Economics 202A, Problem Set 3

Maurice Obstfeld

(Due Tuesday, October 7)

1. *OG Model for the Open Economy.* Consider the overlapping generations model with the following twists: population is constant, the path of output $\{y_t^y, y_t^o\}$is exogenous with $y_t^o \equiv 0$, the country may borrow from foreigners or lend to them at the fixed real interest rate r, and $U\left(c_t^y, c_{t+1}^o\right) = \log c_t^y + \beta \log c_{t+1}^o$. (a) If taxes on the young (old) are τ_t^y (τ_t^o), calculate the consumption functions of the young and the old. (b) What is the intertemporal budget constraint of the government. (c) Assume that initially government debt and taxes are zero. Now consider the following fiscal policy: on date 0, the government makes a gift of $d/2$ in government bonds to the date-0 young and the same gift to the date-0 old. These bonds begin to pay interest (at rate r) on date 1. Taxes on date 0 population do not change, but taxes on everyone rise by $rd/2$ from $t = 1$ onward. Show that this policy is consistent with the government's intertemporal budget constraint. (d) Calculate the effect of the fiscal policy (as a function of d) on *aggregate* consumption for every date $t = 0, 1, 2, 3$, etc. (e) What is the effect on long-run aggregate consumption? This effect cannot be due to the crowding out of capital, because there is no capital. Explain intuitively what has happened.

2. *Barro versus Feldstein.* In a critique of Barro's famous paper "Are Government Bonds Net Wealth?" Martin Feldstein (*JPE*, April 1976) argued that government debt may be net wealth in a growing economy. His case went as follows: Suppose the government gives an amount of debt D_0 to people and taxes them for all interest paid on this and other government debt issued in the future. Let the interest rate in the economy be fixed at r, the growth rate of *total* (not per capita) output g, and suppose $r > g$ (ruling out Ponzi games). Suppose the government taxers people to cover $(r - g)D_0$ each period but simply issues new debt in the amount gD_0 to cover the balance of the total interest bill rD_0. Then, Feldstein argued, the public debt-to-output ratio will remain constant, the government's intertemporal budget constraint will still hold, but the people who receive the gift D_0 from the

government will enjoy a net increase in wealth equal to $(g/r)D_0$, which is proportional to the portion of the debt rolled over each period. [Use the continuous-time government budget constraint

$$D_0 = \int_0^\infty \mathrm{e}^{-rt} T(t)\mathrm{d}t$$

for this one, where $T(t)$ denotes total taxes collected on date t.] (a) Do you agree with Feldstein's analysis for an economy with a constant number of identical immortal agents? (b) How would this argument fare if the demographics were different? Specifically, assume that people are immortal, but that *new* immortal people are born each period and are taxed to pay for past government liabilities as well as those incurred during their own lifetimes.

3. *Intertemporal Tax Smoothing.* Here is a problem related to F. P. Ramsey's *other* great contribution to economics, his paper on optimal taxation. (The ramifications of the Ramsey tax principle throughout economics are many.) The government seeks to maximize the utility of a typical consumer,

$$U = \int_0^\infty \mathrm{e}^{-rt} \left\{ \log[c(t)] - \frac{\phi}{2}\ell(t)^2 + v\left[g(t)\right] \right\} \mathrm{d}t,$$

where ℓ is the labor that the individual devotes to production and g is a public good that the government provides (perhaps national defense). The path of g is exogenous. (I am assuming here that the subjective discount rate equals the market real rate of interest r.) Output is given by $y = A\ell$; however, the government uses a tax on labor to finance itself, so that the worker's perceived return to working is instead

$$y = (A - \tau)\,\ell.$$

This results in a divergence between the private and social returns to effort, a tax *distortion.* If initially the government has no debt, one can view the government as solving the intertemporal problem

$$\max_{\{c(t),\tau(t)\}} U$$

subject to the worker's first-order condition for labor supply [where does it come from? – this is part (a) of this problem],

$$\frac{A-\tau(t)}{c(t)} = \phi\ell(t), \tag{1}$$

the output constraint

$$c(t)+g(t) = y(t) = A\ell(t), \tag{2}$$

and the intertemporal government budget constraint

$$\int \mathrm{e}^{-rt}\left[g(t)-\tau(t)\right]\mathrm{d}t = 0.$$

This last budget condition can, alternatively, be represented by the set of equations for government debt

$$\dot{d}(t) = rd(t)+g(t)-\tau(t),\ d(0)=0,\ \lim_{t\to\infty}\mathrm{e}^{-rt}d(t)=0. \tag{3}$$

(b) Explain that by combining (*) with (**), we get the following equation showing how government tax policy effectively controls consumption:

$$c = \frac{-g+\sqrt{g^2+4A(A-\tau)/\phi}}{2} \equiv \omega(\tau),\ \omega'(\tau)<0.$$

(c) Show why labor supply can be controlled by

$$\ell = \frac{A-\tau}{\phi\omega(\tau)}.$$

(d) Now argue that the government's problem can be expressed as

$$\max_{\{\tau(t)\}} \int_0^\infty \mathrm{e}^{-rt}\left(\log\left\{\omega\left[\tau(t)\right]\right\} - \frac{\phi}{2}\left\{\frac{A-\tau(t)}{\phi\omega\left[\tau(t)\right]}\right\}^2 + v\left[g(t)\right]\right)\mathrm{d}t$$

subject to (***).

(e) Apply the Maximum Principle to show that the path of taxes will be *constant* over time, even if the path of expenditures $\{g(t)\}$ is not (This is where Ramsey taxation comes in.) Can you interpret this result? [Hint: When will the government run surpluses, when will it run deficits, and why?] Comment on the proposition: if taxes distort economic activity, then the Ricardian equivalence prediction that the time-path of taxes and public deficits is irrelevant may not hold.