

Economics 202A

Problem 7: Asset Bubbles, Capital Inflows, and Price Collapse*

First consider a one-good closed economy. There are two periods, periods 1 and 2, and there are two assets available, a safe asset offering a predictable gross return of r between periods 1 and 2 and a risky asset (think of office buildings or shopping malls) offering a random gross return of R , where R is distributed on $[0, R_{MAX}]$ with c.d.f $H(R)$ and mean \bar{R} .

Competitive risk-neutral *banks* have an exogenously determined amount of output B that they lend inelastically to risk-neutral *entrepreneurs* at the gross interest rate, which equals r in equilibrium. (We will not model the determination of B , but one interpretation is suggested in an open-economy context in the last part of the question.) Banks lend to entrepreneurs because they themselves lack the know-how to invest their resources in assets on date 1. The total supply of the risky asset on date 1 is fixed and normalized at 1.¹ Any entrepreneur who holds x units of the risky asset at the end of period 1 pays a *nonpecuniary* period 2 cost $c(x)$, where $c(0) = c'(0) = 0$, $c'(x), c''(x) > 0$. (Think of this extra cost as the labor or stress involved in running a risky project; because it is nonpecuniary, it does not have to be paid out of the entrepreneur's funds.) If entrepreneurs invest x output units in the safe asset, the total gross return is $f(x)$ output units on date 2, where the function $f(\cdot)$ has the usual properties, i.e., it is nonnegative, strictly concave, makes $f(0) = 0$, and satisfies the Inada conditions. The date 1 price of the safe asset is always 1, under the assumption of a costless technology for transforming output into safe assets. In equilibrium, of course, $f'(x) = r$.

Finally, banks cannot observe how entrepreneurs invest borrowed resources, and can enter only into debt contracts with entrepreneurs. These contracts are similar to those discussed in David Romer's macro text: borrowers default if the (random) value of their date 2 portfolio is below what they owe the bank, but keep any value in excess of what is owed. (We do not, however, explicitly model costs of state verification.)

¹Imagine that in the background there is an overlapping-generations structure in which old owners of the risky asset supply it inelastically. You may assume that entrepreneurs themselves have no wealth to invest.

(a) Imagine that a representative entrepreneur buys X_R units of the risky asset and X_S units of the safe one on date 1. Let P be the date 1 price of the risky asset. Show that the date 2 (pecuniary) payoff to the entrepreneur under the contract just described is:

$$\Pi(R) = \max \{RX_R - rPX_R, 0\}.$$

Graph this payoff function (i.e., graph Π against R).

(b) What happens to the expected payoff $E\Pi(R)$ as the variance of R rises, given \bar{R} and X_R ? What is the intuition?

(c) Show why a representative entrepreneur maximizes

$$\int_{rP}^{R_{MAX}} (RX_R - rPX_R) dH(R) - c(X_R).$$

Derive his/her first-order optimality condition w.r.t. X_R .

(d) The model's other equilibrium conditions are: $X_R = 1$, $X_S + P = B$, and $r = f'(X_S)$. Explain each of these. Show that a unique equilibrium exists when $\bar{R} > c'(1)$. [Hint: Write the first-order condition from part (c) with 1 substituted for X_R . Show that in equilibrium, $rP > 0$ if and only if $\bar{R} > c'(1)$. Graph, in the (P, r) plane, the equilibrium first-order condition. Finish by graphing the last two equilibrium conditions listed in this part of the question.] Show that in equilibrium, banks earn an expected return strictly below r on their loans. (They have no choice but to pay a rent to entrepreneurs.)

(e) Prove that the locus defining the downward-sloping schedule in the diagram from part (d) can be expressed as:

$$P = \frac{1}{r} \left[\frac{\int_{rP}^{R_{MAX}} R dH(R) - c'(1)}{\Pr \{R \geq rP\}} \right].$$

(f) We may define the *fundamental* level of the risky asset's price as the price P^* that would prevail if entrepreneurs financed asset purchases entirely out of their own wealth B (rather than borrowing the same amount B). (This price will not reflect an overvaluation due to entrepreneurs' increased

propensity to gamble on the risky asset and default in low-return states.) Show that in equilibrium,

$$P^* = \frac{1}{r} \left[\int_0^{R_{MAX}} R dH(R) - c'(1) \right] = \frac{1}{r} [\bar{R} - c'(1)]$$

Interpret this relationship.

(g) Assume that the risk-free interest rate r in the formula of part (f) is the same as the one in the formula of part (e). Show that in that case, $P > P^*$. One can think of the difference as a bubble in the asset's price. (The proof is not completely trivial. Work out an example if you prefer.)

(h) Show that in an economy where entrepreneurs finance investment entirely out of their own wealth B , the equilibrium interest rate r' actually must be *below* the one determined in part (d). Show that, nonetheless, the fundamentals asset price $P^{*'}$ is still below the bubble-ridden price P in part (d).

(i) Returning to the diagram in part (d), show how a rise in B (think of it as an infusion of credit from the banking system) affects r and P . Explain these effects intuitively.

(j) Suppose we have an open economy and that banks are all foreign and willing to supply loans provided the expected return on the loans equals a given (world) interest rate r^w . Show that for a given capital inflow B , the values of r and P are determined as in part (d). Show that, however, B is now endogenous and is determined to equate the expected return on domestic lending (given r and P) to r^w . Prove that $r > r^w$: there is a country premium in the domestic interest rate. Let the safe technology be given by $f(x) = x^\alpha$, where $0 < \alpha < 1$. Show how a fall in the world interest rate r^w leads to a rise in B (higher capital inflows), a fall in r , and a rise in P .

*Problem inspired by F. Allen and D. Gale, "Bubbles and Crises," *Economic Journal* 110 (January 2000): 236-55.