excessive capital. A public debt acts like a scheme of transfers from young to



Diamond Model with Public Debt

old – the young pay taxes to the government, which transfers them to the old in the form of interest payments on government debt. So it works just like the hypothetical Pareto-improving scheme we discussed above – with the debt providing a way for generations not alive at the same time effectively to trade with each other. In this setting, the promise that the government will always honor its debt works like a compact between present and unborn generations. That compact can be broken, however, if the government decides to default on its debt.

Social Security

Unfunded social security – the prevailing arrangement nowadays in the United States and most other countries – is exactly like public debt in its effects. Government taxes the young (social security taxes) and makes transfers to the old (social security payments). The scheme reduces the capital stock. Capital-stock reduction is beneficial, of course, only in the dynamically inefficient case.

In the case of *fully funded* social security the government taxes the young but invests the proceeds in capital k, using the return on the capital to pay the old. Because in this scheme the savings of the young are not diverted into government paper, crowding out can be avoided.

The Possibility of Asset Bubbles under Dynamic Inefficiency

Suppose the government issues an asset that pays no dividend. Think of it as a piece of paper carrying George W. Bush's portrait. In a dynamically efficient economy the paper will have no value. In the dynamically inefficient economy, however, there can be a Bush bubble: the paper will have value (and its value will even rise through time) if every generation believes that future generations will value it.

Let the number of Bush portraits be D and the price of each one (in terms of output), p. Savers will be willing to hold the paper provided its price rises at the (gross) rate of interest:

$$\frac{p_{t+1}}{p_t} = 1 + r.$$

This means, also, that the supply of the asset, $p_{t+1}D/p_tD$, rises at rate $1 + r$. The supply of savings in the economy, however, grows at the gross rate $1 + n > 1 + r$. So as long as p_0D does not exceed the initial savings of the young, the young will always be able to buy the available supply of Bush portraits, and will be willing to do so because they yield the same return as does capital.

Furthermore, the Bush asset will have the beneficial effect of crowding out some excess capital. In effect, we are looking at an equilibrium in which future generations "promise" to purchase the paper at a specific price, and the resulting expectation takes the place of a hypothetical (but infeasible) contract among unborn generations.

This bubble is not sustainable if $r > n$ because in that case, the value of the artificial asset eventually comes to exceed the savings of the young, at which point a price collapse is inevitable. As a result of this terminal infeasibility, the only possible equilibrium is $p_0 = 0$ in the dynamically efficient case.

For more details, see the paper by Tirole in *Econometrica* (November 1986).