

# International Risk Sharing, Costs of Trade, and Portfolio Choice: Some Recent Developments

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## ***Plan of these lectures***

1. Documentation of the explosion in international asset trade.
2. Home biases in currencies and equities, asset-price movements, and the international adjustment process. Relation to Feldstein-Horioka.
3. Need for a general-equilibrium portfolio-balance model.
4. Basics of risk-sharing: complete asset markets, trade costs, and the Backus-Smith condition.
5. Trade costs in a model of risk sharing: Coeurdacier's local approach.

6. Comparison with the analysis of "Six puzzles": Can small welfare gains help to explain portfolio volatility?
7. Nontraded goods and nontraded-industry equities: Home bias reinstated? Relation to the local approach of Baxter, Jermann, and King.
8. Sticky prices, monetary shocks, home equity bias, and home currency preference.
9. A general framework with multiple shocks.
10. Beyond complete markets: The consumption-real exchange rate anomaly, its causes and its implications.

# **1. The explosion in international asset trade**

Since the early 1990s, an unparalleled expansion in private international asset trade has taken place. It has largely been a developed-country phenomenon, but developing countries, especially the more open “emerging markets,” also have participated.

The characteristics of this phenomenon call out for explanation, as there are critical implications for the nature of international risk sharing and adjustment.

The reasons behind this development are not the main topic of my lectures, though at a basic level, it can be attributed to four main factors:

1. Technology, including advances in financial engineering.
2. Policy competition among governments, including the spread of investor-friendly institutions.
3. Domestic politics.
4. Ideology and advances in economic knowledge.

See Obstfeld and Taylor (2004) for an account of international financial deregulation that focuses on the desire for monetary autonomy in democratic societies.

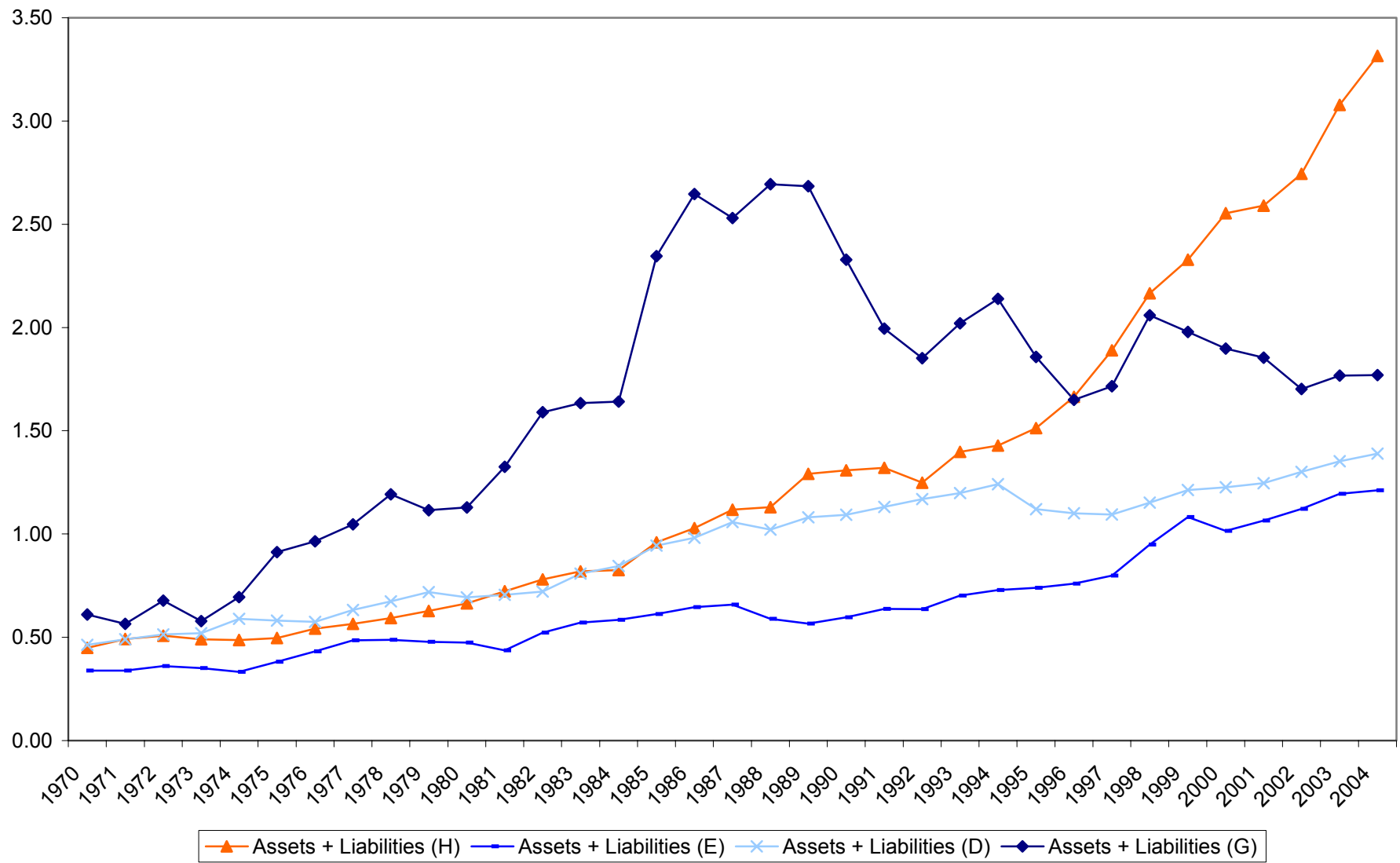
## Basic trends

Important data assembled by Lane and Milesi-Ferretti (2006) show that levels of gross external assets and liabilities have risen sharply relative to GDP in recent years.

The figure shows the extent of asset trade, measured as  $\frac{A + L}{GDP}$ , for the following country groupings:

- High-income countries ( $H$ )
- Emerging market countries ( $E$ )
- Developing countries ( $D$ ) — generally poorer and less open financially than  $E$ , but including some oil exporters with positive reported net foreign assets, such as Brunei and Venezuela
- Persian gulf oil exporters ( $G$ )

Assets plus liabilities, 1970-2004 (ratio to group GDP)



- For *H* countries, the volume of asset trade accelerated sharply after the early 1990s.
- Despite being relatively open financially, *E* countries still have less asset trade relative to GDP than *D* countries — reflecting both the very high liabilities of developing countries, and their much lower levels of GDP.
- If we examine assets and liabilities separately, *E* foreign assets (as a GDP share) caught up with *D* foreign assets (as a GDP share) in the late 1990s. *E* liabilities remain higher than *E* assets, but *D* liabilities are much higher (again, relative to GDP).
- The trend of both assets and liabilities is generally upward, most sharply for the *H* countries. For *D* countries as a group, though, foreign liabilities relative to GDP are essentially flat since the mid-1990s. Patterns for *G* countries largely reflect factors specific to energy markets.



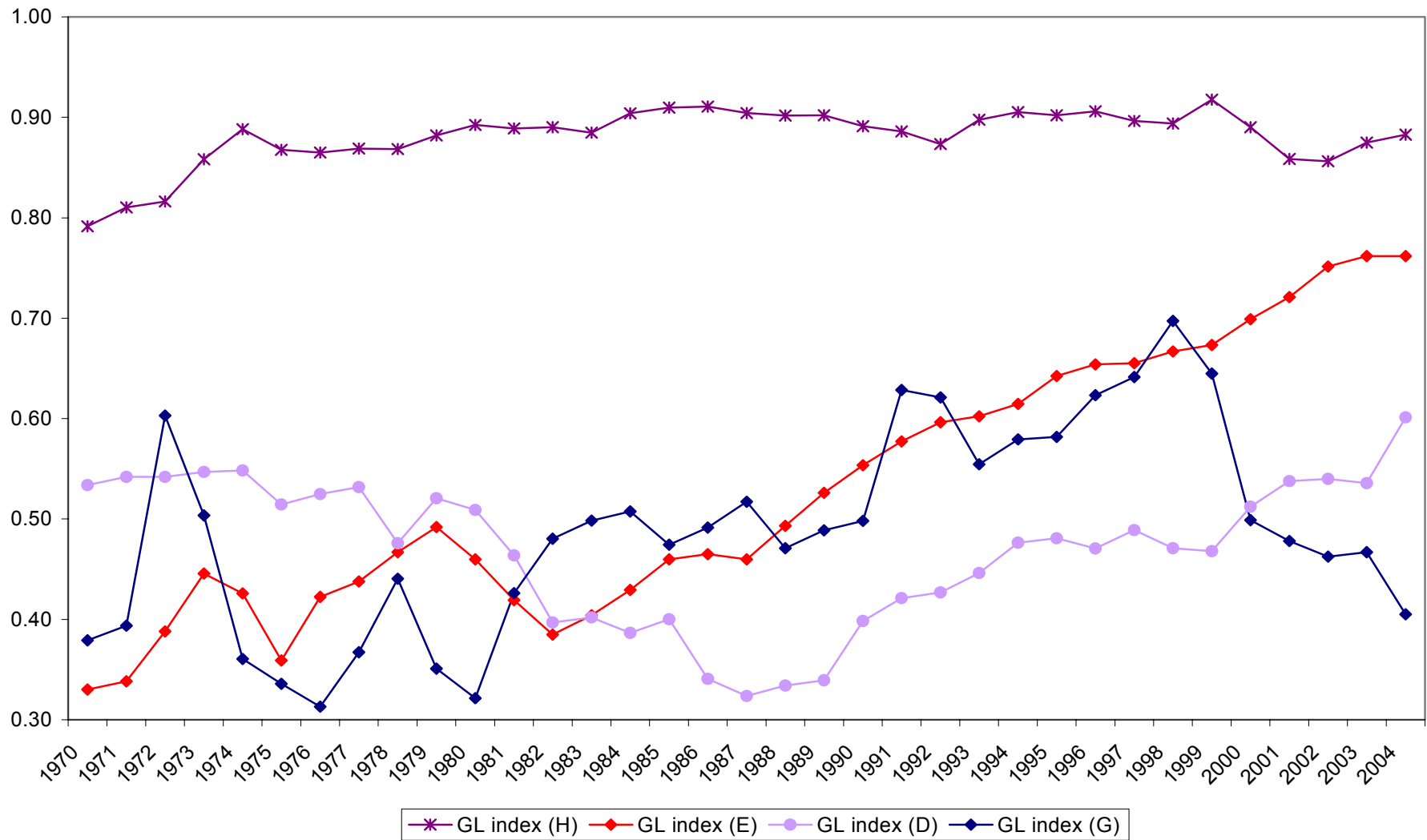
To assess changes over time in the extent of risk diversification, as contrasted with intertemporal trade, I have proposed (2004) the Grubel-Lloyd index, originally used as a measure of inter-industry trade,

$$GL \equiv 1 - \frac{|A - L|}{A + L}.$$

If  $A = L$  (no net external position, pure diversification)  $GL = 1$ ; if  $A$  or  $L = 0$  (pure one-way asset-trade),  $GL = 0$ .

- For  $H$  countries,  $GL$  is basically flat at a high level. Little intertemporal trade.
- Trend is sharply upward for  $E$ , but part of the recent increase reflects reserve accumulation.
- $D$  countries currently low, but rising from the trough reached in 1980s debt crisis due to reserve losses.

**G-L index, 1970-2004 (GDP weighted average for each group)**



## **2. Home biases and international adjustment**

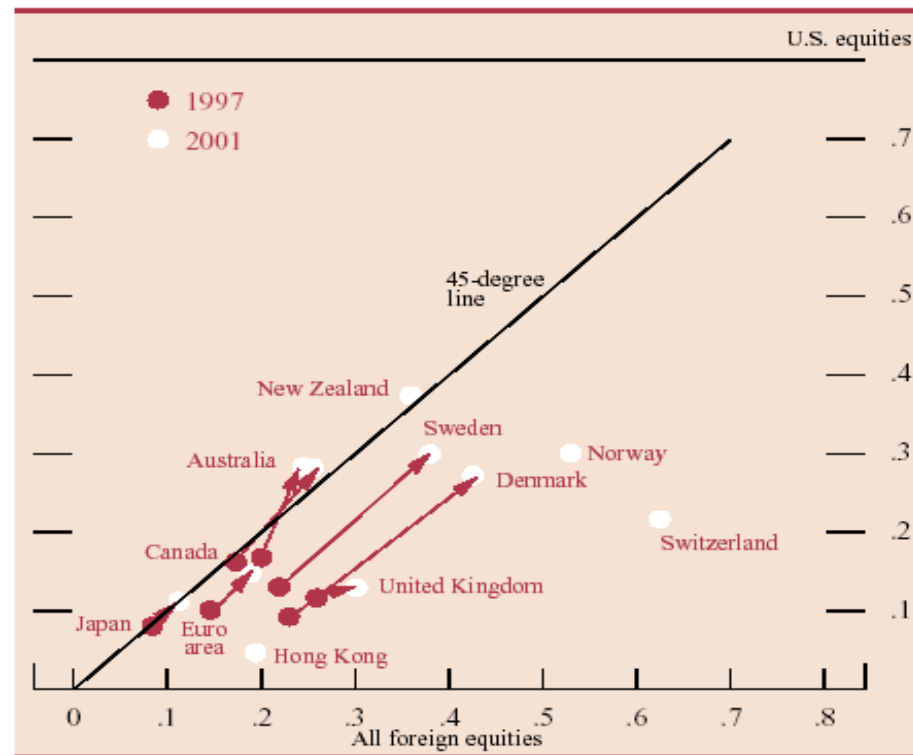
There are three types of home bias that figure prominently in discussion of the international adjustment mechanism:

1. Home bias in equity holdings.
2. Home bias in currency holdings.
3. Home bias in consumption.

The three are interrelated and help to determine the speed and nature of the adjustment mechanism. I will say little about explaining consumption home bias, which is heavily related to costs of international trade (Anderson and van Wincoop 2004). An important question, however, is the role of international (commodity) trade costs in causing the asset biases.

- Regarding home bias in *equities*, most countries' investors hold an outsize equity position in domestic shares.
- A simple ICAPM benchmark suggests that for any country  $i$ , the  $i$  equity portfolio share occupied by country  $j$  equity should equal country  $j$ 's share in the global equity market. Thus, if publicly traded U.S. companies account for 50 percent of global equity-market capitalization, all investors worldwide should hold half of their equity in the U.S.
- But global investors hold shares of foreign equities that are proportionally smaller than the respective world market shares. This is the *home bias*. The next chart shows how much less. A country at (1,1) displays no home bias (with respect to non-U.S. foreign or U.S. equities). A country at (0,0) holds only domestic shares.
- Home equity bias remains substantial, though falling over time.
- It can potentially help explain the Feldstein-Horioka regularity – see Kraay and Ventura (2000).

5. Portfolio weights of U.S. equities and of all foreign equities for selected countries, December 31, 1997 and 2001



Source: Bertaut and Grier (2004), figure 5, p. 22, based on CPIS data.

- Regarding home bias in *currencies*, nominal equity returns are heavily influenced by nominal exchange-rate movements, so to some degree, home equity bias entails home currency bias.
- But there is overwhelming home-currency bias in bond holdings, too. For the United States, as reported in Bertaut and Grier (2004), even debt claims on foreigners are mostly U.S. dollar-denominated.
- Because U.S. debt claims on U.S. residents are almost entirely in dollars, the share of U.S. bond holdings in dollars is overwhelming.

1. Distribution of U.S. holdings of foreign debt securities, by currency of denomination, December 31, 1997 and 2001

Billions of dollars except as noted

Currency	1997		2001			
	Long-term		Long-term		Short-term	
	Amount	Percent	Amount	Percent	Amount	Percent
U.S. dollar .....	315	58	334	67	123	84
Euro <sup>1</sup> .....	75	14	90	18	7	5
Yen .....	30	5	25	5	12	8
Canadian dollar .....	42	8	22	4	1	1
U.K. pound .....	26	5	16	3	3	2
Other .....	39	7	15	3	1	0
Unknown .....	20	4	*	0	*	0
<b>Total .....</b>	<b>547</b>	<b>100</b>	<b>502</b>	<b>100</b>	<b>147</b>	<b>100</b>

NOTE. Here and in the following tables, components may not sum to totals because of rounding.

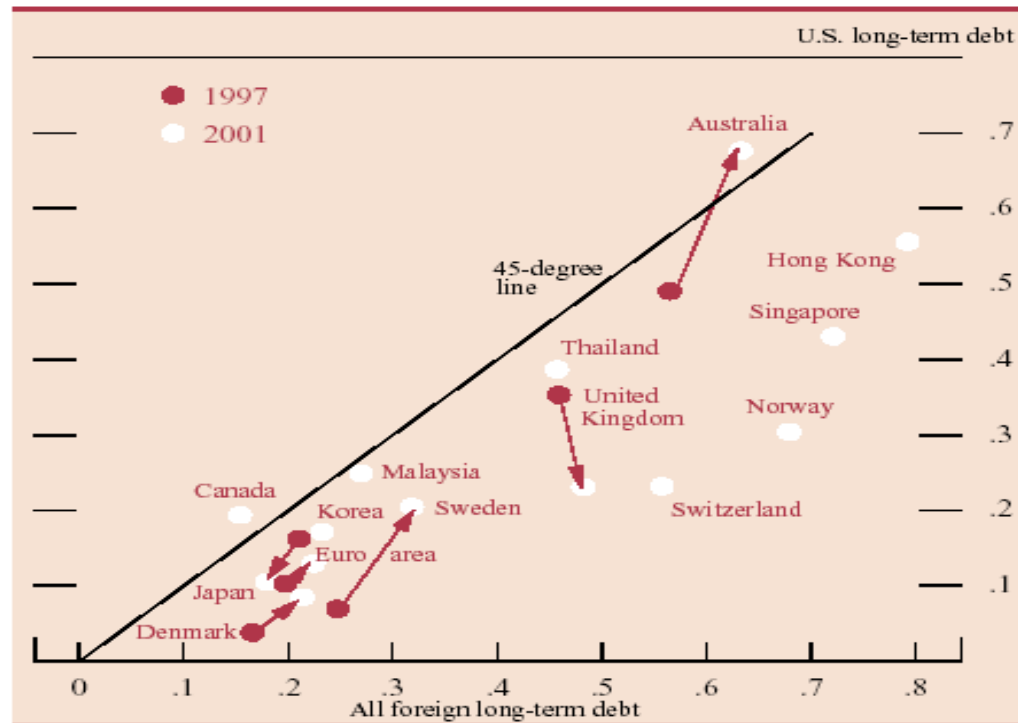
1. Amount for 1997 is denominated in the former national currencies of countries now in the euro area (for those countries, see general note to chart 4).

\* Less than \$500 million.

SOURCE. U.S. Department of the Treasury, *Report on U.S. Holdings of Foreign Securities*, Foreign Portfolio Investment Benchmark Surveys (May 2003), p. 11 ([www.treas.gov/tic/shc2001r.pdf](http://www.treas.gov/tic/shc2001r.pdf)).

- Another factor suggestive of home currency preference is the concentration of long-term bond holdings in domestic bond markets, as indicated by the next chart.

6. Portfolio weights of U.S. long-term debt and of all foreign long-term debt for selected countries, December 31, 1997 and 2001



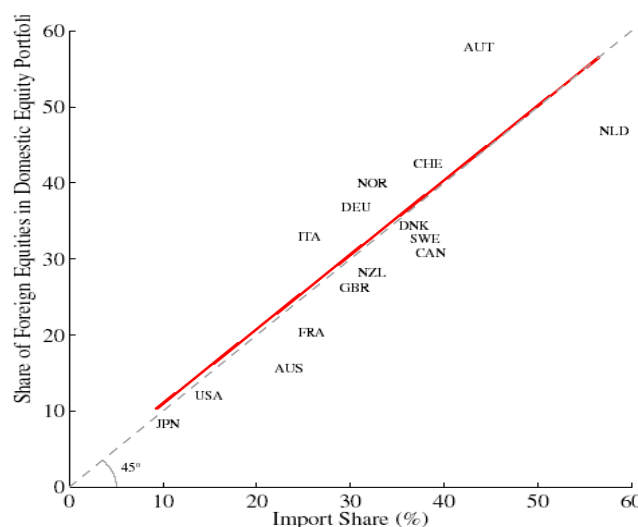
Source: Bertaut and Grier (2004), figure 6, p. 23, based on CPIS data.



# Home equity bias and trade openness

Empirically there seems to be a strong relationship between international diversification of equity holdings and trade openness. See the chart below (from Collard et al. 2007):

Figure 1: Portfolio equity assets and imports (share of GDP, average 1995–2004, developed countries)



Note: Foreign equity assets and liabilities from Lane–Milesi-Feretti, 2006. Market capitalization from the World Development Indicators, 2006. The regression line is  $PEA_i = 1.2256 + 0.9785 IM_i$  and has  $R^2 = 0.71$ , where  $PEA_i$  is the foreign equity asset measure ( $s_{ij}$ ) and  $IM_i$  is the import share (standard errors in parenthesis).

## **Asset prices in international adjustment**

Consider a country with a current-account deficit. By definition, it is reducing its net foreign wealth, or increasing its net foreign liability.

Through what natural mechanism might this deficit decline over time, as might be required for long-term national solvency? This is a classical question, going back (at least) to Hume, who stressed the expenditure changing and switching effects of the international distribution of precious metal hoards.

In this process, commodity and asset preferences play the key roles:

- As wealth is transferred to foreigners with home equity preference, their equity prices rise relative to ours. The value of our foreign equity holdings rises, while the value of foreign holdings of our equities falls. This reduces our net external debt, easing the required subsequent trade balance adjustment needed to achieve solvency. *Further*, the fall in the value of foreign-held domestic equities increases foreigners' appetite for domestic equities, other things equal. (This is the *portfolio-balance effect*.) They are thereby induced to continue providing us with finance.
- A similar process occurs in markets for domestic- and foreign-currency bonds, the effect of which is to depreciate our currency relative to foreign currencies. If we hold some foreign-currency assets and foreigners hold domestic-currency assets, there is a resulting pattern of currency-induced capital gains in our favor.

- As wealth is transferred to foreigners, the demand for domestic goods falls relative to that for foreign goods, given home consumption preference. This induces a fall in the relative price of domestic goods – a terms of trade deterioration for us. If price levels are being targeted by central banks, then a nominal depreciation of our currency effects this relative price change.
- This last “transfer effect” on the terms of trade was, of course, central to John Maynard Keynes’s position in his 1929 *Economic Journal* debate with Bertil Ohlin.

Declines in our overall wealth (due to both financial flows and capital losses on the assets in which our wealth is concentrated)  $\Rightarrow$  expenditure reduction.

Relative price change  $\Rightarrow$  expenditure switching.

Asset-price effects on net foreign wealth  $\Rightarrow$  less demanding, quicker adjustment. (Not in the case of so-called “original sin.”)

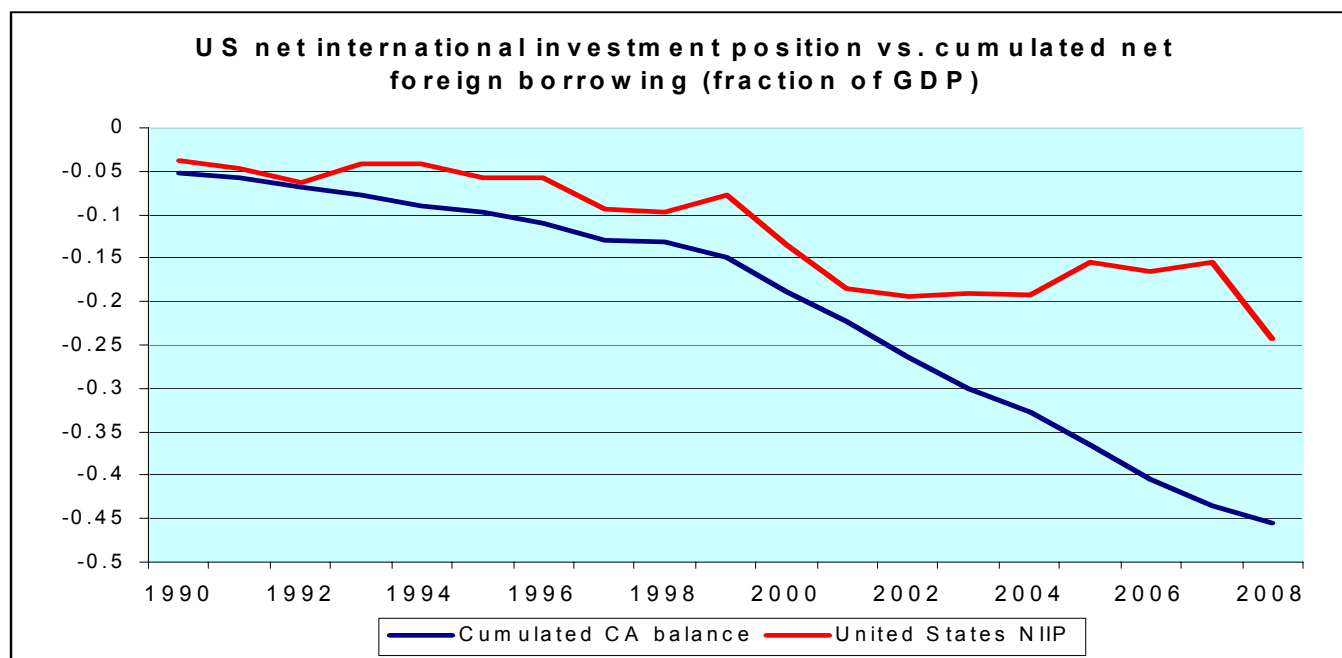
This last feature has become important recently with the growth of asset trade and is stressed by Kim (2002), Gourinchas and Rey (2007), Lane and Milesi-Ferretti (2005), and Tille (2003).

Numerical example:

- Right now, U.S. net external debt about 25% GDP.
- Gross foreign assets = 139% U.S. GDP.
- Gross foreign liabilities = 164% U.S. GDP.
- About 65% of U.S. assets in foreign currencies.
- About 95% of U.S. liabilities in dollars.
- Effect of a 1% balanced dollar depreciation:  $(0.01)(0.65)(1.39) - (0.01)(0.05)(1.64) = 0.8\%$  GDP, or about \$114 billion transfer to the United States.

For a lengthy period after 1990, valuation effects kept the U.S. net external debt moderate despite growing current account deficits.

Why? Several factors. U.S. borrows to hold foreign equity. Curcuru, Dvorak, Warnock (2009) suggest poor foreign timing in reallocation between bonds and equity.



But in 2008, U.S. lost nearly \$1 trillion due to equity crash and dollar appreciation.

Gourinchas and Rey quantify the contribution to U.S. adjustment of capital gains and losses on the net external asset position (through 2004).

They find that these are a substantial fraction of the total adjustment in recent years. In particular:

- Over 31% of stabilizing U.S. external adjustment can be expected through capital gains/losses.
- Deviations from trend in the ratio  $NX/NFA$  predicts asset returns 1 quarter to 2 years ahead and  $NX$  at longer horizons.
- Exchange-rate change is forecastable by  $NX/NFA$  out of sample, one quarter out and beyond (compare Meese-Rogoff result).
- IMF *World Economic Outlook*, April 2005: Related results for some industrial countries, but not for emerging markets.

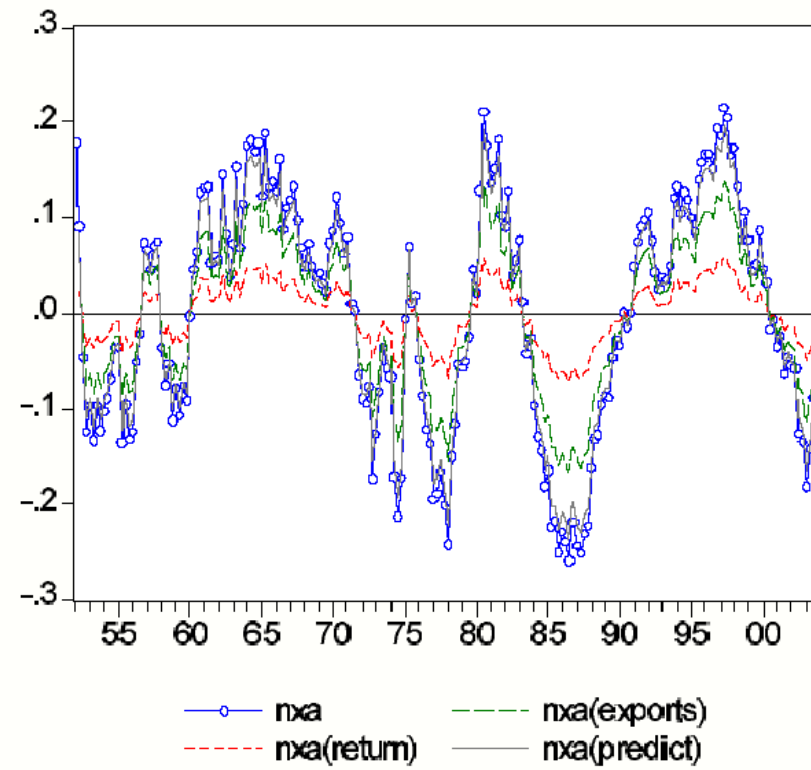
The following figure (borrowed from their paper) illustrates the Gourinchas-Rey (GR) results.

Below, the variable  $nxa$  is their constructed stationary portion of the net external imbalance.

GR show that if  $nxa < 0$ , then either returns on the net external asset position are expected to be high, or net exports are expected to rise.

They estimate a VAR in returns, change in net exports, and  $nxa$  in order to decompose calculate the theoretically predicted  $nxa$  as the sum of (i) the p.d.v. of expected returns and (ii) the p.d.v. of expected net export changes (with a discount factor of 0.95). Predicted and actual  $nxa$  are very close.





Decomposition of  $\text{nxa}(\text{predict})$  into  $\text{nxa}(\text{return})$  and  $\text{nxa}(\text{exports})$

### **3. Wanted: A general-equilibrium portfolio-balance model**

In light of these important implications of international portfolios, it is imperative to understand how investors make asset allocation decisions for different asset classes across countries and currencies.

In the 1980s, scholars including Branson, Henderson, and Kouri proposed a “portfolio-balance” approach in which (i) bonds denominated in different currencies are imperfect substitutes and (ii) there is home-currency preference in bond demand.

Partial-equilibrium accounts of microfoundations partially rationalized the approach. See, e.g., Adler and Dumas (1983), and, for a survey tightly focused on the portfolio-balance model, Branson and Henderson (1985). A key partial-equilibrium result, expounded, e.g., by Krugman (1981), suggested that home consumption preference translates into home-currency portfolio bias if and only if the coefficient of relative risk aversion,  $\rho > 1$ . See also Stulz (1983).

The portfolio-balance approach fell out of favor for a number of reasons: lack of a general-equilibrium rationale, failure in modeling risk premia as stable functions of asset stocks, weak evidence on sterilized intervention efficacy. Yet the need for such an approach has become acute as asset trade has expanded.

Among the questions of current relevance that one would like to answer:

- How elastic is the “appetite” of foreign investors for U.S.-issued assets?
- What are the implications of reserve diversification by central banks?
- Under floating rates, and notably when the policy nominal interest rate is zero, can large FX market interventions such as Japan’s “great intervention” of 2003-04 move exchange rates?
- Are the large international redistributions caused by exchange-rate changes envisioned by investors and part of an intelligible global risk-sharing mechanism?
- Do fiscal deficits have exchange-rate effects through asset markets?

There have been some recent attempts at general-equilibrium models of portfolios in a world economy. Among other notable contributions of the last few years are the following:

- 1.** Coeurdacier (2009)
- 2.** Collard, Dellas, Diba, and Stockman (2007)
- 3.** Devereux and Saito (2006)
- 4.** Devereux and Sutherland (2006)
- 5.** Engel and Matsumoto (2006, 2008)
- 6.** Evans and Hnatkovska (2005a, 2005b)
- 7.** Ghironi, Lee, and Rebucci (2006)
- 8.** Heathcote and Perri (2004)
- 9.** Kollmann (2006)
- 10.** Kumhof and Van Nieuwerburgh (2005)
- 11.** Pavlova and Rigobon (2003).
- 12.** van Wincoop and Warnock (2006).

- 13.** Tille and van Wincoop (2008).
- 14.** Coeurdacier, Kollmann, and Martin (2007)
- 15.** Coeurdacier and Gourinchas (2009)

I will discuss a few of these analyses below.

## 4. Fundamentals of international risk sharing

Let  $P$  and  $P^*$  be Home and Foreign price levels, measured in a costlessly tradable global numeraire currency.

Let  $C = \Omega(c_1, \dots, c_N)$  be the (linear homogenous) domestic consumption index,  $C^*$  the corresponding index for Foreign. Denote period utility by  $u(C)$ , or  $u^*(C^*)$  for Foreign.

Suppose there is an integrated global market in Arrow-Debreu contingent securities, payable in the numeraire currency. In particular, there are no costs of *asset* trade. Invoking the inter-temporal Euler conditions for the Arrow-Debreu securities, one finds that for every state of nature on date  $t + 1$ ,

$$\frac{u'(C_{t+1})/P_{t+1}}{u'(C_t)/P_t} = \frac{u^{*'}(C_{t+1}^*)/P_{t+1}^*}{u^{*'}(C_t^*)/P_t^*}.$$

This is the risk sharing condition proposed and tested by Backus and Smith (1993), and it holds even in the presence of various costs in goods markets that can drive wedges between national price levels.

Under purchasing power parity (PPP),  $P = P^*$  always and therefore the preceding condition reduces to

$$\frac{u'(C_{t+1})}{u'(C_t)} = \frac{u^{*'}(C_{t+1}^*)}{u^{*'}(C_t^*)}.$$

For two ex ante symmetrical countries, the Backus-Smith condition takes the form

$$\frac{u'(C_t)}{P_t} = \frac{u'(C_t^*)}{P_t^*},$$

which holds in every state of nature, and which I use liberally below.



Even without complete markets, but with costless trade in bonds, the Backus-Smith condition holds in expectation,

$$\mathbb{E}_t \left\{ \frac{u'(C_{t+1})/P_{t+1}}{u'(C_t)/P_t} \right\} = \mathbb{E}_t \left\{ \frac{u^{*'}(C_{t+1}^*)/P_{t+1}^*}{u^{*'}(C_t^*)/P_t^*} \right\}.$$

Using data for the U.S., Japan, and Germany, I found limited empirical support for this condition (in my 1989 paper). In general, intermediate degrees of market incompleteness imply similar expectational equalities: the appropriate condition is

$$\mathbb{E} \left\{ \frac{u'(C_{t+1})/P_{t+1}}{u'(C_t)/P_t} \middle| I_{t+1} \right\} = \mathbb{E} \left\{ \frac{u^{*'}(C_{t+1}^*)/P_{t+1}^*}{u^{*'}(C_t^*)/P_t^*} \middle| I_{t+1} \right\},$$

where the conditioning information in  $I_{t+1}$  reflects date- $t + 1$  events upon which date- $t$  contracts may be written (Obstfeld 1994).

The empirical support for fully complete markets is exceedingly weak, as I discuss further below.

For isoelastic  $u(C)$  with coefficient of relative risk aversion  $\rho$ , the Backus-Smith condition is (using Jonesian "hats" for % changes):

$$\widehat{C} - \widehat{C}^* = -\frac{1}{\rho} \left( \widehat{P} - \widehat{P}^* \right).$$

An equivalent expression is sometimes a bit more intuitive:

$$\widehat{PC} - \widehat{P^*C^*} = \left( 1 - \frac{1}{\rho} \right) \left( \widehat{P} - \widehat{P}^* \right).$$

According to this latter rendition, when  $\rho > 1$  — the empirically reasonable case — total consumption spending should rise in the country experiencing the real currency appreciation (the one with faster CPI inflation comparing in a common numeraire).

How might valuation effects of the type discussed earlier help to implement efficient risk sharing?

- In general the answer is model-specific.
- If a country, as does the U.S., borrows mostly in domestic currency and holds mostly foreign currency abroad, it will receive unexpected net payments from abroad when its currency unexpectedly depreciates in nominal terms. To enhance risk sharing (for  $\rho > 1$ ), these must also be states when the domestic currency appreciates in real terms. Empirically, this seems not to happen. A puzzle for the complete-markets model? I will return to this below.
- Neumeyer (1998) studies welfare implications of nominal bonds with incomplete markets in a very general flexible-price setting. See Benigno (2006) for an incomplete-markets model focusing on monetary policy.

## 5. Coeurdacier's local approach: Tradables only

- Basic model takes its assumptions from Lucas (1982) and Obstfeld and Rogoff (2000a).
- There are two symmetric countries, Home and Foreign, each specialized in a perishable but tradable "fruit" whose endowments each period,  $Y_H$  and  $Y_F$ , are random.
- The representative Home consumer chooses portfolio shares ex ante to maximize expected utility

$$U = E \left\{ (1 - \rho)^{-1} C_T^{1-\rho} \right\}$$

(nontradables to come later), where

$$C_T = \left[ \alpha^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1 - \alpha)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

while Foreign has mirror-symmetric preferences,

$$C_T^* = \left[ \alpha^{\frac{1}{\eta}} C_F^{*\frac{\eta-1}{\eta}} + (1 - \alpha)^{\frac{1}{\eta}} C_H^{*\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}.$$

- Fruit depreciates entirely after a period, but if the fruit is shipped within a period, a fraction  $t < 1$  of it is lost in transit.
- *Implication*: the cost of one unit of an output f.o.b. is  $\frac{1}{1-t}$  units, c.i.f.
- In this model, there are two sources of trade impediment: transport costs (obviously) but also the asymmetry in preferences ( $\alpha \geq \frac{1}{2}$ , which could be viewed as reflecting differential costs not captured in conventional measures of  $t$  including law of one price deviations).
- Given these impediments, PPP will not hold, nor will the law of one price. Indeed, for the prices of the individual goods, measured in a hypothetical world unit of account,<sup>1</sup>

$$P_F = \frac{P_F^*}{1-t}, \quad P_H^* = \frac{P_H}{1-t}.$$

- Equilibrium in the Home output market requires the following:

$$Y_H = \alpha \left( \frac{P_H}{P_T} \right)^{-\eta} C_T + \left( \frac{1-\alpha}{1-t} \right) \left( \frac{P_H^*}{P_T^*} \right)^{-\eta} C_T^*,$$

where the (tradables) price indexes are

$$P_T = \left[ \alpha P_H^{1-\eta} + (1-\alpha) P_F^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

$$\begin{aligned} P_T^* &= \left[ \alpha (P_F^*)^{1-\eta} + (1-\alpha) (P_H^*)^{1-\eta} \right]^{\frac{1}{1-\eta}} \\ &= \left\{ \alpha [(1-t) P_F]^{1-\eta} + (1-\alpha) [(1-t)^{-1} P_H]^{1-\eta} \right\}^{\frac{1}{1-\eta}}. \end{aligned}$$

- Define the inverse index of trade barriers (*à la* Coeurdacier),

$$\beta(\alpha, t) = \left[ \frac{\frac{1}{2} - (\alpha - \frac{1}{2})}{\frac{1}{2} + (\alpha - \frac{1}{2})} \right] (1-t)^{\eta-1}.$$

Assuming  $\eta > 1$ , this index falls (trade becomes more compressed) as  $\alpha$  or  $t$  rises. At the extreme, as  $\alpha \rightarrow 1$  or as  $t \rightarrow 1$ ,  $\beta \rightarrow 0$ .

- Again following Coeurdacier, consider the market-clearing condition, expressed in a relative form. This form makes clear the role of trade barriers in generating "transfer" effects of national expenditure levels, as invoked in the Keynes-Ohlin exchange:

$$\frac{Y_H}{Y_F} = \left( \frac{P_F^*}{P_H} \right)^\eta \left[ \frac{1 + \beta(\alpha, t) \left( \frac{P_T^*}{P_T} \right)^\eta \left( \frac{C_T^*}{C_T} \right)}{\beta(\alpha, t) + \left( \frac{P_T^*}{P_T} \right)^\eta \left( \frac{C_T^*}{C_T} \right)} \right].$$

- Above,  $P_F^*/P_H \equiv \tau$  denotes the f.o.b. terms of trade (ratio of f.o.b. prices of the Home and Foreign traded goods).
- It is useful to note that by logarithmically differentiating the national price level definitions near the symmetric equilibrium with all relative prices equal to 1, we obtain:

$$\hat{P}_T = \frac{1}{1+\beta} \hat{P}_H + \frac{\beta}{1+\beta} \hat{P}_F^*, \quad \hat{P}_T^* = \frac{1}{1+\beta} \hat{P}_F^* + \frac{\beta}{1+\beta} \hat{P}_H.$$

## Implications of complete markets

- Backus-Smith risk-sharing condition:

$$\hat{C}_T - \hat{C}_T^* = -\frac{1}{\rho} \left( \hat{P}_T - \hat{P}_T^* \right).$$

- Using the definitions of price levels above, relative consumption growth therefore is linked to terms-of-trade change by

$$\hat{C}_T - \hat{C}_T^* = -\left( \frac{1}{\rho} \right) \left( \frac{1 - \beta}{1 + \beta} \right) \left( \hat{P}_H - \hat{P}_F^* \right) = \frac{\lambda}{\rho} \hat{\tau},$$

where  $\lambda \equiv \frac{1-\beta}{1+\beta}$ . As in Coeurdacier (2009),  $\lambda$  (like  $\beta$ , a function of  $\alpha$  and  $t$ ) is between 0 and 1 and is *increasing* in trade barriers.



- Differentiating the relative goods-market equilibrium condition (at the symmetric equilibrium) and substituting the preceding risk-sharing condition, we find the difference in gross nominal (i.e., numeraire-denominated) returns on the equity claims to Home and Foreign outputs,

$$\widehat{P_H Y_H} - \widehat{P_F^* Y_F} = (\psi - 1)\hat{\tau},$$

$$\text{where } \psi \equiv (1 - \lambda^2)\eta + \lambda^2\left(\frac{1}{\rho}\right).$$

- When  $\psi > 1$ , a terms of trade deterioration due to an increase in relative Home output (the only shock in this model) is associated with a higher relative return on home equity. When trade barriers are nil ( $\lambda = 0$ ),  $\psi > 1$  depends only on the trade elasticity,  $\eta$ , exceeding 1: relatively price-elastic demand is necessary and sufficient for a cut in price to cause a rise in total revenue absent Keynesian transfer effects.

- With trade barriers ( $\lambda > 0$ ), however, there are deviations from PPP — namely, a fall in the relative domestic *price level* — when relative home output rises. Under complete markets, relative home consumption rises (with an elasticity of  $1/\rho$ ), shifting world demand toward Home output and augmenting Home equity returns.
- Correspondingly, the presence of trade barriers reduces the influence of the pure trade elasticity  $\eta$  by sheltering the market from the demand effects of terms-of-trade changes.

## Optimal portfolios

Optimal portfolios give countries the income they need to purchase optimal consumption bundles in every state of nature.

- Let a Home consumer hold a share  $\phi_T$  of the Home revenue process and  $1 - \phi_T$  of the Foreign process.
- Given initial symmetry, the Foreign consumer holds a share  $\phi_T$  of the Foreign process, as the pretrade equity prices are equal.

- Formally, the feasibility conditions for the Home and Foreign consumers are

$$P_T C_T = \phi_T (P_H Y_H) + (1 - \phi_T) (P_F^* Y_F),$$

$$P_T^* C_T^* = \phi_T (P_F^* Y_F) + (1 - \phi_T) (P_H Y_H).$$

- Logarithmically differentiating at the initial symmetric equilibrium (where  $P_H Y_H = P_F^* Y_F$ ), and differencing, we get:

$$\widehat{P_T C_T} - \widehat{P_T^* C_T^*} = (2\phi_T - 1) \widehat{P_H Y_H} + (1 - 2\phi_T) \widehat{P_F^* Y_F}.$$

- Backus-Smith, however, says that

$$\widehat{P_T C_T} - \widehat{P_T^* C_T^*} = \left(1 - \frac{1}{\rho}\right) (\hat{P}_T - \hat{P}_T^*).$$

- Because

$$\hat{P}_T - \hat{P}_T^* = -\lambda \hat{\tau},$$

the last two equations, plus  $\widehat{P_H Y_H} - \widehat{P_F^* Y_F} = (\psi - 1) \hat{\tau}$ , derived earlier, imply that to support the optimal allocation,  $\phi_T$  must satisfy:

$$(2\phi_T - 1)(\psi - 1) \hat{\tau} = -\left(1 - \frac{1}{\rho}\right) \lambda \hat{\tau}.$$

- Thus, the optimal share of initial wealth invested in one's own home equities is:

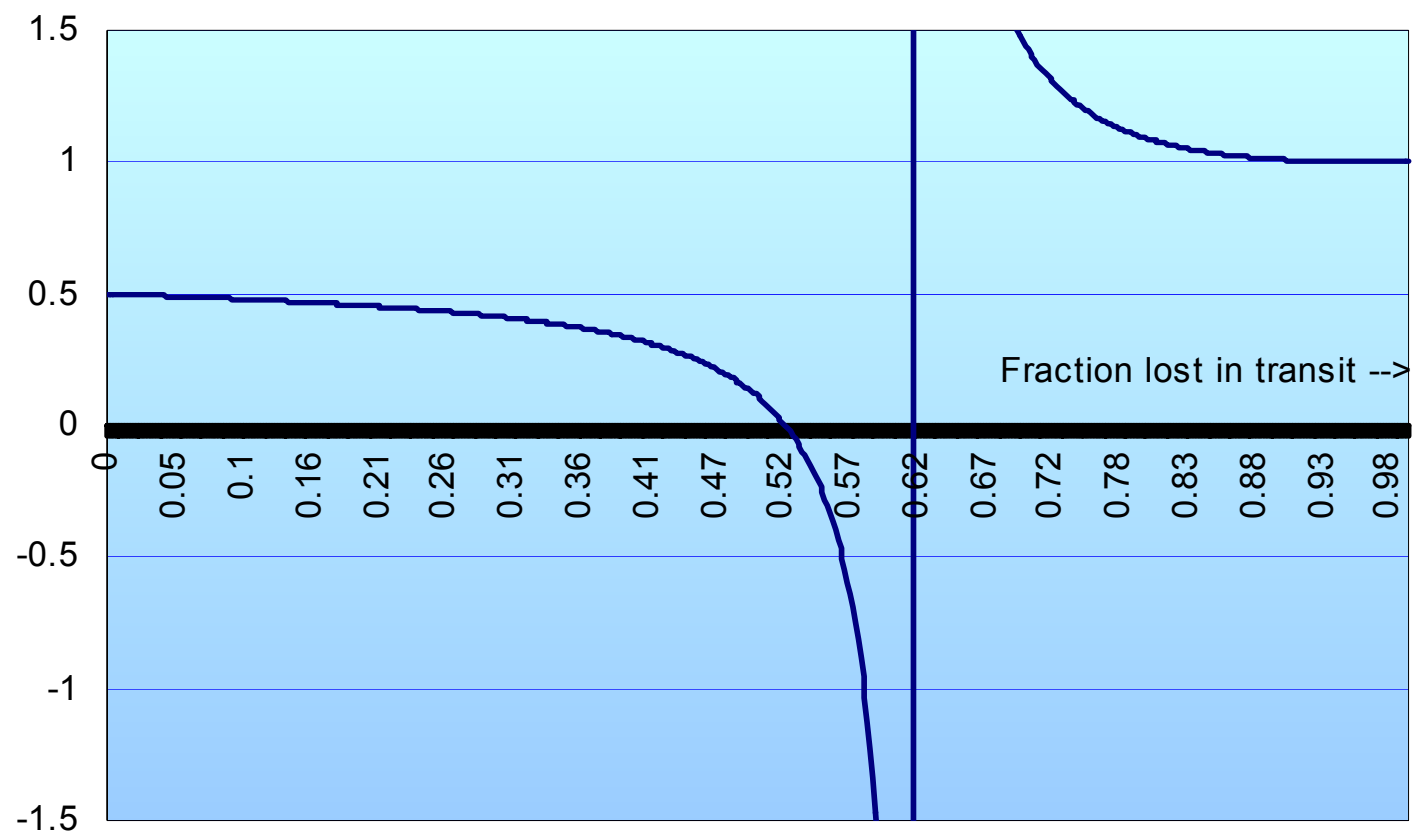
$$\phi_T = \frac{1}{2} \left[ 1 - \frac{(1 - \frac{1}{\rho}) \lambda}{\psi - 1} \right].$$

- When  $\lambda = 0$  (no trade barriers),  $\phi_T = \frac{1}{2}$ , as in Lucas (1982).
- Most analyses would assume  $\rho > 1$ , say  $\rho$  in the range of 2 to 4.
- Since  $\lambda \geq 0$ , home bias can occur only if  $\psi < 1$ .

- What is the likely range of values of  $\psi = (1 - \lambda^2)\eta + \lambda^2(\frac{1}{\rho})$ ?
- For reasonable values of  $\rho$ ,  $t$ , and  $\eta$ , it is likely that  $\psi > 1$ .
- However, for large values of trade costs  $t$  or large trade elasticities  $\eta$  — the latter in effect amplifying the effects of trade costs via  $\beta(\alpha, t)$  — we can have  $\psi < 1$ .
- But this actually is not the central issue. The problem, as I now show, is that even when  $\psi < 1$ , for example, because trade costs are very big, the model will generate ***too much*** home bias!

- The following diagram is generated from the model when  $\alpha = 0.5$ ,  $\rho = 4$ , and  $\eta = 4$ , but it is quite typical. (See picture.)
- The vertical asymptote is at the level of trade cost (about 0.62) where  $\psi$  reaches 1 from above, so that portfolios become indeterminate — essentially due to Cole-Obstfeld (1991) price-effect insurance. All very sensitive to parameters.
- The main point is that for  $\psi < 1$ , home bias is too great, with countries actually shorting the foreign equity. As  $t \rightarrow 1$ ,  $\phi_T \rightarrow 1$ , of course, but from above. *This is not what we observe*: in practice there is at least some international diversification:  $0.5 < \phi_T < 1$ .

Home equity share as a function of trade cost



## Intuition

- When  $\rho > 1$ , total consumption spending is optimally higher when the price level is higher. When  $\psi > 1$  as well, Home equities have relatively high payoffs when Home output is relatively high and Home's terms of trade  $\tau$  are relatively unfavorable (i.e.,  $\tau = P_F^*/P_H$  relatively high).
- But  $\tau = P_F^*/P_H$  relatively high  $\Rightarrow P$  relatively low. To have high spending when  $P$  is high (i.e., when  $\tau$  is low), the Home consumer skews the portfolio toward Foreign equities.
- Note, though, that as  $\psi \rightarrow 1$  from above, the quantitative link between terms of trade and relative revenues, while constant in sign, becomes smaller in absolute value, so Home investors must hold progressively more Foreign equities to support the optimum. Eventually they short Home equities.



- On the other hand, once  $\psi$  decreases enough that it crosses the threshold of 1 from above, the correlation between relative firm revenues and terms of trade is reversed. Now, with  $\psi < 1$ , higher relative output depresses terms of trade so much that relative revenues fall (as in the case of immiserizing growth).
- Initially, though, the absolute value of this effect is so small that the Home investor needs massively to short Foreign equities. As the trade cost  $t \rightarrow 1$ , the Home investor reduces the short position in Foreign equities, asymptotically reaching the autarkic portfolio ( $\phi_T = 1$ ).

## 6. Comparison with "Six puzzles"

Obstfeld and Rogoff (2000a) claimed that trade costs can explain home bias in equity holdings (along with other international macro anomalies).

In the formula

$$\phi_T = \frac{1}{2} \left[ 1 - \frac{(1 - \frac{1}{\rho})\lambda}{\psi - 1} \right], \quad \psi = (1 - \lambda^2)\eta + \lambda^2 \left( \frac{1}{\rho} \right),$$

let  $\eta = 1/\rho$ . Obstfeld and Rogoff showed that, in this case, the complete markets allocation can be supported by equity trade alone, not only locally (for tiny shocks), but globally.

In this special case,  $\psi = 1/\rho$  and therefore, provided  $\lambda > 0$  (positive trade impediments),  $\phi_T = \frac{1}{2}(1 + \lambda) > \frac{1}{2}$ .

However, such cases require either that  $\rho$  be implausibly low (and, in particular, below 1), so that under complete markets, countries with high price levels spend less; or that  $\eta$  be implausibly low, so that relative tradable returns are *positively* correlated with favorable terms of trade movements.

Heathcote and Perri (2004) look at a closely related special case involving production and investment.

### **Consumption allocations**

Obstfeld and Rogoff (2000a) calculated the complete-markets consumption allocations, for various parameter settings and realizations of the output shocks (Home-Foreign output ratios).

The starred (\*) rows below correspond to cases in which complete markets can be replicated globally by equity trade alone.

State, $y_H \equiv Y_H/Y_F$ :	0.8	0.9	1.0	1.1	1.2
<u>Parameters</u>	<u>Home consumption shares (<math>c_H, c_F</math>)</u>				
$t = 0.1, \eta = 2, \rho = 2$	0.53, 0.43	0.53, 0.43	0.53, 0.43	0.53, 0.43	0.52, 0.42
$t = 0.1, \eta = 3, \rho = 2$	0.56, 0.41	0.55, 0.40	0.55, 0.40	0.55, 0.40	0.55, 0.40
$t = 0.1, \eta = 5, \rho = 2$	0.61, 0.37	0.61, 0.36	0.60, 0.36	0.60, 0.35	0.60, 0.35
$^*t = 0.1, \eta = 6, \rho = \frac{1}{\eta}$	0.63, 0.33	0.63, 0.33	0.63, 0.33	0.63, 0.33	0.63, 0.33
$t = 0.1, \eta = 6, \rho = 2$	0.64, 0.35	0.63, 0.34	0.63, 0.33	0.62, 0.33	0.62, 0.32
$t = 0.1, \eta = 6, \rho = 5$	0.64, 0.35	0.64, 0.34	0.63, 0.33	0.62, 0.33	0.62, 0.32
$t = 0.2, \eta = 2, \rho = 2$	0.56, 0.36	0.56, 0.36	0.56, 0.36	0.55, 0.35	0.55, 0.35

---

$*t = 0.2, \eta = 6, \rho = \frac{1}{\eta}$	0.75, 0.20	0.75, 0.20	0.75, 0.20	0.75, 0.20	0.75, 0.20
$t = 0.2, \eta = 6, \rho = 2$	0.78, 0.22	0.76, 0.21	0.75, 0.20	0.74, 0.19	0.73, 0.18
$t = 0.2, \eta = 6, \rho = 5$	0.78, 0.22	0.77, 0.21	0.75, 0.20	0.74, 0.18	0.73, 0.18
$t = 0.3, \eta = 6, \rho = 2$	0.89, 0.13	0.87, 0.11	0.86, 0.10	0.84, 0.09	0.83, 0.08
$t = 0.3, \eta = 8, \rho = 2$	0.95, 0.08	0.94, 0.07	0.92, 0.05	0.91, 0.04	0.89, 0.04

---

OR reasoned that, in other cases, the consumption allocations do not vary too much across the states of nature, so that portfolios with considerable home equity bias would come close to replicating complete markets, and therefore be approximately correct representations of actual equity choices. Coeurdacier's analysis has shown that this intuition was wrong.

As a first step in developing some perspective, let us calculate the consumption shares implied by Coeurdacier's approximate complete-markets portfolio allocations. It is instructive to compare these to the exact complete-markets shares reported above. The consumption allocations in this second case can be found simply by assuming optimal portfolios ( $\phi_T$ , reported below), which then define the Home and Foreign income endowments of the two goods in the various states of nature. For brevity, only the final five rows of the preceding table are considered.

These calculations all assume (as in OR) that  $\alpha = 0.5$  (internationally symmetric preferences over tradables):

State, $y_H \equiv Y_H/Y_F$ :	0.8	0.9	1.0	1.1	1.2	$\phi_T$
<u>Parameters</u>	<u>Home consumption shares (<math>c_H, c_F</math>), percent</u>					
$*t = 0.2, \eta = 6, \rho = \frac{1}{\eta}$	75.3, 19.7	75.3, 19.7	75.3, 19.7	75.3, 19.7	75.3, 19.7	0.75
$t = 0.2, \eta = 6, \rho = 2$	77.7, 21.8	76.5, 20.7	75.3, 19.7	74.2, 18.9	73.2, 18.2	0.46
$t = 0.2, \eta = 6, \rho = 5$	77.9, 22.0	76.6, 20.8	75.3, 19.7	74.1, 18.8	73.1, 18.0	0.44
$t = 0.3, \eta = 6, \rho = 2$	88.6, 12.6	87.1, 11.2	85.6, 10.1	84.1, 9.1	82.7, 8.3	0.42
$t = 0.3, \eta = 8, \rho = 2$	95.4, 8.6	94.0, 6.7	92.4, 5.3	90.6, 4.3	88.7, 3.5	0.37

These are very close to the (globally) complete markets numbers. They provide excellent approximations even for substantial ( $\pm 20$  percent) uncertainty.



- A striking feature of the table is how different the optimal portfolio and consumption positions can be, even though a portfolio that was (in some sense) close to the first-best consumption positions would enable the investor to come close to them in equilibrium, simply by consuming (approximately) his or her endowment.
- For example, let us consider the third row of the preceding table. We could imagine endowing the Home country investor with portfolios corresponding to the consumption ratios  $\phi_T = 0.779, \phi_T = 0.766, \dots, \phi_T = 0.731$  (and, correspondingly endowing the Foreign investor with  $\phi_T^* = 1 - \phi_T = 0.221, 0.234, \dots, 0.269$ ). Given these endowments, we can easily calculate the resulting equilibrium consumption levels, which define a symmetric matrix as follows:<sup>7</sup>

State, $y_H \equiv Y_H/Y_F$ :	0.8	0.9	1.0	1.1	1.2
<u>Assigned portfolio</u>	<u>Home consumption shares (<math>c_H, c_F</math>), percent</u>				
$\phi_T = 0.779$	75.1, 19.6	75.2, 19.7	75.3, 19.7	75.4, 19.8	75.5, 19.9
$\phi_T = 0.766$	75.2, 19.7	75.3, 19.7	75.3, 19.7	75.4, 19.8	75.4, 19.8
$\phi_T = 0.753$	75.3, 19.7	75.3, 19.7	75.3, 19.7	75.3, 19.7	75.3, 19.7
$\phi_T = 0.741$	75.4, 19.8	75.4, 19.8	75.3, 19.7	75.3, 19.7	75.2, 19.7
$\phi_T = 0.731$	75.5, 19.9	75.4, 19.8	75.3, 19.7	75.2, 19.7	75.1, 19.6

Note: The underlying parameters are  $\alpha = 0.5$ ,  $t = 0.2$ ,  $\eta = 6$ ,  $\rho = 5$ .

- These consumption levels do not differ too much from those in the previous table, although they vary less across states of nature
- But what is the *welfare significance* of the difference?
- Let  $U$  be realized (normalized) utility under an assigned portfolio allocation from the last table, given by

$$U = (1 - \rho)^{-1} \mathbb{E} \left\{ \left[ \alpha^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1 - \alpha)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta(1-\rho)}{\eta-1}} \right\}.$$

Let  $\tilde{U}$  be utility under the complete-markets allocation.

- Then the percentage welfare gain moving from the assigned to the complete-markets allocation is measured by

$$\zeta = \left[ \left( \frac{\tilde{U}}{U} \right)^{\frac{1}{1-\rho}} - 1 \right] \times 100.$$

- The degree of output uncertainty in the preceding tables is much larger than what is typical for the industrial countries. To construct a more realistic example, I assume five states of nature with the following output levels and probabilities of occurrence:

State:	1	2	3	4	5
Probability	0.125	0.25	0.25	0.25	0.125
$Y_H$	0.974358974	0.987341772	1	1.012658228	1.025641026
$Y_F$	1.025641026	1.012658228	1	0.987341772	0.974358974
$y_H \equiv Y_H/Y_F$	0.95	0.975	1	1.025641026	1.052631579

- Note the imposed *symmetry* of the two countries (with respect to outcomes and probabilities).

- Take the assigned portfolio to be  $\phi_T = 0.753193536$ , which entails considerable home bias. The *optimal* portfolio (for  $\alpha = 0.5, t = 0.2, \eta = 6, \rho = 5$ ) instead sets  $\phi_T \sim 0.44$ .
- The implied consumption levels for Home under the assigned and optimal portfolios, respectively, are then:

State:	1	2	3	4	5
$C_H$	0.733880881	0.74365944	0.753193536	0.762727631	0.772506191
$C_F$	0.202507868	0.199944478	0.197445172	0.194945866	0.192382475
$\tilde{C}_H$	0.739874773	0.746670949	0.753193536	0.759613026	0.766089299
$\tilde{C}_F$	0.207641381	0.202436162	0.197445172	0.192536659	0.187587361

- The consumption levels are in all cases very close, but in the first-best allocation relative to the assigned one, expenditure is smoother across states of nature.

- In this case, the welfare loss from the portfolio discrepancy turns out to be minuscule:

$$\zeta = 0.014 \text{ percent,}$$

that is, a welfare loss equivalent to only a bit over *one one-hundredth of a percent of GDP*. It corresponds to a fractional tax of  $0.00014 = 1.4 \times 10^{-4}$ . And this is for a fairly generous level of relative risk aversion ( $\rho = 5$ ).

- Cole and Obstfeld (1991) observed that the welfare gains from international diversification can be very small indeed, at least in a homogeneous-agent setting with complete markets.
- This again illustrates essentially the same point, on which there is now a large literature. In the example, dissimilar portfolios make very little difference to investor welfare.

- Coeurdacier (in the manuscript version) makes the important point that very small transaction costs — in his model, a tax on returns on the order of  $10^{-3}/\rho$  — generate significant home bias. This strikingly confirms the Cole-Obstfeld conjecture: “When the gains from diversification abroad are small, even minor impediments to asset trade can wipe them out.”
- These impediments need not reflect explicit taxes or transaction costs. Any possibility of internationally asymmetric payouts will do. E.g., a potential political event that has differential effects on the returns to home and foreign investors, such that foreign investors lose proportionally more than home investors, generates home bias.

A final important implication, is that small changes in expected returns may generate huge volatility in portfolio positions, and in capital flows. Potential diversification gains, when small, are not much of a brake on the pursuit of return.

Coeurdacier's results are not surprising in view of Uppal's (1993) earlier discussion, in the finance literature, of transport costs and international diversification.

Uppal finds bias toward foreign equities iff  $\rho > 1$ . (He also offers some interesting observations on currency preference.)

Uppal, however, assumes perfect substitution ( $\eta \rightarrow \infty$ ) between national outputs, allowing no analysis of the terms-of-trade effects that are so important above. He assumes complete markets, and that investors hold physical capital that can be transferred between countries at a cost.

All these results are potentially sensitive to the presence of equities in nontraded-industry firms.



## 7. The key importance of nontraded goods

Nontraded goods can be added to Coeurdacier's framework. Indeed, Baxter, Jermann, and King (BJK, 1998) analyzed the role of nontradable goods and nontraded-industry equities (which themselves *are* tradable) in generating home bias for traded-industry shares. They did so using a local methodology essentially identical to that of Coeurdacier.

But they assumed that there were no trade barriers for tradables, and that different countries' tradables were perfect substitutes ( $\eta \rightarrow \infty$ , though only the trade-barrier assumption matters). They concluded that  $\phi_T$  should always equal  $\frac{1}{2}$ .

*Bottom-line result of a more general analysis:* The extended model can plausibly generate reasonable degrees of home equity bias, including home bias for traded-good shares.<sup>2</sup>

## Model with nontraded goods: Assumptions

- The representative Home consumer chooses portfolio shares ex ante to maximize expected utility

$$(1 - \rho)^{-1} E\{C^{1-\rho}\},$$

where

$$C = \left[ \gamma^{\frac{1}{\theta}} C_T^{\frac{\theta-1}{\theta}} + (1 - \gamma)^{\frac{1}{\theta}} C_N^{\frac{\theta-1}{\theta}} \right],$$

traded consumption  $C_T$  is as specified earlier, and  $C_N$ , nontraded consumption, must equal nontraded supply  $Y_N$  in equilibrium.

- The key parameter  $\theta$  is the elasticity of substitution between traded and nontraded goods.
- A key approximate relationship is that, in a neighborhood of a symmetric initial position,

$$\hat{P} = \frac{1}{1 + \omega} \hat{P}_T + \frac{\omega}{1 + \omega} \hat{P}_N,$$

where

$$\omega \equiv \frac{1-\gamma}{\gamma} \overline{P_N/P_T}^{1-\theta}.$$

Here,  $\overline{P_N/P_T}$  is the relative price of nontradables at the internationally symmetric point of linearization.

- In the present context, if markets in nominal claims are complete, the Backus-Smith condition applies to overall consumption and price levels:

$$\widehat{PC} - \widehat{PC}^* = \left(1 - \frac{1}{\rho}\right) \left(\widehat{P} - \widehat{P}^*\right).$$

## Output markets' equilibrium

- For the nontradables market

$$P_N C_N = P_N Y_N = \left(\frac{P_N}{P}\right)^{1-\theta} PC.$$

- With complete markets, differential returns for nontraded- industry

shares therefore follow

$$\widehat{P_N Y_N} - \widehat{P_N^* Y_N^*} = (1 - \theta)(\hat{P}_N - \hat{P}_N^*) + \left(\theta - \frac{1}{\rho}\right)(\hat{P} - \hat{P}^*).$$

- The parameter difference  $\theta - \frac{1}{\rho}$  is a key one in this literature. (See BJK 1998.) Why? The answer comes from Backus-Smith.
- Fixing nontradables prices and expenditures, a rise in the overall price level raises nontraded firms' revenues with elasticity  $\theta - 1$ .
- At the same time, however, due to Backus-Smith, the rise in the overall price level also raises total expenditure with an elasticity of  $1 - \frac{1}{\rho}$ . The sum of these two elasticities is  $\theta - \frac{1}{\rho}$ . If  $\theta$  or  $\rho$  is very low, the effect could be negative.
- We would like to know how both relative nontradables prices and terms of trade affect nontraded firms' revenues, so we must decompose the overall price levels into their components. Let  $p \equiv P_N^*/P_N$ , and recall that  $\tau \equiv P_F^*/P_H$ .

- Putting together earlier results yields

$$\begin{aligned}\widehat{P} - \widehat{P}^* &= \frac{1}{1 + \omega} \left( \widehat{P}_T - \widehat{P}_T^* \right) - \frac{\omega}{1 + \omega} \widehat{p} \\ &= - \left( \frac{1}{1 + \omega} \lambda \widehat{\tau} + \frac{\omega}{1 + \omega} \widehat{p} \right),\end{aligned}$$

and so, the key international revenue differential:

$$\begin{aligned}\widehat{P_N Y_N} - \widehat{P_N^* Y_N^*} &= -(1 - \theta) \widehat{p} - \left( \theta - \frac{1}{\rho} \right) \left( \frac{1}{1 + \omega} \lambda \widehat{\tau} + \frac{\omega}{1 + \omega} \widehat{p} \right) \\ &= (\nu - 1) \widehat{p} - \frac{\lambda (\theta - \frac{1}{\rho})}{1 + \omega} \widehat{\tau},\end{aligned}$$

where

$$\nu \equiv \frac{1}{1 + \omega} \theta + \frac{\omega}{1 + \omega} \left( \frac{1}{\rho} \right).$$

- It will turn out that whether  $\nu$  is above or below 1 is a central question in understanding portfolio positions.

- The intuition for the equation is driven by Backus-Smith.
- Consider, first, the component involving the terms of trade,

$$- \left( \theta - \frac{1}{\rho} \right) \left( \frac{\lambda}{1 + \omega} \right) \hat{\tau}.$$

When  $\hat{\tau} > 0$  (Home's terms of trade worsen), Foreign's price level rises relative to Home's. Foreign's total relative spending rises with the elasticity  $1 - \frac{1}{\rho}$ , whereas, given nontraded prices, relative demand for Foreign nontradables rises with an elasticity  $\theta - 1$  due to the price response of demand. The sum of the two effects is a decline in the relative revenue of Home nontraded firms of  $\theta - 1 + 1 - \frac{1}{\rho} = \theta - \frac{1}{\rho}$ .

- What about the term

$$(v - 1)\hat{p}?$$

When  $\hat{p} > 0$  (Home's relative nontradable price falls), then, given tradables' prices, there is a demand shift toward Home nontradables mediated by the price elasticity  $\theta$ . In addition, because Home's relative overall price level  $P/P^*$  falls,  $C/C^*$  rises, further raising demand for home nontradables. This latter effect is stronger the more important are nontradables in the Home CPI (the higher is  $\omega$ ). At the same time, a higher  $\omega$  lowers the pure demand-shift effect of the fall in  $P_N$  relative to  $P_N^*$ , which depends on the change in the ratio  $P_N/P$  relative to that in  $P_N^*/P^*$ . When  $v$ , the sum of these effects, exceeds 1, a positive value of  $\hat{p}$  (a decline in the Home “virtual” terms of trade in nontradables) implies a rise in the relative return to Home nontraded-industry equities.



- The next job is to derive an analogous equation describing the relative return on Home traded-industry equities,  $\widehat{P_H Y_H} - \widehat{P_F^* Y_F}$ .
- The equilibrium condition for tradables is still given by

$$\frac{Y_H}{Y_F} = \left( \frac{P_F^*}{P_H} \right)^\eta \left[ \frac{1 + \beta(\alpha, t) \left( \frac{P_T^*}{P_T} \right)^\eta \left( \frac{C_T^*}{C_T} \right)}{\beta(\alpha, t) + \left( \frac{P_T^*}{P_T} \right)^\eta \left( \frac{C_T^*}{C_T} \right)} \right].$$

However, it is now the case that

$$\widehat{C}_T - \widehat{C}_T^* = \theta \left( \widehat{P} - \widehat{P}^* \right) - \theta \left( \widehat{P}_T - \widehat{P}_T^* \right) + \left( \widehat{C} - \widehat{C}^* \right).$$

- Log-linearizing and combining terms gives the equation we seek:

$$\widehat{P_H Y_H} - \widehat{P_F^* Y_F} = (\psi - 1)\hat{\tau} - \frac{\lambda\omega(\theta - \frac{1}{\rho})}{1 + \omega}\hat{p},$$

where now,  $\psi$  has the modified definition

$$\begin{aligned}\psi &\equiv (1 - \lambda^2)\eta + \lambda^2\left[\frac{\omega}{1+\omega}\theta + \frac{1}{1+\omega}\left(\frac{1}{\rho}\right)\right] \\ &= (1 - \lambda^2)\eta + \lambda^2\left(\theta + \frac{1}{\rho} - \nu\right).\end{aligned}$$

- Notice that when  $\omega = 0$ , this reduces to the relationship in the tradables-only model. Parameter  $\theta$  enters the preceding formula because expenditure switches from nontradables to tradables enhance tradable-industry revenues.

## Portfolios

- Introduce the matrix of relative-price factors that drive sectoral returns,

$$\mathbf{M} = \begin{bmatrix} \psi - 1 & -\omega\xi \\ -\xi & \nu - 1 \end{bmatrix},$$

$$\text{where } \xi \equiv \frac{\lambda(\theta - \frac{1}{\rho})}{1 + \omega}.$$

- Denote the column vectors of relative returns and relative prices by

$$\mathbf{r} \equiv \begin{bmatrix} \widehat{P_H Y_H} - \widehat{P_F^* Y_F} \\ \widehat{P_N Y_N} - \widehat{P_N^* Y_N^*} \end{bmatrix}, \quad \mathbf{q} \equiv \begin{bmatrix} \hat{\tau} \\ \hat{p} \end{bmatrix}.$$

- A compact representation of the relationship between returns and output market relative prices is

$$\mathbf{r} = \mathbf{M}\mathbf{q}.$$

- Denote the portfolio-share vector by

$$\boldsymbol{\phi} \equiv \begin{bmatrix} 2\phi_T - 1 \\ 2\phi_N - 1 \end{bmatrix},$$

where  $\phi_N$  is the share of Home nontraded-sector equities held by the Home country.

- The Backus-Smith condition takes the form

$$\widehat{PC} - \widehat{PC}^* = -\left(1 - \frac{1}{\rho}\right) \left( \frac{\lambda}{1 + \omega} \hat{\tau} + \frac{\omega}{1 + \omega} \hat{p} \right).$$

If we define

$$\mathbf{\Omega} \equiv \begin{bmatrix} \frac{1}{1+\omega} & 0 \\ 0 & \frac{\omega}{1+\omega} \end{bmatrix}, \quad \boldsymbol{\ell} \equiv \begin{bmatrix} \lambda \\ 1 \end{bmatrix},$$

the condition is

$$\widehat{PC} - \widehat{PC^*} = -(1 - \frac{1}{\rho}) \boldsymbol{\ell}' \mathbf{\Omega} \mathbf{q}.$$

- Given symmetric countries, the relative budget constraint is

$$\frac{PC}{P^*C^*} = \frac{\phi_T(P_H Y_H) + (1-\phi_T)(P_F^* Y_F) + \phi_N(P_N Y_N) + (1-\phi_N)(P_N^* Y_N^*)}{(1-\phi_T)(P_H Y_H) + \phi_T(P_F^* Y_F) + (1-\phi_N)(P_N Y_N) + \phi_N(P_N^* Y_N^*)},$$

which has the log approximation<sup>3</sup>

$$\widehat{PC} - \widehat{P^*C^*} = \frac{(2\phi_T-1)(\widehat{P_H Y_H} - \widehat{P_F^* Y_F}) + \omega(2\phi_N-1)(\widehat{P_N Y_N} - \widehat{P_N^* Y_N^*})}{1+\omega}.$$

Using matrix notation, this last equation is

$$\widehat{PC} - \widehat{P^*C^*} = \boldsymbol{\varphi}' \mathbf{\Omega} \mathbf{r} = \boldsymbol{\varphi}' \mathbf{\Omega} \mathbf{M} \mathbf{q}.$$

- To support the efficient allocation, the equation  $\boldsymbol{\varphi}' \boldsymbol{\Omega} \mathbf{M} \mathbf{q} = - (1 - \frac{1}{\rho}) \boldsymbol{\ell}' \boldsymbol{\Omega} \mathbf{q}$  must hold for all  $\mathbf{q}$ , or, equivalently, the portfolio shares  $\boldsymbol{\varphi}$  must satisfy:

$$\boldsymbol{\varphi}' \boldsymbol{\Omega} \mathbf{M} = - (1 - \frac{1}{\rho}) \boldsymbol{\ell}' \boldsymbol{\Omega}.$$

To solve, notice that  $\boldsymbol{\Omega}$  and  $\boldsymbol{\Omega} \mathbf{M}$  are symmetric, so that

$$\boldsymbol{\varphi}' = - (1 - \frac{1}{\rho}) \boldsymbol{\ell}' \mathbf{M}'^{-1}.$$

- Because

$$\mathbf{M}'^{-1} = \frac{1}{(\psi-1)(\nu-1)-\omega\xi^2} \begin{bmatrix} \nu-1 & \xi \\ \omega\xi & \psi-1 \end{bmatrix},$$

we find that

$$\boldsymbol{\varphi}' = \frac{-\left(1-\frac{1}{\rho}\right)}{(\psi-1)(\nu-1)-\omega\xi^2} \begin{bmatrix} \lambda(\nu-1) + \omega\xi & \psi-1 + \lambda\xi \end{bmatrix}.$$

- In terms of more basic variables, we have:

$$\phi_T = \frac{1}{2} \left[ 1 - \frac{\lambda(1 - \frac{1}{\rho})(\theta - 1)}{(\psi - 1)(\nu - 1) - \frac{\omega\lambda^2(\theta - \frac{1}{\rho})^2}{(1+\omega)^2}} \right],$$

$$\phi_N = \frac{1}{2} \left\{ 1 + \frac{(1 - \frac{1}{\rho})[1 - (1 - \lambda^2)\eta - \lambda^2\theta]}{(\psi - 1)(\nu - 1) - \frac{\omega\lambda^2(\theta - \frac{1}{\rho})^2}{(1+\omega)^2}} \right\}.$$

## Special cases

- Notice that when there are no nontraded goods ( $\omega = 0$ ), the traded portfolio share reduces to

$$\phi_T = \frac{1}{2} \left[ 1 - \frac{\lambda(1 - \frac{1}{\rho})}{(\psi - 1)} \right],$$

with  $\psi = (1 - \lambda^2)\eta + \lambda^2(\frac{1}{\rho})$ . This is Coeurdacier's formula.

- When  $\lambda = 0$  (no trade impediments for tradables), the nontraded portfolio share reduces to

$$\begin{aligned} \phi_N &= \frac{1}{2} \left[ 1 - \frac{(1 - \frac{1}{\rho})}{(\nu - 1)} \right] \\ &= \frac{1}{2} \left[ 1 + \frac{1 - \frac{1}{\rho}}{1 - (\frac{1}{1+\omega} \cdot \theta + \frac{\omega}{1+\omega} \cdot \frac{1}{\rho})} \right]. \end{aligned}$$



- This formula can be derived directly from those in BJK (and it holds even when countries' tradables are imperfect substitutes, provided preferences over tradables are the same in all countries). That the assumption of perfectly substitutable tradables is irrelevant to this case is apparent from the analysis in Lucas (1982).
- Thus, when  $\theta = \frac{1}{\rho}$  and  $\lambda = 0$  (the case of separable utility),  $\phi_N = 1$ , the finding of Stockman and Dellas (1989). (In general, even when  $\lambda > 0$ ,  $\phi_N = 1$  when  $\theta = \frac{1}{\rho}$ , but  $\phi_T$  will not necessarily equal  $\frac{1}{2}$ .)
- When  $\lambda = 0$ ,  $\phi_T = \frac{1}{2}$  even with nontradables, as BJK claimed. There is no home bias in tradables. Deviations from the perfectly pooled tradables portfolio are intimately linked to trade barriers (possibly preference-driven) for tradable goods.

- What explains the BJK condition? Notice that when  $\lambda = 0$ , the relationship between relative nontraded-industry returns and relative international nontradable prices is simply

$$\widehat{P_N Y_N} - \widehat{P_N^* Y_N^*} = (\nu - 1)\hat{p},$$

where  $p = P_N^*/P_N$ . So the intuition Coeurdacier offers for tradables still applies.<sup>4</sup> When  $\nu > 1$ , a rise in Home nontraded output depresses Home relative price but raise Home relative revenues in nontradables. Since Home's relative CPI falls in that case, then, if  $\rho > 1$ , Home's relative spending should fall. This can occur if Home, rather than holding the same perfectly pooled global portfolio of nontradables as Foreign, instead skews its holdings toward Foreign nontradables. In that case the BJK formula predicts  $\phi_N < \frac{1}{2}$ .

- It is empirically more plausible, however, that  $v < 1$ . For example, even if  $\theta = 2$ , with  $\rho = 2$  and  $\omega = 3$ , we will have

$$v = \frac{1}{4} \cdot 2 + \frac{3}{4} \cdot \frac{1}{2} = \frac{7}{8}.$$

Thus, it is likely that a rise in the relative price of Home nontradables is associated with an increase in relative home nontradable revenues.

- In that case, again provided  $\rho > 1$ , there will be a home bias in nontradable equity holdings, but with some positive diversification taking place when  $\theta < \frac{1}{\rho}$ . For  $\theta > \frac{1}{\rho}$ , Home will short foreign nontradable equities.<sup>5</sup>

## General case

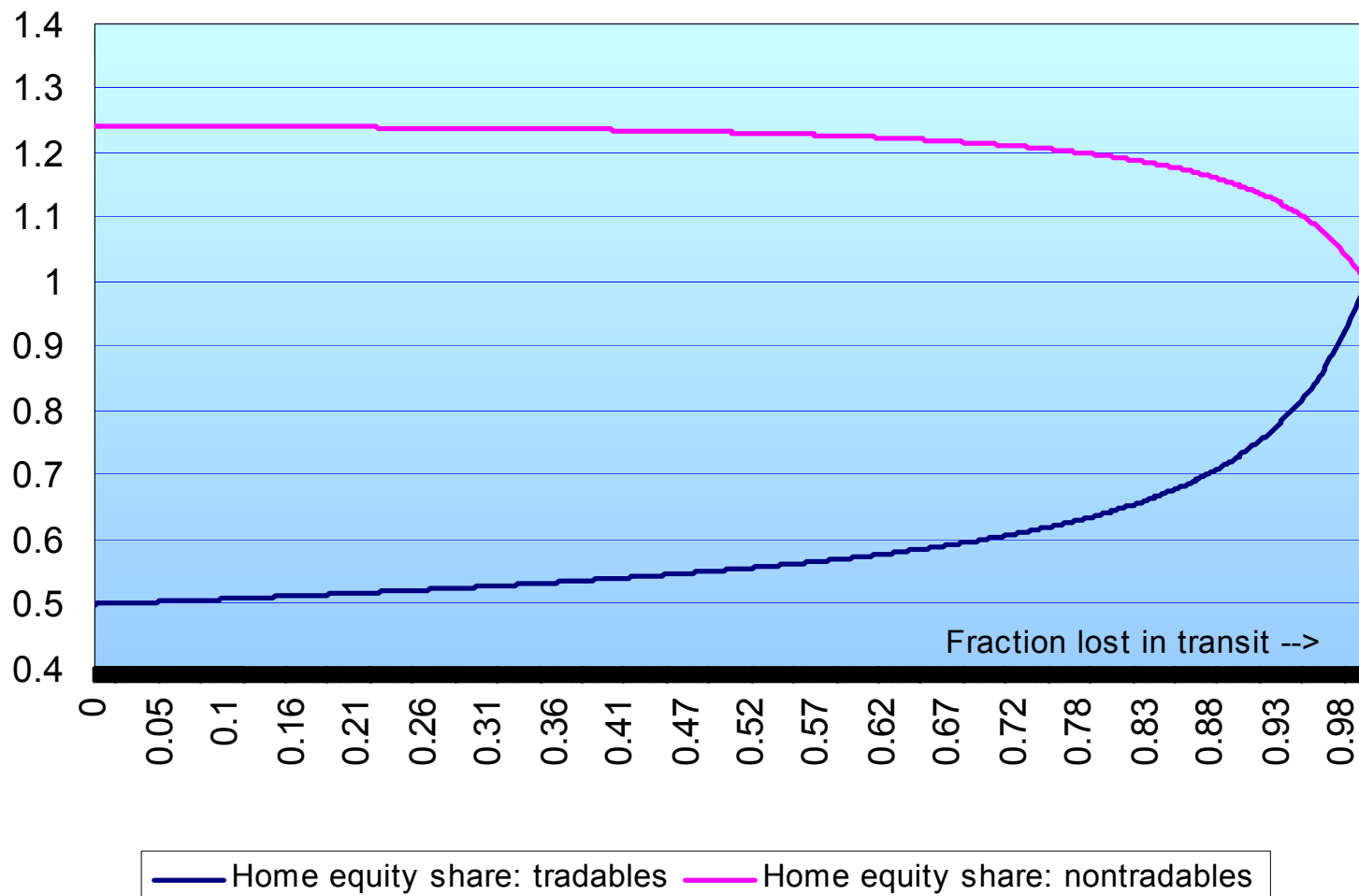
Many possible patterns, depending on specific parameter values. Empirically, it is likely that  $\rho > 1$ ,  $\eta > 1$ , and  $\psi > 1$ , but that  $\nu < 1$ . A value of  $\theta \sim 1.2$  is consistent with the high-end estimates of Ostry and Reinhart (1992) for developing countries.

We would then find that  $\phi_T > \frac{1}{2}$  and  $\phi_N > 1$ : there is home bias in tradables, and a more extreme home bias in nontradables, with investors shorting the foreign nontraded-industry equity.<sup>6</sup> Overall, of course, the domestic portfolio would exhibit a considerable home bias. Some numerical examples:

$\alpha$	$\omega$	$\rho$	$\theta$	$\eta$	$t$	$\phi_T$	$\phi_N$
0.5	3	2	1.2	2	0.2	0.52	1.27
0.5	3	2	1.2	10	0.2	0.53	1.26
0.5	3	2	1.6	2	0.2	0.57	1.60
0.5	3	2	1.2	2	0.3	0.53	1.27
0.6	3	2	1.6	2	0.3	0.73	1.51
0.5	3	2	0.8	2	0.3	0.48	1.09
0.5	3	2	0.4	2	0.3	0.45	0.98

Prediction: Home bias in traded-industry equities increases with trade costs for tradables, home bias in nontradables decreases with that trade cost. The following picture takes  $\alpha = 0.5$ ,  $\omega = 3$ ,  $\rho = 3$ ,  $\theta = 1.2$ , and  $\eta = 2$ :

## Home equity shares as a function of trade cost in tradables



- Of course, as  $t \rightarrow 1$ , tradables become nontraded, autarky develops, and both portfolio shares approach the autarky level of 1.
- For intermediate levels of  $t$ , there is traded- as well as nontraded-equity home bias. The overall (average) level of bias seems reasonable for the U.S., insofar as we can assign empirical counterparts.
- What is the intuition? Observe that there are two proximate sources of real exchange rate fluctuation in the model, fluctuations in the terms of trade  $\tau = P_F^*/P_H$  and in the international relative price of nontradables,  $p = P_N^*/P_N$ .

- To develop the intuition further, start at a point where  $\theta = \frac{1}{\rho}$ , so that utility is separable in tradable and nontradable consumption.
- In that case,  $\widehat{P_N Y_N} - \widehat{P_N^* Y_N^*} = (1 - \frac{1}{\rho})(\widehat{P_N} - \widehat{P_N^*})$ , so relative nontraded returns do not depend on  $\widehat{\tau}$ . Efficient global sharing of nontradable productivity risks is achieved by setting  $\phi_N = 1$ .
- Moreover, relative nontradable prices do not impact the relative returns to traded-industry equities, so Coeurdacier's formula governs the portfolios of tradables.
- Now, let  $\theta$  rise from  $\frac{1}{\rho}$ . In general, as shown earlier,



$$\widehat{P_N Y_N} - \widehat{P_N^* Y_N^*} = (1 - \nu)(\widehat{P_N} - \widehat{P_N^*}) + \frac{\lambda(\theta - \frac{1}{\rho})}{1 + \omega} (\widehat{P_H} - \widehat{P_F^*}),$$

$$\widehat{P_H Y_H} - \widehat{P_F^* Y_F} = \frac{\lambda\omega(\theta - \frac{1}{\rho})}{1 + \omega} (\widehat{P_N} - \widehat{P_N^*}) + (1 - \psi)(\widehat{P_H} - \widehat{P_F^*}).$$

With  $\nu < 1$ ,  $\nu$  rises toward 1 as a result of the rise in  $\theta$ , so  $\phi_N$  must rise to maintain the efficient response to  $p$  shocks. With  $\theta > \frac{1}{\rho}$ , though, Home nontradable returns are also relatively high when the terms of trade are favorable to Home, whereas a rise in  $\theta$  raises  $\psi$  and thereby raises the payoff to Foreign equities when Home's terms of trade move favorably. To avoid an excessive positive response of Home spending to positive  $\tau$  shocks, Home must raise  $\phi_T$  as  $\phi_N$  rises. When eventually  $\theta$  rises above 1,  $\phi_T$  rises above  $\frac{1}{2}$ : a home bias in traded-industry shares emerges.

*Empirical issues:*

**Issue #1** Do we really believe  $\theta > 1$ ?

**Issue #2** As van Wincoop and Warnock (2006) argue, the empirical correlation between relative stock returns and real exchange rates (or terms of trade) is way too low to explain much home bias for tradables equities.

## 8. Sticky money prices, human capital, and international currency positions

To address the issues of this section, I adapt the model of Obstfeld and Rogoff (2000b) to a framework inspired by Engel and Matsumoto's (EM, 2006) recent work. The simplified framework is designed for comparability with the previous results, though it adds some realistic features.

### Assumptions

- In Home (resp. Foreign) there is a *fixed* capital stock  $K$  (resp.  $K^*$ ) and the representative agent's utility is given by

$$U = E \left\{ \frac{1}{1-\rho} C^{1-\rho} - \frac{\kappa}{1+\vartheta} L^{1+\vartheta} + \chi \log \left( \frac{M}{P} \right) \right\}$$

(resp. for Foreign), where  $L$  denotes labor supply.

- As above, we essentially have a one-period model: agents trade assets given wealth and expectations; real and monetary shocks are realized; and production, consumption, and labor supply all occur, based on endowments determined by the portfolio allocation inherited from the prior, asset-trading, stage.
- Output is given by  $Y_H = K^\delta (AL)^{1-\delta}$  (with the corresponding Cobb-Douglas function for Foreign).  $A$ ,  $A^*$  are labor productivity shocks.
- Home output consists of a continuum of symmetric varieties indexed by  $j \in [0,1]$ , each variety monopolistically priced.
- Think of each of the identical firms as operated by an owner of a prorated portion of the economy's total capital  $K$ . An owner  $j \in [0,1]$  owns  $k(j) = K$  and cannot lease that capital to other producers. The owner can, however, sell equity shares, which are claims to the portion of revenue not paid out in wages.

- Separately, of course, the owner sells his or her differentiated and monopolistically priced labor  $\ell(j)$  to all firms on  $[0, 1]$ . We need not model the wage-setting decision here, as we are interested in unexpected shocks.
- Nominal profits for a representative Home firm (say) therefore are  $\Pi = P_H Y_H - WL$ , where  $W$  is the nominal wage, which is pre-set and then sticky. Shares in  $\Pi$  and  $\Pi^*$  are traded ex ante internationally.
- Note: In Engel-Matsumoto prices are sticky, wages flexible; here it is the reverse. This affects the riskiness of factor incomes.
- Let  $\sigma$  be the substitution elasticity among varieties produced by a country. The monopolistic pricing formula in this case is

$$P_H = \frac{\sigma}{(\sigma - 1)(1 - \delta)} \left( \frac{AL}{K} \right)^\delta \frac{W}{A},$$

which implies  $P_H = \left( \frac{\sigma}{\sigma - 1} \right) \frac{W}{A}$  when capital's share  $\delta = 0$ .

- Using this markup formula, total nominal profits are simply<sup>8</sup>

$$\Pi = \frac{1 - \delta + \delta\sigma}{(\sigma - 1)(1 - \delta)} WL.$$

- $\eta$  (which may differ from  $\sigma$ ) will continue to denote the substitution elasticity between the two countries' output aggregates.
- Investors also may take forward exchange positions ex ante: these pay off when shocks are realized. Let  $S$  be the spot exchange rate (Home price of Foreign currency) and  $F$  the corresponding forward exchange rate. The domestic-currency payoff to purchasing a unit of Foreign currency forward at the rate  $F$  is  $S - F$ .
- There again are iceberg costs of international trade. Now, starred prices are measured in Foreign currency. With producer currency pricing (unlike in EM), commodity prices are

$$P_H^* = \frac{P_H}{S(1-t)}, \quad P_F = \frac{SP_F^*}{1-t}.$$

- The output condition in the goods markets can be written as

$$\frac{Y_H}{Y_F} = \left( \frac{SP_F^*}{P_H} \right)^\eta \left[ \frac{1 + \beta(\alpha, t) \left( \frac{SP^*}{P} \right)^\eta \left( \frac{C^*}{C} \right)}{\beta(\alpha, t) + \left( \frac{SP^*}{P} \right)^\eta \left( \frac{C^*}{C} \right)} \right],$$

where  $\beta(\alpha, t) = \left[ \frac{\frac{1}{2} - (\alpha - \frac{1}{2})}{\frac{1}{2} + (\alpha - \frac{1}{2})} \right] (1-t)^{\eta-1}$  as before.

- This has the familiar log-linearization near the symmetric equilibrium,

$$\widehat{P_H Y_H} - \widehat{SP_F^* Y_F} = \widehat{L} - \widehat{S} - \widehat{L^*} = (\psi - 1) \widehat{\tau},$$

where  $\tau \equiv SP_F^*/P_H$  and  $\psi \equiv (1 - \lambda^2)\eta + \lambda^2\left(\frac{1}{\rho}\right)$ .

- Dynamics of overall price levels (near the symmetric equilibrium):

$$\hat{P} = \frac{1}{1+\beta}\hat{P}_H + \frac{\beta}{1+\beta}\left(\hat{S} + \hat{P}_F^*\right), \quad \hat{P}^* = \frac{1}{1+\beta}\hat{P}_F^* + \frac{\beta}{1+\beta}\left(\hat{P}_H - \hat{S}\right).$$

- EM show that international equity trade plus forward exchange trade yield locally complete markets. Here I work backward from the *assumption* of complete markets. The Backus-Smith condition is now

$$\widehat{PC} - \widehat{SP^*C^*} = \left(1 - \frac{1}{\rho}\right)\left(\hat{P} - \hat{S} - \hat{P}^*\right) = -\lambda\left(1 - \frac{1}{\rho}\right)\hat{\tau}.$$

- Key linkages between consumption and real balances:

$$\hat{C} = \frac{1}{\rho}\left(\hat{M} - \hat{P}\right), \quad \hat{C}^* = \frac{1}{\rho}\left(\hat{M}^* - \hat{P}^*\right).$$



- Combination of these with the Backus-Smith conditions gives:

$$\widehat{S} = \widehat{M} - \widehat{M}^*.$$

## Optimal portfolios

- To solve for the optimal portfolio, start with the ex post Home budget constraint:

$$PC = WL + \phi\Pi + (1 - \phi)S\Pi^* + \Omega(S - F),$$

Here,  $\phi$  is the equity portfolio share invested in Home firms and  $\Omega$  is the number of units of foreign currency bought forward.<sup>9</sup> Foreign's constraint, expressed in domestic prices, is

$$SP^*C^* = SW^*L^* + \phi S\Pi^* + (1 - \phi)\Pi - \Omega(S - F).$$

- Log differentiation (using  $\Pi = \frac{1-\delta+\delta\sigma}{(\sigma-1)(1-\delta)}WL$  and remembering that money wages are predetermined) yields

$$\begin{aligned}\widehat{PC} - \widehat{SP^*C^*} &= \frac{(\sigma-1)(1-\delta)}{\sigma} \left( \widehat{L} - \widehat{SL^*} \right) + \frac{(2\phi-1)(1-\delta+\sigma\delta)}{\sigma} \left( \widehat{\Pi} - \widehat{S\Pi^*} \right) + 2\widetilde{\Omega}\widehat{S} \\ &= \left[ \frac{(\sigma-1)(1-\delta) + (2\phi-1)(1-\delta+\sigma\delta)}{\sigma} \right] \left( \widehat{L} - \widehat{SL^*} \right) + 2\widetilde{\Omega}\widehat{S},\end{aligned}$$

where  $\widetilde{\Omega}$  is the share of initial consumption expenditure devoted to forward foreign exchange purchases.

- On the other hand, from the preceding pricing equation,

$$\widehat{P_H} = \delta\widehat{L} - (1-\delta)\widehat{A}, \quad \widehat{P_F^*} = \delta\widehat{L^*} - (1-\delta)\widehat{A^*}.$$

- So

$$\begin{aligned}\widehat{L} - \widehat{SL^*} &= (\psi - 1)\widehat{\tau} \\ &= (\psi - 1) \left[ (1-\delta)\widehat{S} + \delta \left( \widehat{SL^*} - \widehat{L} \right) - (1-\delta) \left( \widehat{A^*} - \widehat{A} \right) \right],\end{aligned}$$

- Solving,

$$\widehat{L} - \widehat{SL}^* = \frac{(\psi - 1)(1 - \delta) \left[ \widehat{S} + \left( \widehat{A} - \widehat{A}^* \right) \right]}{1 + \delta(\psi - 1)}.$$

- Thus, Backus-Smith implies

$$\begin{aligned} \widehat{PC} - \widehat{SP^*C^*} &= -\lambda \left( 1 - \frac{1}{\rho} \right) \left[ (1 - \delta) \widehat{S} + \delta \left( \widehat{SL}^* - \widehat{L} \right) \right. \\ &\quad \left. + (1 - \delta) \left( \widehat{A} - \widehat{A}^* \right) \right] \\ &= -\lambda \left( 1 - \frac{1}{\rho} \right) \frac{(1 - \delta) \left[ \widehat{S} + \left( \widehat{A} - \widehat{A}^* \right) \right]}{1 + \delta(\psi - 1)}. \end{aligned}$$

- Also, from the budget constraints,

$$\widehat{PC} - \widehat{SP^*C^*} = \left[ \frac{(\sigma-1)(1-\delta) + (2\phi-1)(1-\delta+\sigma\delta)}{\sigma} \right] \left\{ \frac{(\psi-1)(1-\delta) \left[ \widehat{S} + (\widehat{A} - \widehat{A}^*) \right]}{1+\delta(\psi-1)} \right\} + 2\widetilde{\Omega}\widehat{S}.$$

- The portfolio solution is  $\widetilde{\Omega} = 0$  and

$$\phi = \frac{1}{2} \left[ 1 - \frac{\sigma\lambda(1 - \frac{1}{\rho}) + (\sigma-1)(1-\delta)(\psi-1)}{(\psi-1)(1-\delta+\sigma\delta)} \right].$$

## Special cases

- Suppose there are neither trade barriers ( $\lambda = 0$ ) nor capital ( $\delta = 0$ ). Then

$$\phi = \frac{1}{2} - \frac{(\sigma - 1)}{2}.$$

There is no home bias; instead,  $\sigma > 1 \Rightarrow$  home investors overweight in foreign equities. Foreign profits are a hedge against labor-income ( $WL$ ) risk because domestic profits are perfectly correlated with  $WL$ . The bigger is  $\sigma$ , the lower are profits, so the bigger the foreign equity position has to be. The case  $\lambda = 0$ ,  $\delta > 0$  generalizes:

$$\phi = \frac{1}{2} \left[ 1 - \frac{(\sigma - 1)(1 - \delta)}{1 - \delta + \sigma\delta} \right].$$

In the limit as  $\sigma \rightarrow \infty$ ,  $\phi \rightarrow 1 - \frac{1}{2\delta}$ . For example, if  $\delta = \frac{1}{2}$ ,  $\phi = 0$ : there is a full equity swap to hedge labor income risk. These findings recall the flexible-price Baxter-Jermann (1997) findings.

- Next take  $\delta = 0$  again but let the trade costs become very large:  $\lambda \rightarrow 1$ . This can be thought of as approaching the case studied by EM, in that the pricing-to-market they assume presupposes complete segmentation of product markets at the consumer level. In this case  $\psi \rightarrow \rho^{-1}$  and  $\phi \rightarrow 1$ . There is full home bias, as EM predict based on their model.
- For the “reasonable” intermediate cases with  $\psi > 1$  and  $\rho > 1$ , the formula predicts  $\phi < \frac{1}{2}$ . The graph of  $\phi$  against trade costs looks just like the one in Coeurdacier’s flex-price endowment model. Nontradables, with sector-specific cost shocks, would make a difference.
- Because the only shock affecting relative income and returns is the monetary/real composite  $\hat{S} + (\hat{A} - \hat{A}^*)$ , there is no role for forward trades in achieving complete markets. Thus,  $\tilde{\Omega} = 0$ .

## The Engel-Matsumoto assumptions

Now we have PTM with local-currency pricing, flexible wages, full consumer-market segmentation, but equal weights of  $\frac{1}{2}$  on the Home and Foreign goods in the total consumption index  $C$ . I continue to assume fixed stocks of capital.

- With fixed CPIs the Backus-Smith condition is very simple:

$$\widehat{PC} - \widehat{SP^*C^*} = -\left(1 - \frac{1}{\rho}\right)\widehat{S}, \quad \text{where } \widehat{S} = \widehat{M} - \widehat{M^*}.$$

- Labor-market clearing conditions are

$$\widehat{A} + (1 - \delta)\widehat{L} = \frac{1}{2}(\widehat{C} + \widehat{C^*}) = \widehat{A^*} + (1 - \delta)\widehat{L^*}.$$

- Profit dynamics (around an initial symmetric equilibrium) are

$$\hat{\Pi} = \left( \frac{\sigma}{1-\delta+\sigma\delta} \right) \frac{\hat{C}}{2} + \left( \frac{\sigma}{1-\delta+\sigma\delta} \right) \frac{\hat{S} + \hat{C}^*}{2} - \frac{(\sigma-1)(1-\delta)}{(1-\delta+\sigma\delta)} \left( \hat{W} + \hat{L} \right),$$

$$\begin{aligned} \hat{S}^* + \hat{\Pi} &= \left( \frac{\sigma}{1-\delta+\sigma\delta} \right) \frac{\hat{C}}{2} + \frac{1}{2} \left( \frac{\sigma}{1-\delta+\sigma\delta} \right) \frac{\hat{S} + \hat{C}^*}{2} \\ &\quad - \frac{(\sigma-1)(1-\delta)}{(1-\delta+\sigma\delta)} \left( \hat{S}^* + \hat{W} + \hat{L} \right). \end{aligned}$$

- From the money markets,

$$\hat{C} = \frac{1}{\rho} \hat{M}, \quad \hat{C}^* = \frac{1}{\rho} \hat{M}^*.$$

- Finally, from the condition for optimal wage setting,

$$\hat{W} = \vartheta \hat{L} + \rho \hat{C}, \quad \hat{W}^* = \vartheta \hat{L}^* + \rho \hat{C}^*.$$



- The Home and Foreign budget constraints are as in the last model. However, wages are no longer predetermined:

$$\begin{aligned}
\widehat{PC} - \widehat{SP^*C^*} &= \frac{(\sigma-1)(1-\delta)}{\sigma} \left( \widehat{W} + \widehat{L} - \widehat{SW^*} - \widehat{L^*} \right) \\
&\quad + \frac{(2\phi-1)(1-\delta+\sigma\delta)}{\sigma} \left( \widehat{\Pi} - \widehat{S\Pi^*} \right) + 2\widetilde{\Omega}\widehat{S} \\
&= \frac{2(1-\phi)(\sigma-1)(1-\delta)}{\sigma} \left( \widehat{W} + \widehat{L} - \widehat{S} - \widehat{W^*} - \widehat{L^*} \right) \\
&\quad + 2\widetilde{\Omega}\widehat{S}.
\end{aligned}$$

- *Note:* The revenue component of profits can never differ between countries' firms in this model, because consumption preferences are internationally uniform and PTM with local-currency pricing precludes an expenditure-switching role for the exchange rate. Only labor costs can differ ex post.

- Solving for consumption and wage changes in terms of money and labor input yields:

$$\widehat{PC} - \widehat{SP^*C^*} = \frac{2(1 - \phi)(\sigma - 1)(1 - \delta)(1 + \vartheta)}{\sigma} \left( \widehat{L} - \widehat{L^*} \right) + 2\widetilde{\Omega}\widehat{S}.$$

- Compare this expression with this model's Backus-Smith conditions. It is immediate that

$$\widetilde{\Omega} = -\frac{1}{2} \left( 1 - \frac{1}{\rho} \right),$$

$$\phi = 1,$$

the portfolio derived by EM.

- *Intuition for  $\phi$* : In this model, as pointed out on the last slide, Home and Foreign firms' *revenues* are perfectly correlated. So if a Home investor chooses  $\phi = 1$ , no diversification benefits are lost from failing to diversify the revenue component or firm profits, whereas *labor costs* are perfectly correlated with wage income, so that the domestic firm provides a perfect hedge for human capital risk. Contrary to Baxter and Jermann (1997), as well as the last model, profits and the wage bill are perfectly negatively correlated in the EM model.<sup>10</sup>
- *Intuition for  $\tilde{\Omega}$* : The forward position transfers just enough income between countries to optimally share the risk of relative price-level changes. (These occur only through exchange-rate changes here.) Home's long position in domestic currency (for  $\rho > 1$ ) finances the requisite rise in Home spending when  $S$  falls (a domestic currency appreciation, which lowers Foreign's relative price level).

- As EM observe, because relative domestic consumption falls when  $M$  falls (recall that  $\hat{C} = \frac{1}{\rho}\hat{M}$ ), and the Home currency appreciates in that state of nature, a forward position that is long in domestic currency will yield a positive insurance payoff to Home investors when their consumption is relatively low. This correlation pattern makes domestic bonds an effective hedge against domestic consumption risk — giving rise to extreme home-currency bias when  $\rho > 1$ .
- **However**, related to an apparent puzzle raised above, this prediction contradicts the fact that, empirically, the U.S. receives an unexpected money transfer from abroad when the dollar unexpectedly depreciates, not when it appreciates. As a nation, the U.S. is short on dollars and long on foreign exchange, notwithstanding home currency preference. Not so for "original sinners."

## 9. A general framework with multiple shocks

A useful general framework for understanding the implications of risk sharing through both bonds and equity is provided by Coeurdacier and Gourinchas (2009), hereafter CG. It is critical from a theoretical perspective, as well as for realism, to allow for *additional shocks beyond productivity shocks*, as indeed the sticky-price models do (by incorporating monetary alongside real disturbances).

In the CG model, agents may trade equity shares as well as bonds indexed to the CPI.

Key finding: with multiple shocks, real exchange rate hedging is done primarily through the *bond* portfolio.

**Assumptions**

The same as in the Coeurdacier model (with all goods tradable) but with the following modifications:

- 1. For simplicity trade costs  $t = 0$ , with home consumption bias coming entirely from mirror-symmetric preferences such that  $\alpha > \frac{1}{2}$ .
- 2. A share  $\delta$  of the output process is tradable; the fraction  $1 - \delta$  must be retained at home.
- 3. People can trade CPI-indexed bonds.
- 4. Shocks other than productivity shocks potentially influence asset returns.

First consider the implications of assumptions 1 – 3 above.  
(Because all goods will be tradable, I now omit  $T$  subscripts.)

If  $t = 0$ , then  $P_H = P_H^*$  and  $P_F = P_F^*$ , so the overall CPIs for Home and Foreign are

$$P = \left[ \alpha P_H^{1-\eta} + (1 - \alpha) P_F^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad P^* = \left[ \alpha P_F^{1-\eta} + (1 - \alpha) P_H^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

If Home residents (for example) hold a share  $\phi$  of the Home endowment process (by symmetry) a share  $1 - \phi$  of the Foreign process, they receive equity dividends  $\phi \delta P_H Y_H$  and  $(1 - \phi) \delta P_F Y_F$  and "human capital dividends"  $(1 - \delta) P_H Y_H$ .

Home (for example) holds bonds  $B_H$  indexed to the Home CPI. These pay out  $B_H$  units of the Home CPI. By symmetry,  $B_H = B_F^*$ . By market clearing,  $B_H + B_H^* = 0 = B_F + B_F^*$ . Thus, in equilibrium

$$B_H = -B_F = B_F^* = -B_H^* \equiv B.$$

As in the Coeurdacier model we derive the goods market equilibrium condition:

$$\frac{Y_H}{Y_F} = \tau^\eta \left[ \frac{1 + \beta(\alpha, 0) \left( \frac{P^*}{P} \right)^\eta \left( \frac{C^*}{C} \right)}{\beta(\alpha, 0) + \left( \frac{P^*}{P} \right)^\eta \left( \frac{C^*}{C} \right)} \right],$$

where  $\beta(\alpha, 0) = (1 - \alpha)/\alpha$ . Log-differentiation around an initial symmetric steady state with  $Y_H = Y_F = 1$  implies that  $\hat{P} = \alpha \hat{P}_H + (1 - \alpha) \hat{P}_F$  and  $\hat{P}^* = \alpha \hat{P}_F + (1 - \alpha) \hat{P}_H$ , so that if the real exchange rate is defined as

$$e \equiv P^*/P,$$

$$\hat{e} = (2\alpha - 1) \hat{\tau}.$$



By log-differentiating the goods-market equilibrium as before, we get

$$\widehat{P_H Y_H} - \widehat{P_F^* Y_F} = (\psi - 1)\hat{\tau},$$

where  $\psi \equiv (1 - \lambda^2)\eta + \lambda^2\left(\frac{1}{\rho}\right)$  but now,  $\lambda = 2\alpha - 1$ .

Let  $y \equiv Y_H/Y_F$ . We can re-write the last in terms of the differential (Home less Foreign) equity return  $R_S$ :

$$\begin{aligned}\hat{R}_S &= \hat{y} - \hat{\tau} = (\psi - 1)\hat{\tau} \\ &= \left(1 - \frac{1}{\psi}\right)\hat{y}.\end{aligned}$$

Next consider the Home budget constraint. It takes the form

$$PC = \phi\delta P_H Y_H + (1 - \phi)\delta P_F Y_F + (1 - \delta)P_H Y_H + PB - P^*B,$$

with a symmetric constraint for Foreign. The differential form is

$$\widehat{PC} = (\phi\delta + 1 - \delta)(\widehat{P}_H + \widehat{Y}_H) + (1 - \phi)\delta(\widehat{P}_F + \widehat{Y}_F) - B\widehat{e};$$

subtracting this from its Foreign counterpart yields

$$\widehat{PC} - \widehat{P^*C^*} = [\delta(2\phi - 1) + 1 - \delta](\psi - 1)\widehat{\tau} - 2B(2\alpha - 1)\widehat{\tau}.$$

We are almost done once we recall Backus-Smith,

$$\widehat{PC} - \widehat{P^*C^*} = -\left(1 - \frac{1}{\rho}\right)\widehat{e} = -\left(1 - \frac{1}{\rho}\right)(2\alpha - 1)\widehat{\tau},$$

because we can derive optimal portfolios by equating the last two expressions.

***Note a critical point, however: the system is underdetermined, in that the portfolio position  $(\phi, B)$  that creates optimal risk sharing is not unique.***

For example, assume, as in Coeurdacier, that risk sharing is accomplished through equities only, so that  $B = 0$ . In that case,

$$\phi = \frac{1}{2} \left[ 1 - \frac{(1 - \frac{1}{\rho})(2\alpha - 1)}{\delta(\psi - 1)} - \frac{1 - \delta}{\delta} \right].$$

The main difference here is that the nontradable position  $1 - \delta$  is hedged through a correspondingly bigger position in Foreign equity, as in Baxter-Jermann.

On the other hand, let us postulate that  $\phi = 1$  (full home bias) and solve for the endogenous value of  $B$ . Then

$$B = \left( \frac{\psi - 1}{2\alpha - 1} \right) \cdot \frac{1}{2} \left[ 1 + \frac{(1 - \frac{1}{\rho})(2\alpha - 1)}{\psi - 1} \right]$$

(Observe that  $B$  should be interpreted as a proportion of initial  $Y_H$ .)

Notice that when  $\psi > 1$ , an increase in relative domestic output entails a relatively high return on domestic equity, a real currency depreciation, and therefore, a relatively low return on domestic bonds. Indeed, The relative Home bond return is

$$\hat{R}_B = -\hat{e} = -(2\alpha - 1)\hat{\tau} = -\frac{2\alpha - 1}{\psi - 1}\hat{R}_S.$$

When  $\rho > 1$ , relative domestic expenditure should decline when  $\hat{y} > 0$  according to Backus-Smith, so Home must go long in domestic bonds and short in Foreign bonds by just the amount needed to replicate the payoffs from the optimal (long) Foreign equity position when  $B$  is constrained to zero.

Full diversification through bonds alone (cf. Engel-Matsumoto): but again, the false implication (for the U.S.) that we should make a transfer to foreigners as a result of unexpected real depreciation.

## Additional shocks

CG propose an extended model with a general structure in which the relative returns to stocks, bonds, and human capital are subject to an additional shock  $\hat{\varepsilon}$ .

Define the return on nonfinancial wealth (human capital) to be  $(1 - \delta)P_H Y_H$ . Let  $\hat{R}_N$  denote the relative international return on nonfinancial wealth. GC postulate the general structure

$$\hat{R}_S = (\bar{\psi} - 1)\hat{\tau} + \gamma_S \hat{\varepsilon},$$

$$\hat{R}_B = -(2\alpha - 1)\hat{\tau} + \gamma_B \hat{\varepsilon},$$

$$\hat{R}_N = (\bar{\psi} - 1)\hat{\tau} + \gamma_N \hat{\varepsilon},$$

in which the parameter  $\bar{\psi}$  depends on the specific model being studied.

**Example:** If  $\bar{\psi} = \psi$ ,  $\gamma_E = 1$ ,  $\gamma_B = 0$ , and  $\gamma_N = -\frac{\delta}{1-\delta}$ , we have a pure redistributive shock.

GC propose a convenient representation of the model. By taking a linear combination of the equity and bond equations to eliminate  $\hat{\varepsilon}$ , one expresses the real exchange rate change as

$$\hat{e} = (2\alpha - 1)\xi\hat{R}_B - (2\alpha - 1)\frac{\gamma_B}{\gamma_S}\xi\hat{R}_S, \text{ where } \xi^{-1} \equiv \frac{\gamma_B}{\gamma_S}(1 - \bar{\psi}) - (2\alpha - 1).$$

Doing the same with the equity and nonfinancial wealth equations and substituting for  $\hat{\tau}$  using  $\hat{e} = (2\alpha - 1)\hat{\tau}$  leads to

$$\hat{R}_N = (\bar{\psi} - 1)\left(1 - \frac{\gamma_N}{\gamma_S}\right)\xi\hat{R}_B + \left[\frac{\gamma_N}{\gamma_S} - (\bar{\psi} - 1)\left(1 - \frac{\gamma_N}{\gamma_S}\right)\frac{\gamma_B}{\gamma_S}\xi\right]\hat{R}_S.$$

As GC point out, it is illuminating to think of the equilibrium asset return loadings above as regression coefficients. Thus, let

$$\beta_{e,B} = \text{cov}\left(\hat{e}, \hat{R}_B \mid \hat{R}_S\right) / \text{var}\left(\hat{R}_B \mid \hat{R}_S\right),$$

$$\beta_{e,S} = \text{cov}\left(\hat{e}, \hat{R}_S \mid \hat{R}_B\right) / \text{var}\left(\hat{R}_S \mid \hat{R}_B\right),$$

$$\beta_{N,B} = \text{cov}\left(\hat{R}_N, \hat{R}_B \mid \hat{R}_S\right) / \text{var}\left(\hat{R}_B \mid \hat{R}_S\right),$$

$$\beta_{N,S} = \text{cov}\left(\hat{R}_N, \hat{R}_S \mid \hat{R}_B\right) / \text{var}\left(\hat{R}_S \mid \hat{R}_B\right).$$

Then we may rewrite the expressions for  $\hat{e}$  and  $\hat{R}_N$  as:

$$\hat{e} = \beta_{e,B} \hat{R}_B + \beta_{e,S} \hat{R}_S, \quad \hat{R}_N = \beta_{N,B} \hat{R}_B + \beta_{N,S} \hat{R}_S.$$



The advantage of this formulation is seen by plugging it into the equality between the Backus-Smith condition and the relative budget constraint:

$$-\left(1 - \frac{1}{\rho}\right)\hat{e} = \delta(2\phi - 1)\hat{R}_S + (1 - \delta)\hat{R}_N + 2B\hat{R}_B.$$

To make this equality hold for all shock realizations the coefficients on the terms in  $\hat{R}_S$  and the terms in  $\hat{R}_B$  must cancel out separately. Implies two equations in the two unknowns  $B$  and  $\phi$ , with solutions:

$$B = -\frac{1}{2}\left[\left(1 - \frac{1}{\rho}\right)\beta_{e,B} + (1 - \delta)\beta_{N,B}\right],$$

$$\phi = \frac{1}{2}\left[1 - \frac{1}{\delta}\left(1 - \frac{1}{\rho}\right)\beta_{e,S} - \frac{1}{\delta}(1 - \delta)\beta_{N,S}\right].$$

- Both equations show that asset demand depends on the desires to hedge real exchange rate risk and human capital risk.
- The strength of these effects depend on conditional correlations of the asset returns with the real exchange rate and return on human capital. Empirical question – one that can be answered.
- For example: Assuming  $\rho > 1$ , if Home real depreciation is associated with a negative relative return on Home bonds *conditional on the relative return to equity*, then  $\beta_{e,B} < 0$ , leading to a long Home position in Home bonds.
- Equity demand depends on the correlation between the real exchange rate and relative equity returns *conditional on the relative return to bonds*. If the latter is zero, as shown by van Wincoop and Warnock (which requires  $\gamma_B = 0$  here), equities will not be used to hedge real exchange rate risk – it is more efficient to use bonds for that purpose as equities will have a comparative advantage as a hedge for human capital risk.

- If  $\beta_{N,B} > 0$  it is possible that  $B < 0$ , as in the U.S. case. GC look at empirical evidence suggesting this is the case. For example, a story in which high domestic aggregate demand is associated with real appreciation and high output growth, conditional the return on equities.

**Example** Take  $\gamma_B = 0$ . Then  $\hat{e} = -\hat{R}_B$ ,  $\beta_{e,S} = 0$ , and

$$\phi = \frac{1}{2} \left[ 1 - \frac{1}{\delta} (1 - \delta) \frac{\gamma_N}{\gamma_S} \right].$$

Equity holdings depart from the Lucas perfectly pooled benchmark because of the possibility that  $\gamma_N \neq 0$ . In the case of pure redistributive shocks  $\gamma_N/\gamma_S = -\delta/(1 - \delta)$  and  $\phi = 1$ . Holding all the Home equity neutralizes the redistributive shocks; bonds are used to hedge the real exchange rate/terms of trade shocks. Engel-Matsumoto yields basically this result.

Importantly, the instabilities in asset demands (as a function of parameters) are not an issue in this model.

GC show their results are robust to adding nontraded goods. Nontraded goods still help explain the correlation between openness and equity diversification.



## **10. Beyond complete markets**

The assumption of complete markets provides a simple (and therefore useful) benchmark for understanding the elements of international portfolio behavior in a variety of settings. But there is a strong empirical case for relaxing the assumption of complete global asset markets.

Real-world asset markets do not appear to be complete and the Backus-Smith condition (as those authors documented in their 1993 paper) appears to be grossly violated in aggregate data.

In addition, we would like to integrate portfolio behavior with models of current account imbalances and adjustment, models such as those I discussed at the outset of these lectures.

*I maintain that models with complete markets are not well suited to help us understand many aspects of real-world current-account dynamics and adjustment, at least not at the present stage of financial-market evolution. Such imbalances often do seem to involve long-term permanent movements in relative national wealth levels.*

What are the implications of incomplete asset markets? Portfolio choice under incomplete markets had long been *terra incognita*, but now some progress is being made.<sup>11</sup> For example, Gourinchas and Coeurdacier show their portfolio results are robust to market incompleteness.

[Coeurdacier's profit taxes provide one way to prevent perfect risk-sharing, but for realistic parameters, a small tax has minimal effects on everything but portfolios.]

## Some basic evidence

Backus and Smith (1993) tested predictions of their risk-sharing condition on quarterly seasonally-adjusted 1971:I-1990:IV real consumption spending data from industrial-country pairings.<sup>12</sup>

One telling graph from their paper is reproduced below. In their notation  $e_{ij} \equiv p_j/p_i$ , so a rise in  $e_{ij}$  is a real *depreciation* of currency  $i$  relative to currency  $j$ . Optimal risk-sharing (with CRRA preferences, say) means that  $\hat{C}_i - \hat{C}_j = \frac{\hat{e}_{ij}}{\rho}$ .

Empirically, however, faster consumption growth tends to be associated with real *appreciation*. (Perhaps this reflects a prevalence of demand shocks.) Thus, the predicted positive association between mean aggregate consumption growth rates and mean real depreciation rates simply was not in the data.



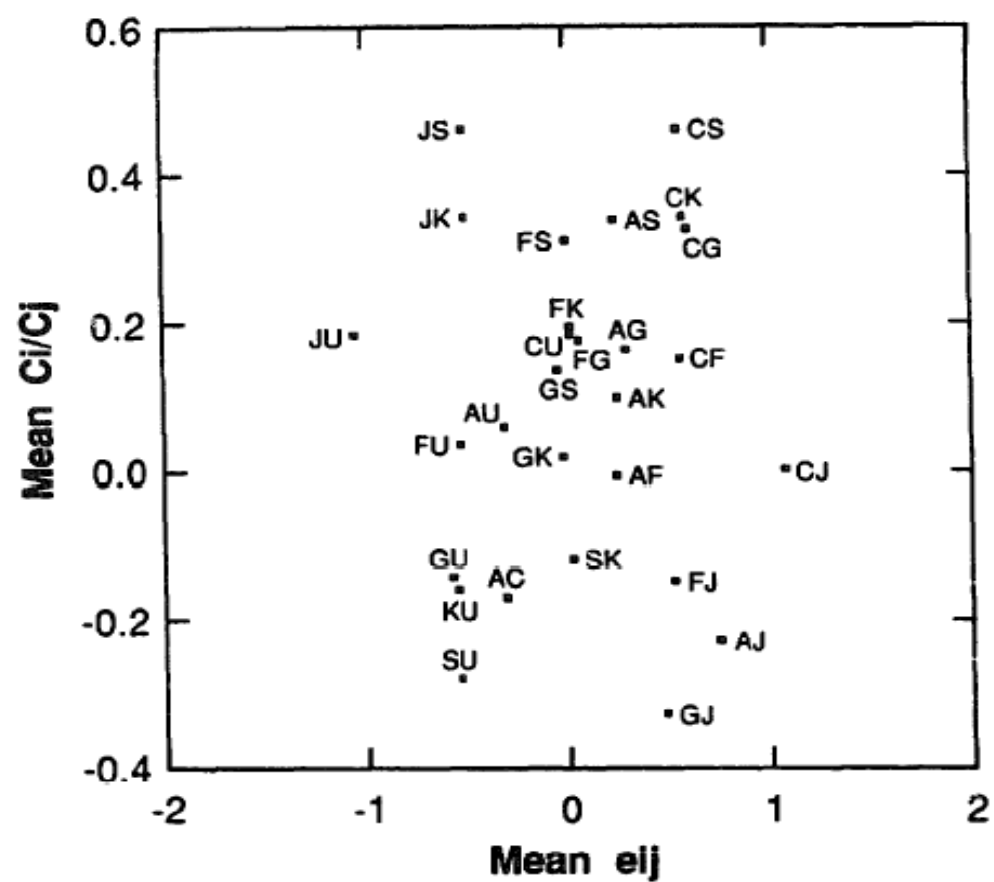
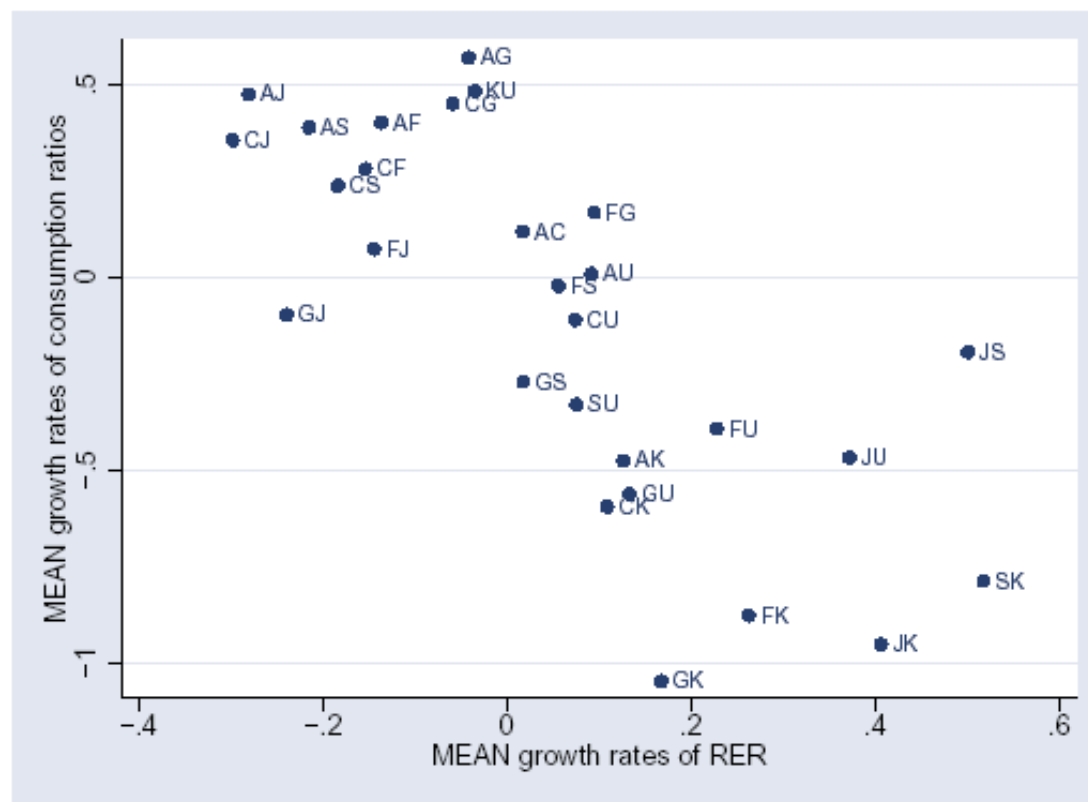
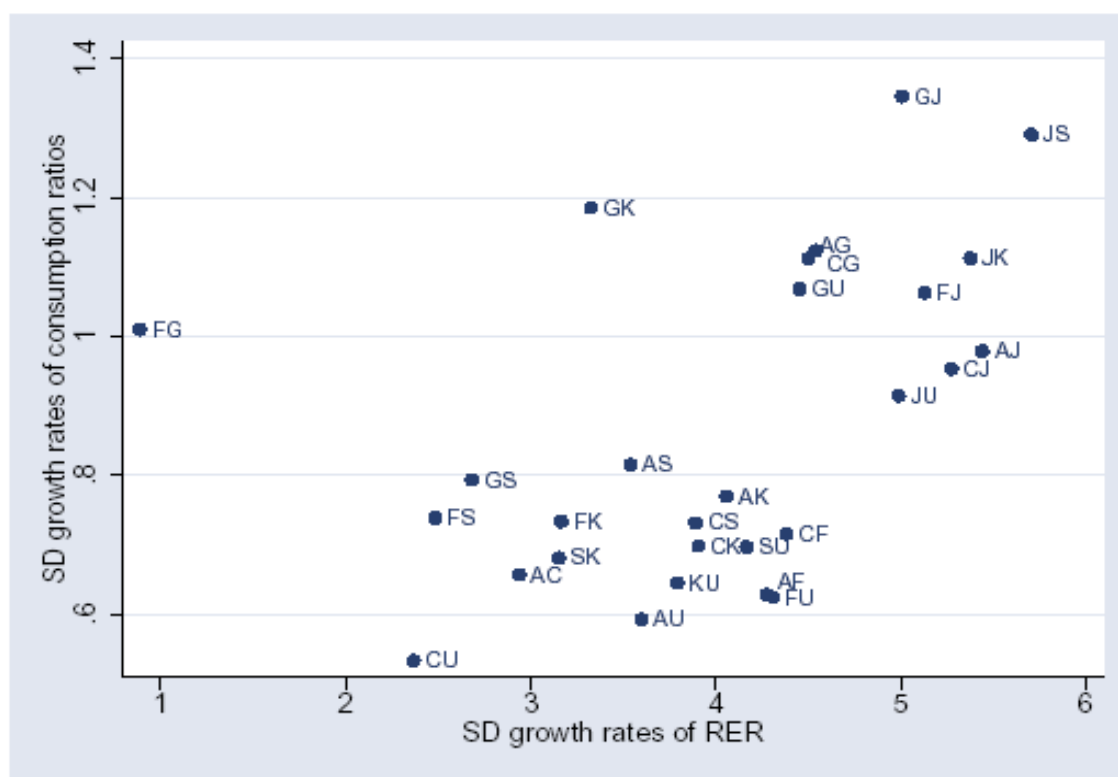


Fig. 3. Means (growth rates).

The risk-sharing condition does no better in more recent data — if anything it does even worse. Instead of the "cloud" of data points that Backus and Smith found, there is a distinct *negative* relationship in data for 1991:I-2006:II (the Swedish data start in 1993:I):



Another empirical failure relates to variability: real exchange rates are much more variable than relative consumption growth rates. While this prediction might be salvaged by assuming a high enough  $\rho$ , the sign reversal documented on the last two slides obviously cannot. (At least the association is positive below!)



## **The consumption-real exchange rate anomaly**

These failures of the Backus-Smith conditions go by the name "consumption-real exchange rate anomaly."

- Proposed resolutions of the anomaly focus on a combination of goods-market and asset-market frictions; Kollmann (1995) is an early study focusing on asset markets.
- In the goods market there may be nontraded goods or less extreme barriers to merchandise trade.
- On the asset side, exogenous limitations on the menu of traded assets (e.g., noncontingent bonds only) or endogenous incentive-compatibility constraints on asset trade volumes.
- Closely related to the “exchange-rate disconnect.”

## **A sampling of recent studies on the anomaly**

- Benigno and Thoenissen (2006)
- Bodenstein (2005)
- Choi (2005)
- Corsetti, Dedola, and Leduc (n.d.)
- Fitzgerald (2006)
- Selaive and Tuesta (2003)
- Coeurdacier, Kollmann, and Martin (2007)

Key challenges are to integrate such realistic models with theories of portfolio behavior, including the relatively unexplored area of currency preference, and with theories of current-account adjustment.

# Notes

\*I thank Pierre-Olivier Gourinchas for helpful discussions and José Antonio Rodríguez-López for excellent research assistance. Research support from UC Berkeley, including support through the Class of 1958 chair and the Coleman Fung Risk Management Center, has been essential and is much appreciated.

1. Coeurdacier assumes instead that  $P_F = (1 + t)P_F^*$ , etc., which will be accurate for an appropriate upscaling of the trade cost  $t$ .
2. An obvious shortcoming of my discussion is that the set of nontradables is taken to be exogenously determined, whereas it might make more sense to think of a range of transport costs for different goods, and the resulting endogenous nontradedness of some of them.
3. To derive the approximation, recall that at the initial symmetric equilibrium,  $P_H Y_H = P_F^* Y_F = P_T C_T$ , while  $P_N Y_N = P_N^* Y_N^*$ . However,

$$\begin{aligned}\frac{P_T C_T}{PC} &= \frac{\gamma \left(\frac{P_T}{P}\right)^{1-\theta}}{\gamma \left(\frac{P_T}{P}\right)^{1-\theta} + (1-\gamma) \left(\frac{P_N}{P}\right)^{1-\theta}} \\ &= \frac{1}{1 + \left(\frac{1-\gamma}{\gamma}\right) \left(\frac{P_N}{P_T}\right)^{1-\theta}},\end{aligned}$$

for example, and by definition,  $\omega = \left(\frac{1-\gamma}{\gamma}\right) \left(\overline{P_N/P_T}\right)^{1-\theta}$ .

4. This argument is, of course, implicit in the BJK paper.

5. At this point there is an apparent contradiction to the intuitive argument in Obstfeld and Rogoff (1996), section 5.5.3. There it is argued that there will be positive diversification into foreign nontradables (i.e.,  $\phi_N < 1$ ) when  $\theta > \frac{1}{\rho}$ , provided that a rise in domestic nontraded output raises the revenue of domestic nontraded firms. We have just observed that when  $\lambda = 0$ , a necessary and sufficient condition for this last assumption to hold is that is that

$$v = \frac{1}{1+\omega} \theta + \frac{\omega}{1+\omega} \left(\frac{1}{\rho}\right) > 1.$$

For  $\rho > 1$ , this does require that  $\theta > 1$ , as stated by Obstfeld and Rogoff, but  $\theta > 1$  is not a sufficient condition. In this case, then, when  $\rho > 1$ , there is indeed positive diversification into Foreign nontradable equities (with  $\phi_N < \frac{1}{2}$ ). When  $\rho > 1$  and  $\theta > \frac{1}{\rho}$  but  $\nu < 1$ , however, the intuitive argument given by Obstfeld and Rogoff goes through in reverse, so Home shorts Foreign equities. For  $\rho < 1$ ,  $\theta > \frac{1}{\rho}$ , there will again be positive diversification into Foreign nontraded equities.

6. A desirable next step is to solve the model under a short-sales constraint.

7. Note the constancy of consumption across the middle row and column of the table below. This is an exact result.

8. Observe that total profits can be expressed as the standard markup over total costs, the latter being the sum of wage costs and shadow capital costs:

$$\Pi = \left( \frac{\sigma}{\sigma - 1} \right) \left[ wL + \left( \frac{\delta}{1 - \delta} \right) wL \right].$$

9. By symmetry, and assuming that  $M = M^*$  initially, it must be true that  $F = 1$ .

10. This perfect negative correlation could be broken by, e.g., home-product



consumption preference. The key role of the correlation between wages and profits in determining the direction of home equity bias has been widely noted. See, for example, Bottazzi, Pesenti, and van Wincoop (1996).

11. The methods described by Evans and Hnatkovska (2005b), Devereux and Sutherland (2006), and Tille and van Wincoop (2008) do, however, offer a promising approach to the exploration of incomplete-market models.

12. As those authors emphasized, because the data points graphed below show multiple partners against each country, the data points cannot be viewed as independent observations. Notwithstanding this fact, as well as other quibbles one might raise about the data and tests, the visual impressions are exceedingly hard to square with any simple form of complete markets. Countries examined: Australia (A), Canada (C), France (F), West Germany (G), Japan (J), Sweden (S), United Kingdom (K), United States (U).

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