

**Econ 230B**  
**Spring 2004**  
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**Problem Set 1**

**DUE DATE: March 8 in Class**

1. Consider an economy where the government sets a flat tax at rate  $\tau$  on earnings to raise revenue. We assume that the economy is static: the total population remains constant and equal to  $N$  over years and there is no overall growth in earnings.

Individual  $i$  earns  $z_i = z_i^0(1 - \tau)^e$  when the tax rate is  $\tau$ .  $z_i^0$  is independent of taxation and is called potential income.  $e$  is a positive parameter equal for all individuals in the economy. The government wants to set  $\tau$  so as to raise as much tax revenue as possible.

a) What is the parameter  $e$ ? Show that the tax rate maximizing total tax revenue is equal to  $\tau^* = 1/(1 + e)$ .

b) The government does not know  $e$  perfectly and thus requests the help of an economist to estimate  $e$ . The government can provide individual data on earnings for two consecutive years: year 1 and year 2. In year 1, the tax rate is  $\tau_1$ . In year 2, the tax rate is *decreased* to level  $\tau_2$ . Suppose that the government can provide you with two cross-section random samples of earnings of the same size  $n$  for each year. This is *not* panel data.

How would you proceed to estimate  $e$  from this data? Provide a formula for your estimate  $\hat{e}$  and a regression specification that would allow you to estimate  $e$  with standard errors.

c) Suppose now that the economy is experiencing exogenous economic growth from year to year at a constant rate  $g > 0$ . The population remains constant at  $N$ . How is the estimate  $\hat{e}$  biased because of growth? Suppose you know  $g$ , how would you correct  $\hat{e}$  to obtain a consistent estimate of  $e$ ? (provide an exact formula of this new estimate).

d) Suppose now that you do not know  $g$  but that the government gives you a new cross-section of data for year 0 in which the tax rate was equal to  $\tau_1$  as in year 1. Using data on year 0 and year 1, provide an estimate of  $g$  and the corresponding regression specification.

Using data for all 3 years, provide a single regression specification and a formula for a consistent estimate  $\hat{e}_R$  of  $e$  that takes into account growth.

e) We now assume again that there is no growth. Suppose that the parameter  $e$  differs across individuals and is equal to  $e_i$  for individual  $i$ . Assume that there are  $N$  individuals in the economy. Individual  $i$  earns  $z_i = (1 - \tau)^{e_i} z_i^0$ . As above,  $z_i^0$  is not affected by taxation.

As in question 1, express the tax rate maximizing tax revenue  $\tau^{**}$  as a function of the  $e_i$  and the realized incomes  $z_i$ . Show that the tax rate  $\tau^{**}$  can be expressed as  $\tau^{**} = 1/(1 + \bar{e})$  where  $\bar{e}$  is an average of the  $e_i$ 's with suitable weights. Give an analytic expression of these weights and provide an economic explanation.

f) Suppose now that the parameter  $e$  is the same for all individuals and that the government redistributes the tax collected as a lump-sum to all individuals. I note  $R$  this lump-sum which is equal to average taxes raised. Suppose that the level of this lump-sum  $R$  affects labor supply through income effects. More precisely, the earnings of individual  $i$  are given by  $z_i = (1 - \tau)^e z_i^0(R)$ . The potential income  $z_i^0(R)$  now depends (negatively) on the lump-sum  $R$ .

Suppose that the government still wants to set  $\tau$  so as to raise as much taxes as possible in order to make the lump-sum  $R$  as big as possible. Should the government set the tax rate  $\tau$  higher or lower than  $\tau^* = 1/(1 + e)$  obtained in question 1?

## 2. Chasing Natural Experiments:

As seen in class, many of the best papers on labor supply responses to taxes and transfers exploit a policy change (a so-called “Natural Experiment”) in order to obtain convincing estimates. This exercise asks you to find a Natural Experiment and propose an estimation methodology.

On the course website, download the pdf copy of the OECD annual publication *Taxing Wages* for years 1998-99 and 2001-02. Part V of this publication describes the tax/benefits systems (including payroll taxes, income taxes, and various benefits) faced by wage income earners for each OECD country. Note that recent changes in the tax/benefit system are explicitly described in Section 4 for each country.

a) Find a reform which took place between those two publications that might allow you to estimate labor supply responses to taxes or transfers for some group of interest in the population.

b) Describe the data you would need and the methodology you would use to estimate such labor supply responses. In particular, make sure to be fully explicit about the assumptions you need to identify the labor supply response parameters. Try to explain whether your estimates capture participation versus intensive elasticities, uncompensated versus compensated elasticities, income effects, etc.

c) In order to improve the confidence in your estimates, explain how you could provide good additional tests of the validity of your method.

d) (FOR FUTURE WORK): If you find a really promising Natural Experiment, the next step is to look for the related literature (you want to be the first to analyze this change!) and then try and get the data to carry out the research project.

3. Consider the taxation problem confronting a utilitarian planner who can levy person-specific taxes (lump-sum taxes) based on ability  $w$ . As in the Mirrlees (1971) model, individual utility depends on consumption  $c$  and leisure  $1 - l$  where  $l$  denotes labor supply. Thus  $u = u(c, 1 - l)$ . Consumption is equal to earnings  $wl$  minus taxes  $T(w)$ . The individual chooses  $l$  to maximize  $u(wl - T(w), 1 - l)$ .

The social planner sets taxes  $T(w)$  so as to maximize the total sum of utilities given by,

$$W = \int u(wl(w) - T(w), 1 - l(w))f(w)dw$$

where  $f(w)$  is the density of people with ability  $w$ . The government budget constraint is given by  $\int T(w)f(w) = 0$ .

a) Set up the Lagrangian for the government maximization problem. Find the first order condition that describes the social planner's optimal choice of  $T(w)$ , and interpret this condition.

b) Now consider how utility varies with changes in individual ability. Differentiate  $u(wl(w) - T(w), 1 - l(w))$  with respect to  $w$ . Show that in the case of additively separable utility function ( $u = a(x) + b(1 - l)$ ),  $du/dw < 0$  iff  $dl/dw > 0$ .

(HINT: In the computation, use the FOC from a) to find an expression for  $T'(w)$ ).

c) Assume that indeed  $dl/dw > 0$ . Explain intuitively why in this case utility is decreasing with ability.