

Optimal Monetary Policy under Asset Market Segmentation¹

AMARTYA LAHIRI

University of British Columbia

RAJESH SINGH

Iowa State University

CARLOS VEGH

University of Maryland and NBER

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Abstract

This paper studies optimal monetary policy in a small open economy under flexible prices. The paper's key innovation is to analyze this question in the context of environments where only a fraction of agents participate in asset market transactions (i.e., asset markets are segmented). In this environment, we study three rules: the optimal state contingent monetary policy; the optimal non-state contingent money growth rule; and the optimal non-state contingent devaluation rate rule. We compare welfare and the volatility of macro aggregates like consumption, exchange rate, and money under the different rules. One of our key findings is that amongst non-state contingent rules, policies targeting the exchange rate are, in general, welfare dominated by policies that allow for some exchange rate flexibility. Crucially, we find that fixed exchange rates are almost never optimal. On the other hand, under some conditions, a non-state contingent rule like a fixed money rule can even implement the first-best allocation.

KEYWORDS: Optimal monetary policy, asset market segmentation

JEL CLASSIFICATION: F1, F2

1 Introduction

The desirability of alternative monetary policies continues to be one of the most analyzed and hotly debated issues in macroeconomics. If anything, the issue is even of greater relevance for emerging markets, which experience far greater macroeconomic volatility than industrial countries. Should emerging markets fix the exchange rate to a strong currency or should they let it float? Should they be targeting inflation and follow Taylor-type rules or should they have a monetary target? In practice, the range of experiences is not only broad but also varies considerably over time. While in the early 1990s many emerging countries were following some sort of exchange rate peg (the 10-year Argentinean currency board that started in 1991 being the most prominent example), most of them switched to more flexible arrangements after the 1994 Mexican crisis and the 1997-98 Asian crises. If history is any guide, however, countries dislike large fluctuations in exchange rates and eventually seek to limit them by interventions or interest rates changes (Calvo and Reinhart (2002)). Hence, it would not be surprising to see a return to less flexible arrangements in the near future. The cross-country and time variation of monetary policy and exchange rate arrangements in emerging countries is thus remarkable and essentially captures the different views of policymakers and international financial institutions regarding the pros and cons of different regimes.

The conventional wisdom derived from the literature regarding the choice of exchange rate regimes is based on the Mundell-Fleming model (i.e., a small open economy with sticky prices and perfect capital mobility). In such a model, it can be shown (see Calvo (1999) for a simple derivation) that if the policymaker's objective is to minimize output variability, fixed exchange rates are optimal if monetary shocks dominate and flexible exchange rates are optimal if real shocks dominate. As Calvo (1999, p. 4) puts it, this is "a result that every well-trained economist carries on [his/her] tongue". The intuition is simple enough: real shocks require an adjustment in relative prices which, in the presence of sticky prices, can most easily be effected through changes in the nominal exchange rate; in contrast, monetary shocks require an adjustment in real money balances that can be most easily carried out through changes in nominal money balances (which

happens endogenously under fixed exchange rates). In fact, most of the modern literature on the choice of exchange rate regimes has considered variations of the Mundell-Fleming model in modern clothes (rechristened nowadays as “new open economy macroeconomics”): for instance, Engel and Devereux (1998) show how the conventional results are sensitive to whether prices are denominated in the producer’s or consumer’s currency and Cespedes, Chang, and Velasco (2000) incorporate liability dollarization and balance sheets effects and conclude that the standard prescription in favor of flexible exchange rates in response to real shocks is not essentially affected. In a similar vein, while the literature on monetary policy rules for open economies is more recent, it has been carried out mostly in the context of sticky-prices model (see, for instance, Clarida, Gali, and Gertler (2001), Ghironi and Rebucci (2001), and Scmitt-Grohe and Uribe (2000)). In particular, Clarida, Gali, and Gertler (2001) conclude that Taylor-type rules remain optimal in an open economy though openness can affect the quantitative magnitude of the responses involved.

The fact that most of the literature on the choice of exchange rate regimes and monetary policy rules relies on sticky prices models raises a fundamental (though seldom asked) question: are sticky prices (i.e., frictions in good markets) more relevant in emerging markets than frictions in asset markets? Given that even for the United States 59 percent of the population (as of 1989) did not hold interest bearing assets (see Mulligan and Sala-i-Martin (2000)) and that, for all the financial opening of recent decades, financial markets in developing countries remain far less sophisticated than in the United States, it stands to reason clear that financial markets frictions are pervasive in developing countries. In this light, it would seem important to understand the implications of models with financial markets frictions for the optimal choice of exchange rate regimes and policy rules. A convenient way of modelling financial market frictions is to assume that, at any point in time, some agents do not have access to asset markets (due to, say, a fixed cost of entry, lack of information, and so forth). This so-called asset market segmentation models have been used widely in the closed macro literature (see, among others, Alvarez and Atkeson (1997), Alvarez, Lucas, and Webber (2001) and Chatterjee and Corbae (1992)). In a first paper (Lahiri, Singh, and Vegh (2006)), we have analyzed the implications of asset market segmentation for the choice of

exchange rate regimes under both complete and incomplete markets (for agents that have access to asset markets). We conclude that the policy prescription is exactly the *opposite* of the one that follows from the Mundell-Fleming model: when monetary shocks dominate, flexible exchange rates are optimal, whereas when real shock dominate, fixed rates are optimal.¹ The punchline is that the choice of fixed versus flexible should therefore not only depend on the *type of shock* (monetary versus real) but also on the *type of friction* (goods markets versus asset markets).

In this paper, we turn to the more general issue of optimal monetary policy rules (of which a fixed exchange rate or pure floating rate are, of course, particular cases). For analytical simplicity, we restrict our attention to the case in which agents that have access to asset markets (called “traders”) face complete markets. Our first result of interest is that there are state contingent rules (based either on the rate of money growth or the rate of devaluation) that can implement the first-best equilibrium.² These rules entail reacting to both output and velocity shocks. Interestingly enough, the optimal reaction to output shocks is procyclical in the sense that either the rate of money growth or the rate of devaluation should be raised in good times and lowered in bad times. Intuitively, this reflects the need to insulate non-traders (i.e., those agents with no access to capital markets) from output fluctuations. In the case of the state-contingent money growth rule, this insulation is achieved by redistributing resources from non-traders to traders in good times and viceversa in bad times. More specifically, by, say, increasing the money supply in good times, traders’ real money balances increase (since they get a disproportionate amount of money while the price level increases in proportion to the money injection), which they can use to buy goods from non-traders. In the case of the state-contingent devaluation rate rule, the insulation is achieved by

¹Not surprisingly, our results are in the spirit of an older – and less well-known literature – that analyzed the choice of exchange rate regimes in models with no capital mobility; see, in particular, Fischer (1977) and Lipschitz (1978). It is worth noting, however, that these early models fail to capture agent heterogeneity and hence miss the role of redistributive policies, a key channel in our model.

²Since there is no distortionary taxation in our model – and in the absence of net initial assets – the first-best equilibrium coincides with the Ramsey outcome. In other words, the Ramsey planner would be able to implement the first-best equilibrium.

devaluing in good times. While such a devaluation does not affect traders' real money balances (since they can always replenish their nominal money balances at the central bank's window), it reduces non-traders' real money balances thus forcing them not to over-consume in good times. In sum, the key to achieving the first best is that the monetary authority's actions hit traders and non-traders asymmetrically.

Since state-contingent rules are, by their very nature, not easy to implement in practice (as they would require the monetary authority to respond to *contemporaneous* shocks), we then proceed to ask the question: what are the optimal policy rules within the class of non-state contingent rules? Since in our model shocks are independently and identically distributed, non-state contingent rules take the form of either a constant money growth rate or a constant rate of devaluation. Our main finding is that, among non-state contingent rules, money-based rules generally welfare-dominate exchange rate-based rules. In fact, a fixed exchange rate is *never* optimal in our model, while a constant money supply rule (i.e., zero money growth) would be optimal if the economy were hit only by monetary shocks. Intuitively, this reflects a fundamental feature of our model: asset market segmentation critically affects the key adjustment mechanism that operates under predetermined exchange rates; namely, the exchange of money for bonds (or viceversa) at the central bank's window. Since only a fraction of agents operate in the asset market, this typical mechanism loses effectiveness in our model. In contrast, the typical adjustment mechanism that operates under flexible rates (adjustments in the exchange rate/price level) is not affected by asset market segmentation. We thus conclude that our model would rationalize monetary regimes where the exchange rate is allowed to (at least partly) respond to various shocks.

An important assumption of the model is that non-traders do not have any financial instruments with which to save (since they only hold nominal money balances and the cash-in-advance constraint binds). While this may be an innocuous assumption for small shocks, it is probably not so for large shocks. To make sure that our results do not critically depend on this assumption, we study in an appendix the case in which non-traders have access to a non-state contingent bond and show that, qualitatively, the same results mentioned above hold.

The paper proceeds as follows. Section 2 presents the model and the equilibrium conditions while Section 3 describes the allocations under alternative exchange rate regimes and compares welfare under the different regimes. Section 5 studies the implications of the different monetary regimes for macroeconomic volatility. Finally, Section 6 concludes. An appendix studies the case in which non-traders have access to a non-contingent bond. Algebraically tedious proofs are also consigned to appendices.

2 Model

The basic model is an open economy variant of the model outlined in Alvarez, Lucas, and Weber (2001). Consider a small open economy perfectly integrated with world goods markets. There is a unit measure of households who consume an internationally-traded good. The world currency price of the consumption good is fixed at one. The households face a cash-in-advance constraint. As is standard in these models, households are prohibited from consuming their own endowment. We assume that a household consists of a seller-shopper pair. While the seller sells the household's own endowment, the shopper goes out with money to purchase consumption goods from other households.

There are two potential sources of uncertainty in the economy. First, each household receives a random endowment y_t of the consumption good in each period. We assume that y_t is an independently and identically distributed random variable with mean \bar{y} and variance σ_y^2 .³ Second, following Alvarez *et al*, we assume that, in addition to the cash carried over from the last period (M_t), the shopper can access a proportion v_t of the household's current period (t) sales receipts to purchase goods for consumption. We assume that v_t is an independently and identically distributed random variable with mean $\bar{v} \in [0, 1]$ and variance σ_v^2 . Henceforth, we shall refer to these v shocks as velocity shocks.⁴ Only a fraction λ of the population – referred to as traders – has access

³We could allow for different means and variances for the endowments of traders and non-traders without changing our basic results.

⁴There are alternative ways in which one can think about these velocity shocks. Following Alvarez, Lucas, and Weber (2001), one can “think of the shopper as visiting the seller's store at some time during the trading day,

to (complete) asset markets, where $0 \leq \lambda \leq 1$.⁵ The remaining fraction, $1 - \lambda$ – referred to as non-traders – can only hold domestic money as an asset.

In each period t , the economy experiences one of the finitely many events $x_t = \{v_t, y_t\}$. Denote by $x^t = (x_0, x_1, x_2, \dots, x_t)$ the history of events up to and including period t . The probability, as of period 0, of any particular history x^t is $\pi(x^t) = \pi(x^t | x^{t-1}) \pi(x^{t-1})$. The households' intertemporal utility function is

$$W_0 = \sum_{t=0}^{\infty} \sum_{x^t} \beta^t \pi(x^t) u(c(x^t)), \quad (1)$$

where β is the households' time discount factor, and $c(x^t)$ is consumption in period t .

The timing runs as follows. First, both the endowment and velocity shocks are realized at the beginning of every period. Second, the household splits. Sellers of both households stay at home and sell their endowment for local currency. Shoppers of the non-trading households are excluded from the asset market and, hence, go directly to the goods market with their overnight cash to buy consumption goods. Shoppers of trading households first carry the cash held overnight to the asset market where they trade in bonds and receive any money injections for the period. They then proceed to the goods market with whatever money balances are left after their portfolio rebalancing. After acquiring goods in exchange for cash, the non-trading-shopper returns straight home while the trading-shopper can re-enter the asset market to exchange goods for foreign bonds. After all trades for the day are completed and markets close, the shopper and the seller are reunited at home.

emptying the cash register, and returning to shop some more". The uncertainty regarding v can be thought of as the uncertainty regarding the total volume of sales at the time that the shopper accesses the cash register. Alternatively, one can think of this as representing an environment where the shopper can purchase goods either through cash or credit. However, the mix of cash and credit transactions is uncertain and fluctuates across periods.

⁵As will become clear below, the assumption of complete markets for traders greatly simplifies the problem. In Lahiri, Singh, and Vegh (2007), we solve the case of incomplete markets for some very simple policy rules (i.e., constant money supply and constant exchange rate) and show that similar results obtain.

2.1 Households' problem

2.1.1 Traders

Traders have access to world capital markets in which they can trade state contingent securities spanning all states. Traders begin any period with assets in the form of money balances and state-contingent bonds carried over from the previous period. Armed with these assets the shopper of the trader household visits the asset market where she rebalances the household's asset position and also receives the lump sum asset market transfers from the government. For any period $t \geq 0$, the accounting identity for the asset market transactions of a trader household is given by

$$\hat{M}^T(x^t) = M^T(x^{t-1}) + S(x^t) f(x^t) - S(x^t) \sum_{x^{t+1}} q(x^{t+1}|x^t) f(x^{t+1}) + \frac{T(x^t)}{\lambda}, \quad (2)$$

where $\hat{M}^T(x^t)$ denotes the money balances with which the trader leaves the asset market under history x^t (which includes the time t state x_t) while $M^T(x^{t-1})$ denotes the money balances with which the trader entered the asset market.⁶ $S(x^t)$ is the exchange rate (the domestic currency price of foreign currency). $f(x^{t+1})$ denotes units of state-contingent securities, in terms of tradable goods, bought in period t at a per unit price of $q(x^{t+1}|x^t)$. A trader receives payment of $f(x^{t+1})$ in period $t+1$ if and only if the history x^{t+1} occurs. T are aggregate (nominal) lump-sum transfers from the government.^{7,8}

After asset markets close, the shopper proceeds to the goods market with \hat{M}^T in nominal money balances to purchase consumption goods. The cash-in-advance constraint for traders is thus given

⁶Note that the money balances with which a trader enters the asset market at time t reflects the history of realizations till time $t-1$. Hence, beginning of period money balances at time t depend on the history x^{t-1} .

⁷We assume that these transfers are made in the asset markets, where only traders are present. Note that since T denotes aggregate transfers, the corresponding per trader value is T/λ since traders comprise a fraction λ of the population.

⁸The assumption of endogenous lump-sum transfers will ensure that any monetary policy may be consistent with the intertemporal fiscal constraint. This becomes particularly important in this stochastic environment where these endogenous transfers will have to adjust to ensure intertemporal solvency for *any* history of shocks. To make our life easier, these transfers are assumed to go only to traders. If these transfers also went to non-traders, then (12) would be affected.

by ⁹

$$S(x^t) c^T(x^t) = \hat{M}^T(x^t) + v_t S(x^t) y_t. \quad (3)$$

Equation (3) shows that for consumption purposes, traders can augment the beginning of period cash balances by withdrawals from current period sales receipts v_t (the velocity shocks). By the law of one price, $S(x^t)$ also denotes the domestic currency price of consumption goods under history x^t . Lastly, period- t sales receipts net of withdrawals become beginning of next period's money balances

$$M^T(x^t) = S(x^t) y_t (1 - v_t). \quad (4)$$

Combining equations (2) and (3) yields

$$\begin{aligned} M^T(x^{t-1}) + \frac{T(x^t)}{\lambda} + v_t S(x^t) y_t &= S(x^t) c^T(x^t) - S(x^t) f(x^t) \\ &+ S(x^t) \sum_{x_{t+1}} q(x_{t+1}|x^t) f(x_{t+1}). \end{aligned} \quad (5)$$

We assume that $M_0^T = \bar{M}$. We also assume that actuarially fair securities are available in international asset markets. By definition, actuarially fair securities imply that

$$\frac{q(x_i^{t+1}|x^t)}{q(x_j^{t+1}|x^t)} = \frac{\pi(x_i^{t+1}|x^t)}{\pi(x_j^{t+1}|x^t)}, \quad (6)$$

for any pair of securities i and j belonging to the set x . Further, no-arbitrage implies that the price of a riskless security that promises to pay one unit next period should equal the price of a complete set of state-contingent securities (which would lead to the same outcome):

$$\frac{1}{1+r} = \sum_{x_{t+1}} q(x_{t+1}|x^t). \quad (7)$$

Using (6) repeatedly to solve for a particular security relative to all others and substituting into (7), we obtain:

⁹Throughout the analysis we shall restrict attention to ranges in which the cash-in-advance constraint binds for both traders and non-traders. In general, this would entail checking the individual optimality conditions to infer the parameter restrictions for which the cash-in-advance constraints bind (see Lahiri, Singh, and Vegh (2007)).

$$q(x^{t+1}|x^t) = \beta\pi(x^{t+1}|x^t), \quad (8)$$

where we have assumed that $\beta = 1/(1+r)$. Note further that the availability of these sequential securities is equivalent to the availability of Arrow-Debreu securities, where all markets open only on date 0. Hence, by the same logic, it must be true for Arrow-Debreu security prices that

$$q(x^t) = \beta^t\pi(x^t). \quad (9)$$

Traders arrive in this economy at time 0 with initial nominal money balances \bar{M} and initial net foreign asset holdings of f_0 . To ensure market completeness, we allow for asset market trade right before period 0 shocks are realized, so that traders can exchange f_0 for state-contingent claims payable after the realization of shocks in period 0. Formally,

$$f_0 = \sum_{x_0} q(x_0) f(x_0), \quad (10)$$

where $q(x_0) = \beta\pi(x_0)$.

Maximizing (1) subject to (5) yields

$$q(x^{t+1}|x^t) = \beta\pi(x^{t+1}|x^t) \frac{u'(c(x^{t+1}))}{u'(c(x^t))}. \quad (11)$$

Equation (11) is the standard Euler equation under complete markets, which relates the expected marginal rate of consumption substitution between today and tomorrow to the return on savings discounted to today. Since $\beta = 1/(1+r)$, it is clear from (8) and (11) that traders choose a flat path for consumption.

2.1.2 Non-traders

As stated earlier, the non-traders in this economy do not have access to asset markets.¹⁰ They are born with some initial nominal money balances \bar{M} and then transit between periods by exchanging

¹⁰In the appendix we analyze the case in which non-traders have access to a non-state contingent bond and show that, qualitatively, results are the same. Qualitatively, then, our results only depend on differential access to asset instruments between traders and non-traders. Quantitatively, however, results will be sensitive to how close the financial instruments that non-traders hold are to a full set of state contingent claims.

cash for goods and vice-versa.¹¹ The non-trader's cash-in-advance constraint is given by:

$$S(x^t)c^{NT}(x^t) = M^{NT}(x^{t-1}) + v_t S(x^t)y_t, \quad (12)$$

where $M^{NT}(x^{t-1})$ stands for beginning of period t nominal money balances (which is dependent on the history x^{t-1}) for non-traders. Their initial period cash-in-advance constraint is

$$S(x^0)c^{NT}(x^0) = \bar{M} + v_0 S(x^0)y_0.$$

Like traders, the non-traders can also augment their beginning of period cash balances by withdrawals from current period sales receipts v_t (the velocity shocks). Money balances at the beginning of period $t + 1$ are given by sales receipts net of withdrawals for period t consumption:

$$M^{NT}(x^t) = S(x^t)y_t(1 - v_t). \quad (13)$$

2.2 Government

The government in this economy holds foreign bonds (reserves) which earn the world rate of interest r . The government can sell nominal domestic bonds, issue domestic money, and make lump sum transfers to traders. Thus, the government's budget constraint is given by

$$S(x^t) \sum_{x_{t+1}} q(x^{t+1}|x^t) h(x^{t+1}) - S(x^t)h(x^t) + T(x^t) = M(x^t) - M(x^{t-1}), \quad (14)$$

where h are foreign bonds held by the government, M is the aggregate money supply, and T denotes government transfers to traders. It is crucial to note that changes in money supply impact only the traders since they are the only agents present in the asset market.

2.3 Equilibrium conditions

Equilibrium in the money market requires that

$$M(x^t) = \lambda M^T(x^t) + (1 - \lambda)M^{NT}(x^t). \quad (15)$$

¹¹Note that we have assumed that the initial holdings of nominal money balances is invariant across the two types of agents, i.e., $M_0^T = M_0^{NT} = \bar{M}$.

The flow constraint for the economy as a whole (i.e., the current account) follows from combining the constraints for non-traders (equations (12) and (13)), traders (equation (5)), and the government (equation (14)) and money market equilibrium (equation (15)):

$$\lambda c^T(x^t) + (1 - \lambda)c^{NT}(x^t) = y_t + k(x^t) - \sum_{x^{t+1}} q(x^{t+1}|x^t) k(x^{t+1}), \quad (16)$$

where $k \equiv h + \lambda f$ denotes per-capita foreign bonds for the economy as a whole.

To obtain the quantity theory, combine (3), (13) and (15) to get:

$$\frac{M(x^t)}{1 - v_t} = S(x^t)y_t. \quad (17)$$

Notice that the stock of money relevant for the quantity theory is end of period t money balances $M(x^t)$. This reflects the fact that, unlike standard CIA models (in which the goods market is open before the asset market and shoppers cannot withdraw current sales receipts for consumption), in this model (i) asset markets open before goods market open (which allows traders to use period t money injections for consumption purposes in that period); and (ii) both traders and non-traders can access current sales receipts.

Combining (12) and (13) yields non-traders' consumption:

$$c^{NT}(x^t) = \frac{S(x^{t-1})}{S(x^t)}(1 - v_{t-1})y_{t-1} + v_t y_t, \quad (18)$$

$$c^{NT}(x^0) = \frac{\bar{M}}{S(x^0)} + v_0 y_0. \quad (19)$$

To obtain the level of constant consumption for traders, we use equation (4) to substitute for M_t^T in equation (5). Then, subtracting $S(x^t)y_t$ from both sides allows us to rewrite (5) as

$$f(x^t) - \sum_{x^{t+1}} q(x^{t+1}|x^t) f(x^{t+1}) + y_t - c^T(x^t) = \frac{M(x^t) - M(x^{t-1})}{S(x^t)} - \frac{T(x^t)}{\lambda},$$

where we have used equation (17) to get

$$M(x^t) - M(x^{t-1}) = S(x^t)y_t - S(x^{t-1})y_{t-1} - (v_t S(x^t)y_t - v_{t-1} S(x^{t-1})y_{t-1}).$$

Using equation (14) in the equation above yields

$$\begin{aligned} & \sum_{x^{t+1}} q(x^{t+1}|x^t) \left(\frac{h(x^{t+1})}{\lambda} + f(x^{t+1}) \right) - f(x^t) - \frac{h(x^t)}{\lambda} \\ &= y_t - c^T(x^t) + \left(\frac{1-\lambda}{\lambda} \right) \left(\frac{M(x^t) - M(x^{t-1})}{S(x^t)} \right), \end{aligned} \quad (20)$$

where h_0 and f_0 are exogenously given. Using (9) and iterating forward on equation (20), it can be checked that under either regime and for any type of shock (i.e., velocity or output shock), consumption of traders is given by:¹²

$$c^T(x^t) = r \frac{k_0}{\lambda} + \bar{y} + r \sum_{x^t} \beta^t \pi(x^t) \left(\frac{1-\lambda}{\lambda} \right) \left(\frac{M(x^t) - M(x^{t-1})}{S(x^t)} \right), \quad t \geq 0, \quad (21)$$

where $k_0 = h_0 + \lambda f_0$. In the following, we shall maintain the assumption that initial net country assets are zero, i.e., $k_0 = 0$.

Finally, we need to tie down the initial period price level, S_0 . From the quantity theory equation, it follows that $S_0 = \frac{M_1}{(1-v_0)y_0}$. In order to keep initial period allocations symmetric across regimes we make the neutral assumption that $M_1 = \bar{M}$. Hence,

$$S_0 = \frac{\bar{M}}{(1-v_0)y_0}. \quad (22)$$

Noting that $S_0 = S(x^0)$, it is easy to check from equation (19) that this assumption implies that

$$c_0^{NT} = y_0. \quad (23)$$

3 First-best policy rules

Having described the model and the equilibrium conditions above, we now derive the first-best policy rules under both flexible and predetermined exchange rates. While these first-best rules depend on contemporaneous shocks – and, hence, would not be operational in practice – they will serve as the benchmark against which we will compare implementable rules. We will first derive

¹²This is accomplished by multiplying each period's flow constraint by $q(x^t)$ and summing it over all possible realizations. Then, summing it over all periods and imposing transversality conditions gives the intertemporal budget constraint.

the first-best rule under flexible exchange rates (i.e., the first-best money growth rule) and then under predetermined exchange rates (first-best devaluation rule).

We shall conduct the welfare analysis by comparing the expectation of lifetime welfare at time $t = 0$ conditional on period 0 realizations (but before the revelation of any information at time 1). Specifically, the expected welfare under any monetary regime is calculated given the initial period shocks x_0 , the initial price level $S_0 = \frac{\bar{M}}{(1-v_0)y_0}$ as well as the associated initial money injection for period 1: $M(x^0) = M_1 = \bar{M}$. To this effect, it is useful to define the following:

$$W^{i,j} = E \left\{ \sum \beta^t \left[u(c_t^{i,j}(x^t)) \right] \right\}, \quad i = T, NT, \quad j = \text{monetary regime}, \quad (24)$$

$$W^j = \lambda W^{T,j} + (1 - \lambda) W^{NT,j}, \quad j = \text{monetary regime}. \quad (25)$$

Equation (24) gives the welfare for each agent under a specific monetary policy regime where the relevant consumption for each type of agent is given by the consumption functions relevant for that regime. Equation (25) is the aggregate welfare for the economy under each regime. It is the sum of the regime specific welfares of the two types of households weighted by their population shares.

3.1 First-best state contingent money growth rule

We have shown above that when traders have access to complete markets, they can fully insure against all shocks. Hence, the only role for policy is to smooth non-traders' consumption, who do not have access to asset markets. Clearly, the first-best outcome for the non-traders would be a flat consumption path (recall that all the welfare losses for non-traders in this model come from consumption volatility).¹³

Recall from equation (18) that consumption of non-traders is given by

$$c^{NT}(x^t) = \frac{S(x^{t-1})}{S(x^t)} (1 - v_{t-1}) y_{t-1} + v_t y_t. \quad (26)$$

¹³The conclusion that the only role for policy is to smooth non-traders' consumption is crucially dependent on the assumptions that (i) the endowment process is the same for both types with the same mean and (ii) initial net country assets are zero. If this were not the case, then an additional goal for policy would be to shift consumption across types in order to equalize marginal utilities.

Using the quantity theory equation $S(x^t)y_t(1 - v_t) = M(x^t)$ in the above and rearranging gives

$$c^{NT}(x^t) = y_t - \left(\frac{M(x^t) - M(x^{t-1})}{S(x^t)} \right).$$

Substituting out for $S(x^t)$ from the quantity theory relationship then yields

$$c^{NT}(x^t) = y_t \left[1 - \left(\frac{M(x^t) - M(x^{t-1})}{M(x^t)} \right) (1 - v_t) \right]. \quad (27)$$

As was assumed earlier, the endowment sequence follows an i.i.d. process with mean \bar{y} and variance σ_y^2 . It is clear that the first-best outcome for non-traders would be achieved if $c_t^{NT} = \bar{y}$ for all t . The key question is thus whether there exists a monetary policy rule which can implement this allocation.

Let $\mu(x^t)$ be the growth rate of money given history x^t .¹⁴ Hence,

$$M(x^t) - M(x^{t-1}) = \mu(x^t)M(x^{t-1}), \quad t \geq 1$$

Substituting this into equation (27) gives

$$c^{NT}(x^t) = y_t \left[1 - \left(\frac{\mu(x^t)}{1 + \mu(x^t)} \right) (1 - v_t) \right].$$

To check if monetary policy can implement the first-best, we substitute $c^{NT}(x^t) = \bar{y}$ in the above to get

$$\frac{\bar{y}}{y_t} = \left[1 - \left(\frac{\mu(x^t)}{1 + \mu(x^t)} \right) (1 - v_t) \right].$$

This expression can be solved for $\mu(x^t)$ as a function of y_t and v_t . Thus,

$$\mu(x^t) = \frac{y_t - \bar{y}}{\bar{y} - v_t y_t}. \quad (28)$$

A few features of this policy rule are noteworthy.¹⁵ First, as long as the monetary authority chooses μ *after* observing the realizations for y and v , this rule is implementable. Second, equation

¹⁴Recall that $M_1 = \bar{M}$ implies that $\mu(x^0) = 0$ by assumption.

¹⁵Notice that since there is no distortionary taxation in our model – and in the absence of net initial assets – the first-best coincides with the Ramsey plan. In other words, the Ramsey problem would also yield the above policy rule which replicates the first-best.

(28) makes clear that when there are no shocks to output, i.e., $y_t = \bar{y}$ for all t , the optimal policy is to choose $\mu_t = 0$ for all t independent of the velocity shock. Hence, under velocity shocks only, a flexible exchange rate regime with a constant money supply implements the first-best allocation.

A third interesting feature of equation (28) is that the optimal monetary policy is procyclical. In particular, it is easy to check that

$$\frac{\partial \mu(x^t)}{\partial y_t} = \frac{\bar{y}(1 - v_t)}{(\bar{y} - v_t y_t)^2} > 0. \quad (29)$$

Note that the latter inequality in (29) follows from the fact that v is strictly bounded above by one. The intuition for this result is that, *ceteris paribus*, an increase in output raises non-traders' consumption through two channels. First, current sales revenue is higher, which implies that there is more cash available for consumption. Second, an increase in output appreciates the currency thereby raising the real value of money balances brought into the period. To counteract these expansionary effects on non-traders' consumption, the optimal monetary policy calls for an expansion in money growth. An expansion in money growth reduces non-traders' consumption by redistributing resources from non-traders to traders. More specifically, since only traders are present in the asset markets, they get a more than proportionate amount of money while the exchange rate (price level) rises in proportion to the money injection. Hence, traders' real money balances increase, which they can use to buy goods from non-traders and exchange for foreign bonds. In bad times, a money withdrawal from the system leaves traders with lower real money balances, which leads them to sell those goods to non-traders. In other words, policymakers are smoothing non-traders' consumption by engineering a transfer of resources from non-traders to traders in good times and viceversa in bad times.

Fourth, the optimal policy response to velocity shocks depends on the level of output relative to its mean level. In particular,

$$\frac{\partial \mu(x^t)}{\partial v_t} = \frac{y_t(y_t - \bar{y})}{(\bar{y} - v_t y_t)^2} \gtrless 0.$$

Thus, when output is above the mean level, an increase in v calls for an increase in money growth while if output is below the mean then the opposite is true. Intuitively, an increase in v_t

has two opposing effects on real balances available for consumption. First, it raises real balances since it implies that a higher proportion of current sales can be used in the current period. Second, a higher v_t depreciates the currency thereby decreasing the real value of money balances brought into the period. When output is equal to the mean level, absent a change in policy, these effects exactly offset each other. On the other hand, when output is above (below) the mean, the current sales effect is stronger (weaker) than the exchange rate effect. Hence, an increase (decrease) in μ provides the appropriate correction through the redistribution channel spelled out above.

3.2 First-best state contingent devaluation rate rule

To derive the first-best state contingent devaluation rate rule, substitute $c_t^{NT} = \bar{y}$ into equation (26) and replace $\frac{S(x^{t-1})}{S(x^t)}$ by $\frac{1}{1+\varepsilon(x^t)}$ to obtain:

$$\varepsilon(x^t) = \frac{(1 - v_{t-1})y_{t-1}}{\bar{y} - v_t y_t} - 1. \quad (30)$$

Again, several features of this rule are noteworthy. First – and as was the case for the money growth rule just discussed – as long as the monetary authority can observe contemporaneous realizations of y and v , this rule is implementable. Second – and unlike the money growth rule just discussed – this rule also depends on past values of output. Intuitively, the reason is that under a peg, non-traders’ consumption depends on last period’s consumption, as follows from (26). Third, if there are no shocks to either output or velocity (i.e., if $y_t = \bar{y}$ and $v_t = \bar{v}$ for all t), then the optimal policy is to keep the exchange rate flat (i.e., $\varepsilon = 0$).

Fourth, this rule is procyclical with respect to output in the sense that, all else equal, a higher realization of today’s output calls for an increase in the rate of devaluation. Intuitively, an increase in today’s output increases today’s non-traders’ consumption because current sales revenue is higher, which implies that there is more cash available for consumption. To keep non-traders’ consumption flat over time, the monetary authority needs to offset this effect. The way to do so is to increase today’s exchange rate (i.e., a nominal devaluation). A nominal devaluation will tend to lower real money balances of both traders and non-traders. Traders, however, can easily undo this by replenishing their nominal money balances at the central bank’s window (as in the standard model).

Non-traders, however, have no way of doing this and hence see their consumption lowered by the fact that they have lower real money balances. In bad times (low realization of output), a revaluation will have the opposite effect. In sum, the monetary authority is able to smooth non-traders' consumption through real balances effect.

Fifth, a high realization of today's velocity shock also calls for an increase in the rate of devaluation. Intuitively, a high value of v implies that both traders and non-traders have a higher level of real cash balances for consumption. Traders, of course, can undo this in the asset markets. Non-traders, however, cannot do this and would be forced to consume too much today. By devaluing, the monetary authority decreases the value of non-traders' real money balances. Conversely, a low value of v would be counteracted by a nominal revaluation.

4 Optimal non-state contingent rule

This section computes the optimal non-state contingent rules. In order to make progress analytically, we shall now specialize the utility function to the quadratic form.¹⁶ Thus, we assume from hereon that the periodic utility of the household of either type is given by:

$$u(c) = c - \zeta c^2. \tag{31}$$

Note that the quadratic utility specification implies that the expected value of periodic utility can be written as

$$E(c - \zeta c^2) = E(c) - \zeta [E(c)]^2 - \zeta Var(c). \tag{32}$$

where $Var(c)$ denotes the variance of consumption.

4.1 Non-state contingent money growth rule

The first non-state contingent rule that we analyze is a time invariant money growth rule. The main exercise is to determine the constant money growth rule which maximizes the joint, share-weighted

¹⁶In Appendix @, we look at the log-normal case and derive reduced forms for both the optimal non-state contingent μ and ε . We choose the quadratic specification as our main set-up because it allows to go farther analytically than a log-normal would.

lifetime welfare of the two types of agents in the economy. Hence, the objective is to choose μ to maximize

$$W^\mu = \lambda W^{T, \mu} + (1 - \lambda) W^{NT, \mu}.$$

In order to compute the optimal constant non-state contingent money growth rule, we first need to determine the consumption allocations for the two agents under this regime (for an arbitrary but constant money growth rate). We use $\bar{\mu}$ to denote the constant money growth. Given a utility specification, μ can be computed by maximizing weighted utilities.

Under the time invariant money growth rule and the quantity theory equation $S_t y_t (1 - v_t) = M_{t+1}$, equations (18) and (21) imply that consumption of nontraders and traders are given by

$$c_t^{NT} = z(1 - v_t) y_t + v_t y_t, \quad t \geq 1, \quad (33)$$

$$c_t^T = r \frac{k_0}{\lambda} + \bar{y} \left[1 + \left(\frac{1 - \lambda}{\lambda} \right) (1 - z) (1 - \bar{v}) \right], \quad (34)$$

where $z \equiv \frac{1}{1 + \mu} \left(= \frac{M_t}{M_{t+1}} \right)$. From here on, we abstract from distributional issues relating to the distribution of initial wealth across agents, by assuming that initial net country assets are zero, i.e., $k_0 = 0$. Since $\lambda E[c^T] + (1 - \lambda) E[c_t^{NT}] = \bar{y}$, under our maintained assumption of quadratic preferences, the optimal z is determined by solving the problem:

$$\min_z \left\{ \lambda (E(c^T))^2 + (1 - \lambda) (E(c^{NT}))^2 + (1 - \lambda) Var[c^{NT}] \right\}. \quad (35)$$

In order to derive the optimal money growth rate we need to know the expected consumption levels of the two types as well as the unconditional consumption variance for the nontrader. The expected consumption is trivial to compute and, from (33), it can be shown that the variance of non-trader's consumption is given by:

$$Var[c_t^{NT}] = z^2 \sigma_y^2 + (1 - z)^2 [\sigma_y^2 \sigma_v^2 + \bar{y}^2 \sigma_v^2 + \bar{v}^2 \sigma_y^2] + 2z(1 - z) \bar{v} \sigma_y^2. \quad (36)$$

Substituting in the relevant expressions for c^T , $E(c^{NT})$, and $Var[c^{NT}]$ into (35) and taking

the first order condition with respect to z yields (after rearranging terms)

$$z = \frac{\sigma_y^2 \sigma_v^2 + \bar{y}^2 \sigma_v^2 - \sigma_y^2 \bar{v} (1 - \bar{v}) + \frac{(1 - \bar{v})^2 \bar{y}^2}{\lambda}}{\sigma_y^2 \sigma_v^2 + \bar{y}^2 \sigma_v^2 + (1 - \bar{v})^2 \left(\sigma_y^2 + \frac{\bar{y}^2}{\lambda} \right)}.^{17} \quad (37)$$

Since $\mu = \frac{1-z}{z}$, the optimal μ that is implied by (37) is

$$\tilde{\mu} = \frac{(1 - \bar{v}) \sigma_y^2}{\sigma_y^2 \sigma_v^2 + \bar{y}^2 \sigma_v^2 + \bar{v}^2 \sigma_y^2 + \frac{(1 - \bar{v})^2 \bar{y}^2}{\lambda} - \bar{v} \sigma_y^2} > 0, \quad (38)$$

where the positive sign follows from the condition $(\sigma_y/\bar{y})^2 < (1 - \bar{v})/\bar{v}\lambda$. Three important features of (38) are worth elaborating on.

First, why is $\tilde{\mu} > 0$? Intuitively, the key is that, as far as the Ramsey planner is concerned, c^T and c^{NT} are gross substitutes. If the relative social price of c^{NT} in terms of c^T were one, then the optimal $\tilde{\mu}$ would be zero. In the presence of uncertainty, however, the relative social price of in terms of c^{NT} in terms of c^T becomes larger than one. Since these two goods are gross substitutes, it is optimal for the Ramsey planner to substitute away from c^{NT} and towards c^T . To put in basic microtheory, c^{NT} and c^T are akin to coffee and tea; when the price of tea goes up, it becomes optimal to consume more tea.

Second, it follows from (38) that

$$\frac{\partial \tilde{\mu}}{\partial \sigma_y^2} = \frac{(1 - \bar{v})}{D^2} \left[\bar{y}^2 \sigma_v^2 + \frac{(1 - \bar{v})^2 \bar{y}^2}{\lambda} \right] > 0,$$

where $D > 0$ is the denominator of expression (38). In other words, the higher is the variability of output the higher are the transfers from non-traders to traders. The intuition follows from the discussion in the previous paragraph. A larger variability increases the social cost of c^{NT} in terms of c^T . Since, from a social point of view, both goods are substitutes, it becomes optimal to increase c^T and the expense of c^{NT} .

¹⁷Notice that while the denominator of z is always positive, the numerator could, in principle, be negative. Given that we want to ensure that $\mu > -1$ (i.e., $z > 0$), we will impose the condition that $(\sigma_y/\bar{y})^2 < (1 - \bar{v})/\bar{v}\lambda$, which ensures that the numerator is positive for any pair of values for σ_y and σ_v . Numerically, this condition is hardly restrictive: even for $\lambda = 1$ and a relatively high value of σ_y/\bar{y} such as 0.1, this condition holds for any $\bar{v} < 0.9901$. (Note that $\bar{v} = 0.99$ implies a velocity of $1/(1 - \bar{v}) = 101$, much higher than empirically observed values.)

Third, it also follows from (38) that

$$\frac{\partial \tilde{\mu}}{\partial \sigma_v^2} = -\frac{(1-\bar{v})\sigma_y^2}{D^2}(\sigma_y^2 + \bar{y}^2) < 0.$$

The higher the variability of monetary shocks, the lower is the optimal μ .

Finally, we should note two special cases. First, when the economy is open to only output shocks, i.e., $\sigma_v^2 = 0$, the optimal rate of money growth implied by equation (38) is¹⁸

$$\tilde{\mu}|_{\sigma_v^2=0} = \frac{1-\bar{v}}{\bar{v}^2 + \frac{(1-\bar{v})^2\bar{y}^2}{\lambda\sigma_y^2} - \bar{v}}.$$

As already established, the optimal $\tilde{\mu}$ is thus an increasing function of the variance of output shocks, σ_y^2 .

Second, when there is no output volatility in the economy so that $\sigma_y^2 = 0$, the optimal constant money growth rate given by (38) is

$$\tilde{\mu} = 0,$$

which implies that a policy of fixed money supply is optimal. Interestingly, we have seen above that the state contingent first-best rule calls for $\mu = 0$ when there are no output shocks. Hence, when there is no output volatility in the economy, the non-state contingent optimal money growth rule coincides with the state contingent first-best rule. In general, however, a fixed money rule does not achieve the first best equilibrium.

4.1.1 Welfare loss relative to the first-best

Under our quadratic preference specification, welfare under the state-contingent rule is

$$W^{fb} = \lambda W^{T,fb} + (1-\lambda) W^{NT,fb} = \bar{y} - \zeta \bar{y}^2 \tag{39}$$

¹⁸Let $k_0 = 0$. Then, a sufficient condition to ensure that $\mu > 0$ is $\bar{v} < \frac{1-\sigma_y^2}{1+\lambda\frac{\sigma_y^2}{\bar{y}^2}}$. Even for $\lambda = 1$, and a relatively high value of $\frac{\sigma_y^2}{\bar{y}^2} = 0.1$ the above holds if $\bar{v} < 0.99$. (Note that a value of 0.99 implies a velocity of $\frac{1}{1-\bar{v}} = 100$, much higher than empirically observed values.)

We now compute the welfare loss under the optimal money growth rule relative to the first best. Define the welfare loss under money growth rate $\tilde{\mu}$, relative to the first-best as¹⁹

$$\Delta W^{\tilde{\mu}} = W^{fb} - W^{\tilde{\mu}}.$$

Observe that the welfare maximizing $\tilde{\mu}$ that is obtained from (35) also minimizes $\Delta W^{\tilde{\mu}}$. In the appendix we show that by substituting in the relevant expressions for $E(c^T)$, $E(c^{NT})$ and $Var[c^{NT}]$ into the welfare loss expression gives

$$\Delta W^{\tilde{\mu}} = \frac{\zeta(1-\lambda)}{1-\beta} \left(\frac{1 + \frac{\lambda}{(1-\bar{v})^2} \sigma_v^2 \left(1 + \frac{\sigma_y^2}{\bar{y}^2}\right)}{1 + \frac{\lambda}{(1-\bar{v})^2} \sigma_v^2 \left(1 + \frac{\sigma_y^2}{\bar{y}^2}\right) + \lambda \frac{\sigma_y^2}{\bar{y}^2}} \right) \sigma_y^2 \quad (40)$$

If only one shock is present at a time, then (40) simplifies to

$$\Delta W^{\tilde{\mu}} = \begin{cases} 0; & \text{only velocity shocks } (\sigma_y^2 = 0) \\ \frac{\zeta(1-\lambda)}{1-\beta} \frac{\sigma_y^2}{1 + \lambda \frac{\sigma_y^2}{\bar{y}^2}}; & \text{only output shocks } (\sigma_v^2 = 0) \end{cases} \quad (41)$$

Equation (41) shows that when there is no output volatility so that $\sigma_y^2 = 0$, the welfare loss from following the optimal money growth rule is zero. This reflects the fact that under no output shocks the optimal state contingent rule and the optimal non-state contingent money growth rule coincide. They both call for a fixed money rule.

Equation (41) also shows that when there is no volatility in the velocity process, i.e., $\sigma_v^2 = 0$, so that the economy is exposed to only output volatility, the welfare losses from following a non-state contingent money growth rule are increasing in the volatility of output and decreasing in the share of traders λ . Both these comparative static effects are intuitive. The higher is σ_y^2 the greater is the loss from not being able to vary the growth rate of money to better accommodate the state of the economy. On the other hand, the greater is the share of traders in the economy (a higher λ), the closer the economy is to full insurance since the traders can completely insure against all risk. Hence, the smaller are the welfare losses relative to the first-best under the fixed money growth rule.

¹⁹Superscript $\tilde{\mu}$ denotes variable values under optimal money growth rule.

A special case of the constant money growth rule is the fixed money supply rule, i.e., $\mu = 0$. Hence, money supply is set to \bar{M} for all t . In this case the welfare loss relative to the first-best is:

$$\Delta W^{\bar{M}} = \frac{\zeta(1-\lambda)}{1-\beta} \sigma_y^2$$

Thus,

$$\frac{\Delta W^{\bar{M}}}{\Delta W^{\bar{\mu}}} = 1 + \lambda \frac{\sigma_y^2}{\bar{y}^2} \geq 1.$$

This expression shows that only in the special case of no output volatility ($\sigma_y^2 = 0$), do we have $\Delta W^{\bar{M}} = \Delta W^{\bar{\mu}} = 0$. In general, a fixed money policy generates welfare losses which are at least as great as those under an optimally chosen constant money growth rule.

4.2 Optimal rate of devaluation

We now turn to our second non-state contingent rule which is a fixed rate of devaluation. This rule is of interest for two reasons. First, a number of developing countries use the exchange rate as a nominal anchor and thereby prefer some sort of exchange rate rule. Second, an exchange rate rule corresponds closely to an inflation targeting policy in this one good world of our model. Needless to say inflation targeting is a policy which is both widely used and discussed in policy circles.

Define $z \equiv \frac{1}{1+\varepsilon} \left(= \frac{S_{t-1}}{S_t} \right)$. As before the optimal z or, equivalently, the rate of devaluation ε is determined by solving (35). Under a constant devaluation rate, equations (18), (23) and (21) imply that consumption of nontraders and traders are given by

$$\begin{aligned} c_t^{NT} &= z(1 - v_{t-1})y_{t-1} + v_t y_t, \quad t \geq 1, \\ c_t^T &= \bar{y} \left[1 + \left(\frac{1-\lambda}{\lambda} \right) (1-z)(1-\bar{v}) \right]. \end{aligned}$$

Hence,

$$Var [c_t^{NT}] = z^2 \sigma_y^2 + (1+z^2) \sigma_{vy}^2 - 2z^2 \bar{v} \sigma_y^2. \quad (42)$$

Given that $\lambda E(c^T) + (1-\lambda)E(c^{NT}) = \bar{y}$, it is still the case that the optimal z (and hence, the optimal rate of devaluation ε) can be derived from the solution to

$$\arg \min_z \left\{ \lambda (E(c^T))^2 + (1-\lambda) (E(c^{NT}))^2 + (1-\lambda) Var [c_t^{NT}] \right\}.$$

The implied optimal rate of devaluation $\hat{\varepsilon}$ is given by

$$\frac{1 - \hat{z}}{\hat{z}} = \tilde{\varepsilon} = \lambda \left[\frac{\sigma_v^2}{(1 - \bar{v})^2} \left(1 + \frac{\sigma_y^2}{\bar{y}^2} \right) + \frac{\sigma_y^2}{\bar{y}^2} \right]. \quad (43)$$

Hence, the optimal rate of devaluation is increasing in the variance of both shocks.

There are two special cases which are worth emphasizing. First, when the economy faces no output uncertainty so that the only uncertainty is regarding the velocity realization, i.e., $\sigma_y^2 = 0$, the optimal rate of devaluation implied by equation (43) is

$$\tilde{\varepsilon} = \frac{\lambda \sigma_v^2}{(1 - \bar{v})^2}.$$

Second, when the only uncertainty is about the output realization, i.e., $\sigma_v^2 = 0$, the optimal devaluation rate is

$$\tilde{\varepsilon} = \frac{\lambda \sigma_y^2}{\bar{y}^2}.$$

It is worth pointing out that equation (43) clearly shows that, in general, it is never optimal to set $\varepsilon = 0$, i.e., fixed exchange rates are never optimal. Only in the uninteresting case of no shocks at all in the economy ($\sigma_v^2 = \sigma_y^2 = 0$) is it optimal to peg the exchange rate.²⁰

4.2.1 Welfare loss relative to the first-best

We next turn to the welfare loss relative to the first best that is implied by following the devaluation rule. The welfare loss expression is $\Delta W^{\tilde{\varepsilon}} = W^{fb} - W^{\tilde{\varepsilon}}$. Substituting the relevant expressions for trader and nontrader consumption, the nontrader variance, and the optimal devaluation rate policy (43) into $\Delta W^{\tilde{\varepsilon}}$ gives

$$\Delta W^{\tilde{\varepsilon}} = \frac{\zeta(1 - \lambda)}{1 - \beta} \left(\frac{\bar{v}^2 \left(1 + \frac{\sigma_v^2}{\bar{v}^2} \left(1 + \frac{\bar{y}^2}{\sigma_y^2} \right) \right) + \left(1 + \frac{\sigma_v^2}{(1 - \bar{v})^2} \left(1 + \frac{\bar{y}^2}{\sigma_y^2} \right) \right)}{(1 - \bar{v})^2 \frac{1 + \left(\frac{\lambda}{(1 - \bar{v})^2} \sigma_v^2 \left(1 + \frac{\bar{y}^2}{\sigma_y^2} \right) + \lambda \frac{\sigma_y^2}{\bar{y}^2} \right)}{1 + \left(\frac{\lambda}{(1 - \bar{v})^2} \sigma_v^2 \left(1 + \frac{\bar{y}^2}{\sigma_y^2} \right) + \lambda \frac{\sigma_y^2}{\bar{y}^2} \right)}} \right) \sigma_y^2. \quad (44)$$

²⁰It would appear from these expressions that a fixed exchange rate is optimal when there are no traders in the economy, i.e., when $\lambda = 0$. However, this conclusion is not valid since the model is discontinuous at $\lambda = 0$. In particular, when there are no traders at all, there is no way for the monetary authority to introduce money into the economy since all money injections, by assumption, are in the asset market. Hence, maintaining a fixed exchange rate by appropriate changes in money supply is not feasible.

If only one shock is present at a time, then (44) simplifies to

$$\Delta W^{\tilde{\varepsilon}} = \begin{cases} \frac{\zeta(1-\lambda)}{1-\beta} \bar{y}^2 \left(1 + \frac{1}{1+\lambda \frac{\sigma_v^2}{(1-\bar{v})^2}} \right) \sigma_v^2; & \text{only velocity shocks } (\sigma_y^2 = 0), \\ \frac{\zeta(1-\lambda)}{1-\beta} \bar{v}^2 \left(1 + \frac{(\frac{1-\bar{v}}{\bar{v}})^2}{1+\lambda \frac{\sigma_y^2}{\bar{y}^2}} \right) \sigma_y^2; & \text{only output shocks } (\sigma_v^2 = 0). \end{cases} \quad (45)$$

A special case of the fixed devaluation rate policy is the policy of a fixed exchange rate, i.e., $\varepsilon = 0$. In this case the welfare loss relative to the first-best is given by

$$\Delta W^{\bar{S}} = \frac{\zeta(1-\lambda)}{1-\beta} \left((1-\bar{v})^2 + \bar{v}^2 + 2\sigma_v^2 \left(1 + \frac{\bar{y}^2}{\sigma_y^2} \right) \right) \sigma_y^2. \quad (46)$$

If only one shock is present at a time, then (46) simplifies to

$$\Delta W^{\bar{S}} = \begin{cases} \frac{\zeta(1-\lambda)}{1-\beta} 2\sigma_v^2 \bar{y}^2; & \text{only velocity shocks } (\sigma_y^2 = 0), \\ \frac{\zeta(1-\lambda)}{1-\beta} \left((1-\bar{v})^2 + \bar{v}^2 \right) \sigma_y^2; & \text{only output shocks } (\sigma_v^2 = 0). \end{cases} \quad (47)$$

4.3 Welfare comparison

It is clear that $\mu = 0$ will always be dominated by an *optimal* μ since the optimal μ is not constrained to be non-zero. Similarly the fixed exchange rate, i.e., $\varepsilon = 0$ will be always be dominated by $\tilde{\varepsilon}$. The question regarding which of these two non-state contingent rules is better from a welfare standpoint still remains to be answered. To address this question it is useful to derive an expression for $\frac{\Delta W^{\tilde{\varepsilon}}}{\Delta W^{\tilde{\mu}}}$. Note that $\frac{\Delta W^{\tilde{\varepsilon}}}{\Delta W^{\tilde{\mu}}} < 1$ implies that a fixed devaluation rate rule will dominate a fixed money growth rule. The opposite holds when $\frac{\Delta W^{\tilde{\varepsilon}}}{\Delta W^{\tilde{\mu}}} > 1$. Using equations (40) and (44) it can be shown that

$$\frac{\Delta W^{\tilde{\varepsilon}}}{\Delta W^{\tilde{\mu}}} \leq 1 \text{ iff} \quad \left(\begin{array}{c} \bar{v}^2 \left(1 + \frac{\sigma_v^2}{\bar{v}^2} \left(1 + \frac{\bar{y}^2}{\sigma_y^2} \right) \right) + \\ (1-\bar{v})^2 \frac{\left(1 + \frac{\sigma_v^2}{(1-\bar{v})^2} \left(1 + \frac{\bar{y}^2}{\sigma_y^2} \right) \right)}{1+\lambda \left(\frac{\sigma_v^2}{(1-\bar{v})^2} \left(1 + \frac{\bar{y}^2}{\sigma_y^2} \right) + \frac{\sigma_y^2}{\bar{y}^2} \right)} \end{array} \right) \leq \frac{1 + \frac{\lambda}{(1-\bar{v})^2} \sigma_v^2 \left(1 + \frac{\sigma_y^2}{\bar{y}^2} \right)}{1 + \frac{\lambda}{(1-\bar{v})^2} \sigma_v^2 \left(1 + \frac{\sigma_y^2}{\bar{y}^2} \right) + \lambda \frac{\sigma_y^2}{\bar{y}^2}} \quad (48)$$

While both sides of equation (48) are increasing (decreasing) in velocity (output) shocks, the LHS increases faster than the RHS when σ_v^2 rises. Hence, $\tilde{\mu}$ will dominate $\tilde{\varepsilon}$ when velocity shocks are relatively dominant. On the other hand, the LHS decreases faster than the RHS when σ_y^2

increases. Hence, the desirability of a optimal ε policy will increase with higher output variance. In the limiting case, when only output shocks are present,

$$\frac{\Delta W^{\tilde{\varepsilon}}}{\Delta W^{\tilde{\mu}}} \leq 1 \text{ if and only if } \bar{v}^2 + \frac{(1 - \bar{v})^2}{1 + \lambda \frac{\sigma_y^2}{\bar{y}^2}} \leq \frac{1}{1 + \lambda \frac{\sigma_y^2}{\bar{y}^2}}$$

Hence, an $\tilde{\varepsilon}$ policy would welfare dominate an $\tilde{\mu}$ policy if and only if

$$\bar{v} \in \left(0, \frac{1}{1 + \frac{\lambda \sigma_y^2}{2 \bar{y}^2}} \right). \quad (49)$$

Figure 1 shows precisely this trade-off through a simulation of the model. It depicts the ratio of the welfare loss (relative to the first best) under optimal μ to the welfare loss under optimal ε . Hence, a value lower than one means that optimal μ delivers higher welfare than optimal ε . The parameters assumed for the simulation are: $\bar{y} = 1$, $\bar{v} = 0.2$, $\lambda = 0.5$, $\beta = 0.97$, $\zeta = 0.15$.²¹ For a given σ_v , $\frac{\Delta W^{\tilde{\mu}}}{\Delta W^{\tilde{\varepsilon}}}$ rises with σ_y . Hence, the relative attraction of the fixed devaluation rate policy rises with the volatility of output. The figure also shows that for a given σ_y , $\frac{\Delta W^{\tilde{\mu}}}{\Delta W^{\tilde{\varepsilon}}}$ falls (the schedule shifts down) as σ_v rises. Hence, the money growth rule becomes more attractive as the relative volatility of velocity increases. To summarize, the model predicts that exchange rate targeting rules begin to welfare dominate money growth rules when output shocks become

²¹We should note that the attempt here is not replicate a specific economy but rather, to determine the qualitative nature of the relationship between the volatility of shocks and the optimal monetary policy regime. We defer till later a discussion about the implications of the model for specific economies.

relatively more important while the opposite is true when velocity shocks are relatively dominant.

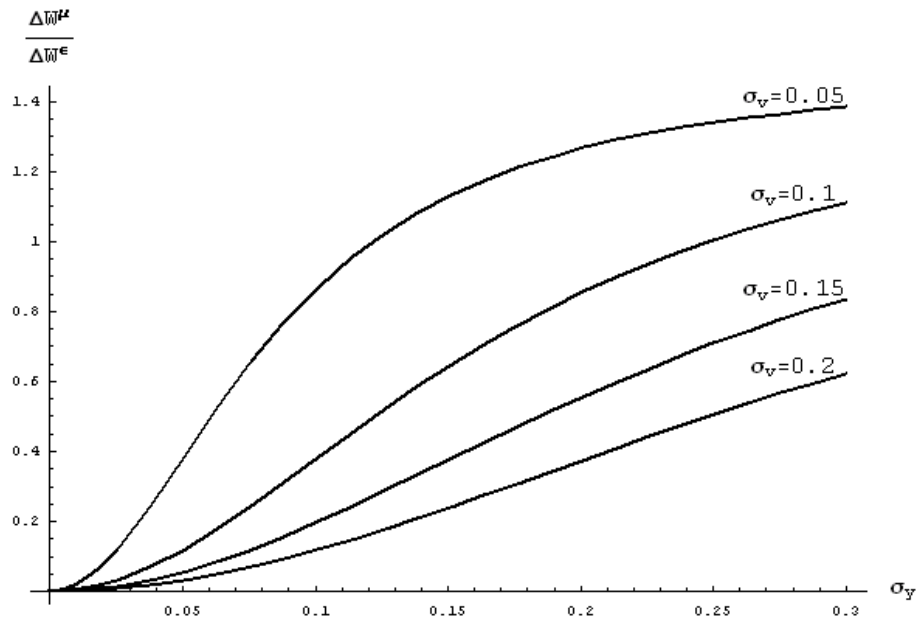


Figure 1

Since Figure 1 indicates that the welfare comparison depends on both output and velocity volatility, it is useful to focus on some actual numbers for illustration purposes. The following table shows output and velocity volatilities (in percentages) for Argentina, Brazil, and, as a benchmark, the United States. We see that, even in highly volatile countries such as Argentina and Brazil, output volatility is less than 5 percent. It is thus clear from Figure 1 that, given the figures presented in the table, all three countries would be better off with a money growth rule (that allows for exchange rate flexibility) than with a devaluation rule.

Country	Output volatility (% std. dev.)	Velocity volatility (M2) (of quarter-to-quarter % changes)
Argentina	4.5	9.9
Brazil	4.5	8.1
United States	2.05	1.2

Table 1: output and veolcity variabilities

These estimates are based on annualized quarterly data for M2 and nominal GDP; for Argentina: 1993Q1 - 2003Q2;

Brazil: 1994Q3 - 2003Q2; US: 1970Q1 - 2003Q2; Source : International Financial Statistics.

For the countries reported in Table 1 the welfare tradeoff between $\tilde{\mu}$ and $\tilde{\varepsilon}$ policies are shown

below in Figure 2

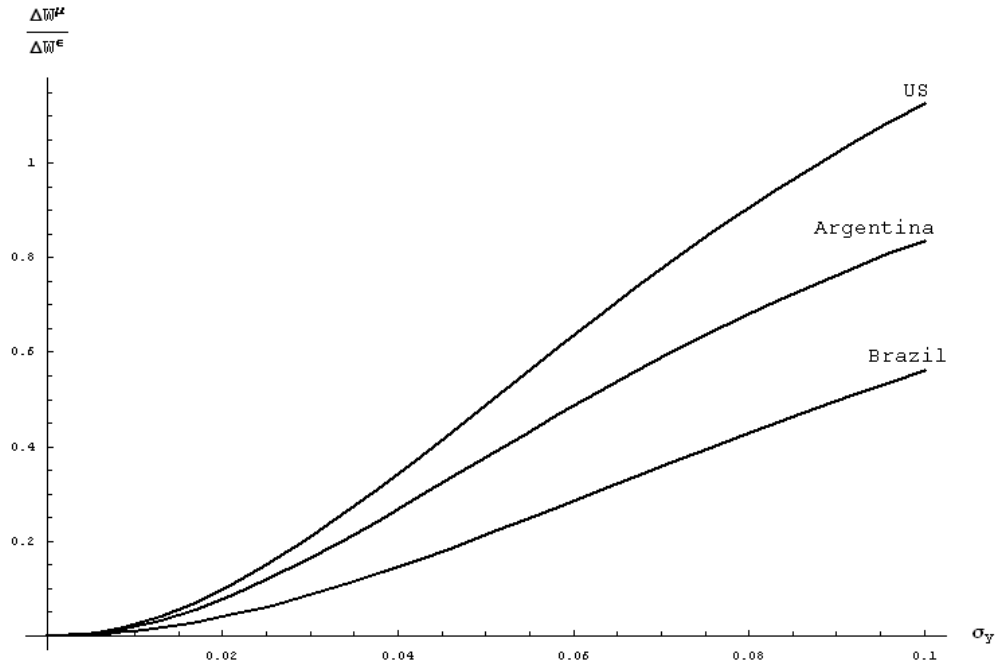


Figure 2

The calulcations assume $\bar{v} = 0.82, 0.09,$ and $0.45,$ and $\sigma_v = 0.05, 0.07,$ and 0.04 for Argentina, Brazil, and US

respectively. These estimates are based on annualized quarterly data for M2 and nominal GDP; for Argentina :

1993Q1 - 2003Q2; Brazil: 1994Q3 - 2003Q2; US: 1970Q1 - 2003Q2; Source : International Financial Statistics.

5 Macroeconomic Volatility

Another issue of interest is the volatility of different macroeconomic variables that is implied by these alternative monetary regimes. This is of interest both from a policy perspective as well as from the perspective of providing us with some testable implications of the structure. There are three key endogenous macroeconomic variables in the model: consumption, money and the exchange rate (or equivalently, the price level). We look at each of these in turn.

5.1 Consumption variances

It can be verified from equations (36) and (42) that the variances of consumption for nontraders under both $\tilde{\mu}$ as well as $\tilde{\varepsilon}$ are increasing in the variances of output and velocity shocks. However, while consumption variability under $\tilde{\varepsilon}$ grows without bounds, under $\tilde{\mu}$ it stays within bounds. On the other hand, the ratio of the consumption variance under $\tilde{\varepsilon}$ to that under $\tilde{\mu}$ decreases with the variance of output. The consumption variance for nontraders under $\tilde{\varepsilon}$ is lower than under $\tilde{\mu}$ if and only if

$$\bar{v} \in \left(0, \frac{1}{1 + \lambda \frac{\sigma_y^2}{\bar{y}^2} + \frac{\lambda^2}{2} \left(\frac{\sigma_y^2}{\bar{y}^2} \right)^2} \right). \quad (50)$$

Although this condition resembles equation (49) which was the relevant condition for comparing welfares, the range under (50) is narrower than in (49). As a result, even though the consumption variance may be higher under $\tilde{\varepsilon}$ in the range $\bar{v} \in \left(\frac{1}{1 + \lambda \frac{\sigma_y^2}{\bar{y}^2} + \frac{\lambda^2}{2} \left(\frac{\sigma_y^2}{\bar{y}^2} \right)^2}, \frac{1}{1 + \frac{\lambda}{2} \frac{\sigma_y^2}{\bar{y}^2}} \right)$, $\tilde{\varepsilon}$ is still preferred to $\tilde{\mu}$. This is because $\tilde{\mu}$ induces a smaller variance at the cost of a larger transfer of consumption to the traders.

5.2 Exchange rate volatilities

A second variable of interest is the volatility of the exchange rate. To get a sense of the implications of different monetary regimes for this volatility, we compare the unconditional variances for the rate of currency depreciation under (1) state-contingent money growth rule, and (2) a fixed money growth rule.

5.2.1 State-contingent rule

Recall that the optimal state contingent money growth rule is given by $\mu_t = \frac{y_t - \bar{y}}{\bar{y} - v_t y_t}$. Substituting this rule into the quantity theory relationship and rearranging the result gives

$$S_t = \frac{M_t}{\bar{y} - v_t y_t}$$

Since $\varepsilon_t = \frac{S_t - S_{t-1}}{S_{t-1}}$, one can use the exchange rate equation derived above to get

$$1 + \varepsilon_t = \frac{S_t}{S_{t-1}} = \frac{(1 - v_{t-1}) y_{t-1}}{\bar{y} - v_t y_t}$$

Taking a second order expansion of this expression around \bar{v} and \bar{y} and then taking expectations yields

$$E \{ \varepsilon_t \} \cong \frac{\sigma_{vy}^2}{\bar{y}^2 (1 - \bar{v})^2} \quad (51)$$

Similarly, a second order approximation for the variance gives

$$Var \{ \varepsilon_t \} \cong \frac{1}{\bar{y}^2 (1 - \bar{v})^2} \left(\sigma_{vy}^2 + (\sigma_{vy}^2 + (1 - 2\bar{v}) \sigma_y^2) \left(\frac{\sigma_{vy}^2}{\bar{y}^2 (1 - \bar{v})^2} + \left(1 + \frac{\sigma_{vy}^2}{\bar{y}^2 (1 - \bar{v})^2} \right)^2 \right) \right) \quad (52)$$

5.2.2 Fixed money growth rule

Under a fixed money growth rate μ , the quantity theory relationship implies that the exchange rate is

$$S_t = \frac{(1 + \mu) M_t}{(1 - v_t) y_t}$$

Hence, the rate of depreciation is given by

$$1 + \varepsilon_t = \frac{S_t}{S_{t-1}} = (1 + \mu) \frac{(1 - v_{t-1}) y_{t-1}}{(1 - v_t) y_t}$$

Taking a second order approximation of this equation around \bar{v} and \bar{y} and then taking expectations yields

$$E \{ \varepsilon_t \} \cong \mu + (1 + \mu) \left(\frac{\sigma_{vy}^2 + (1 - 2\bar{v}) \sigma_y^2}{\bar{y}^2 (1 - \bar{v})^2} \right)$$

Correspondingly, a second order approximation for the variance gives

$$Var \{ \varepsilon_t \} \cong (1 + \mu)^2 \frac{(\sigma_{vy}^2 + (1 - 2\bar{v}) \sigma_y^2)}{\bar{y}^2 (1 - \bar{v})^2} \left(1 + (\sigma_{vy}^2 + (1 - 2\bar{v}) \sigma_y^2) \left(\frac{1}{\bar{y}^2 (1 - \bar{v})^2} + \left(1 + \frac{\sigma_{vy}^2 + (1 - 2\bar{v}) \sigma_y^2}{\bar{y}^2 (1 - \bar{v})^2} \right)^2 \right) \right) \quad (53)$$

5.2.3 Comparison

Under an optimal fixed money growth rate, $\tilde{\mu} > 0$. Hence, from (52) and (53), a sufficient condition for the variance of the depreciation rate under the optimal fixed money growth rule to be larger than that under state-contingent rule is $\bar{v} < 0.5$. If this condition holds, then

$$Var_{t-1} \{\varepsilon_t\}_{\tilde{\mu}} > Var_{t-1} \{\varepsilon_t\}_{sc} > Var_{t-1} \{\varepsilon_t\}_{\bar{\varepsilon}} = 0$$

5.3 Interest rate volatility and its cyclical response

Next, we discuss the response of nominal interest rate to output and velocity shocks, first under the first-best rule, and then under fixed money growth rule.

5.3.1 State-contingent rule

The nominal interest rate is given by the Euler equation (assuming that there is a riskless government bond in the market)

$$\frac{u'(c_t^T)}{S_t} = \beta (1 + i_t) E_t \left\{ \frac{u'(c_{t+1}^T)}{S_{t+1}} \right\} \quad (54)$$

Under market completeness $c_t^T = c^T$ for all t . Using (??), nominal interest rate is derived as

$$(1 + i_t) = (1 + r) \left(E_t \left\{ \frac{\bar{y} - v_{t+1}y_{t+1}}{y_t(1 - v_t)} \right\} \right)^{-1}$$

which under our assumption of i.i.d. shocks becomes

$$1 + i_t = (1 + r) \frac{y_t}{\bar{y}} \frac{1 - v_t}{1 - \bar{v}} \quad (55)$$

Notice that the unconditional expectation of $1 + i_t$ equals $1 + r$. Thus, in terms of % deviation:

$$\frac{\sigma_i^2}{(1 + r)^2} = \frac{\sigma_y^2}{\bar{y}^2} + \frac{\sigma_v^2}{(1 - \bar{v})^2} + \frac{\sigma_y^2}{\bar{y}^2} \frac{\sigma_v^2}{(1 - \bar{v})^2}$$

It directly follows from (55) that $Corr(i, y) = 1$ and $Corr(i, v) = -1$.

5.3.2 Money growth rule

Under a fixed $\mu = \bar{\mu}$, the exchange rate evolves as

$$\frac{S_t}{S_{t+1}} = \frac{1}{1 + \bar{\mu}} \frac{y_{t+1}(1 - v_{t+1})}{y_t(1 - v_t)} \quad (56)$$

Then using (54):

$$1 + i_t = (1 + r)(1 + \bar{\mu}) \frac{y_t}{\bar{y}} \frac{1 - v_t}{1 - \bar{v}} \quad (57)$$

Notice that $E(1 + i_t) = (1 + r)(1 + \bar{\mu})$. Thus the volatility (in % terms) is identical under the first-best and fixed money growth rules.

$$\frac{\sigma_i^2}{(1 + r)^2 (1 + \bar{\mu})^2} = \frac{\sigma_y^2}{\bar{y}^2} + \frac{\sigma_v^2}{(1 - \bar{v})^2} + \frac{\sigma_y^2}{\bar{y}^2} \frac{\sigma_v^2}{(1 - \bar{v})^2}$$

Similarly, it directly follows from (57) that $Corr(i, y) = 1$ and $Corr(i, v) = -1$.

Based on the output and velocity volatilities reported in Table 1, the nominal interest rate and exchange rate volatilities, as implied by the model, are easily computed as:

Country	σ_i (%)	σ_ε under first-best	σ_ε under $\tilde{\mu}$
United States	8.2	11.7	8.3
Argentina	28.2	47.8	29.3
Brazil	8.9	11.9	9.0

Table 2: Interest rate and exchange rate volatilities expressed as % of their mean values.

The calculations assume $\bar{v} = 0.82, 0.09$, and 0.45 , and $\sigma_v = 0.05, 0.07$, and 0.04 for Argentina, Brazil, and US respectively. These estimates are based on annualized quarterly data for M2 and nominal GDP; for Argentina : 1993Q1 - 2003Q2; Brazil: 1994Q3 - 2003Q2; US: 1970Q1 - 2003Q2; Source : International Financial Statistics.

We thus observe that the nominal exchange rate volatility is higher under the first-best optimal monetary policy than under the fixed- μ rule. Intuitively, this reflects the fact that changes in the price level (i.e., the nominal exchange rate) constitute an essential adjustment mechanism in this model as they enable a redistribution from non-traders to traders in good times and from traders to non-traders in bad times.

5.4 Monetary volatility under various rules

A third variable of interest is the volatility of money. Obviously, the cases to look for are: (1) state-contingent rules and (2) fixed exchange rate regime i.e., $\varepsilon \geq 0$. As before, we compare the volatilities of the implied money growth rates.

5.4.1 State-contingent rule

Since the optimal state contingent rule is $\mu_t = \frac{y_t - \bar{y}}{\bar{y} - v_t y_t}$, to a second order approximation we have

$$E \{ \mu_t \} = \frac{\bar{v}}{(1 - \bar{v})^2} \frac{\sigma_y^2}{\bar{y}^2}.$$

Likewise, a second order approximation for the variance yields

$$Var \{ \mu_t \} = \frac{1}{(1 - \bar{v})^2} \frac{\sigma_y^2}{\bar{y}^2}. \quad (58)$$

5.4.2 Optimal devaluation rule

Under a fixed rate of devaluation, ε , the implied money growth rate is

$$1 + \mu_t = (1 + \varepsilon) \frac{(1 - v_t) y_t}{(1 - v_{t-1}) y_{t-1}}.$$

Then, taking a second order approximation gives

$$E \{\mu_t\} = \varepsilon + (1 + \varepsilon) \left(\frac{\sigma_{vy}^2 + (1 - 2\bar{v}) \sigma_y^2}{\bar{y}^2 (1 - \bar{v})^2} \right),$$

while a second order approximation for the variance yields

$$Var \{\mu_t\} \cong (1 + \varepsilon)^2 \frac{(\sigma_{vy}^2 + (1 - 2\bar{v}) \sigma_y^2)}{\bar{y}^2 (1 - \bar{v})^2} \left(\frac{1 + (\sigma_{vy}^2 + (1 - 2\bar{v}) \sigma_y^2) *}{\left(\frac{1}{\bar{y}^2 (1 - \bar{v})^2} + \left(1 + \frac{\sigma_{vy}^2 + (1 - 2\bar{v}) \sigma_y^2}{\bar{y}^2 (1 - \bar{v})^2} \right)^2 \right)} \right). \quad (59)$$

5.4.3 Comparison

From (58) and (59), it is clear that when only velocity shocks are present ($\sigma_y^2 = 0$), the state-contingent rule is $\mu = 0$ for all times, and hence monetary volatility under an optimal devaluation rule is the higher. However, when only output shocks are present

$$Var \{\mu_t\}_{sc} \geq Var \{\mu_t\}_{\bar{\varepsilon}} \quad \text{iff} \\ \left(1 + \lambda \frac{\sigma_y^2}{\bar{y}^2} \right) \left(1 + \frac{\sigma_y^2}{\bar{y}^2} \right) \left(1 + \sigma_y^2 (1 - \bar{v})^2 \left(1 + \frac{\sigma_y^2}{\bar{y}^2} \right) \right) (1 - \bar{v})^2 \leq 1$$

Clearly, for $\bar{v} = 0$, $Var \{\mu_t\}_{sc} < Var \{\mu_t\}_{\bar{\varepsilon}}$. On the other hand, when $\bar{v} = 1$, $Var \{\mu_t\}_{sc} > Var \{\mu_t\}_{\bar{\varepsilon}}$. Since the LHS is decreasing in \bar{v} , there exists \hat{v} , such that for all $\bar{v} > \hat{v}$, the volatility under state-contingent rule is higher than under optimal devaluation rule. Note that the smaller are σ_y^2 , $\frac{\sigma_y^2}{\bar{y}^2}$, or λ , the smaller will be \hat{v} .

6 Conclusion

This paper has examined optimal monetary policy in the context of a small open economy under asset market segmentation. We have also assumed that traders have access to complete markets. In this context, we have shown that there exist state contingent rules based on either the rate of money growth and the rate of devaluation that can replicate the first best equilibrium. These

state contingent rules allow the monetary authority to stabilize non-traders' consumption. While these state contingent rules constitute the natural analytical benchmark, they would be difficult to implement in practice since they require responding to contemporaneous shocks. We therefore examine non-state contingent rules based on either the money supply or the exchange rate and conclude that money supply rules – which allow for exchange rate flexibility – generally dominate exchange rate rules in welfare terms. This would support arrangements that allow for some exchange rate flexibility rather than arrangements based on exchange rate pegs.

Our model has ignored the issue of endogeneity of market segmentation. In particular, one would expect that agents endogenously choose to be traders or non-traders with the choice depending on the cost of participating in asset markets as well as the prevailing exchange rate and/or monetary regime. However, we see no reason to believe that this would change our key results. As should be clear from the intuition provided in the paper, what matters for our results is that, at every point in time, some agents have access to assets market while others do not. What particular agents have access to asset markets and whether this group changes over time should not alter the essential arguments. A formal check of this conjecture is left for future work.

7 Appendix

7.1 Optimal non-state contingent rules for the CRRA case

This appendix derives the optimal fixed- μ and optimal fixed- ε rules for the case in which preferences are CRRA. The overall message is that, under CES preferences, the optimal fixed- μ and optimal fixed- ε can be zero, positive, or negative depending on the value of σ .

Let preferences for both traders and non-traders be given by:

$$u(c) = \begin{cases} c^{1-\frac{1}{\sigma}} - 1 \\ 1 - \frac{1}{\sigma} \end{cases}. \quad (60)$$

where $1/\sigma$ is the coefficient of relative risk aversion ($\sigma > 0$).

We begin by deriving the fixed- μ rule and then turn to the fixed- ε rule.

7.1.1 Optimal fixed- μ rule

Recall that, regardless of preferences, consumption of traders and non-traders are given by, respectively:

$$\begin{aligned} c^T &= \left[1 + \frac{1-\lambda}{\lambda} \frac{\bar{\mu}}{1+\bar{\mu}} (1-\bar{v}) \right] \bar{y}, \\ c_t^{NT} &= \left[1 - \frac{\bar{\mu}}{1+\bar{\mu}} (1-v_t) \right] y_t, \end{aligned}$$

where $\bar{\mu} > -1$. The optimal $\bar{\mu}$ is obtained by solving the following problem:

$$\max_{\bar{\mu}} \left\{ \lambda E_0 \sum_{t=0}^{\infty} \beta^t u \left(\bar{y} \left(1 + \frac{1-\lambda}{\lambda} \frac{\bar{\mu}}{1+\bar{\mu}} (1-\bar{v}) \right) \right) + (1-\lambda) E_0 \sum_{t=0}^{\infty} \beta^t u \left(\left(1 - \frac{\bar{\mu}}{1+\bar{\mu}} (1-v_t) \right) y_t \right) \right\},$$

We now study, in turn, the case of velocity shocks and then the case of output shocks only.

Velocity shocks only First, consider only velocity shocks. Here, $y_t = \bar{y}$ for all t . The optimal $\bar{\mu}$ is obtained, after simplifying and using (60), from the following first-order condition:

$$\left(1 + \frac{1-\lambda}{\lambda} \frac{\bar{\mu}}{1+\bar{\mu}} (1-\bar{v}) \right)^{-\frac{1}{\sigma}} (1-\bar{v}) \sum_{t=0}^{\infty} \beta^t \bar{y}^{1-\frac{1}{\sigma}} = E_0 \sum_{t=0}^{\infty} \beta^t \left(1 - \frac{\bar{\mu}}{1+\bar{\mu}} (1-v_t) \right)^{-\frac{1}{\sigma}} (1-v_t) \bar{y}^{1-\frac{1}{\sigma}},$$

where the expectations operator has been dropped from the LHS since there are no stochastic variables. It is easy to verify that $\bar{\mu} = 0$ solves the above equation, and hence fixing money supply is optimal under only velocity shocks. This is of course the same result that we obtained for the quadratic case.

Output shocks only Next, consider output shocks; here $v_t = \bar{v}$ for all t . The first order condition now is

$$\left(1 + \frac{1-\lambda}{\lambda} \frac{\bar{\mu}}{1+\bar{\mu}} (1-\bar{v})\right)^{-\frac{1}{\sigma}} \sum_{t=0}^{\infty} \beta^t \bar{y}^{1-\frac{1}{\sigma}} = \left(1 - \frac{\bar{\mu}}{1+\bar{\mu}} (1-\bar{v})\right)^{-\frac{1}{\sigma}} E_0 \sum_{t=0}^{\infty} \beta^t y_t^{1-\frac{1}{\sigma}},$$

Rearranging terms, we can rewrite this condition as:

$$\frac{1 + \frac{1-\lambda}{\lambda} \frac{\bar{\mu}}{1+\bar{\mu}} (1-\bar{v})}{1 - \frac{\bar{\mu}}{1+\bar{\mu}} (1-\bar{v})} = \left(\frac{\sum_{t=0}^{\infty} \beta^t \bar{y}^{1-\frac{1}{\sigma}}}{E_0 \sum_{t=0}^{\infty} \beta^t y_t^{1-\frac{1}{\sigma}}} \right)^{\sigma}. \quad (61)$$

From Jensen's inequality, it follows that

$$\sum_{t=0}^{\infty} \beta^t \bar{y}^{1-\frac{1}{\sigma}} \begin{cases} \geq \\ \leq \end{cases} E_0 \sum_{t=0}^{\infty} \beta^t y_t^{1-\frac{1}{\sigma}} \quad \text{iff } \sigma \begin{cases} \geq \\ \leq \end{cases} 1. \quad (62)$$

Hence, from (61) and (62),

$$\bar{\mu} \begin{cases} < 0 \text{ iff } \sigma < 1, \\ = 0 \text{ iff } \sigma = 1, \\ > 0 \text{ iff } \sigma > 1. \end{cases} \quad (63)$$

Hence, if $\sigma = 1$ (the logarithmic case), the optimal $\bar{\mu} = 0$, which means that the optimal policy is not to implement any transfers between traders and non-traders. If $\sigma < 1$ (coefficient of risk aversion greater than one), then the optimal $\bar{\mu}$ is negative (i.e., it is optimal to transfer resources from traders to non-traders). If $\sigma > 1$ (coefficient of risk aversion smaller than one), then the optimal $\bar{\mu}$ is positive (i.e., it is optimal to transfer resources from non-traders to traders).

What is the intuition behind these results? The key is to realize that the parameter σ determines whether, from a social point of view, c^T and c^N are gross substitutes or gross complements. If $\sigma < 1$,

these two “goods” are gross complements in the sense that an increase in the relative price of, say, c^N , will reduce demand for c^T . If $\sigma > 1$, they are gross substitutes in the sense that an increase in the relative price of c^N will increase demand for c^T . If there were no uncertainty in the model (and hence the endowment streams were constant over time), the “relative price” of both goods would be unity and optimal transfers would be zero. When uncertainty is present, the relative price of c^N becomes larger than one because, given that non-traders cannot smooth consumption over time, it takes more resources for them to achieve a certain level of utility. In response to this higher relative price, it becomes optimal to consume more c^T if the two goods are gross substitutes (which implies a transfer from non-traders to traders) and to consume less c^T if both goods are gross complements (which implies a transfer from traders to non-traders). In the logarithmic case, the higher relative price does not affect consumption of c^T (optimal response is no transfers).

An example with log-normal distribution To fix ideas, it proves illuminating to derive a closed-form solution for the case of in which y follows a log-normal distribution. Specifically, suppose that $\log(y)$ follows a normal distribution with with mean 0 and variance σ_y^2 . As is well-known,

$$\bar{y} \equiv E(y) = e^{\frac{1}{2}\sigma_y^2}. \quad (64)$$

Furthermore, if $\ln y$ follows a normal distribution with mean 0 and variance σ_y^2 then $(1 - \frac{1}{\sigma}) \ln y = \ln y^{1 - \frac{1}{\sigma}}$ follows a normal distribution with mean zero and variance $(1 - \frac{1}{\sigma})^2 \sigma_y^2$. Hence,

$$E\left(y^{1 - \frac{1}{\sigma}}\right) = e^{\frac{1}{2}(1 - \frac{1}{\sigma})^2 \sigma_y^2}. \quad (65)$$

From (61), (64), and (65), it then follows that

$$\frac{1 + \frac{1-\lambda}{\lambda} \frac{\bar{\mu}}{1+\bar{\mu}} (1 - \bar{v})}{1 - \frac{\bar{\mu}}{1+\bar{\mu}} (1 - \bar{v})} = e^{\frac{\sigma-1}{2\sigma^2} \sigma_y^2},$$

which, after some manipulations, yields

$$\bar{\mu} = \frac{e^{\frac{\sigma-1}{2\sigma^2} \sigma_y^2} - 1}{\frac{1}{\lambda} - \bar{v} \left(e^{\frac{\sigma-1}{2\sigma^2} \sigma_y^2} - 1 \right)}. \quad (66)$$

As a particular case, notice that if $\bar{v} = 0$, we obtain a particularly simple expression:

$$\bar{\mu} = \lambda \left(e^{\frac{\sigma-1}{2\sigma}\sigma_y^2} - 1 \right). \quad (67)$$

Expression (66) is, of course, consistent with what was shown above for any general distribution: $\bar{\mu} > 0$ (< 0) if $\sigma > 1$ (< 1). We also see that as $\sigma \rightarrow 0$, $\bar{\mu} \rightarrow -1/(1/\lambda + \bar{v})$.²² A new insight provided by expression (66) is how σ_y^2 and λ affect the optimal $\bar{\mu}$. Suppose that $\sigma > 1$ in which case $\bar{\mu} > 0$. Then the higher is σ_y^2 (i.e., the larger is output volatility), the larger is $\bar{\mu}$ (i.e., the higher are the transfers from non-traders to traders). Conversely, if $\sigma < 1$ and $\bar{\mu} < 0$, a higher output volatility implies a larger transfer from traders to non-traders. Larger output volatility thus always makes the optimal monetary policy more “aggressive”. The same is true of the fraction of traders, λ .

7.1.2 Optimal fixed- ε rule

Under a fixed devaluation rule, the two consumptions are given by

$$\begin{aligned} c_t^T &= \bar{y} \left[1 + \left(\frac{1-\lambda}{\lambda} \right) \frac{\bar{\varepsilon}}{1+\bar{\varepsilon}} (1-\bar{v}) \right]. \\ c_t^{NT} &= \frac{1-v_{t-1}}{1+\bar{\varepsilon}} y_{t-1} + v_t y_t, \quad t \geq 1. \end{aligned}$$

The optimal $\bar{\varepsilon}$ is then obtained by solving the following problem:

$$\max_{\bar{\varepsilon}} \left\{ \lambda E_0 \sum_{t=0}^{\infty} \beta^t u \left(\bar{y} \left(1 + \left(\frac{1-\lambda}{\lambda} \right) \frac{\bar{\varepsilon}}{1+\bar{\varepsilon}} (1-\bar{v}) \right) \right) + (1-\lambda) E_0 \sum_{t=0}^{\infty} \beta^t u \left(\frac{1-v_{t-1}}{1+\bar{\varepsilon}} y_{t-1} + v_t y_t \right) \right\},$$

which, after simplifying and using (60), yields the following first-order condition:

$$\left[1 + \frac{1-\lambda}{\lambda} \frac{\bar{\varepsilon}}{1+\bar{\varepsilon}} (1-\bar{v}) \right]^{-\frac{1}{\sigma}} (1-\bar{v}) \sum_{t=0}^{\infty} \beta^t \bar{y}^{1-\frac{1}{\sigma}} = E_0 \sum_{t=0}^{\infty} \beta^t (1-v_{t-1}) y_{t-1} \left(\frac{1-v_{t-1}}{1+\bar{\varepsilon}} y_{t-1} + v_t y_t \right)^{-\frac{1}{\sigma}}. \quad (68)$$

Velocity shocks only Consider the case of velocity shocks only with $y_t = \bar{y}$ for all t . Then the (68) can be rewritten as

$$\left[1 + \frac{1-\lambda}{\lambda} \frac{\bar{\varepsilon}}{1+\bar{\varepsilon}} (1-\bar{v}) \right]^{-\frac{1}{\sigma}} (1-\bar{v}) \sum_{t=0}^{\infty} \beta^t = E_0 \sum_{t=0}^{\infty} \beta^t (1-v_{t-1}) \left(\frac{1-v_{t-1}}{1+\bar{\varepsilon}} + v_t \right)^{-\frac{1}{\sigma}}. \quad (69)$$

²²This is valid as long as the Friedman rule is not reached. If it were reached, the cash-in-advance constraints are not necessarily binding and hence the optimality conditions derived in the text no longer hold.

For simplicity, we will just focus on the logarithmic case and show that optimal $\bar{\varepsilon}$ can be zero, positive, or negative depending on the value of \bar{v} .²³ Suppose then that $\sigma = 1$ and evaluate the LHS at $\bar{\varepsilon} = 0$ to obtain $(1 - \bar{v}) / (1 - \beta)$ and the RHS to obtain $E_0 \sum_{t=0}^{\infty} \beta^t [(1 - v_{t-1}) / (1 - v_{t-1} + v_t)]$. Clearly, for $\bar{v} = \frac{1}{2}$, $\bar{\varepsilon} = 0$. For $\bar{v} < \frac{1}{2}$ ($> \frac{1}{2}$), the RHS is greater (smaller) than the LHS, and for (69) to hold $\bar{\varepsilon} < 0$ (> 0).

Output shocks only Consider now the case of output shocks only. In this case, $v_t = \bar{v}$ for all t .

Then

$$\left(1 + \frac{1-\lambda}{\lambda} \frac{\bar{\varepsilon}}{1+\bar{\varepsilon}} (1-\bar{v})\right)^{-\frac{1}{\sigma}} \sum_{t=0}^{\infty} \beta^t \bar{y}^{1-\frac{1}{\sigma}} = E_0 \sum_{t=0}^{\infty} \beta^t y_{t-1}^{1-\frac{1}{\sigma}} \left(\frac{1-\bar{v}}{1+\bar{\varepsilon}} + \bar{v} \frac{y_t}{y_{t-1}}\right)^{-\frac{1}{\sigma}},$$

To simplify, suppose $\bar{v} = 0$ and $y_{-1} = \bar{y}$. Then

$$\frac{\left(1 + \frac{1-\lambda}{\lambda} \frac{\bar{\varepsilon}}{1+\bar{\varepsilon}} (1-\bar{v})\right)}{\frac{1}{1+\bar{\varepsilon}}} = \left(\frac{\sum_{t=0}^{\infty} \beta^t \bar{y}^{1-\frac{1}{\sigma}}}{E_0 \sum_{t=0}^{\infty} \beta^t y_t^{1-\frac{1}{\sigma}}}\right)^{\sigma}$$

Following the same logic as under optimal fixed money growth rule:

$$\bar{\varepsilon} \begin{cases} < 0 \text{ iff } \sigma < 1, \\ = 0 \text{ iff } \sigma = 1, \\ > 0 \text{ iff } \sigma > 1. \end{cases} \quad (70)$$

For values of $\bar{v} \in (0, 1)$, in general, the above may not hold. To see this consider the log case, i.e., $\sigma = 1$. Evaluating the LHS of (68) at $\bar{\varepsilon} = 0$ gets $1/(1 - \beta)$ while the RHS equals $E_0 \sum_{t=0}^{\infty} \beta^t [1/(1 - \bar{v} + \bar{v}y_t/y_{t-1})]$. But $E\{1/(1 - \bar{v} + \bar{v}y_t/y_{t-1})\} \gtrless 1$ iff $\bar{v} \lesseqgtr 1/2$. Then, if $\bar{v} = 1/2$ (70) will continue to hold. If $\bar{v} < 1/2$ ($> 1/2$), however, the RHS in (68) is less (greater) than $1/(1 - \beta)$ and for (68) to hold for $\sigma = 1$, $\bar{\varepsilon} > 0$ (< 0). By continuity there exists a $\hat{\sigma} < 1$ (> 1) such that for all $\sigma \lesseqgtr \hat{\sigma}$, $\bar{\varepsilon} \lesseqgtr 0$.

An example with log-normal distribution In the fixed- ε case, a reduced form for the case of log-normal distribution of y is only possible with no velocity shocks and $\bar{v} = 0$. In this case, and proceeding as before, it can be shown that

²³It can also be shown that the same is true for values of $\sigma \neq 1$.

$$\bar{\varepsilon} = \lambda \left(e^{\frac{\sigma-1}{2\sigma^2} \sigma_y^2} - 1 \right).$$

This expression, of course, parallels the case of output shocks only and $\bar{v} = 0$ derived above (recall equation (67)).

7.2 Alternative formulation with price of risk

In this appendix, we reformulate the problem of finding the optimal fixed- μ discussed in the text so as to highlight the intuitive interpretation of “the price of risk” discussed in the text.²⁴

Preliminaries As a preliminary step, rewrite equilibrium consumption of traders and non-traders as

$$c_t^{NT} = \alpha y_t, \tag{71}$$

$$c^T = \frac{1 - \alpha(1 - \lambda)}{\lambda} \bar{y}, \tag{72}$$

where α is some positive constant. The maximization problem can then be stated as

$$\max_{\alpha} \left\{ \lambda \sum_{t=0}^{\infty} \beta^t u \left(\frac{1 - \alpha(1 - \lambda)}{\lambda} \bar{y} \right) + (1 - \lambda) E_0 \sum_{t=0}^{\infty} \beta^t u(\alpha y_t) \right\}.$$

The first-order condition with respect to α is given by:(68)

$$\sum_{t=0}^{\infty} \beta^t u' \left(\frac{1 - \alpha(1 - \lambda)}{\lambda} \bar{y} \right) \bar{y} = E_0 \sum_{t=0}^{\infty} \beta^t u'(\alpha y_t) y_t. \tag{73}$$

This condition determines the optimal α which will, of course, depend on the specific preferences.

A planner facing a price of risk Imagine now a Ramsey-type planner that must choose consumption for traders and non-traders subject to the constraint that the allocation be implementable as a competitive equilibrium. This planner has access to complete markets which he/she can use to transform all uncertain income into its expected present discounted value. Further, the planner

²⁴We will deal with the case of output shocks only and set $\bar{v} = 0$.

can also give the non-traders a certain (i.e., certainty equivalent) consumption but it has to be purchased at a relative price p . Formally,

The planner's problem is then

$$\max_{\{c^T, \bar{c}^N\}} \left\{ \lambda \sum_{t=0}^{\infty} \beta^t u(c^T) + (1 - \lambda) \sum_{t=0}^{\infty} \beta^t u(\bar{c}^N) \right\},$$

subject to

$$\lambda c^T + p(1 - \lambda) \bar{c}^N = \bar{y},$$

where \bar{c}^N is the certain amount of consumption that the planner now gives to the non-traders by paying an implicit relative price (in terms of the consumption of traders), denoted by p . The term \bar{y} on the RHS of the above budget constraint shows that with complete markets the planner manages to convert its entire stochastic output into its expected value. The first-order condition for this (essentially static) problem is:

$$\frac{u'(c^T)}{u'(\bar{c}^N)} = \frac{1}{p}. \quad (74)$$

By definition and using (71), \bar{c}^N will be given by

$$\sum_{t=0}^{\infty} \beta^t u(\bar{c}^N) = E \sum_{t=0}^{\infty} \beta^t u(\alpha y_t). \quad (75)$$

Given the optimal value of α characterized by equation (73) and expression (72), equations (74) and (75) will allow us to solve for p and verify our intuitive conjecture that p is greater than one, reflecting the fact that, from a social point of view, consumption of non-traders is more expensive than that of traders because the latter have access to perfect capital markets.

We now solve for p first under quadratic preferences and then under CES preferences.

Quadratic preferences In the quadratic case ($u(c) = c - \zeta c^2$), condition (73) becomes

$$\frac{1 - \alpha(1 - \lambda)}{\lambda} = \alpha \left(1 + \frac{\sigma_y^2}{\bar{y}^2} \right).$$

Substituting the latter into (72), we obtain

$$c^T = \alpha \left(1 + \frac{\sigma_y^2}{\bar{y}^2} \right) \bar{y}.$$

The first-order condition (74) becomes

$$p = \frac{1 - 2\zeta\bar{c}^N}{1 - 2\zeta c^T}. \quad (76)$$

On the other hand, from (75), it follows that

$$\bar{c}^N - \zeta (\bar{c}^N)^2 = \alpha\bar{y} - \zeta\alpha^2\bar{y}^2 \left(1 + \frac{\sigma_y^2}{\bar{y}^2} \right).$$

Multiplying both sides of this equation by $1 + \frac{\sigma_y^2}{\bar{y}^2}$ to obtain:

$$\left(1 + \frac{\sigma_y^2}{\bar{y}^2} \right) (\bar{c}^N - \zeta (\bar{c}^N)^2) = c^T - \zeta (c^T)^2.$$

Since the utilities are assumed to be increasing in consumption (i.e., marginal utilities are positive), it follows that $c^T > \bar{c}^N$. Hence, from (76),

$$p = \frac{1 - 2\zeta\bar{c}^N}{1 - 2\zeta c^T} > 1.$$

CRRRA preferences Let preferences be given by (60) and consider first the case of $\sigma \neq 1$.

Condition (73) takes the form:

$$\frac{1 - \alpha(1 - \lambda)}{\alpha\lambda} \bar{y} = \frac{\bar{y}^{\sigma-1}}{\left[E \left(y_t^{1-\frac{1}{\sigma}} \right) \right]^\sigma}. \quad (77)$$

This condition is, of course, the same as condition (61) with $\bar{v} = 0$ and $\alpha = 1/(1 + \mu)$

Further, in the CES case, conditions (74) and (75) become, respectively,

$$\frac{c^T}{\bar{c}^N} = p^\sigma, \quad (78)$$

$$\bar{c}^N = \alpha \left[E \left(y_t^{1-\frac{1}{\sigma}} \right) \right]^{1-\frac{1}{\sigma}}. \quad (79)$$

Substituting (72) and (79) into (78), we obtain

$$\frac{\frac{1-\alpha(1-\lambda)\bar{y}}{\lambda}}{\alpha \left[E \left(y_t^{1-\frac{1}{\sigma}} \right) \right]^{\frac{\sigma}{\sigma-1}}} = p^\sigma. \quad (80)$$

Finally, combining (77) and (80), we obtain

$$p = \frac{\bar{y}}{\left(E \left(y_t^{1-\frac{1}{\sigma}} \right) \right)^{\frac{\sigma}{\sigma-1}}}.$$

If $\sigma > 1$, $E \left(y_t^{1-\frac{1}{\sigma}} \right) < \bar{y}$ by Jensen's inequality and, thus, $\left[E \left(y_t^{1-\frac{1}{\sigma}} \right) \right]^{\frac{\sigma}{\sigma-1}} < 1$ and $p > 1$. Similarly, when $\sigma < 1$, $E \left(y_t^{1-\frac{1}{\sigma}} \right) > \bar{y}$ by Jensen's inequality and thus $\left[E \left(y_t^{1-\frac{1}{\sigma}} \right) \right]^{\frac{\sigma}{\sigma-1}} < \bar{y}$ and $p > 1$.

For the logarithmic case ($\sigma = 1$), it is easy to check that $\alpha = 1$ and that

$$p = \frac{\bar{y}}{\exp \{ (E \{ \ln y \}) \}}.$$

From Jensen's inequality $(E \{ \ln y \}) < \ln \bar{y}$ and therefore $\exp \{ (E \{ \ln y \}) \} < \bar{y}$; hence, $p > 1$.

7.3 Welfare loss relative to the first-best under $\tilde{\mu}$

The expected consumption and variances of traders and nontraders are obtained from (33), (34), and (36) as

$$\begin{aligned} c^T &= \left(1 + \left(\frac{1-\lambda}{\lambda} \right) (1-\bar{v}) \frac{\mu}{1+\mu} \right) \bar{y}; \\ E [c_t^{NT}] &= \frac{1+\mu\bar{v}}{1+\mu} \bar{y}; \\ Var [c_t^{NT}] &= \left(\frac{1}{1+\mu} \right)^2 (\sigma_y^2 (1+2\bar{v}\mu) + \mu^2 \sigma_{vy}^2). \end{aligned} \quad (81)$$

Using (81) and (39), we obtain the welfare loss, under $\tilde{\mu}$ as

$$\begin{aligned} \Delta W^{\tilde{\mu}} &= W^{fb} - W^{\tilde{\mu}} \\ &= \frac{\zeta}{1-\beta} (1-\lambda) \left(\frac{1}{1+\tilde{\mu}} \right)^2 \bar{y}^2 \left(\frac{(1-\bar{v})^2}{\lambda} \tilde{\mu}^2 + \frac{\sigma_y^2}{\bar{y}^2} (1+2\bar{v}\tilde{\mu}) + \tilde{\mu}^2 \frac{\sigma_{vy}^2}{\bar{y}^2} \right). \end{aligned}$$

Using (38) in the above expression, and after some algebra we obtain

$$\Delta W^{\bar{\mu}} = \frac{\zeta}{1-\beta} (1-\lambda) \frac{1 + \frac{\lambda}{(1-\bar{v})^2} \sigma_v^2 \left(1 + \frac{\sigma_y^2}{\bar{y}^2}\right)}{1 + \frac{\lambda}{(1-\bar{v})^2} \sigma_v^2 \left(1 + \frac{\sigma_y^2}{\bar{y}^2}\right) + \lambda \frac{\sigma_y^2}{\bar{y}^2}} \sigma_y^2.$$

which is equation (40) in the main text.

7.4 Riskless bonds for non-traders

This appendix discusses the case in which non-traders are able to access a non-state contingent bond.

We first derive the optimal non-state contingent money growth rule and then the corresponding exchange rate rule.

7.4.1 Optimal non-state contingent money growth rule

Allowing non-traders to trade in riskless bonds implies

$$c_t^{NT} = \frac{M_t^{NT}}{S_t} + v_t y_t + (1+r) b_t - b_{t+1}. \quad (82)$$

Now, non-traders maximize (1) subject to (82), and hence, their first order condition implies

$$c_t = E_t \{c_{t+1}\} \quad (83)$$

With a fixed money growth rule (82) can be rewritten as

$$c_t^{NT} = z(1-v_t)y_t + v_t y_t + (1+r) b_t - b_{t+1}. \quad (84)$$

Taking (83) into account and iterating (84) forward, it can be shown that

$$c_t^{NT} = r b_t + (1-\beta) Z_t + \beta \bar{Z} \quad (85)$$

where $Z_t = z(1-v_t)y_t + v_t y_t$, $\bar{Z} = z(1-\bar{v})\bar{y} + \bar{v}\bar{y}$. Substituting (85) into (84) yields

$$b_{t+1} = b_t + \beta (Z_t - \bar{Z}). \quad (86)$$

For convenience, let $b_0 = 0$. Then, it is easy to verify that

$$\begin{aligned} E \{c_t^{NT}\} &= \bar{Z}, \quad \forall t; \\ VAR \{c_t^{NT}\} &= r^2 VAR \{b_t\} + (1-\beta)^2 VAR \{Z\}. \end{aligned} \quad (87)$$

Further, using (86), it can be shown that

$$VAR\{b_t\} = t\beta^2 VAR\{Z\}.$$

Substituting the above expression in (87), the variance component of non-trader's lifetime utility can be computed as

$$-\zeta VAR\{Z\}.$$

Under a μ rule, the trader's consumption is same as that given by (34)

$$c_t^T = \bar{y} \left[1 + \left(\frac{1-\lambda}{\lambda} \right) (1-z)(1-\bar{v}) \right],$$

where we have assumed that $k_0 = 0$. The optimal money growth rule is now obtained by solving

$$\min_z \frac{1}{1-\beta} \left\{ \lambda (E(c^T))^2 + (1-\lambda) (E(c^{NT}))^2 + (1-\beta)(1-\lambda) Var[Z] \right\},$$

where

$$Var[Z] = z^2 \sigma_y^2 + (1-z)^2 \sigma_{vy}^2 + 2z(1-z) \bar{v} \sigma_y^2,$$

and where $\sigma_{vy}^2 = \sigma_y^2 \sigma_v^2 + \bar{y}^2 \sigma_v^2 + \bar{v}^2 \sigma_y^2$. Then

$$\tilde{z} = \frac{\sigma_{vy}^2 - \bar{v} \sigma_y^2 + \frac{(1-\bar{v})^2}{\lambda(1-\beta)} \bar{y}^2}{\sigma_{vy}^2 + (1-2\bar{v}) \sigma_y^2 + \frac{(1-\bar{v})^2}{\lambda(1-\beta)} \bar{y}^2},$$

and

$$\tilde{\mu} = \frac{(1-\bar{v}) \sigma_y^2}{\sigma_{vy}^2 + \frac{(1-\bar{v})^2 \bar{y}^2}{\lambda(1-\beta)} - \bar{v} \sigma_y^2}. \quad (88)$$

7.4.2 Optimal non-state contingent exchange rate rule

Using (82) obtain

$$c_t^{NT} = z(1-v_{t-1})y_{t-1} + v_t y_t + (1+r)b_t - b_{t+1}. \quad (89)$$

Using (83) it can be shown that

$$c_t^{NT} = r b_t + (1-\beta) z(1-v_{t-1})y_{t-1} + (1-\beta) Z_t + \beta \bar{Z}, \quad (90)$$

where $Z_t = v_t y_t + \beta z(1 - v_t)y_t$, and $\bar{Z} = E\{Z\}$. Combining (90) with (89) yields

$$b_{t+1} = b_t + \beta z((1 - v_{t-1})y_{t-1} - (1 - v_t)y_t) + \beta(Z_t - \bar{Z}). \quad (91)$$

From (90):

$$\begin{aligned} VAR\{c_t^{NT}\} &= r^2 VAR\{b_t\} + (1 - \beta)^2 (z^2 VAR\{(1 - v)y\} + VAR\{Z\}) \\ &\quad + 2r(1 - \beta)z COV\{b_t, (1 - v_{t-1})y_{t-1}\}. \end{aligned} \quad (92)$$

From (91):

$$\begin{aligned} VAR\{b_0\} &= 0; \\ VAR\{b_1\} &= \beta^2 VAR\{vy - (1 - \beta)z(1 - v)y\}; \\ VAR\{b_t\} &= VAR\{b_1\} + (t - 1)\beta^2 VAR\{Z\} \end{aligned} \quad (93)$$

Combining (93) with (92) yields

$$VAR\{c_t^{NT}\} = (t + 1)(1 - \beta^2) VAR\{Z\}, \quad \forall t.$$

Then, the variance component of non-trader's lifetime utility can be computed as

$$-\zeta VAR\{Z\}.$$

Under an ε rule, the trader's consumption is given by

$$c_t^T = \bar{y} \left[1 + \left(\frac{1 - \lambda}{\lambda} \right) (1 - z)(1 - \bar{v}) \right],$$

$\tilde{z} \left(\equiv \frac{1}{1 + \varepsilon} \right)$ is now obtained by solving

$$\min_z \frac{1}{1 - \beta} \left\{ \lambda (E(c^T))^2 + (1 - \lambda) (E(c^{NT}))^2 + (1 - \beta)(1 - \lambda) Var[Z] \right\},$$

where

$$Var[Z] = \beta^2 z^2 \sigma_y^2 + (1 - \beta z)^2 \sigma_{vy}^2 + 2\beta z(1 - \beta z) \bar{v} \sigma_y^2,$$

and where $\sigma_{vy}^2 = \sigma_y^2 \sigma_v^2 + \bar{y}^2 \sigma_v^2 + \bar{v}^2 \sigma_y^2$. Then

$$\begin{aligned}\tilde{z} &= \frac{\beta (\sigma_{vy}^2 - \bar{v} \sigma_y^2) + \frac{(1-\bar{v})^2}{\lambda(1-\beta)} \bar{y}^2}{\beta^2 (\sigma_{vy}^2 + (1-2\bar{v}) \sigma_y^2) + \frac{(1-\bar{v})^2}{\lambda(1-\beta)} \bar{y}^2}, \\ \tilde{\varepsilon} &= \frac{\beta^2 (1-\bar{v}) \sigma_y^2 - \beta (1-\beta) (\sigma_{vy}^2 - \bar{v} \sigma_y^2)}{\beta (\sigma_{vy}^2 - \bar{v} \sigma_y^2) + \frac{(1-\bar{v})^2}{\lambda(1-\beta)} \bar{y}^2}.\end{aligned}\tag{94}$$

With velocity shocks only²⁵

$$\tilde{\varepsilon} = \frac{-\beta (1-\beta) \frac{\sigma_v^2}{(1-\bar{v})^2}}{\beta \frac{\sigma_v^2}{(1-\bar{v})^2} + \frac{1}{\lambda(1-\beta)}} < 0.$$

With output shocks only²⁶

$$\tilde{\varepsilon} = \frac{\beta \frac{\bar{v}}{1-\bar{v}} + \beta^2 \frac{\sigma_y^2}{\bar{y}^2}}{\frac{1}{\lambda(1-\beta)} - \beta \frac{\bar{v}}{1-\bar{v}} \frac{\sigma_y^2}{\bar{y}^2}} > 0.$$

7.5 Welfare Comparison

Qualitatively – and as Figure 2 makes clear – the results are the same as for the case without riskless bonds for non-traders. The higher the monetary volatility, the more desirable is $\tilde{\mu}$ for any given σ_y , while the higher the output volatility the more desirable is $\tilde{\varepsilon}$ for any given σ_v . Quantitatively, however, a comparison of Figures 1 and 2 clearly reveals that the relative attractiveness of exchange rate rules increases. This is to be expected because as non-traders become more “similar” to traders in terms of access to asset markets, the two regimes (money rules versus exchange rate rules) should also lead to smaller welfare differences. We would view this case, however, as an extreme upper bound for the kind of access that non-traders may have in reality. Unfortunately, more realistic arrangements (like just allowing cash to be carried over from period to period) are analytically much less tractable.

²⁵Note that $-r\beta < \varepsilon < 0$. To see this, look at the first order condition:

$$-\frac{(1-z)(1-v)^2}{\lambda(1-\beta)} - \beta(1-\beta z) \sigma_v^2 = 0,$$

which implies that $1 < z < 1+r$.

²⁶The inequality holds for $\bar{v} < \frac{1}{1+\beta(1-\beta)\lambda\frac{\sigma_y^2}{\bar{y}^2}}$. Even for $\frac{\sigma_y}{\bar{y}} = 0.1$, $\lambda = 1$, and $\beta = 0.5$ (which maximizes $\beta(1-\beta)$), this requires $\bar{v} < 0.9975$.

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