UNIVERSITY OF CALIFORNIA
DEPARTMENT OF ECONOMICS

Economics 202A
Fall 2011
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Problem Set 8
Due in lecture, Thursday, December 1

1. Consider a consumer who lives for two periods. The consumer's lifetime utility is $U = u(c_1) + u(c_2)$, $u'(\cdot) > 0$, $u''(\cdot) < 0$. The consumer's period-1 labor income, Y_1, is certain, and is equal to $\bar{Y}$. The consumer's period-2 labor income, Y_2, is uncertain, with mean $\bar{Y}$ and variance σ_Y^2. The consumer's initial wealth is zero.

a. Assume the consumer can borrow and lend at an interest rate of $r = 0$, and that there are no other financial assets. What is the first-order condition for c_1?

For parts (b)-(d), assume that in addition to the safe asset with a real return of zero, there is a second, risky asset whose return has a mean $\bar{r} > 0$ and variance $\sigma_r^2 > 0$. The payoff to the risky asset is uncorrelated with Y_2.

b. Without using any math, explain in a sentence or two whether the consumer will purchase a strictly positive amount, a strictly negative amount, or none of this asset.

c. Without using any math, explain in a sentence or two why the following argument is wrong: "Because the return on the risky asset is uncorrelated with Y_2 and has a mean that is greater than that on the riskless asset, the consumer would like to borrow as much as possible at the riskless interest rate and purchase as much as possible of the risky asset."

d. Set up the consumer's maximization problem, and find the first-order conditions.

2. (The risk-free rate puzzle.) There is considerable evidence that individuals are quite impatient and quite risk averse. In light of this, consider the standard Euler equation relating consumption in periods t and t+1 under certainty: $U'(C_t) = [(1 + r)/(1 + \rho)]U'(C_{t+1})$. Suppose that ρ is 5 percent, the coefficient of relative risk aversion is 4, and that the growth rate of consumption is 1.5 percent. What must r be for consumers to be satisfying their Euler equation?

3. Romer, Problem 8.10.

(OVER)

4. Consider an asset that has potentially stochastic payoffs in multiple periods, Z_1, Z_2, …. Suppose that its price is given by:

$$P_t = E_t\left[\sum_{k=1}^{\infty} \frac{1}{(1+\rho)^k} \frac{U'(C_{t+k})}{U'(C_t)} Z_{t+k}\right] \qquad \text{for all t.}$$

Show that this implies

$$P_t = E_t\left[\frac{1}{1+\rho} \frac{U'(C_{t+1})}{U'(C_t)} (Z_{t+1} + P_{t+1})\right].$$

(Note: As always, we assume that C_t is known as of time t.)

EXTRA PROBLEMS (NOT TO BE HANDED IN/ONLY SKETCHES OF ANSWERS WILL BE PROVIDED)

5. Romer, Problem 9.14.

6. If the expected return on Asset A exceeds the expected return on Asset B:

A. Financial markets cannot be in equilibrium.

B. Financial markets can be in equilibrium, but only if there are noise traders.

C. A rational investor would choose to hold Asset B only if the covariance of Asset B's return with the marginal utility of consumption is greater than the covariance of Asset A's return with the marginal utility of consumption.

D. A rational investor would choose to hold Asset B only if the covariance of Asset B's return with the marginal utility of consumption is less than the covariance of Asset A's return with the marginal utility of consumption.

7. Consider the model of investment under asymmetric information in Section 9.9 of Romer. Suppose that initially the entrepreneur is undertaking the project, and that $(1 + r)(1 - W)$ is strictly less than R^{MAX}. Describe how each of the following affects D:

a. A small increase in W.

b. A small increase in r.

c. A small increase in c.

d. Instead of being distributed uniformly on $[0,2\gamma]$, the output of the project is distributed uniformly on $[\gamma - b,\gamma + b]$, and there is a small increase in b.

e. Instead of being distributed uniformly on $[0,2\gamma]$, the output of the project is distributed uniformly on $[b,2\gamma + b]$, and there is a small increase in b.