Economics 131
Section Notes
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Microeconomics Review in a Two Good World

Note: These notes are not meant to be a substitute for attending section. It may in fact be difficult to understand these notes without attending section, where this material is embellished with graphs and student questions. If attendance in section falls, I will stop providing these notes. 2nd Note: There are a few calculus tricks you may need to review as they will be used again and again. The footnotes are more for your own edification and general understanding and not a strict requirement.

- Two goods: *leisure* l (or *labor* L) with price w and *consumption* x with price p. Only the ratio w/p will matter ultimately
- Firms have a *production function* $x = f(L)$ and wish to maximize *profit* $\pi = px - wL$ subject to $x \geq 0$. Assume $f(0) = 0$, $f'(L) > 0$ and $f''(L) < 0$ for all L (i.e. f is concave).
- Conusumers have a *budget constraint* $px \leq wL + M$ and a *time constraint* $l + L = T$ where T is *time available* and M is *non-labor income* (e.g. share of profit if it owns part of a firm) They maximize *utility* $U(l, x)$ which is increasing in both arguments and concave in (l, x). Also $x \geq 0$ and $0 \leq l \leq T$.
- Competition in this world is *"perfectly" or "purely" competitive* so both firms and workers are price takers, i.e. w and p are fixed for them. This may be justified by postulating a "large" number of both consumers and firms. Market equilibrium will determine the ratio w/p.
- For simplicity we assume that all firms and consumers are the same and that there are an equal number N (a large number) of each, so that each person owns one firm. This is much like a "Robinson Crusoe" economy repeated over and over again, where the Robinson Crusoe's are allowed to trade with each other.

1 Firm's Problem

1.1 Cost Minimization

We will start be deriving the cost function from the production function, pointing out the "*duality*" of cost and production functions. For a given output x the firm will wish to minimize the cost

$$\min_L wL \qquad \text{s.t.} \quad x = f(L) \tag{Cost Min}$$

With one input the solution to this problem is simply $C(w, x) = wf^{-1}(x)$ where $f^{-1}(x)$ is the inverse function of x. Some math shows that the concavity of f assures the convexity of C in x.

1.2 Profit Maximization

Now the firm is seen to maximize

$$\max_x px - C(w, x) \tag{Firm Objective}$$

which is an unconstrained maximization problem (except that we need $x \geq 0$) The first order condition for a local maximum is now

$$\frac{d\pi}{dx} = p - \frac{\partial C\left(w, x^S\right)}{\partial x} \leq 0 \text{ with "=" if } x^S > 0 \tag{Firm FOC}$$

Assuming $x^S > 0$ is optimal (no corner solution) this is just the familiar condition $p = MC_w\left(x^S\right)$ the marginal cost of x for a given wage w. [1].

[1] Note a firm will not want to produce if it makes negative profits, i.e. if $\pi = px - C(w, x) < 0$ or $p < C(w, x)/x = AC(w.x)$ The minimum efficient scale x^{MES} is where

$$AC\left(w, x^{MES}\right) = MC_w\left(x^{MES}\right) = p^{shutdown}$$

1.3 Consumption Supply

Let $MC_w^{-1}(p)$ be the inverse of the MC function in x for a given w. Then, ignoring the corner solution the supply of x is given by

$$x^S(w,p) = MC_w^{-1}(p) \tag{Supply of x}$$

i.e. the supply curve is the marginal cost curve above the average cost curve. Remember that now we are treating p parametrically.[2]

1.4 Marginal Productivity and Labor Demand

A fact from calculus supports the following

$$MC_w\left(x^S\right) = w\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{w}{f'\left[f^{-1}\left(x^S\right)\right]} = \frac{w}{f'\left(L^D\right)}$$

where $L^D = f^{-1}\left(x^S\right)$. The first order condition implies

$$p = \frac{w}{f'\left(L^D\right)} \Rightarrow pf'\left(L^D\right) = w \Longleftrightarrow f'\left(L^D\right) = \frac{w}{p}$$

This is the familiar $MRP\left(L^D\right) = w$ or $MP\left(L^D\right) = w/p$ condition which you could also get by taking the first order condition for that alternate formulation for profit maximization $\min_L \{pf(L) - wL\}$ which is written in terms of *input* rather than *output*. Inverting the marginal productivity condition gives the familiar labor demand curve[3]

$$L^D(w,p) = MP^{-1}(w/p) \tag{Demand for L}$$

1.5 Marginal Rate of Transformation

Notice that $f'(L)$ is the number of units of consumption transformed per unit of labor, i.e. it is the marginal rate of transformation MRT of labor into consumption. We would like to find the marginal rate of transformation of leisure into consumption, as both are goods. To do this just let $L = T - l$ and substitute $x = f(T-l)$. Treating x as a function of l, $x^{PF}(l)$, we get $\frac{dx^{PF}}{dl} = -f'(T-l). = -MRT_{lx}$. In other words $MRT_{lx} = f'(T-l) = f'(L)$ and so it follows that at the profit maximum $MRT_{lx} = w/p$ [4]

1.6 Second Order Condition

The second order condition is just that at the optimum

$$\frac{d^2\pi}{dx^2} = -\frac{\partial^2 C\left(w,x^S\right)}{\partial x^2} < 0 \Longleftrightarrow \frac{\partial^2 C\left(w,x^S\right)}{\partial x^2} > 0 \tag{Firm SOC}$$

i.e. that marginal costs are increasing in x. This is assured by the convexity of C in x, by virtue of the concavity of $f(L)$. This concavity of f is what assures us that the supply curve for x slopes up in p and

The supply curve is positive only above $p^{shutdown}$ but we don't need to worry about this here since f is everywhere concave so $x_{MES} = 0$ and $p^{shutdown} = MC_w(0)$.

[2] This review does not take into account the entry and exit of firms. With entry and exit, firms will enter so long as positive profits can be made. In the long run firms will end up producing at x_{MES} at minimum average cost $p_{shutdown}$ and make zero profits. A sensible analysis along these lines would require introducing a more complicated production function, intially convex, and then concave, so that $x_{MES} > 0$.

[3] This can also give us a simpler supply curve for x in terms of just the ratio w/p as $x^S(w,p) = f\left[L^D(w,p)\right]$ and so

$$x^S(w,p) = f\left[MP^{-1}(w/p)\right]$$

[4] If we call the marginal cost of labor $MC_w\left(L^D\right) = w$ the wage, then we see the marginal rate of transformation is ratio of marginal costs.

$$MRT_{lx} = f'(L) = \frac{w}{MC_w\left(x^S\right)} = \frac{MC_w\left(L^D\right)}{MC_w\left(x^S\right)}$$

that the the demand for labor slopes down in w, i.e. $\partial x^S/\partial p > 0$ and $\partial L^D/\partial w < 0$ which can be found implicitly differentiating the FOC.[5]

2 Consumer's Problem

2.1 Indifference Curves

An *indifference curve* is simply a locus of any points (l, x) such that utility is constant, i.e., $U(l, x) = \bar{u}$. The implicit function theorem from calculus allows us to then treat x as a function of l again (ignoring the budget constraint), $x^{IC}(l)$ which satisfies $U\left[l, x^{IC}(l)\right] = \bar{u}$. Differentiating this with respect to l we have

$$\frac{\partial U\left[l, x^{IC}(l), l\right]}{\partial l} + \frac{\partial U\left[l, x^{IC}(l)\right]}{\partial x} \frac{dx^{IC}(l)}{dl} = 0$$

so solving for the slope of the indifference curve is

$$\frac{dx^{IC}(l)}{dl} = -\frac{\frac{\partial U\left[l, x^{IC}(l)\right]}{\partial l}}{\frac{\partial U\left[l, x^{IC}(l)\right]}{\partial x}} = -MRS_{lx}$$

The MRS_{lx} is the negative of the slope of the indifference curve at $U(l, x)$

2.2 Time and Budget Constraint

Solving the time constraint in terms of $L = T - l$ and substituting it into the budget constraint gives

$$px \leq w(T - l) + I \Longleftrightarrow px + wl \leq wT + M \qquad \text{(Budget constraint)}$$

now the left hand side is in terms of goods while the right hand side is in terms of endowments and depends partly on w. Setting the inequality to equality (a small assumption) and solving for x in terms of l i.e.

$$x^{BC}(l) = \frac{M}{p} + \frac{w}{p}(T - l)$$

which we can substitute into the utility function. Notice the slope of the budget constraint $\frac{dx^{BC}(l)}{dl} = -\frac{w}{p}$ for $l < T$.

2.3 Utility Maximization

Now we want to solve a one-dimensional problem[6]

$$\max_l U\left(l, x^{BC}(l)\right) = \max_l U\left(l, \frac{M}{p} + \frac{w}{p}(T - l)\right) \qquad \text{(Consumer Objective)}$$

The first order condition for l is now

$$\frac{\partial U\left(l^D, x^{BC}\left(l^D\right)\right)}{\partial l} - \frac{\partial U\left(l^D, x^{BC}\left(l^D\right)\right)}{\partial x}\frac{w}{p} \leq 0 \text{ with } \text{"="} \text{ if } l^D > 0 \qquad \text{(Consumer FOC)}$$

[5] The FOC implies that $\frac{\partial C\left[w, x^S(w,p)\right]}{\partial x} = p$. Differentiating implicitly with respect to p implies

$$\frac{\partial^2 C\left[w, x^S(w,p)\right]}{\partial x^2}\frac{\partial x^S}{\partial p} = 1 \Rightarrow \frac{\partial x^S}{\partial p} = \left(\frac{\partial^2 C\left[w, x^S(w,p)\right]}{\partial x^2}\right)^{-1} > 0$$

[6] Another way of solving this problem would be to set up a Lagrangean

$$L(l, x, \lambda) = U(l, x) + \lambda\left[M + w(T - l) - px\right]$$

take the three FOC $\partial L/\partial l = 0$, $\partial L/\partial x = 0$ and $\lambda\left[M + w(T - l) - px\right] = 0$, and solve for $\left(l^D, x^D, \lambda^D\right)$ directly. $\lambda^D \geq 0$ gives the marginal value of an extra dollar added to M. The last FOC implies that when the the budget constraint does not hold with equality $\lambda^D = 0$.

Notice that it may not be optimal to work at all, which means there is a corner solution (there is also a corner solution where $l^D = T$ but that does not usually seem economically relevant). If there is no corner solution then letting $x^D = x^{BC}\left(l^D\right)$,

$$MRS_{lx} = \frac{MU\left(l^D\right)}{MU\left(x^D\right)} = \frac{\frac{\partial U\left(l^D,x^D\right)}{\partial l}}{\frac{\partial U\left(l^D,x^D\right)}{\partial x}} = \frac{w}{p} \qquad \text{(Tangency)}$$

i.e. the marginal rate of substitution equals the ratio of factor prices. This means that the bundle demanded $\left(l^S, x^S\right)$ will occur where the constraint and the indifference curves are tangent as the above condition implies $\frac{dx^{IC}(l)}{dl} = \frac{dx^{BC}(l)}{dl}$. We'll ignore the second order condition here - it is complicated.

2.4 Product Demand and Labor Supply

With a more concrete functional form we could solve for the optimal demand for leisure $l^D = l^D(w,p,M)$ which implies labor supply $L^S(w,p,M) = T - l^S(w,p,M)$ and consumption $x^D = x^D(w,p,M)$. Note there are only 2 independent arguments $(w/p, M/p)$.

2.5 Income and Substitution Effects

Income effects concern how demand reacts to changes in income M (or possibly T). We'll assume that both leisure and labor are normal goods, i.e. $\partial l^D/\partial M > 0$ and $\partial x^D/\partial M > 0$. *Substituion effects* always imply that an increase in price will cause substitution away from that good. If consumption is a normal good, then its demand must be downward sloping, i.e. $\partial x^D/\partial p < 0$ - all Giffen goods are inferior goods $\left(\partial x^D/\partial M < 0\right)$. Labor supply is typically upward sloping, but it may "bend back": if the wage increases the substitution effect will cause the consumer to work more (leisure is more expensive) while the income effect will cause the consumer to work less (the consumer purchases more leisure) so $\partial l^D/\partial w \lesseqgtr 0$ as $\partial L^S/\partial w \gtreqless 0$.[7]

3 Partial Equilibrium

3.1 Aggregation

Total demand for consumption x^{TD} is given by adding up all of the individual demands horizontally, i.e. for a given p, which in this case is just $x^{TD} = \sum_i x_i^D(w,p,M_i) = Nx^D(w,p,M)$ assuming $M_i = M$ is the same for everyone. Total Industry supply of consumption goods is $x^{TS} = Nx^S(w,p)$. Labor demand and supply are given by $L^{TD} = L^D(w,p)$ and $L^{TS} = L^S(w,p,M)$.

3.2 Market Equilibrium

In consumption partial equilibrium occurs at the price p^* where supply equals demand, i.e. $x^{TD} = x^{TS}$, which using the aggregation above and cancelling out N just means

$$x^D(w,p^*,M) = x^S(w,p^*) = x^* \qquad \text{(P Eq'm)}$$

Such a price should exist so long as the two curves intersect. Equilibrium may not occur if, say, any price high enough to elicit positive supply $x^S(w,p) > 0$ is too high for anyone to demand the good $x^D(w,p) = 0$. For labor supply the appropriate equation to determine w^* is $L^D(w^*,p) = L^S(w^*,p,M)$.

[7] The *Slutsky equation* gives the exact relationship that

$$\frac{\partial x^D}{\partial p} = s_x - x^D \frac{\partial x^D}{\partial M}$$

where $s_x < 0$ is the substitution effect for good x and $-x^D \frac{\partial x^D}{\partial M} < 0$ is the income effect. For labor supply this equation is

$$\frac{\partial L^S}{\partial w} = s_L + L^S \frac{\partial L^S}{\partial M} = -s_l - \left(T - l^D\right)\frac{\partial l^D}{\partial M}$$

where $s_L = -s_l > 0$ ($s_l < 0$) is the substitution effect for labor (leisure) and $L^S \frac{\partial L^E}{\partial M} = -\left(T - l^D\right)\frac{\partial l^D}{\partial M} < 0$ is the income effect. These effects have the same sign for x^D and opposite signs for L^S

3.3 Consumer and Producer Surplus

In this case we should invert the demand function and solve for $p^D(x,w,M)$, the individual's inverse demand. The inverse demand function gives the consumer's marginal willingness to pay for each additional unit of x. The difference $p^D(x,w,M) - p^*$ gives the difference between how much the good is worth to the consumer and what it is worth to her. Summing up this "surplus" over all goods bought by all N individuals with an integral gives the consumer surplus

$$CS = N \int_0^{x^*} \left[p^D(x,w,M) - p^* \right] dx \tag{CS}$$

Similarly for producers, the difference between the market price p^* and the marginal cost of providing it $MC_w(x)$ gives it's surplus, which adding up gives the producer's surplus

$$PS = N \int_0^{x^*} \left[p^* - MC_w(x) \right] dx = N\pi(w,p^*) \tag{PS}$$

which is just N times the profit made by an individual firm. Total surplus is the sum of the two $TS = CS + PS$ and is maximized at the p^*.[8]

4 General Equilibrium

4.1 Market Equilibrium

Here we take into account all of the interactions of the market and find the set of prices (w^*, p^*) that clear *both* markets. However, because we have only two markets, and by a fact known as Walras' Law[9], we know if one market clears, so must the other one. So if the labor market clears $L^D = L^S$ so will the consumption market, $x^S = x^D$ (or vice-versa) and so we will focus on the labor market. Since only the ratio of prices matters we can set in advance $p^* = 1$ and look for the right $w^* = w^*/p^*$ that will clear the labor market. Another consideration in general equilibrium is that now non-labor income is equal to the profits of the firm the individual owns $M = \pi(w,p) = \pi(1,w) = \pi(w)$, which depends on w.

An equilibrium wage w^* will satisfy

$$\begin{aligned} L^D(w^*) &= L^S\left[w, \pi(w^*)\right] \\ &= L^S\left[w^*, f\left[L^D(w^*)\right] - w^* L^D(w^*)\right] \end{aligned} \tag{G Eq'm}$$

where $\pi(w) = f\left[L^D(w)\right] - wL^D(w)$. Together $w^*, p^* = 1, x^* = x^S(w^*), L^* = L^D(w^*)$ are a *Walrasian* equilibrium. Since everyone here has the same endowments, has the same utility, and faces the same constraints and prices, everyone will get the same allocation of goods, i.e. $(l_i^*, x_i^*) = (l^*, x^*)$ for all i.

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[8] **An interesting tangency:** For the sake of exposition set $w = 1$, so by the budget constraint $l^{BC}(x) = M + T - px$ and substitute into the budget constraint to get utility in terms of x and p rather than x and l,

$$\tilde{U}(x,p) = U(M + T - px, x)$$

We can now find an indifference curve in (x,p) space setting $U(M+T-px, x) = \bar{u}$, treating p as a function of x and differentiating with respect to x

$$\frac{\partial U}{\partial x} - \frac{\partial U}{\partial l}\left(x\frac{dp}{dx} + p\right) = 0 \Rightarrow \frac{dp}{dx} = \frac{\left(\frac{\partial U}{\partial x} - p\frac{\partial U}{\partial l}\right)}{x\frac{\partial U}{\partial l}} \tag{IC Slope}$$

The consumer FOC implies that at $x = x^D$, $\frac{\partial U}{\partial x} - p\frac{\partial U}{\partial l} = 0$ which means $\frac{dp}{dx}\left(x^D\right) = 0$, i.e. the indifference curve crosses the demand curve with slope zero.

Similarly for the firm the isoprofit curve is set by $px - C(x) = \bar{\pi}$. Differentiating implicitly implies

$$x\frac{dp}{dx} + p - C'(x) = 0 \Rightarrow \frac{dp}{dx} = \frac{p - C'(x)}{x} \tag{Isoprofit Slope}$$

and $x = x^S$ the firm FOC implies $p = C'\left(x^S\right)$ so $\frac{dp}{dx}\left(x^S\right) = 0$. So at (x^*, p^*) the indifference curve and the isoprofit curves will be tangent with slope zero, so that there is no other combination (x,p) that would make *both* individual *and* firm better off.

[9] Walras' Law states that if all but one market clears, the last market must also clear. This law comes from the budget constraint.

4.2 Pareto Optimality

A feasible allocation of goods $\left\{\left(l_i^P, x_i^P\right)\right\}_{i=1}^N$ is Pareto optimal if there does not exist another feasible allocation of goods $\left\{\left(\tilde{l}_i, \tilde{x}_i\right)\right\}_{i=1}^N$ such that

$$U_i\left(\tilde{l}_i, \tilde{x}_i\right) \geq U_i\left(l^P, x_i^P\right) \quad \text{for all } i = 1, ..., N \tag{Pareto}$$

with at least one inequality strict. This means that of all the leisure-consumption bundles which are technically feasible, no individual can be made better off without making someone else worse off. As we will see later this means we are on the outer portion of the utility possibility frontier. Note that in cases where everyone will have the same outcome, Pareto optimality just requires that our representative agent be at her highest feasible utility.

4.3 First Fundamental Theorem of Welfare Economics

This theorem states that if $\left(p^*, w^*, \{l_i^*, x_i{}^*\}_{i=1}^N\right)$ is a Walrasian equilibrium then $\{l_i^*, x_i{}^*\}_{i=1}^N$ is Pareto optimal, i.e. the (l^*, x^*) calculated above maximizes the agent's utility out of all production possibilities. This result is more interesting in situations where individuals and firms are not all the same. However, for this theorem to apply, there must be a market for *each and every good* that enters the production function or the utility function. Some restrictions on the utility function must also apply - a sufficient but not necessary condition is that each good always has a positive marginal utility.

Note that at the equilibrium

$$MRS_{l^*x^*} = w^* = f'(L^*) = MRT_{l^*x^*} \tag{MRS=MRT}$$

This result is known as *production efficiency* as there is no other production plan which will make agents better off. Another welfare property of these types of equilibria is *allocative efficiency* where the marginal rates of substitution for everyone is equal, i.e. $MRS_{lx}^i = MRS_{lx}^j$ for all individuals i and j. This is achieved in this economy since $MRS_{l^*x^*}^i = w^* = MRS_{l^*x^*}^j$.

4.4 Planner's Problem Equivalence

Here we forget about markets and prices and instead imagine a perfectly informed and benevolent planner who sets (l^{**}, x^{**}) to maximize the utility of the representative agent along all the feasible allocations determined by the production function and tje time constraint (the budget constraint is irrelevant)

$$\max_{l,x} U(l, x) \qquad \text{s.t. } x = f(L) = f(T-l)$$

substituting in the constraint the problem is just

$$\max_{L} U\left[l, f(T-l)\right] \tag{Planner's Objective}$$

taking the FOC and substituting back $x^{**} = F(T - l^{**})$ and $L^{**} = T - l^{**}$

$$\frac{\partial U(l^{**}, x^{**})}{\partial l} - \frac{\partial U(l^{**}, x^{**})}{\partial x} f'(L^{**}) = 0 \tag{Planner's FOC}$$

and rearranging

$$MRS_{l^{**}x^{**}} = \frac{\frac{\partial U(l^{**}, x^{**})}{\partial l}}{\frac{\partial U(l^{**}, x^{**})}{\partial x}} = f'(L^{**}) = MRT_{l^{**}x^{**}} \tag{MRS=MRT}$$

which is the same condition we got for the Walrasian equilibrium, implying the same allocation of goods $(l^{**}, x^{**}) = (l^*, x^*)$. Note that in cases with more than one type of consumer, the planner's outcome and the equilibrium outcome may no longer always coincide.