# Economics 202A Lecture Outline \#6 (version 1.1) <br> Maurice Obstfeld 

## The Open Economy and the Current Account

Let's define (all in real terms)

$$
\begin{aligned}
Y & =G D P \\
C & =\text { consumption } \\
I & =\text { investment } \\
G & =\text { government purchases } \\
N X & \equiv X-M=\text { net exports } \\
F & =\text { net foreign income earnings }(+ \text { transfers }) .
\end{aligned}
$$

The flow $F$ is basically the interest, dividend, and profit income we earn on our gross foreign assets less the income foreigners earn on their gross assets located in our country.

The national income identity states that

$$
Y=C+I+G+N X
$$

The determination of $N X$ is complicated in reality. In simple models, $N X$ is a function of (among other variables) the real exchange rate, $E P^{*} / P$, where the nominal exchange rate $E$ is the price of foreign currency in terms of home currency, so that a rise in $E$ is a relative depreciation of home currency. (In general $N X$ also depends on domestic spending, with more spending raising imports.) In many models $N X$ is the aggregate demand component that is sensitive to the real exchange rate, with

$$
\frac{\partial N X}{\partial\left(E P^{*} / P\right)}>0
$$

Define the current account balance as

$$
C A=N X+F
$$



From the identity $Y=C+I+G+N X$, we derive

$$
\text { national income }=Y+F=C+I+G+C A .
$$

National saving is defined as

$$
S=Y+F-C-G
$$

Then we can see that

$$
C A=Y+F-(C+I+G)=S-I
$$

The current account equals saving less investment. Savings in excess of domestic investment needs are invested abroad. Investment needs in excess of the home supply of savings must be borrowed from foreigners.

Relation to current events: U.S. saving has been very low. Thus, the U.S. ran historically high external deficits in the 2000s. Thanks to the subprime crisis, private saving has been rising and investment falling, cutting the current account deficit of the U.S. in half.

## Real Interest Rates

So far we have not talked about interest rates. But the United States is part of a global capital market - its current account deficit, which represents its net demand for foreign savings, must be matched, in equilibrium, by the supply of savings by non-U.S. countries. That means that if the U.S. is runing a big external deficit, the rest of the world must be running corresponding external surpluses. Real interest rates serve to bring about the global equilibrium.

The first point to make is that even under the nominal interest parity condition that would hold with risk-neutral investors (it follows from perfect risk-neutral arbitrage in international financial markets), national real interest rates need not be equal. Let a hat over a variable denote an expected percentage change. Then (nominal) interest parity states that

$$
i=i^{*}+\widehat{E}
$$

The domestic and foreign real rates of interest are

$$
\begin{aligned}
r & =i-\widehat{P} \\
r^{*} & =i^{*}-\widehat{P}^{*}
\end{aligned}
$$



So we can rewrite the nominal interest parity formula as

$$
\begin{aligned}
r & =i^{*}+\widehat{E}-\widehat{P} \\
& =r^{*}+\widehat{E}+\widehat{P}^{*}-\widehat{P} \\
& =r^{*}+E \widehat{P^{*}} / P .
\end{aligned}
$$

In words, the home real interest rate equals the foreign rate plus the expected real depreciation rate of home against foreign currency.

Notwithstanding this refinement, I will, in what follows, talk about "the" world real interest rate. You can think of this as a sort of average world rate to which individual country rates are moving. Because the following discussion is fairly long run, this is hopefully a permissible simplification. Some actual real interest rates (long-term, from inflation indexed government debt) are plotted below.

## Global Imbalances

The next picture below shows the evolution of global imbalances in recent years; it is striking.

There has been a lot of debate about the forces behind this picture. A very influential viewpoint holds that a global glut of saving starting in the late 1990s pushed world real interest rates down. Partially as a result, the U.S. current account deficit grew. For an exceptionally clear exposition see Ben Bernanke, "The Global Saving Glut and the U.S. Current Account Deficit," March 10, 2005, at:
http://www.federalreserve.gov/boarddocs/speeches/2005/200503102/default.htm Chairman Bernanke updated his views in another speech, "Global Imbalances: Recent Developments and Prospects," September 11, 2007, at:
http://www.federalreserve.gov/newsevents/speech/bernanke20070911a.htm Both articles are well worth reading and pondering.
To understand the theoretical framework they use, we need to consider a model of global intertemporal equilibrium. Within that context, we can also begin to understand aspects of modern consumption theory.



## Saving, Investment, and Global Equilibrium

The basic material here comes from Obstfeld and Rogoff, Foundations, section 1.3. They provide more detail than I can in lectures.

## Endowment Economy

Start with a pure endowment economy - i.e., there is no investment and two countries. There are two dates, $t=1,2$. There is only one good on each date - so we do not worry about real exchange rates, terms of trade, etc. On each date goods-market equilirbium means that

$$
C_{t}+C_{t}^{*}=Y_{t}+Y_{t}^{*} .
$$

Alternatively, but equivalently,

$$
S_{1}+S_{1}^{*}=\left(Y_{1}-C_{1}\right)+\left(Y_{1}^{*}-C_{1}^{*}\right)=C A_{1}+C A_{1}^{*}=0 .
$$

Suppose the home country borrows the (possibly negative) amount $B_{1}$ from abroad on date 1. Then $S_{2}=Y_{2}-r B_{1}-C_{2}=C A_{2}, S_{2}^{*}=Y_{2}^{*}+r B_{1}-C_{2}^{*}=$ $C A_{2}^{*}$, and once again, equilibrium in the goods markets means that

$$
S_{2}+S_{2}^{*}=\left(Y_{2}-C_{2}\right)+\left(Y_{2}^{*}-C_{2}^{*}\right)=C A_{2}+C A_{2}^{*}=0 .
$$

The intertemporal allocation problem of a representative domestic individual individual is to find consumption levels $C_{1}$ and $C_{2}$ that solve:

$$
\max U\left(C_{1}, C_{2}\right)
$$

subject to

$$
\begin{equation*}
C_{1}+\frac{C_{2}}{1+r}=Y_{1}+\frac{Y_{2}}{1+r}, \tag{1}
\end{equation*}
$$

where $r$ is the real rate of interest. Interpretation of this constraint: If my income is $Y_{1}$ and consumption $C_{1}$, I must borrow an amount (possibly negative) on date 1,

$$
B_{1}=C_{1}-Y_{1} .
$$

On date 2 I must repay the principal plus interest, $(1+r) B_{1}$, so my consumption has to be

$$
C_{2}=Y_{2}-(1+r) B_{1} .
$$

If I substitute $C_{1}-Y_{1}$ for $B_{1}$ above, I get the intertemporal constraint in (1). I will assume the (correct) answer that the budget constraint holds with equality, so as to avoid Kuhn-Tucker inequalities.

Let us assume, in line with the macro consumption literature, that $U\left(C_{1}, C_{2}\right)=$ $u\left(C_{1}\right)+\beta u\left(C_{2}\right), 0<\beta<1$. The Lagrangian is

$$
u\left(C_{1}\right)+\beta u\left(C_{2}\right)-\lambda\left[C_{1}+\frac{C_{2}}{1+r}-\left(Y_{1}+\frac{Y_{2}}{1+r}\right)\right]
$$

with associated first-order conditions

$$
\begin{aligned}
u^{\prime}\left(C_{1}\right) & =\lambda, \\
u^{\prime}\left(C_{2}\right) & =\frac{\lambda}{(1+r) \beta} .
\end{aligned}
$$

If we elimate $\lambda$ from these, we get an expression known as the (deterministic) consumption Euler equation,

$$
u^{\prime}\left(C_{1}\right)=(1+r) \beta u^{\prime}\left(C_{2}\right) .
$$

the Euler equation and the budget constraint give two equations in two unknowns ( $C_{1}$ and $C_{2}$ ).

Interpretation: At an optimum, the consumer cannot gain from reducing $C_{1}$ by $\varepsilon$ and raising $C_{2}$ by the additional consumption $(1+r) \varepsilon$ that this change would permit. For $\beta=(1+r)^{-1}$, which means that the consumer's subjective discount rate equals the market discount rate, this indifference occurs when $C_{1}=C_{2}$. This is the basic idea of "consumption smoothing" theories of consumption.

The idea of smoothing consumption over a finite lifetime of variable income leads to the "life-cycle" theory of Modigliani and Brumberg. We will also look later look at how consumption smoothing works under uncertainty, which is closely related both to Milton Friedman's "permanent income" hypothesis and Bob Hall's "random walk" hypothesis.

When $C_{1}=C_{2}=\bar{C}$, we can solve for consumption from the budget constraint $C_{1}+\frac{C_{2}}{1+r}=Y_{1}+\frac{Y_{2}}{1+r}$ to get

$$
\bar{C}=\frac{Y_{1}+\beta Y_{2}}{1+\beta}
$$

## Saving and the Real Interest Rate

If we use the last expression, we can only tell the effect of a change in $\beta$ on consumption if we assume that $r$ changes at the same time to maintain the equality $\beta=(1+r)^{-1}$. In general, however, there is reason when $\beta$ has to equal $\frac{1}{1+r}$. So it is useful to solve for $C_{1}$ and $C_{2}$ under more general assumptions.

For that purpose we need a specific functional form for the utility function $u(C)$. A very convenient one is the form

$$
\begin{equation*}
u(C)=\frac{C^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}} \tag{2}
\end{equation*}
$$

called the isoelastic utility function. You can show that for this form, $\lim _{\sigma \rightarrow 1} u(C)=$ $\log (C)$ (meaning the base $e$ or natural logarithm).

The parameter $\sigma$ is known as the intertemporal elasticity of substitution. Notice that the Euler equation for the isoelastic case is $C_{1}^{-1 / \sigma}=$ $(1+r) \beta C_{2}^{-1 / \sigma}$. So the intrepretation of $\sigma$ comes from the equality

$$
\mathrm{d} \log \left(\frac{C_{1}}{C_{2}}\right)=-\sigma \mathrm{d} \log (1+r)
$$

where $1+r$ is interpreted as the relative price of date 1 consumption in terms of date 2 consumption. The higher is $\sigma$, the easier it is to substitute between periods [think of $u(C)$ as more nearly linear], so the greater is the relative consumption reponse to a change in relative price.

If we solve for consumption explicitly using function (2), the answer is

$$
C_{1}=\frac{1}{1+(1+r)^{\sigma-1} \beta^{\sigma}}\left(Y_{1}+\frac{Y_{2}}{1+r}\right) .
$$

(Solve for $C_{2}$ as an exercise.)
To get back to the current account: how does a rise in the interest rate affect saving $S_{1}=Y_{1}-C_{1}$ ? Interestingly (and importantly), it is not automatically true that a rise in the real interest rate $r$ depresses consumption today, thereby raising saving in the present. Instead, the total response reflects the interplay of three distinct effects:

1. Substitution effect. A rise in $r$ raises the return to saving, generating more of it. Alternatively, a rise in $r$ makes future consumption cheaper
to obtain in terms of consumption forgone today. So the substitution effect of a rise in the real interest rate is to reduce $C_{1}$ and raise $C_{2}$ with a response mediated by $\sigma$. The power $\sigma$ attached to $1+r$ in the last expression for $C_{1}$ reflects this effect.
2. Income effect. Substitution is not the entire story. If you are a saver, and the interest rate rises, you could, if you wished to, reduce saving but still have higher consumption in both periods. This possibility is an outward shift in the entire intertemporal budget line, an income effect. Other things equal, it leads to a rise in $C_{1}$, and hence, a fall in saving. The term $(1+r)^{\sigma-1}$ in the denominator of the last expression shows the tension between the substitution and the income effects. For the $\log$ case they cancel each other exactly, but for $\sigma>1$, the substitution effect dominates. $\sigma=1$ is probably the upper bound for most empirical estimates of $\sigma$, implying that, in practice, the income effect may dominate. This does not mean that a rise in the interest rate lowers saving, however, because there is a third effect.
3. Wealth effect. A rise in $r$ lowers discounted lifetime income $Y_{1}+Y_{2} /(1+$ $r$ ), which in this model is the same as wealth. That change reinforces the substitution effect, likewise promoting higher saving.

## Global Equilibrium for the Endowment Economy

The next step is to think about two economies that can borrow and lend with each other, and that possibly differ in their intertemporal patterns of income. For example, one country might have low income today and high income tomorrow (an emerging market), while the other has a more balanced intertemporal income pattern (an already industrialized economy), or even falling income (an aging economy). How will the global equilibrium interest rate and pattern of current account imbalances look?

For each of the two economies in the world, we can define the autarky rate of interest. It is the rate of interest that would prevail in the loan market if, counterfactually, the economy were completely closed off from the rest of the world. Since we have no investment, each economy would, in autarky, simply consume its endowments, and the equilibrium interest rate would adjust so that people chose this consumption pattern voluntarily. Thus, for the home country the autarky rate of interest, $r^{A}$, is defined by

$$
u^{\prime}\left(Y_{1}\right)=\left(1+r^{A}\right) \beta u^{\prime}\left(Y_{2}\right)
$$

or, equivalently, by

$$
r^{A}=\frac{u^{\prime}\left(Y_{1}\right)}{\beta u^{\prime}\left(Y_{2}\right)}-1
$$

Notice that if people are more impatient ( $\beta$ is lower), $r^{A}$ is higher. If $Y_{2}$ is higher, then because $u^{\prime \prime}(C)<0$ (diminishing marginal utility), $r^{A}$ is higher. Intuitively, if people think they will have more income tomorrow, they will attempt to borrow against their future income so as to consume more today. In an endowment economy closed to the rest of the world, this is not technologically feasible, so the interest rate is driven up until people are happy to simply consume $Y_{1}$. For the foreign country, naturally, $r^{A *}=\frac{u^{\prime}\left(Y_{1}^{*}\right)}{\beta^{*} u^{\prime}\left(Y_{2}^{*}\right)}-1$.

To be very concrete, let us assume that the autarky interest rates have the alignment

$$
r^{A}<r^{A *}
$$

This could be because foreigners are comparatively impatient $\left(\beta^{*}<\beta\right)$ or because the foreign country is growing rapidly compared to the home country (perhaps the foreign country is emergent, like China). In that case, which country will run a current account deficit in the first period, and which will run a surplus? Since the equilibrium world interest rate will be between the two autarky rates, it is easy to see that in period 1 , the foreign country will borrow and the home country will lend.

For example (do this as an exercise), for the isoelastic utility function, and with $\beta$ and $\sigma$ identical in the two countries, the equilibrium world interest rate $r$ is:

$$
1+r=\left[\frac{Y_{1}}{Y_{1}+Y_{1}^{*}}\left(1+r^{A}\right)^{\sigma}+\frac{Y_{1}^{*}}{Y_{1}+Y_{1}^{*}}\left(1+r^{A *}\right)^{\sigma}\right]^{\frac{1}{\sigma}}
$$

So $1+r$ (the price of present consumption in terms of future consumption; also the gross rate of interest) is a CES aggregator of the autarky prices, with weights representing the respective importance of the two countries in the period 1 output market.

For the home country, say, the saving function is

$$
S_{1}=Y_{1}-C_{1}=Y_{1}-\frac{1}{1+(1+r)^{\sigma-1} \beta^{\sigma}}\left(Y_{1}+\frac{Y_{2}}{1+r}\right)
$$

using. Differentiating, one can show that

$$
\frac{\mathrm{d} S_{1}}{\mathrm{~d} r}=\frac{\sigma C_{2} /(1+r)-S_{1}}{1+r+\left(C_{2} / C_{1}\right)} .
$$

(Exercise; or see Obstfeld-Rogoff, p. 29.) This derivative is unambiguously positive if $\sigma \geq 1$ [because $Y_{2} \geq 0$, the intertemporal budget constraint implies that $\left.C_{2} \geq(1+r) S_{1}\right]$. But if $\sigma<1$, it can be negative, when income and wealth effects dominate substitution effects.

Also, for the derivative to be negative, we obviously need $S_{1}>0$. This terms represents the net effect of the income effect (which is proportional to $C_{2}$, according to the Slutsky equation) and the wealth effect - for internationaltrade enthusiasts, it represents the net effect of the intertemporal terms of trade on consumption opportunities. For a saving (current-account surplus) country, a rise in the interest rate expands dynamic consumption possibilities.

## Adding Investment

Let us know assume that for period 2,

$$
Y=A F(K), \quad Y^{*}=A^{*} F^{*}\left(K^{*}\right)
$$

with the production functions strictly concave. Here, $K$ and $K^{*}$ are accumulated through investment in the first period. It is easy to show that in this economy with no capital installation costs, $K=I$ and investment is determined by the familiar condition

$$
F^{\prime}(K)=r .
$$

(Similarly for the foreign country.) So investment is a declining function of $r$.

Taking account of the fact that now, $C A_{1}=S_{1}-I_{1}=I_{1}^{*}-S_{1}^{*}=-C A_{1}^{*}$, we get the picture of a global equilibrium that follows.

We can now analyze global equilibrium as in terms of this diagram. For the home country, with the lower autarky interest rate, the availability of a lending opportunity leads it to cut consumption today and send resources abroad. It runs a current-account surplus in period 1 (and it consumes the rewards in period 2 , running a deficit). The foreign country, on the other hand, would have liked to consume more in autarky, but its residents were deterred from borrowing by the very high autarky interest rate. When trade becomes available, the autarky interest rate falls to the world level and residents indeed do borrow, running an external deficit in period 1. They must run a corresponding surplus in period 2 to repay their debt. The figure shows how the world equilibrium interest rate is determined by the equality of the supply of savings and the demand for savins. Clearly, as per the Bernanke
speech, an outward shift in the savings schedule, for either country, lowers the world real rate of interest. Bernanke's point is that precautionary factors, plus high commodity prices, may have led to such shifts for the developing countries.

Also, however, an inward shift of the investment function (as happened in East Asia after the late-1990s financial crisis) will lower the real interest rate throughout the world. It is also likely to reduce global saving.


