UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2010
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Problem Set 5
Due in Vico Vanasco's mailbox (in the fifth floor hallway), Friday, November 12, 5:00

1. Romer, Problem 7.14.

**2. Habit formation and serial correlation in consumption growth.** Suppose the utility of the representative consumer, individual i, is $\sum_{t=1}^{T} \frac{1}{(1+\rho)^t} \frac{1}{1-\theta} \left(\frac{C_{it}}{Z_{it}}\right)^{1-\theta}$, $\rho > 0$, $\theta > 0$, where $Z_{it}$ is the "reference" level of consumption. Assume the interest rate is constant at some level, r, and that there is no uncertainty.

a. **External habits.** Suppose $Z_{it} = C_{t-1}^{\phi}, 0 \leq \phi \leq 1$. Thus the reference level of consumption is determined by aggregate consumption, which individual i takes as given.

i. Find the Euler equation for the experiment of reducing $C_{it}$ by dC and increasing $C_{i,t+1}$ by $(1 + r)$dC. Express $C_{i,t+1}/C_{i,t}$ in terms of $C_t/C_{t-1}$ and $(1 + r)/(1 + \rho)$.

ii. In equilibrium, the consumption of the representative consumer must equal aggregate consumption: $C_{it} = C_t$ for all t. Use this fact to express current consumption growth, $\ln C_{t+1} - \ln C_t$, in terms of lagged consumption growth, $\ln C_t - \ln C_{t-1}$, and anything else that is relevant. If $\phi > 0$ and $\theta = 1$, does habit formation affect the behavior of consumption? What if $\phi > 0$ and $\theta > 1$? Explain your results intuitively.

b. **Internal habits.** Suppose $Z_t = C_{i,t-1}$. Thus the reference level of consumption is determined by the individual's own level of past consumption (and the parameter $\phi$ is fixed at 1).

i. Find the Euler equation for the experiment considered in part (a)(i). (Note that $C_{it}$ affects utility in periods t and t + 1, and $C_{i,t+1}$ affects utility in t + 1 and t + 2.)

ii. Let $g_t \equiv (C_t/C_{t-1}) - 1$ denote consumption growth from t - 1 to t. Assume that $\rho = r = 0$ and that consumption growth is close to zero (so that we can approximate expressions of the form $(C_t/C_{t-1})^{\gamma}$ with $1 + \gamma g_t$, and can ignore interaction terms). Using your results in (i), find an approximate expression for $g_{t+2} - g_{t+1}$ in terms of $g_{t+1} - g_t$ and anything else that is relevant. Explain your result intuitively.

(OVER)

3. Romer, Problem 8.5.

4. In the q-theory model where the initial value of K exceeds its long-run equilibrium value, as the economy moves toward the long-run equilibrium:

A. The $\dot{q} = 0$ locus is shifting to the right and the $\dot{K} = 0$ locus is shifting down.
B. The $\dot{q} = 0$ locus is shifting to the right and the $\dot{K} = 0$ locus is not shifting.
C. The $\dot{K} = 0$ locus is shifting down and the $\dot{q} = 0$ locus is not shifting.
D. None of the above.

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

5. Romer, Problem 7.13.

6. Consider the continuous-time consumption problem discussed in lecture: an individual lives from 0 to T; has initial wealth A(0); and a path of labor income given by Y(t). The path of the instantaneous interest rate is given by r(t). There is no uncertainty.

Suppose the individual's instantaneous utility function is logarithmic. That is, lifetime utility is $\int_{t=0}^{T} e^{-\delta t} \ln[C(t)]\, dt.$ Derive an expression for C(t) as a function of things the individual takes as given.

7. Consider the set-up in Problem 6. Suppose the instantaneous interest rate is constant and equal to r, and that the instantaneous utility function, instead of being logarithmic, takes the constant-relative-risk-aversion form, $u(C) = C(t)^{1-\theta}/(1-\theta)$, $\theta > 0$. Derive an expression for C(t) as a function of things the individual takes as given.

8. Consider the q-theory model where K is converging to its long-run equilibrium level from below. Over time, K is rising, and:

A. q is falling, and investment is positive but falling.
B. q is falling, and investment is positive but can be sometimes rising and sometimes falling.
C. q is falling, and investment can be sometimes positive and sometimes negative.
D. q can be sometimes rising and sometimes falling.