Optimal Stabilization Policy in a Model with Endogenous Sudden Stops:*

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Abstract

We develop a framework to study optimal stabilization policy in a small open economy with endogenous "sudden stops" induced by the presence of an occasionally binding credit constraint. The objective of the paper is to characterize the optimal policy outside of the crisis period, but with a possibility that such a crisis may indeed occur (i.e., with a positive probability to run into a sudden stop). In the model, the policy instrument of the government is a distortionary tax wedge on consumption of non-tradable goods that affects directly the real price on nontradable goods (which is the internal component of the real exchange rate in the model). We find that, for a plausible calibration of the model, the optimal policy is highly nonlinear. If the constraint is not binding, the optimal tax rate is zero, as in an economy without credit constraint. If the constraint is binding, the optimal tax rate is negative, meaning that the government subsidizes non tradable consumption in a sudden stop.

1 Introduction

Emerging market countries have experienced periodic crises that cause significant economic turmoil. These episodes, labeled "sudden stops" (Calvo, 1998), are characterized by a sharp reversal in private capital flows, large drops in output and consumption, coupled with large declines in asset prices and the real exchange rate. Progress has been made in understanding optimal policy responses in models in which the economy *is* in a sudden stop.[1] In this paper we address the complementary issue of optimal stabilization policy for an economy that *might be* subject to a sudden stop. Our model can in principle provide direction on how stabilization policy should be designed for both the tranquil periods in which a crisis is only a possibility, as well as periods when the economy actually is in a crisis. There are two distinguishing features of our approach to characterizing optimal policy for actual or potential sudden stops. First, we focus on the precautionary component to optimal policy. That is, at what point before a possible crisis should the government intervene? Should the government wait that the crisis strikes or should it intervene before that point? Second, how does the government's commitment to the optimal policy affect the private sector behavior outside the crisis? Specifically, does this commitment increase welfare even if the crisis never occurs?

To answer these two main questions we investigate optimal stabilization policy in an environment in which access to international capital markets is not only incomplete but might also be suddenly curtailed. To do this we must sacrifice a certain amount of quantitative accuracy for the states of the world when the borrowing constraint binds because of the technical difficulties of solving such a model. That is, while our model is capable of capturing the core features of a sudden stop, it is not rich enough to match the full range of empirical facts typically associated with sudden stops. Fortunately, the existing literature (see footnote 1) has already provided models of policy responses to a variety of existing crises that match

[1] See Braggion, Christiano, Roldos (2007), Caballero and Panageas (2007), Christiano, Gust, and Roldos (2004), Cúrdia (2007), Caballero and Krishnamurthy (2005) and on the optimal (monetary) policy response to these crisis periods.

many features of those particular crises.[2] Our framework, however, is uniquely suited to inform on how to design optimal policies for the normal times in which sudden stops are only a possibility, though we are equally interested in how the commitment to an optimal policy response to an actual sudden stops affects private sector behavior outside of the crisis period.

To our knowledge, in fact, there are no contributions on the analysis of optimal stabilization policy in such an environment, and our work aims at filling this important gap in the literature. Durdu and Mendoza (2005) analyze broad alternative policy strategies in such an environment, but don't characterize optimal policy. Adams and Billi (2006a and b) study optimal monetary policy in a very simple new Keynesian model in which the zero bound constraint is occasionally binding, but their zero-bound constraint is not evolving endogenously as ours. Bordo and Jeanne (2002) and Devereaux and Poon (2004) investigate precautionary components of optimal monetary policy responses to asset prices and sudden stops, respectively, but are not fully specified DSGE models.

In modelling the possibility of a sudden stop, we follow the contributions by Mendoza (2000, 2002) and consider a two-sector (tradable and non-tradable) small open economy in which financial markets are not only incomplete, but also imperfect because access to foreign financing is intermittent and occasionally constrained. We assume that international borrowing cannot be made state-contingent, because the asset menu is restricted to a one period risk-less bond paying off the exogenously given foreign interest rate. In addition, we assume that the international borrowing is constrained by a fraction of households' total income. Therefore, the actual credit limit is endogenous since domestic agents' ability to borrow from foreigners is limited by the evolution of income and prices. In this setting, the occurrence of a 'sudden stop' (i.e. the situation in which the international borrowing constraint becomes binding) is an endogenous outcome of the model depending on the history and state of the economy. In other words, our framework is a DSGE in which, for given initial level of external debt, a long enough sequence of small bad shocks can occasionally push the

[2] The longer run goal of providing a unifying model for policy analysis in emerging markets is to combine our policy framework, which allows for the possibility of a sudden stop, with the richer models of how to respond to a sudden stop that has occured.

economy against a credit constraint and hence into a sudden stop. This credit constraint itself evolves endogenously as income fluctuates due to both exogenous productivity shocks and endogenous choices of labor and capital.

In this class of models, agents self insure against the low-probability but high cost possibility of the sudden stop generated by the occasionally binding credit constraint. This is through precautionary saving and associated accumulation of net foreign assets. Our goal is to explore both the precautionary component in the optimal policy response and how the commitment to optimal policy affects the precautionary savings of the private sector.

To provide clear intuition and insight into both precautionary policy behavior and private sector response to optimal policy we begin with a version of our endogenous sudden stop model with only one source of shocks (i.e. to the tradable goods endowment) that abstracts from capital accumulation and also allows for a non-distortionary financing of the optimal policy through lump-sum transfers. The last section of the paper then extends our analysis to the case of endogenous capital accumulation and distortionary financing of the optimal policy through capital income tax. In both versions of the model, we focus on a tax wedge on non-tradable consumption, with a balanced budget.

There are two main results. First, the optimal stabilization policy is highly non-linear. If the credit constraint is not binding, optimal policy would mimic the one that would arise in an economy without a credit constraint (zero tax rate in our model). Therefore, in the simpler version of the model with no capital accumulation and lump-sum financing, there is no precautionary component in the optimal policy. If instead the credit constraint is binding, the optimal tax rate is negative, meaning that the government subsidizes non-tradables consumption, thereby supporting both the demand and the supply of non-tradable goods in the economy, increase the amount of collateral in the economy.

Second, the commitment to implement the optimal policy has important welfare implications. When comparing the solution of the model with and without the optimal policy in the presence of the credit constraint the differences in the private sector behavior are not large. With optimal policy, on average, agents accumulate 3.4 percent more debt than in the economy without the optimal policy. However, this small difference has important welfare implications. This additional debt allows agents to increase consumption, save less (accumulate more foreign debt), and hence forgo less consumption to self insure. In welfare terms, the gain from committing to the optimal policy is non-trivial: the amount one would pay in consumption equivalents to move from a world with the constraint to one without, is about 0.5 percent of consumption, in line with Mendoza (2002); our calculations show that roughly 40 percent of this gain can be captured by committing to the optimal policy. Relative to the size of welfare gains reported in the business cycle literature, this is a significant number.

The paper is also related to other broader literatures that have developed separately. One focuses on financial frictions that may help replicate the main features of the business cycle in emerging market economies—e.g., Mendoza (1991, 2002), Neumeyer and Perri (2005) and Oviedo (2006). A second focuses on the analysis of optimal fiscal and monetary policy in dynamic general equilibrium models (see, for example, Chari and Kehoe, 1999; Schmitt-Grohé and Uribe, 2004). While studies of emerging market business cycles can provide a realistic description of the economic environment in which these economies operate, the question of how policy should be set in such environments, particularly outside of the crisis period, remains open. In contrast, characterization of optimal fiscal and monetary policy in standard open economies may not be appropriate to provide insight on how policy should be set in the environment faced by emerging market economies. Fianlly, our analysis is also related to the growing literature on the interaction between house prices, borrowing constraints and the role of monetary policy (see for example Iacoviello, 2005, and Monacelli, 2007). Most of these works assume that the collateral constraint expressed in terms of the value of the house stock is always binding, while the solution methods we implement would allow for examining the situation in which a borrowing constraint might not bind in equilibrium.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 discusses its calibration and solution. Section 4 contains the main results of the paper and presents the optimal policy rules and associated implications. Section 5 discuses the welfare implications of these results. Section 6 reports on the sensitivity of the results to key parameters. Section 7 develops a model with capital and distortionary financing. Section

8 concludes. Technical details, including on the numerical algorithm we use to solve the model, are in appendix.

2 Model

This section simplifies the two-good, small open, production economy, with a form of liability dollarization and an occasionally binding credit constraint, originally proposed by Mendoza (2002). Compared to that model, we consider only one source of disturbance, to aggregate productivity in the tradable sector of the economy, and we allow for distortionary tax rate on non-tradable consumption. The specification of endogenous discounting is also simplified by assuming that the agents' discount rate depends on aggregate consumption as opposed to the individual one, as in Schmitt-Grohé and Uribe (2003).[3]

2.1 Households

There is a continuum of households $j \in [0,1]$ that maximize the utility function

$$U^j \equiv E_0 \left\{ \sum_{t=0}^{\infty} \exp\left(-\theta_t\right) u\left(C_t^j - z\left(H_t^j\right)\right) \right\}, \tag{1}$$

with C^j denoting the individual consumption basket and H^j the individual supply of labor. We assume that the expected utility includes an endogenous discount factor as:

$$\theta_t = \theta_{t-1} + \beta \ln\left(1 + C\left(C_t^T, C_t^N\right) - z\left(H_t\right)\right)$$
$$\theta_0 = 1,$$

with C denoting aggregate per capita consumption that the individual household takes as given.[4]

[3]So, our formulation corresponds to what Schmitt-Grohé and Uribe (2003) call "endogenous discount factor without internalization. Due to precautionary savings it may not be necessary in the stochastic model.

[4]Endogenous discounting pins down a well defined net foreign asset position in the deterministic steady state of the model.

The functional form of the period utility function is,

$$u\left(C\left(C_t^T,C_t^N\right)-z\left(H_t\right)\right)\equiv\frac{1}{1-\rho}\left(C_t-\frac{H_t^\delta}{\delta}\right)^{1-\rho}, \tag{2}$$

omitting for simplicity the superscript j, and where δ is the elasticity of labor supply with respect to the real wage, and ρ is the coefficient of relative risk aversion. The consumption basket C is a composite of tradable and non-tradables goods:

$$C_t\equiv\left[\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{\frac{\kappa-1}{\kappa}}+(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}}. \tag{3}$$

The parameter κ denotes the elasticity of intratemporal substitution between consumption of tradable and nontradable goods, while ω represents a weighting factor. The corresponding aggregate price index is given by

$$P_t=\left[\omega+(1-\omega)\left(P_t^N\right)^{1-\kappa}\right]^{\frac{1}{1-\kappa}};$$

the price of tradables is normalized to 1.

Households maximize utility subject to the following period budget constraint expressed in units of tradable consumption (where again for simplicity we omit the superscript j):

$$C_t^T+\left(1+\tau_t^N\right)P_t^NC_t^N=\pi_t+W_tH_t-B_{t+1}-(1+i)\,B_t-T_t^T-P_t^NT^N, \tag{4}$$

where W_t is the real wage, B_{t+1} denotes the amount of bonds issued with gross real return $1+i$, τ_t^N is a distortionary taxes on non-tradables consumption, and T^T and T^N are lump sum taxes in units of tradables and non-tradables, respectively. π_t represents per capita firm profits and W_tH_t represents the household labor income.

International financial markets are incomplete and access to them is also imperfect. International borrowing cannot be made state-contingent, because the asset menu includes only a one period bond denominated in units of tradable consumption, paying off the exogenously given foreign interest rate. In addition, we assume that the amount that each individual can borrow internationally is limited by a fraction of his current total income:

$$B_{t+1}\geqslant-\frac{1-\phi}{\phi}\left[\pi_t+W_tH_t\right]. \tag{5}$$

This constraint (5) depends endogenously on the current realization of profits and wage income. One important feature of (5), is that it captures the effects of liability dollarization since foreign borrowing is denominated in units of tradables while part of the income that serves as a collateral comes from the nontradables sector. Roughly speaking, this constraint assumes that only a fraction of current income can be effectively claimed in the event of default, so lenders are unwilling to permit borrowing beyond that limit.

As emphasized in Mendoza (2002), this form of liquidity constraint shares some features, namely the endogeneity of the risk premium, that would be the outcome of the interaction between a borrower and a risk-neutral lender in a contracting framework as in Eaton and Gersovitz (1981). However, it is not derived as the outcome of an optimal credit contract.

Households maximize (1) subject to (4) and (5) by choosing C_t^N, C_t^T, B_{t+1}, and H_t. The first order conditions of this problem are the following:[5]

$$\frac{C_{C_t^N}}{C_{C_t^T}} = \left(1+\tau_t^N\right) P_t^N, \tag{6}$$

$$u_{C_t} C_{C_t^T} = \mu_t, \tag{7}$$

$$\mu_t + \lambda_t = \exp\left(-\beta \ln\left(1 + C\left(C_t^T, C_t^N\right) - z\left(H_t\right)\right)\right)\left(1+i\right) E_t\left[\mu_{t+1}\right], \tag{8}$$

and

$$z_H(H_t) = C_{C_t^T} W_t \left[1 + \frac{\lambda_t}{\mu_t}\frac{1-\phi}{\phi}\right]. \tag{9}$$

μ_t and λ_t are the multipliers on the budget and liquidity constraint, respectively. As usual, the relevant transversality conditions are assumed to be satisfied. Equation (6) determines the optimal allocation of consumption across tradable and nontradable goods by equating the marginal rate of substitution between C_t^N and C_t^T with the relative price of non-tradable, and distortionary taxation in the nontradable sector (7) determines the multiplier μ_t in terms

[5]We denote with $C_{C_t^N}$ the partial derivative of the consumption index C with respect to non-tradable consumption. u_C denotes the partial derivative of the period utility function with respect to consumption and z_H denotes the derivative of labor disutility with respect to labor.

of the marginal utility of tradable consumption. Equation (8) is obtained from the optimal choices of foreign bonds. Note when the multiplier on the international borrowing constraint is positive (i.e. the constraint is binding), the standard Euler equation incorporates a term λ_t that can be interpreted as country-specific risk premium on external financing. Finally (9) determines the optimal supply of labor as a function of the relevant real wage and the multipliers. It is important to note that the presence of the international borrowing constraint increases the marginal benefit of supplying one unit of labor since this improves agents' borrowing capacity.

2.2 Firms

Our small open economy is endowed with a stochastic stream of tradable goods, $\exp(\varepsilon_t^T)Y^T$, where ε_t^T is a random Markov disturbance, and produces non-tradable goods, Y^N. Unlike Mendoza (2002), we assume that ε^T follows a standard autoregressive process of the first order, AR(1). However, we abstract from other sources of macroeconomic uncertainty, such as shocks to the technology for producing non-tradables, the world interest rate, and the tax rate for simplicity.

Firms produce non-tradables goods Y_t^N based on the following Cobb-Douglas technology

$$Y_t^N = AK^\alpha H_t^{1-\alpha},$$

where K is a constant level of capital stock, and A is a scaling factor.[6] Since capital stock is given, the firm's problem is static and current-period profits (π_t) are:

$$\pi_t = \exp\left(\varepsilon_t^T\right) Y^T + P_t^N AK^\alpha H_t^{1-\alpha} - W_t H_t.$$

The first order condition for labor demand, in equilibirum , gives:

$$W_t = (1-\alpha)\, P_t^N AK^\alpha H_t^{-\alpha}, \tag{10}$$

so the real wage (W_t) is equal to the value of the marginal product of labor.

[6]We analyze a model with endogenous capital accumulation in the last section of the paper.

2.3 Government

The government runs a balanced budget in each period, so that the consolidated government budget constraint is given by

$$\exp(G_t^T) + P_t^N \exp\left(G_t^N\right) = \tau_t^N P_t^N C_t^N + T_t^T + P_t^N T_t^N.$$

Stabilization policy is implemented by means of a distortionary tax rate τ_t^N on private domestic non-tradables consumption.[7]

Movements in the primary fiscal balance are offset via lump-sum rebates or taxes. Specifically, we assume that the government keeps a constant level of non-tradable expenditure financed by a constant lump-sum tax (i.e. $\exp\left(G^N\right) = T^N$). Thus, changes in the policy variable τ^N are financed by a combination of changes in the lump-sum transfer on tradables, T_t^T, and the endogenous response of the relative prices, for given public expenditures on tradable and non-tradables. This simplifying assumption implies that we abstract from the important practical issue of how to finance changes in the tax rate in the case in which they are negative (i.e., $\tau^N < 0$ is a subsidy). This allow us to focus on the implications of the occasionally binding constraint for the design of the tax policy abstracting from other optimal tax policy considerations. We study the implication of distortionary financing of the optimal policy in the last section of the paper.

2.4 Aggregation and equilibrium

We now consider the aggregate equilibrium conditions. Combining the household budget constraint, government budget constraint, and the firm profits we have that the aggregate constraint for the small open economy can be rewritten as

$$C_t^T + P_t^N C_t^N + B_{t+1} = \exp\left(\varepsilon_t^T\right) Y^T + P_t^N Y_t^N + (1+i)\, B_t - \exp\left(G_t^T\right) - P_t^N \exp\left(G_t^N\right),$$

[7] Mendoza and Uribe (2000) emphasize how movements in this tax rate can approximate some of the effects induced by currency depreciation in monetary models of exchange rate determination. As such, it captures one important aspect of monetary policy in emerging markets, which is distinct from the more conventional role of monetary policy in the presence of nominal rigidities.

and the equilibrium condition in the non-tradeable good sector is

$$C_t^N + \exp\left(G_t^N\right) = Y_t^N = AK^\alpha H_t^{1-\alpha}. \tag{11}$$

Combining these two equations we have

$$C_t^T = Y_t^T - \exp(G_t^T) - B_{t+1} + (1+i)\,B_t, \tag{12}$$

that represents the evolution of the net foreign asset position as if there were no international borrowing constraint. In this model though, using the definitions of firm profit and wages, the liquidity constraint implies that the amount that the country as a whole can borrow is constrained by a fraction of the value of its GDP:

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi}\left[\exp\left(\varepsilon_t^T\right) Y^T + P_t^N Y^N\right]. \tag{13}$$

3 Calibration and solution

In this section we discuss the calibration of model parameters and the solution method.

3.1 Calibration

The calibration of the model is reported in Table 1 and largely follows Mendoza (2002), who calibrates his model to the Mexican economy.[8] We normalize the calibration by setting $Y_T = 1$ and $p_N = 1$. We follow Mendoza in setting the interest rate at $i = 0.0159$, which yields an annual real rate of interest of 6.5 percent). The elasticity of intratemporal substitution between tradables and nontradables follows from Ostry and Reinhart (1992) who estimates a value of $\kappa = 0.760$ for developing countries.[9] The elasticity of labor supply for non-tradable sector is unitary so that $\delta = 2$ while the elasticity of intertemporal substitution is set to $\rho = 2$

[8] We use mostly his calibration despite the fact that our analysis does not aim at replicating the feature of Mexican business cycle as Mendoza (2002) but rather at using the model for examining policy implications.

[9] These values are comparable to the ones adopted by Kehoe and Ruhl (2005) in their calibration on Mexico. The real interest rate in Kehoe and Ruhl (2005) is set to 5% per year while the elasticity of intratemporal substituion is $\kappa = 0.5$.

which is a value compatible with many studies in the real business cycle literature.[10] The value of the liquidity parameter determines the tightness of the constraint in the deterministic steady state: as in Mendoza (2002) ϕ is set at $\phi = 0.74$ so that the economy is close to the binding region in the deterministic steady state.[11]

The labor share of production in the non tradable output sector is $\alpha = 0.636$. We then set β, ω, and AK^{α} to obtain the steady state foreign borrowing to GDP ratio of 35 percent (this corresponds to the estimates obtained by Lane and Milesi Ferretti (1999) for the period 1970-1997), a steady state ratio of tradable to non-tradables output of 64.8 percent, and the steady state relative price of non-tradables equal to one. The implied discount factor is $\beta = 0.0177$, slightly lower than in Mendoza (2002) because of the different specification of endogenous discounting. Government spending is set as 1.7 percent of output in the tradable sector and 14.1 percent of output in the nontradable sector.[12] While the tax rate on nontradable consumption in the steady state, and in the model without optimal policy, is fixed at $\tau = 0.0793$.

In our analysis, for simplicity, we focus on the behavior of the economy following a stochastic shock to tradeable output, which we model as an AR(1) process. Specifically, the shock process is ε_t,

$$\varepsilon_t = \rho_{\varepsilon}\varepsilon_{t-1} + \sigma_n n_t,$$

where n_t is an iid N(0,1) innovation, and σ_n is a scaling factor. The parameters of the AR(1) process are chosen to match the standard deviation and serial correlation of tradeable output in Mexico of 3.36 percent and 0.553, respectively. These are the same moments that Mendoza (2002) matched with a discrete two state Markov chain.

[10] Since estimates for the elasticity of labor supply in the non-tradable sectore are not available for Mexico, Mendoza (2002) sets it to unitary value.

[11] As in Mendoza (2002), for the borrowing constraint to bind in the deterministic steady state ϕ needs to be bigger than 0.741.

[12] To maintain consistency with the model that has capital accumulation later in the paper, a fixed fraction of output in every period is assumed to be investment.

3.2 Solution

In order to compute the competitive equilibrium of the economy without optimal policy (i.e., $\tau_t^N = 0$ for all t), we solve a quasi-planner problem that satisfies the following Bellman equation:

$$V(B_t, \varepsilon_t^T) = \max_{B_{t+1}} \left\{ u\left(C_t - z\left(H_t\right)\right) + \exp\left(-\beta \ln\left(C_t - z\left(H_t\right)\right)\right) E\left[V(B_{t+1}, \varepsilon_{t+1}^T)\right] \right\}. \tag{14}$$

The constraints on this problem are the competitive equilibrium conditions (**??**)-(**??**), the aggregate consumption definition (3), and the credit constraint (5). To solve the constrained problem, we use a spline parameterization for the value function, solve the maximization using feasible sequential quadratic programming methods, and solve for the fixed point using value iteration with Howard improvement step.[13]

In Figure 1 and 2 we compare the policy function for the constrained and unconstrained economy (i.e. the real business cycle small open economy case). Figure 1 plots the equilibrium decision rule (or policy function) for gross foreign borrowing, $B_{t+1} = g\left(B_t, \varepsilon_t^T\right)$, conditional on the value of the tradable shock that corresponds to the negative of the standard deviation of its marginal distribution.[14] This intersects the 45° line at the boundary of the constrained region; that is, if the economy perpetually received this realization of the shock, it would converge to a level of external debt for which the credit constraint is just binding. If the economy happened to find itself in the interior of the constrained region, it would diverge to $B = -\infty$, violating the implicit trasversality condition that requires long-run solvency. Therefore the decision rules must be truncated at the boundary. This divergence would occur in any state in which there exists a positive probability of entering the interior of the constrained region, and this probability is always positive with an AR(1) process.

Figure 2 compares the equilibrium decision rules with and without the credit constraint for the real wage (W_t), the relative price of non tradable (P_t^N), aggregate consumption (C_t), employment (H_t). The first point to observe is that both employment and consumption

[13]The algorithm for solving the problem is described in Appendix B.

[14]As there is only one asset, gross and net foreign liabilities or assets (NFA) coincide.

are lower in the presence of the credit constraint, reflecting precautionary savings driven by the possibility of hitting the credit constraint. Consumption is lower because households save a higher fraction of their income to accumulate foreign assets. Even if the constraint is not binding, the reduction in consumption drives down the relative price of non-tradables because demand of non-tradables falls more than the supply of non tradables since agents save more for precautionary reasons. Supply of non-tradables falls as the negative effect of a reduction in labor demand dominates the positive effect on labor supply of a decline in the relative price of nontradables.

Equilibrium real wages, relative price and labor fall sharply as the economy approaches the region in which the constraint becomes binding since as NFA deteriorates the precautionary motive determines a bigger drop in consumption and the possibility of hitting the constraint amplifies the equilibrium response.

Precautionary saving induced by the occasionally binding credit constraint is quantitatively significant in the model. For instance, the average NFA position in the ergodic distribution of the economy with no collateral constraint is $B = -3.0$ (this corresponds to about -30 percent of annual average GDP), while in the economy with the constraint $B = -2.37$ (or -22 percent of annual average GDP).

This difference is large, considering the small shocks that hit this economy and the relatively low degree of risk aversion. In contrast, Aiyagari (1994) finds that measured uninsurable idiosyncratic earnings risk, which is an order of magnitude larger than the shocks considered here, generates only a 3 percent increase in the aggregate capital stock. The main reason why precautionary savings is larger here is that the return on saving increases as the price of non-tradables falls, since gross real interest rate in terms of consumption good increases. So additional saving does not reduce its return, a mechanism that tends to weaken precautionary balances in a closed economy setting, such as the one studied by Aiyagari (1994).

4 Optimal stabilization policy

Once we have examined the implications of endogenous borrowing constraint for the small open economy, it is natural to ask how stabilization policy should be designed in this environment. Does optimal policy exhibit any precautionary motive? And, how does optimal policy affect private agents' decisions?

To address these questions and solve the optimal tax problem, the only change needed to the solution method discussed in the previous section is to introduce a second control into the problem to (14) —the tax rate on non-tradable consumption, τ^N—yielding the functional equation

$$\widehat{V}_0(B_t, \varepsilon_t^T) = \max_{B_{t+1}, \tau_t^N} \left\{ u(C_t - z(H_t)) + \exp(-\beta \ln(1 + C_t - z(H_t))) E\left[V^n\left(B_{t+1}, \varepsilon_{t+1}^T\right)\right] \right\}. \tag{15}$$

The decision rule for τ^N, as well as the implied lump sum transfer over GDP ($P_t^N T_t^N / Y$), which adjust in order to satisfy the balanced budget rule by the government, are plotted in Figure 3. Figure 4 plots the same decision rules reported in Figure 2, with and without optimal policy for τ^N. Figure 5 plots the policy function for the net foreign asset position with and without the optimal policy, while Figure 6 reports the ergodic distribution of net foreign asset position for the economy without borrowing constraint, the one with borrowing constraint, and the one in which policy expressed in terms of τ^N is optimally set.

Optimal policy has two possible roles in the model.[15] The first role is related to the existence of the occasionally binding constraint, and there are two goals for policy due to the presence of the credit constraint (though not exclusive)—to reduce the probability of reaching the region in which the constraint is binding and to minimize the effects when it binds by increasing the value of the collateral. The second possible role, as in other incomplete market models (such as Aiyagari 1995), is to increase welfare by choosing policies that reduce agents precautionary savings. This role for policy is independent of the presence of the credit constraint and relies only on the general inefficiency of incomplete market models. As we

[15] Our model also features an externality—the endogenous discount factor depends on aggregate consumption, and therefore agents do not internalize the effect of current consumption and labor supply on discounting. But this effect should be minor since the discount factor is nearly inelastic.

will see, there is no scope for this second role for policy in the simple version of the model without capital accumulation and distortionary financing of the optimal policy.

As we can see from Figure 3, the optimal policy schedule, the equilibrium decision rule for the policy instrument τ^N is highly non-linear: in states of the world in which the constraint is not binding (in normal or tranquil times) the optimal policy is "no policy action", i.e., $\tau^N = 0$, while in states of the world in which the constraint is binding (in the "sudden stop" region), the optimal policy is a subsidy to non-tradable consumption, i.e. $\tau^N < 0$.

This result shows that there is no precautionary motive for the optimal policy either related to market incompleteness or to the presence of the borrowing constraint. When the constraint is not binding, policy is set as to minimize the distortion associated with the use of the policy instrument so that $\tau^N = 0$, like in the unconstrained economy. In the model without collateral constraint, there is no policy trade off, and setting $\tau^N = 0$ is always optimal, despite the incompleteness of the international asset market. This is because the tax wedge, τ^N does not affect agents' intertemporal decisions and hence has no role to play to mitigate the consequence of market incompleteness.

If there is the constraint and it is binding, there is a trade-off between efficiency (i.e., to minimize marginal distortions by setting $\tau^N = 0$), and the need to mitigates the effects of the credit constraint. The planner does so by subsidizing non tradable consumption, which increases the value of the collateral in the sudden stop region, "lifting" the decision rules for any level of foreign borrowing, thus relaxing the borrowing constraint. Specifically, with such a subsidy, demand and to a lesser extent supply for non-tradable goods increases, as a result the relative price of non-tradables goods rises, so that the value of the collateral increases. The worsen is the state of the world (in terms of negative net foreign asset position), the bigger is the subsidy required to rise the value of the collateral.

Nonetheless, optimal policy affects private agents' behavior even when the constraint is not binding. Figure 5 shows that optimal policy shrinks the region in which the constraint is binding so that, for a given realization of the shock and initial net foreign asset position, the amount of foreign borrowing allowed before the constraint starts to bind is higher. This effect is due to the interaction between private agents behavior and optimal policy: private agents anticipate policy response in the binding region and they reduce their precautionary saving by increasing consumption, so that P_N is higher in the non-binding region compared to the non optimal policy case. Due to this interaction, optimal policy lowers the likelihood of entering the binding region for each pair (B_t, ε_t^T). As we can see from Figure 4, agents consume more and the equilibrium wages, relative prices and labor are higher than in the non-optimal policy case.

Figure 5 also shows that the optimal policy of τ^N is such that the liquidity constraint becomes "just binding"; that is, the policy function for B_t is tangent to the binding region and the corresponding multiplier λ_t of the liquidity constraint remains 0. The goal of optimal policy is to distort the economy as little as possible, and any deviation of the shadow price of foreign borrowing from zero is costly. Therefore the planner relaxes the constraint just enough to make it non-binding. But the constraint is not relaxed beyond this, because that involves additional distortions that are welfare-reducing.

As Figure 6 shows, the average NFA position in the ergodic distribution of the economy is not affected significantly by optimal policy—the welfare implications, however, are significant as we shall see below. Relative to the no-stabilization case (with the credit constraint), the average net foreign debt in terms of the deterministic steady state GDP increases by 3.4 percent, to B=-2.45 (or 22 percent of average GDP), under the optimal policy. However, the probability of hitting the constraint in the ergodic distribution decreases by about 15 percent, from 0.6 percent without optimal tax policy to 0.5 percent with it.

[The optimal policy is state-contingent, requiring knowledge of the unobservable shocks and net foreign asset position for its implementation. We therefore also explore the impact of simple, constant subsidy rules that are not state contingent, and can be easily financed (meaning relatively small). Figure 9 shows how the constrained regions change moving from the economy without policy to the ones with non-state contingent policy that we consider ($\tau = 0$, $\tau = -0.01$, $\tau = -0.05$). This shows that the fixed tax rule moves the economy in the direction of the optimal policy. Interestingly, this suggests that a small overvalaution, which effectively subsidizes consumption of non-tradable goods, may be a desirable policy option.]

5 Sensitivity analysis

In this section we explore the robustness of the optimal policy results to alternative values of key structural parameters, as well as of the stochastic process for the tradable endowment. From the outset, it is important to mention that none of these changes affect the main result, namely the absence of a precautionary component in the optimal policy. This suggests that the result is a robust qualitative feature of the model. The inner working of the model, as illustrated by the decision rules for the main endogenous variables, is also fairly robust to alternative parameterization.

As the optimal policy hinges on the labor effort behavior and the substitutability between tradable and non-tradable goods in consumption, it is important to consider alternative values for κ and δ. A second set of parameters potentially affecting the working of the model include the degree of risk aversion (ρ), the tightness of the credit constraint in the deterministic steady state (ϕ), and finally the parameters governing the stochastic process for the tradable endowment (ρ_ε and σ_n respectively).

We consider four alternative cases for κ and δ, two higher values and two lower values than assumed in the baseline, changing only one parameter at a time. Specifically, we consider the following alternative cases: $\kappa = .3$ or $\kappa = 0.9$ (less or more substitutability between tradable and non tradable goods in consumption than in the baseline) and $\delta = 1.2$ or $\delta = 5$ (higher and lower labor elasticity than in baseline); and four cases for $\rho = 5$ (more risk aversion), $\phi = 0.5$ (looser constraint in the deterministic steady state and less likely to be occasionally binding), $\rho_\varepsilon = 0.95$ and $\sigma_n = 0.05$ (more persistent or more volatile AR1 process).

The results are summarized in Figure 7. The results are robust, except in the case of a lower labor supply elasticity. When tradables and non-tradables goods become closer substitutes ($\kappa = 0.9$), optimal policy would cut taxes less aggressively compared to the baseline specification. The general principle of optimal policy is to relax the borrowing constraint by increasing the value of collateral when the constraint becomes binding (i.e. by raising $P_t^N Y_t^N$). When the intratemporal elasticity of substitution between tradables and non tradables is higher, it is more efficient to do so by increasing the relative price of non-tradables and decreasing non tradables production compared to the baseline parametrization. Indeed, for a given relative price of non tradables and a given subsidy, a higher substitutability between tradables and non tradables will push the demand for tradable goods higher. Since tradable output is exogenously given, demand needs to be decreased in order to clear the tradable goods market if the economy cannot borrow from abroad. For a relatively higher κ this could be achieved with a relatively lower subsidy. Non-tradables demand will rise relatively more than with a lower κ so that the relative price of non tradables is higher, real wages are lower, and non-tradables production is lower since agents will decrease their labor supply compared to the baseline case. The opposite logic applies in the case in which $\kappa = .3$.

When labor supply becomes more elastic ($\delta = 1.2$), optimal policy would cut taxes more aggressively compared to the baseline specification. In this case it is efficient to relax the borrowing constraint by increasing non-tradables production and decreasing the relative price of non-tradables compared to our parametrization. Indeed, for a given real wage the more elastic is labor supply the higher is production of non-tradables. Equilibrium in the non-tradables goods market is achieved by decreasing the relative price of non-tradables and increasing demand by subsidizing non-tradables consumption more aggressively than in the baseline parametrization. The opposite logic applies in the case in which labor supply is less elastic ($\delta = 5$).

When the constraint is looser ($\phi = 0.5$), the probability that the constraint tends to zero and the economy tends to behave as the unconstrained one.

Higher risk aversion ($\rho = 5$) than the baseline parametrization doesn't make any significant difference in terms of the policy function. On the other hand higher persistence and volatility of the shock of the tradable shock would both imply a higher subsidy in the binding region and a lower level of debt beyond which the constraint starts to bind. Recall here that we are plotting the policy functions conditional on the value of the tradable shock that corresponds to the negative of the standard deviation of its marginal distribution. Increased volatility of the shock for the given state of net foreign asset position requires bigger subsidy since agents rise precautionary saving due to higher uncertainty. Similarly when the shock is more persistent, given a bad realization of the shock the economy is more likely to hit the borrowing limit. This increases precautionary saving and by reducing consumption requires a higher subsidy to relax the constraint in the binding region.

6 Welfare Gains of Optimal Policy

In order to quantify the welfare gains associated with the optimal stabilization policy we compute a "consumption equivalent" measure of welfare in the spirit of Lucas (1987). Specifically, we compute the percent change in the average lifetime consumption, at every date and state, that would leave the stand-in household indifferent between the economy with optimal policy and the benchmark economy.[16] We then compute an overall summary measure by weighting the welfare gain at each state by the probability of being in that state, using the ergodic distribution.

Note that there are two sources of potential welfare gain from the optimal policy in our model. The first is the efficiency gain or loss from altering the tax distortion. The second is welfare gain from mitigating the effects of the credit constraint and reducing the probability of its occurrence. And the former is one order of magnitude larger than latter. To illustrate this we report results from three experiments in which we either remove the credit constraint, or tax distortion, or both.

Table 3 reports the results of various welfare experiments. The gain from eliminating altogether the credit constraint while retaining the tax distortion is 0.5 percent in consumption equivalent terms, consistent with the welfare gain reported by Mendoza (2002). The gain from moving to a zero tax rate regime while retaining the credit constraint is 3.41 percent in consumption equivalent terms. The gain from the joint removal of the tax distortion and the credit constraint, which is an upper bound for the welfare gain from optimal policy, is 3.84 percent.

The gains associated with the optimal stabilization policy are significant. Moving from the benchmark economy to the economy with the optimal policy, average lifetime consump-

[16] To recall, the benchmark economy has a fixed tax rate of 7.93 percent and the occasionally binding credit constraint. Appendix B provides technical details on these calculations. Also note that the computed gain includes the costs and gains associated with the transition from one state to the other.

tion increases by 3.6 percent. Subtracting then the gain from eliminating the tax distortion, which is 3.41 percent, we obtain a 0.2 percent gain from using the optimal policy to mitigate the impact of the credit constraint. This gain represents about 40 percent of the gain from the complete elimination of the constraint.

It is important to note that our welfare gain, which is large by the standards of the cost of business cycle literature, is accounting only part of the potential benefits from optimal policy. The true gain could be even bigger. In our model in fact there is no idiosyncratic risk, only aggregate risk, as markets are complete with respect to risk sharing across agents within the country. Chatterjee and Corbae (2007), who do account for idiosyncratic risk, show that the gains to eliminating the possibility of a crisis state can be as large as seven percent of annual consumption. In their model, households face idiosyncratic risk that is correlated with the aggregate shock, as in İmrohoroğlu (1989). In a crisis state, there is a large increase in the variance of idiosyncratic risk; eliminating this possibility generates enormous welfare gains.[17] Chatterjee and Corbae (2007) focus on measuring the gains to eliminating the risk of a crisis: they use the İmrohoroğlu (1989) environment as a measurement tool without explaining how one would eliminate the crisis.[18] In contrast, we ignore the idiosyncratic risk component and instead explicitly model the policy response used to eliminate the crisis. Properly accounting for the welfare gains documented in Chatterjee and Corbae (2007) would likely increase the number we report in Table 2 substantially. We believe that idiosyncratic risk is important in emerging markets, hence we view the welfare numbers in Table 2 as a lower bound on the welfare gains.[19]

While Table 3 provides valuable measures of the welfare gains from various policies, it obscures the fact that the welfare gain also has a state contingent dimension. The amount the stand-in household would pay to change policy is in fact dependent on the current state

[17] The probability of the crisis state in their model is calibrated to the probability of a great depression in the United States.

[18] Storesletten, Telmer, and Yaron (2001) show that eliminating normal business cycles generates larger gains in a model with countercyclical idiosyncratic risk.

[19] Mendoza, Quadrini, and Ríos-Rull (2007) show how the large current imbalances in global asset markets can be reproduced using a model where developing countries have less-developed markets for idiosyncratic risk.

of the world. In states near the binding constraint the gains can be much larger. Figure 8 plots the welfare gain in each state for the optimal policy rule, and the gains from moving to the optimal rule are much larger for debt levels that are close to the constrained region. One way to interpret this graph is that economies that spend more time near the constraint are going to have bigger gains from the optimal policy, since these states will get larger weights in the ergodic set. Alternatively, the gains from adopting the optimal policy are larger for those countries with higher debt levels, as the conditional probability of a crisis is higher.

7 Capital accumulation and distortionary financing

This section investigates the role that capital may have in affecting the decision rules and the optimal government policy. The model we use has a fixed capital stock. Allowing for investment and endogenous capital accumulation may require an optimal tax response even when the credit constraint is not binding, and hence introduce a precautionary component in the optimal policy.

The introduction of capital also allows us to investigate alternative specifications of the budget rule and the financing of the optimal policy. In equilibrium, the amount of financing needed to implement the optimal policy is potentially large, and in our model the financing entailed by the (large) consumption subsidy is costless. Hence, a large subsidy can be applied right when the constraint binds. If the budget is balanced by a distortionary capital tax rather then a lump-sum transfer, the government may find it optimal to start using the consumption subsidy before the credit constraint is hit. If there is an increasing cost of the consumption subsidy, however, the government may want to start with a smaller subsidy away from the constraint that increases as the constraint is approached. Financing the subsidy through a distortionary capital tax is one way to capture this increasing cost.[20]

[20] In practice, the financing of the expansionary government policy in response to a sudden stop is likely to come from the drawdown of accumulated official reserves, thereby supporting the real exchange rate and the non-tradable sector of the economy. Vice versa, a small precautionary component would result in the accumulation of official reserves over time. However, studying the optimal level of official reserves requires and multiple asset framework, which is behind the scope of this paper.

7.1 Model changes

To incorporate capital accumulation and capital income taxation, we modify the household and firm problem as follows. Households continue to maximize (1), subject to a modified period budget constraint (still expressed in units of tradable consumption, and omitting the superscript j for simplicity):

$$\begin{aligned} & C_t^T + \left(1+\tau_t^N\right) P_t^N C_t^N + \left(K_t - \left(1+\Phi\left(\frac{x_t}{K_{t-1}}\right)\right) K_{t-1}\right) \\ = \; & \left(1-\tau_t^K\right)\left(r_t-\delta\right) K_{t-1} + W_t H_t - B_{t+1} - (1+i) B_t - T_t^T - P_t^N T^N \end{aligned}$$

where τ_t^K is a distortionary tax on capital income, and K_{t-1} represents per capita stock of capital that the household owns and rents to firms in period t, δ is the depreciation rate, and $\Phi(.)$ is a cost of installing investment goods, with x_t representing gross investment.[21]

The credit constraint becomes:

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi}\left[\left(1-\tau_t^K\right)\left(r_t-\delta\right) K_{t-1} + W_t H_t\right]. \tag{16}$$

The international borrowing constraint depends on profits as it did in the model without capital accumulation. The difference is that the return to capital is now potentially time varying.

Adding capital accumulation adds a first order condition for K_t, the amount of capital to carry to next period:

$$\begin{aligned} \mu_t\left(1+\phi'\left(\frac{x_t}{K_{t-1}}\right)\right) &= \exp\left(-\beta \ln\left(1+C\left(C_t^T, C_t^N\right)-z\left(H_t\right)\right)\right) \\ &\left\{\begin{array}{c} E_t\left[\mu_{t+1}\left(\left(1-\tau_{t+1}^K\right)\left(r_{t+1}-\delta\right)+1+\phi\left(\frac{x_{t+1}}{K_t}\right)-\phi'\left(\frac{x_{t+1}}{K_t}\right)\frac{K_{t+1}}{K_t}\right)\right] \\ +E_t\left[\frac{1-\phi}{\phi}\lambda_{t+1}\left(\left(1-\tau_{t+1}^K\right)\left(r_{t+1}-\delta\right)\right)\right] \end{array}\right\}, \end{aligned}$$

where $\Omega_{t+1} = (1-\delta) + \phi\left(\frac{x_{t+1}}{K_t}\right) - \phi'\left(\frac{x_{t+1}}{K_t}\right)\left(\frac{x_{t+1}}{K_t}\right)$. All other first order conditions are unchanged.

[21] The function $\phi(.)$ is such that, in the deterministic, steady state $\phi(.) = x/k$, $\phi'(.) = 1$, and $\phi''(.) < 0$.

The firms' production functions are unchanged, but capital is now endogenous. So, the typical firm maximises its profit by choosing the amount of labor and capital to demand:

$$\max_{K_t, H_t} \left[\exp\left(\varepsilon_t^T\right) Y^T + P_t^N Y_t^N - W_t H_t - r_t K_{t-1}\right]$$

subject to the production function:

$$Y_t^N = A K_{t-1}^{\alpha} H_t^{1-\alpha}$$

The corresponding first order conditions are now given by:

$$W_t = (1-\alpha) P_t^N A_t \left(\frac{K_{t-1}}{H_t}\right)^{\alpha}$$

$$r_t = \alpha P_t^N A_t \left(\frac{K_{t-1}}{H_t}\right)^{\alpha-1}$$

As before, firms are wholly owned by domestic households.

Finally the government continues to runs a balanced budget in each period, so that the budget constraint is:

$$\exp(G_t^T) + P_t^N \exp\left(G_t^N\right) = \tau_t^N P_t^N C_t^N + T_t^T + P_t^N T_t^N + \tau_t^K \left(r_t - \delta\right) K_{t-1}.$$

As before stabilization policy is implemented by means of a distortionary tax rate τ_t^N on non-tradables consumption, but now it is financed through capital income taxation (i.e. through changes in $\tau_t^K \left(r_t - \delta\right) K_{t-1}$), while we keep both lump-sum transfers fixed at their (deterministic) steady state values.

When we calibrate the model with capital we minimize distance between model with and without capital. Specifically, we keep all the calibration settings here the same as in the model without capital, except that we do not impose the size of nontradable investment in the economy. Additionally, we set the capital stock in the steady state to be 1.45 times the annual GDP and the capital depreciation to be 21.7 percent of GDP (Parameter and the steady state values are in appendix.)

7.2 Results

<To be completed>

8 Conclusions

In this paper we study the optimal stabilization problem for a small open economy subject to an occasionally binding borrowing constraint. In our benchmark economy, we characterize policy in terms of a distortionary tax on non-tradable consumption allowing for costless financing through lump sum transfers. The main result is that optimal policy is non linear: when the constraint is not binding, the tax rate should be set equal to zero while in the binding region optimal policy would subsidize non -tradable consumption in order to relax the borrowing constraint. The implications of this result is that, in our benchmark case, optimal policy does not exhibit any precautionary motive. The commitment to optimal policy affects private agents' behavior even when the constraint is not binding: agents consume more and accumulate more debt. In welfare terms the gain from optimal policy are non- trivial and account up to 40 percent of the gain that would arise from eliminating the borrowing constraint.

A Appendix

This appendix reports the model steady state and provides the details of the numerical algorithm we use to solve the model and compute optimal policy.

A.1 Steady state

The deterministic steady state equilibrium conditions are given by the following set of equations. The first four correspond to the first order conditions for the household maximization problem,

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\left(\frac{C^T}{C^N}\right)^{\frac{1}{\kappa}} = (1+\tau^N)P^N,$$

$$\left(C-\frac{H^\delta}{\delta}\right)\omega^{\frac{1}{\kappa}}\left(\frac{C^T}{C}\right)^{-\frac{1}{\kappa}} = \mu,$$

$$\left(1+\frac{\lambda}{\mu}\right) = \exp\left(-\beta\ln\left(1+C-\frac{H^\delta}{\delta}\right)\right)(1+i),$$

$$H^{\delta-1} = W\omega^{\frac{1}{\kappa}}\left(\frac{C^T}{C}\right)^{-\frac{1}{\kappa}}\left[1+\frac{\lambda}{\mu}\frac{1-\phi}{\phi}\right],$$

and the fifth is the definition of the consumption index:

$$C \equiv \left[\omega^{\frac{1}{\kappa}}\left(C^T\right)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}}\left(C^N\right)^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}}.$$

The other equilibrium conditions are given by the liquidity constraint

$$B \geqslant -\frac{1-\phi}{\phi}\left[Y^T + P^N Y^N\right]$$

and the equilibrium condition in the tradable sector that will determine the level of tradable consumption in the case in which the liquidity constraint is binding (i.e. $\lambda > 0$)

$$C^T + B = Y^T + (1+i)B - G^T.$$

We then have the production function for the non-tradeable sector and the good market equilibrium for non tradeables.

$$\begin{aligned} Y^N &= AK^\alpha H^{1-\alpha} \\ Y^N &= C^N + G^N. \end{aligned}$$

MODIFY TO ADD CAPITAL HERE

A.2 Solution algorithm

The solution algorithm we follows is a standard value iteration approach augmented with Howard improvement steps, also known as policy function iteration.[22] We initialize the algorithm by guessing a value function on the right-hand-side of equation (14). This guess consists of a vector of numbers over a fixed set of nodes in the space $\left(B, \epsilon^T\right)$. We then extend the value function to the entire space for B by assuming it is parameterized by a linear spline.

To perform the maximization we use feasible sequential quadratic programming. We first proceed by assuming that the collateral constraint (**??**) is not binding and solving the optimization problem, obtaining the values for all variables other than B, and using the competitive equilibrium equations (3), (**??**), (**??**), (**??**), (10), (**??**) (this step involves solving one equation numerically, which we do using bisection). After the maximization step has obtained a candidate solution we check whether it violates the credit constraint. If it does not, we have computed the maximum for that value in the state space. If the constraint is violated, we replace (**??**) with (**??**) holding with equality and solve as before.[23] Thus, we have computed

$$\widehat{V}_0(B_t, \varepsilon_t^T) = \max_{B_{t+1}} \left\{ u(C_t - z(H_t)) + \exp(-\beta \ln(1 + C_t - z(H_t))) E\left[V^n\left(B_{t+1}, \varepsilon_{t+1}^T\right)\right] \right\}. \tag{17}$$

We then take several Howard improvement steps, each of which involves the functional equation

$$\widehat{V}_{n+1}(B_t, \varepsilon_t^T) = u(C_t - z(H_t)) + \exp(-\beta \ln(1 + C_t - z(H_t))) E\left[\widehat{V}_n(B_{t+1}, \varepsilon_{t+1}^T)\right], \tag{18}$$

[22] Judd (1999) and Sargent (1987) contain references for the Howard improvement step, which is also referred to as policy iteration.

[23] In some cases, particularly if $\kappa > 1$, there may exist multiple solutions to the equilibrium conditions for given values of (B, B') when the constraint is binding. In these cases, one can use (**??**) to compute a value for λ and choose the solution where $\lambda \geq 0$.

where the difference between this equation and (17) is the absence of a maximization step. After N iterations on this equation, we obtain the updated value function $V^{n+1}\left(B_t, \varepsilon_t^T\right) = \widehat{V}_N\left(B_t, \varepsilon_t^T\right)$, and we continue iterating until the value function converges.

In our implementation we set $N = 40$, although a much smaller number of policy maximization steps is usually sufficient to achieve convergence. The number of nodes on the grid for B is 25, and we place most of them in the constrained region where the value function displays more curvature.

Solving the model with capital is similar – the only change is that we use bilinear splines to parameterize the value function in the space (K_t, B_t). To solve the optimal tax problem, the only change needed is to introduce a second control into the problem – the tax rate – yielding the functional equation

$$\widehat{V}_0(B_t, \varepsilon_t^T) = \max_{B_{t+1}, \tau_t^N} \left\{ u(C_t - z(H_t)) + \exp(-\beta \ln(1 + C_t - z(H_t))) E\left[V^n\left(B_{t+1}, \varepsilon_{t+1}^T\right)\right] \right\}. \tag{19}$$

The process for ε^T is a continuous state-space AR(1) with normal innovations. We compute the expectation in the right-hand-side of (17) and (18) using Gauss-Hermite quadrature, which converts the integral into a weighted sum where the nodes are the zeros of a Hermite polynomial and the weights are taken from a table in Judd (1998):

$$E\left[V^n\left(B_{t+1}, \varepsilon_{t+1}^T\right)\right] = \frac{1}{\sqrt{\pi}} \sum\nolimits_{i=1}^{m} \omega_i V^n\left(B_{t+1}, \rho_\varepsilon \varepsilon_t^T + \sqrt{2}\sigma_n n_i\right).$$

Linear splines are used to evaluate the value function at $\varepsilon_{t+1}^T = \rho_\varepsilon \varepsilon_t^T + \sqrt{2}\sigma_n n_i$ points that are not on the grid.

A.3 Computing consumption equivalents

To compute welfare gains from optimal policy, we consider the functional equations

$$\begin{aligned} V_{PO}\left(B_t, \varepsilon_t^T\right) \;=\;& u(C_{PO}\left(B_t, \varepsilon_t^T\right) - z(H_{PO}\left(B_t, \varepsilon_t^T\right))) + \\ & \exp(-\beta \ln(C_{PO}\left(B_t, \varepsilon_t^T\right) - z(H_{PO}\left(B_t, \varepsilon_t^T\right)))) E\left[V_{PO}\left(B_{PO}\left(B_t, \varepsilon_t^T\right), \varepsilon_{t+1}^T\right)\right] \end{aligned}$$

and

$$V_{CE}\left(B_t,\varepsilon_t^T\right) = u(C_{CE}\left(B_t,\varepsilon_t^T\right) - z(H_{CE}\left(B_t,\varepsilon_t^T\right))) + \exp(-\beta\ln(C_{CE}\left(B_t,\varepsilon_t^T\right) - z(H_{CE}\left(B_t,\varepsilon_t^T\right))))E\left[V_{CE}\left(B_{CE}\left(B_t,\varepsilon_t^T\right),\varepsilon_{t+1}^T\right)\right] \tag{20}$$

the first corresponds to the value function in the optimal allocation and the second to the value function in the competitive economy without stabilization policy. We then inflate total consumption in (20) by a fraction χ, keeping the decision rules fixed, so that

$$V_{CE}\left(B_t,\varepsilon_t^T;\chi\right) = u((1+\chi)\,C_{CE}\left(B_t,\varepsilon_t^T\right) - z(H_{CE}\left(B_t,\varepsilon_t^T\right))) + \exp(-\beta\ln((1+\chi)\,C_{CE}\left(B_t,\varepsilon_t^T\right) - z(H_{CE}\left(B_t,\varepsilon_t^T\right))))E\left[V_{CE}\left(B_{CE}\left(B_t,\varepsilon_t^T\right),\varepsilon_{t+1}^T\right)\right]. \tag{21}$$

For each state $\left(B_t,\varepsilon_t^T\right)$, we set

$$V_{PO}\left(B_t,\varepsilon_t^T\right) = V_{CE}\left(B_t,\varepsilon_t^T;\chi\right)$$

and solve this nonlinear equation for χ, which yields the welfare gain from switching the optimal policy conditional on the current state. To obtain the average gain, we simulate using the decision rules from (20) and weight the states according to the ergodic distribution.

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Table 1. Calibrated parameters and steady state values for the model without capital

Structural parameters	**Values**
Elasticity of substitution between tradable and non-tradable goods	$\kappa = 0.760$
Intertemporal substitution and risk aversion	$\rho = 2$
Labor supply elasticity	$\delta = 2$
Credit constraint parameter	$\phi = 0.74$
Labor share in production	$\alpha = 0.636$
Relative weight of tradable and non-tradable goods	$\omega = 0.344$
Discount factor	$\beta = 0.0177$
Production factor	$AK^{\alpha} = 1.723$
Tax rate on non-tradable consumption	$\tau = 0.0793$
Endogenous variables	**Steady state values**
Per capita NFA	$B = -3.562$
Relative price of non-tradable	$P_N = 1$
World real interest rate	$i = 0.0159$
Tradable government consumption	$\exp(G_T) = 0.0170$
Nontradable government consumption	$\exp(G_N) = 0.218$
Per capita tradable consumption	$C_T = 0.607$
Per capita non-tradable consumption	$C_N = 1.093$
Per capita consumption	$C = 1.698$
Per capita tradable GDP	$Y_T = 1$
Per capita non-tradable GDP	$Y_N = 1.543$
Per capita GDP	$Y = 2.543$
Productivity process	
Persistence	$\rho_{\varepsilon} = 0.553$
Volatility	$\sigma_n = 0.028$

Table 2. Calibrated parameters and steady state values for the model with capital

Structural parameters	**Values**
Elasticity of substitution between tradable and non-tradable goods	$\kappa = 0.760$
Intertemporal substitution and risk aversion	$\rho = 2$
Labor supply elasticity	$\delta = 2$
Credit constraint parameter	$\phi = 0.74$
Labor share in production	$\alpha = 0.509$
Relative weight of tradable and non-tradable goods	$\omega = 0.344$
Discount factor	$\beta = 0.0185$
Production factor	$A = 0.4248$
Tax rate on non-tradable consumption	$\tau = 0.0793$
Capital depreciation rate	$\delta_k = 0.0374$
Endogenous variables	**Steady state values**
Per capita NFA	$B = -3.561$
Relative price of non-tradable	$P_N = 1$
World real interest rate	$i = 0.0159$
Tradable government consumption	$\exp(G_T) = 0.0170$
Nontradable government consumption	$\exp(G_N) = 0.218$
Per capita tradable consumption	$C_T = 0.607$
Per capita non-tradable consumption	$C_N = 1.094$
Per capita consumption	$C = 1.699$
Per capita tradable GDP	$Y_T = 1$
Per capita non-tradable GDP	$Y_N = 1.543$
Per capita GDP	$Y = 2.543$
Capital Stock	$K = 14.75$
Productivity process	
Persistence	$\rho_\varepsilon = 0.553$
Volatility	$\sigma_n = 0.028$

Table 3. Welfare Gains of Moving from Benchamerk Economy 1/

Gain 2/	Tax/Subsidy	Credit Constraint
0.50	0.079	NO
3.41	0.000	YES
3.84	0.000	NO
3.60	Optimal Policy	YES

1/ Benchmark is economy with fixed tax rate at 7.93 percent and credit constraint.
2/ Percent increase in average lifetime consumption.

Figure 1: Policy function of net foreign asset with and without the liquidity constraint and baseline $\tau_t^N \equiv 0.0793$

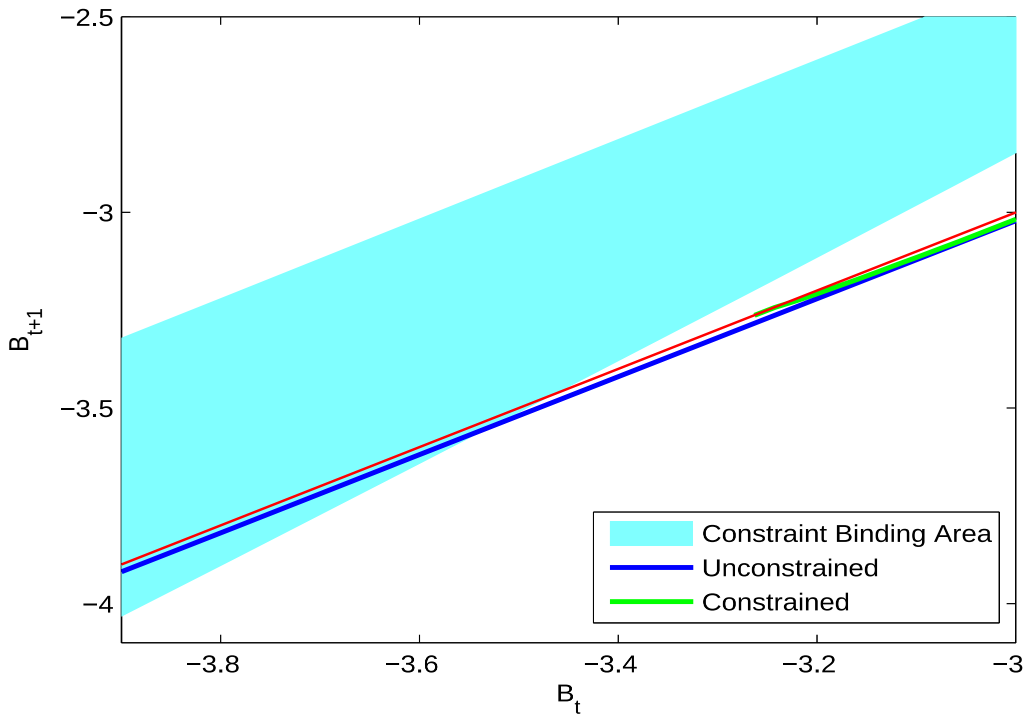
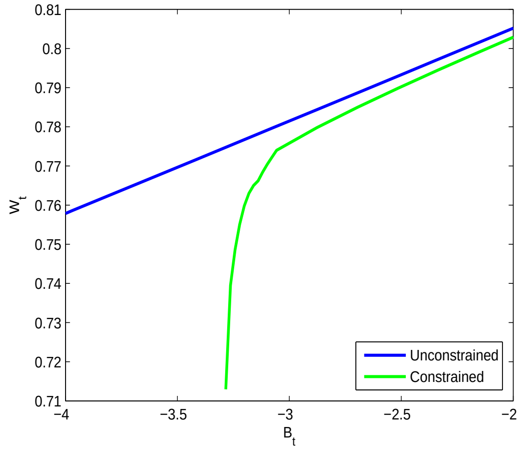
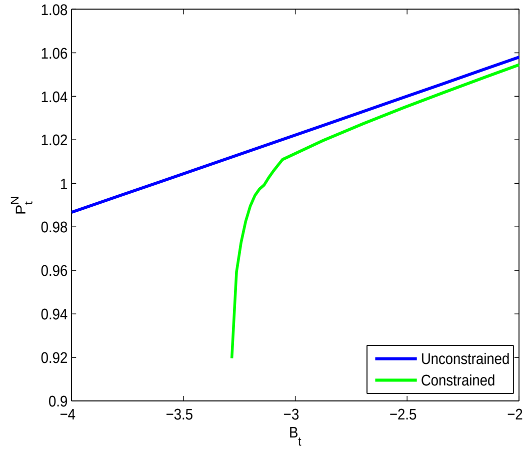


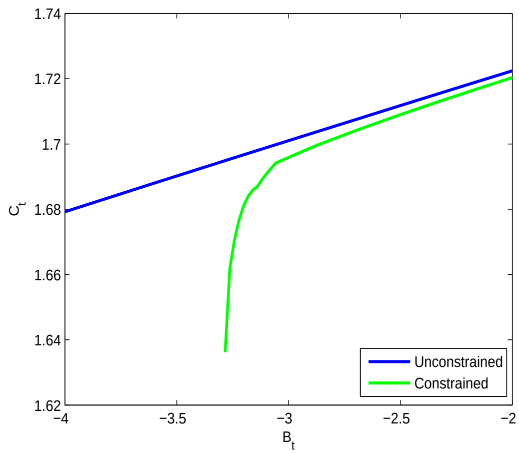
Figure 2: Policy functions for key endogenous variables with and without the liquidity constraint and baseline $\tau_t^N \equiv 0.0793$



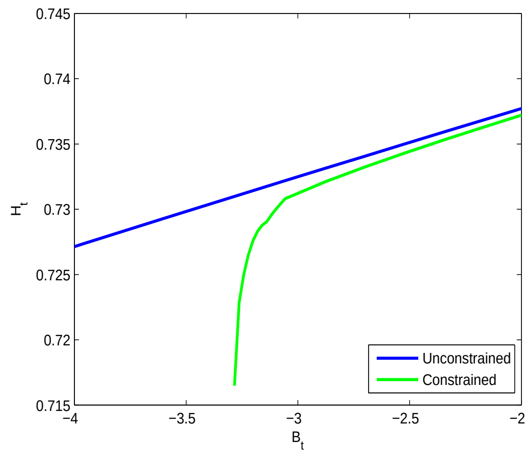
(a) W



(b) P^N

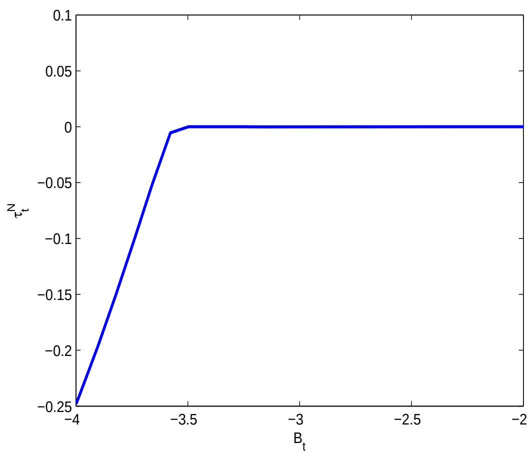


(c) C

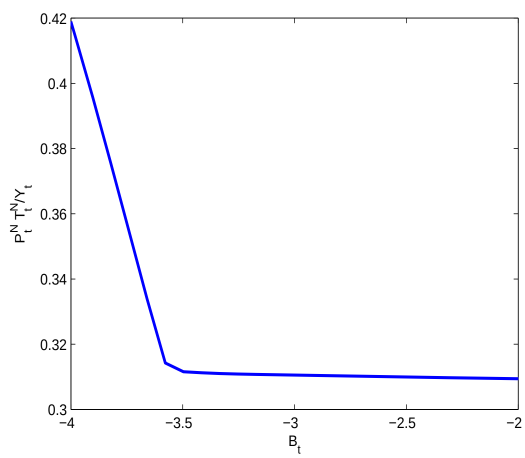


(d) H

Figure 3: Optimal policy for tax rate and lump-sum tax

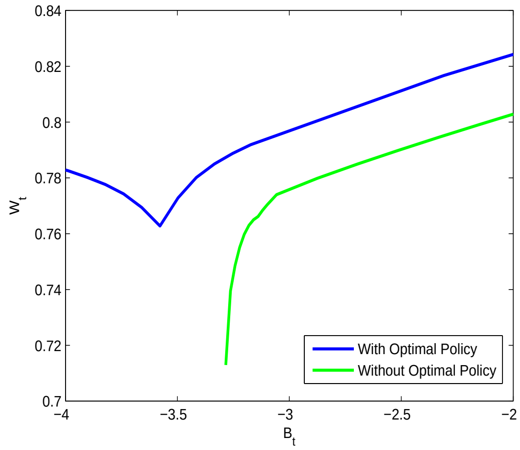


(a) τ^N

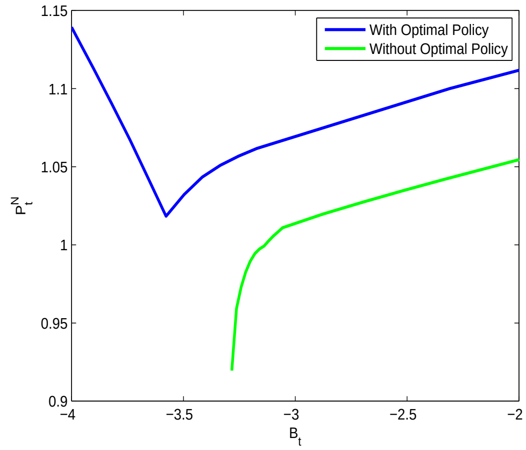


(b) $\frac{P^N T^N}{Y}$

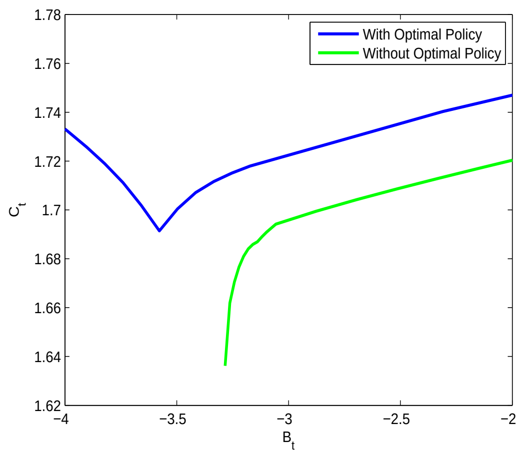
Figure 4: Policy functions for key endogenous variables with and without optimal tax policy



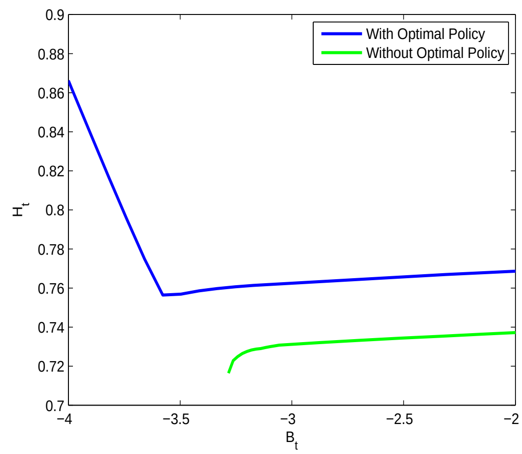
(a) W



(b) P^N



(c) C



(d) H

Figure 5: Optimal policy function for net foreign asset with the liquidity constraint

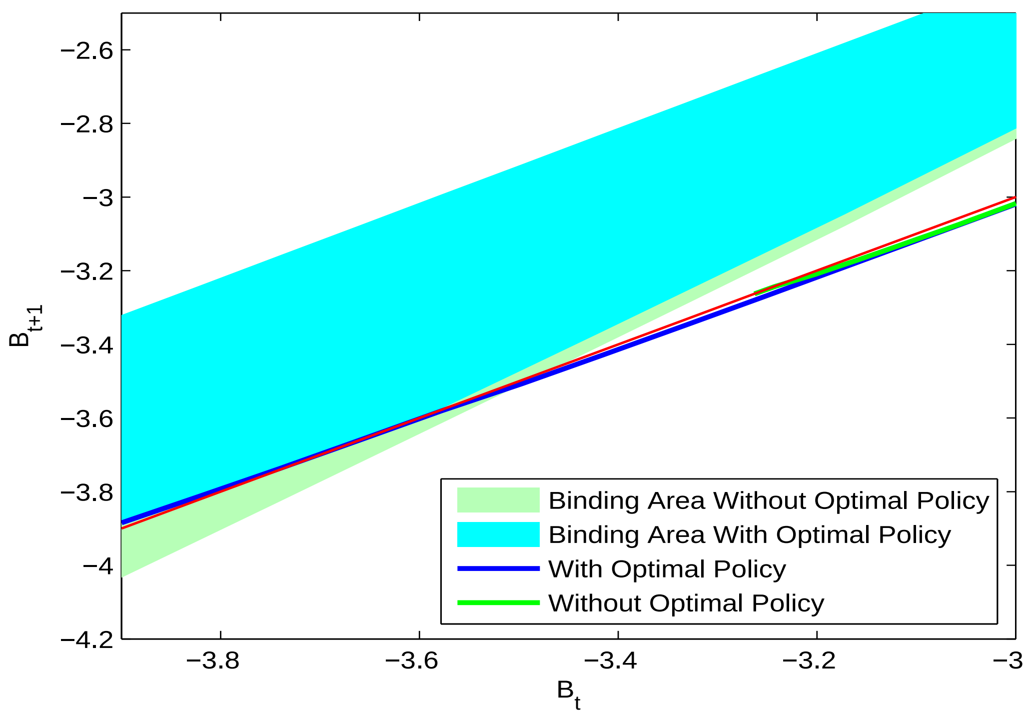


Figure 6: Ergodic distribution of net foreign asset

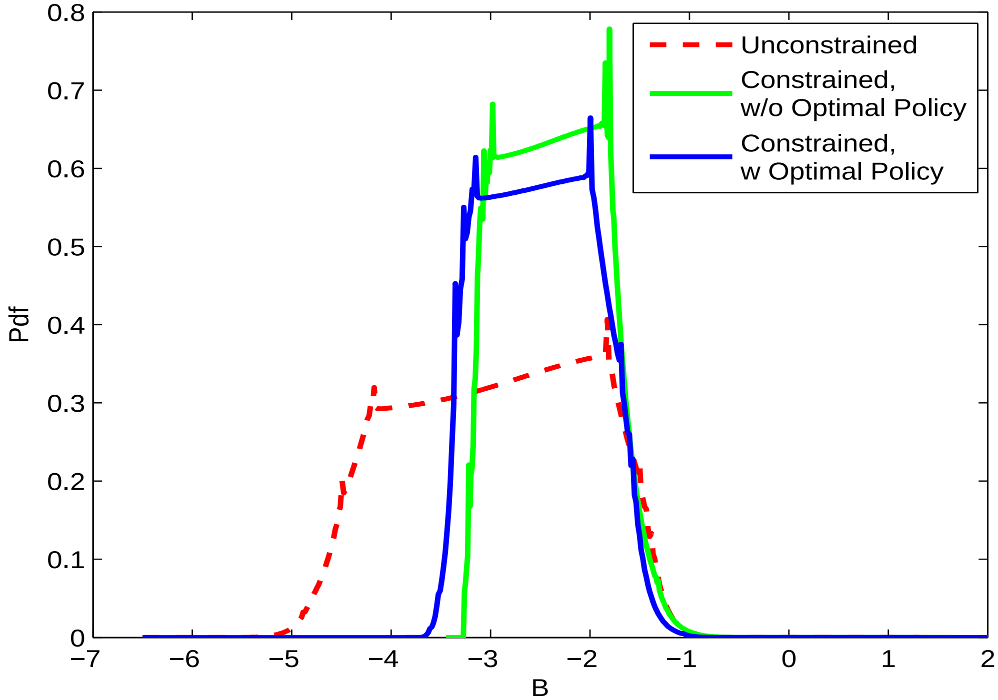
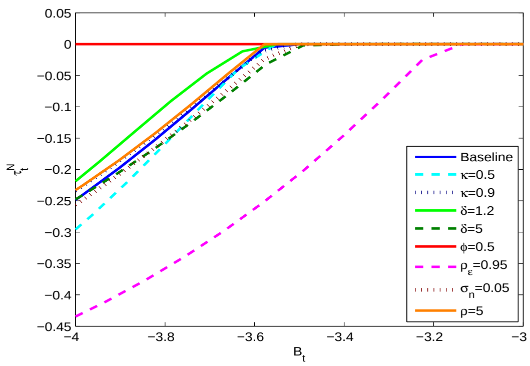
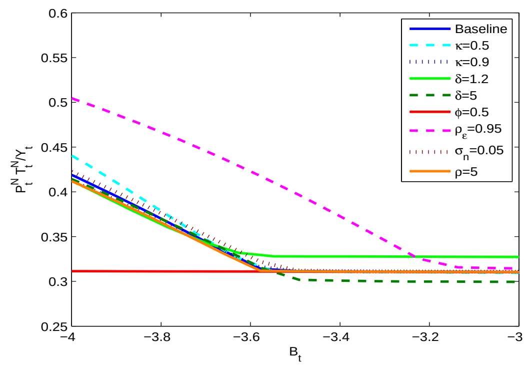


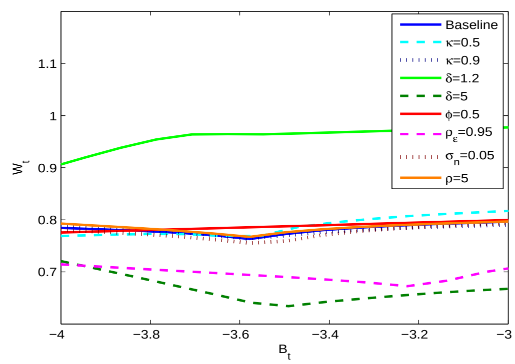
Figure 7: Sensitivity Analysis



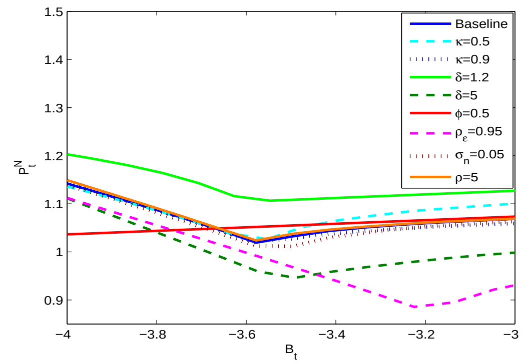
(a) τ^N



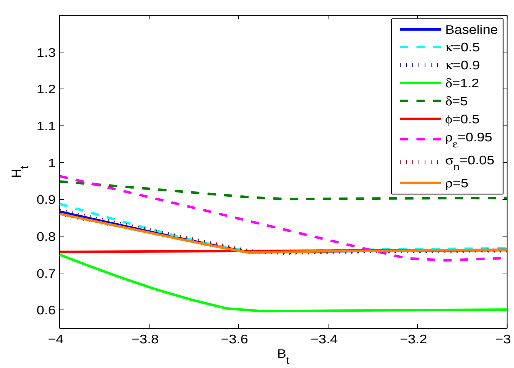
(b) $\frac{P^N T^N}{Y}$



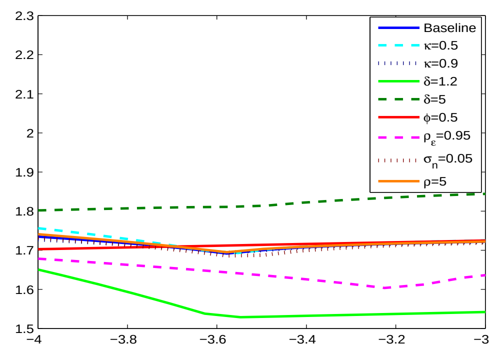
(c) W



(d) P^N



(e) H



(f) C

Figure 8: DISTRIBUTION OF WELFARE GAIN FROM CONSTRAINED ECONOMY WITH $\tau_t^N \equiv 0$ TO OPTIMAL POLICY ON TAX

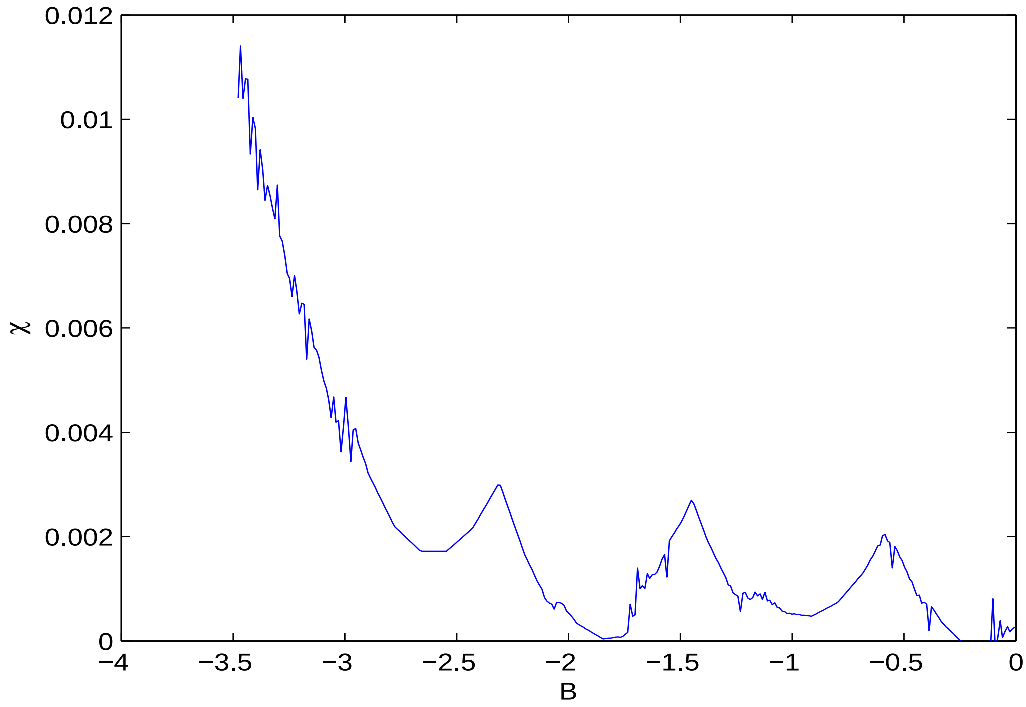


Figure 9: Credit Constraint Binding Regions with Different Rules on Tax