Economics 202A Midterm Exam

October 16, 2008

Instructions: You have 1 hour and 15 minutes. Answer both questions, which carry equal weights. Take the first 5 minutes to look over both pages of the exam before you start. That way you can better pace yourself. If you get stuck, go on to something you can answer more easily and return to the difficult bits later on.

1. This question is about a Solow model with human capital. Let H denote the stock of human capital, and let the production function be

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta},$$

where $\dot{A}/A = g$ is the rate of labor-augmenting technical change and $\dot{L}/L = n$. I assume that the accumulation of physical capital is given by

$$\dot{K} = s_K Y$$

(there is no depreciation) and of human capital by

$$\dot{H} = s_H Y.$$

(a) Show how to write the (two) dynamic equations of the model in terms of y, k, and h, where $y = Y/AL$, $k = K/AL$, and $h = H/AL$.
(b) Does the model have a balanced growth path? Describe its properties in words.
(c) Solve for the steady-state ratios $\bar{k}$ and $\bar{h}$.
(d) How do these steady-state values depend on s_K, s_H, and $g + n$, and why?
(e) Consider the linear first-order Taylor approximation (around the steady state) to the differential-equation system you have derived above, which can be written as $\dot{h} = \Phi(h, k)$, $\dot{k} = \Psi(h, k)$. Recall that it takes the form

$$\begin{bmatrix} \dot{h} \\ \dot{k} \end{bmatrix} \approx \begin{bmatrix} \Phi_h(\bar{h}, \bar{k}) & \Phi_k(\bar{h}, \bar{k}) \\ \Psi_h(\bar{h}, \bar{k}) & \Psi_k(\bar{h}, \bar{k}) \end{bmatrix} \begin{bmatrix} h - \bar{h} \\ k - \bar{k} \end{bmatrix},$$

where $\Phi_h \equiv \frac{\partial \Phi}{\partial h}$, etc., and these partial derivatives are evaluated at $(h, k) = (\bar{h}, \bar{k})$. Show that the preceding approximation is:

$$\begin{bmatrix} \dot{h} \\ \dot{k} \end{bmatrix} \approx \begin{bmatrix} (g+n)(\beta-1) & \frac{s_H}{s_K}\alpha(g+n) \\ \frac{s_K}{s_H}\beta(g+n) & (g+n)(\alpha-1) \end{bmatrix} \begin{bmatrix} h-\bar{h} \\ k-\bar{k} \end{bmatrix}.$$

(f) (*Extra credit*) Draw the phase diagram for this system, placing h on the vertical axis and k on the horizontal axis. You need only draw the diagram locally, in the neighborhood of the steady state. Is adjustment stable? Note that the locus along which $\dot{k} = 0$ is steeper than that along which $\dot{h} = 0$.

2. Consider a version of the Ramsey-Cass-Koopmans model in which a representative dynastic family that grows in size at rate $n < \theta$ maximizes

$$\int_0^\infty \mathrm{e}^{-(\theta-n)t} u\left[c(t), k(t)\right] \mathrm{d}t$$

subject to an initial k_0 and

$$\dot{k}(t) = f\left[k(t)\right] - c(t) - nk(t)$$

(capital does not depreciate, and there is no technical progress). Here, people derive direct utility from holding capital, i.e., $u_k > 0$. The production function is strictly concave and satisfies the Inada conditions.

(a) Use the maximum principle to derive the necessary conditions for an optimal consumption/saving plan.

(b) Assume that $u(c, k) = \ln(c) + \eta k$, where η is a very small positive number. Derive the economy's equations of motion for c and k.

(c) Solve for the economy's steady-state values $\bar{c}$ and $\bar{k}$, assuming that these values exist and are both positive.

(d) Draw the phase diagram for the system. (Hint: The locus along which $\dot{c}/c = 0$ is no longer vertical, and it lies to the right of the vertical $\dot{c}/c = 0$ schedule that applies in the "usual" model from class.)

(e) Is the Golden Rule satisfied? Show why or why not and explain your answer, being sure to define the Golden Rule level of capital. What is the implication for dynamic efficiency/inefficiency?