## Econ 230B

## Spring 2004

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## Problem Set 2

## DUE DATE: March 29 in Class

1. Consider a standard infinite horizon model in continuous time where each individual supplies inelastically one unit of labor at each instant for an exogenous and constant (over time and across individuals) wage $w$. Assume that the pre-tax interest tax rate $r$ is exogenous and constant. The government imposes a non-linear tax on capital income $T($.$) . Let us denote by$ $a_{t}^{i}$ wealth holding of individual $i$ at time $t$, and $c_{t}^{i}$ consumption of individual $i$ at time $t$. Taxes paid at time $t$ are equal to $T\left(r a_{t}^{i}\right)$.
a) Find the differential equation describing the evolution of wealth $a_{t}^{i}$ for a given path of consumption $c_{t}^{i}$.
b) Individual $i$ maximizes the following utility function:

$$
U^{i}=\int_{0}^{\infty} u\left(c_{t}^{i}\right) e^{-\rho t} d t
$$

where $\rho$ is the discount rate (the same for each individual) and instantaneous utility is $u(c)=$ $c^{1-\sigma} /(1-\sigma)$. Individual $i$ starts with initial wealth $a_{0}^{i}$.

Show that the optimal path of consumption of individual $i$ satisfies (use a standard Hamiltonian technique):

$$
\frac{\dot{c}_{t}^{i}}{c_{t}^{i}}=\frac{1}{\sigma}\left(r\left(1-T^{\prime}\left(r a_{t}^{i}\right)-\rho\right)\right.
$$

c) Assume that $r>\rho$. Assume that $T$ is progressive so that $T^{\prime}$ is strictly increasing with $T^{\prime}(0)=0, T(0)=0$, and $T^{\prime}(\infty)=1$. Show that, in the long-run, wealth holdings $a_{t}^{i}$ is converging to a unique value $a^{*}$. Find an implicit expression for $a^{*}$ is terms of $r, \rho$, and $T($.$) .$
d) What does this model predict for the impact of progressive capital income taxation on wealth inequality? Is this a realistic outcome?
2. Consider an overlapping generation economy with constant population and identical individuals. Each individual lives for two periods and has a utility function of the form,

$$
U\left(c_{1}, c_{2}, l\right)=\log \left(c_{1}-l^{k+1} /(k+1)\right)+\theta \log \left(c_{2}\right)
$$

where $c_{1}$ is period 1 consumption, $c_{2}$ is period 2 consumption, and $l$ is labor supply when young. The old do not work and finance their consumption with savings. Assume that the wage rate is $w$ and the interest rate on savings is $r$. The government imposes a linear tax $t_{L}$ on labor income and a linear tax on interest income $t_{K}$. Assume also that the government provides a lumpsum amount $R$ to the young. To simplify computations, we normalize the price of $c_{1}$ to one and we note $p=1 /(1+r)$ the pre-tax price of $c_{2}$ and $q=1 /\left(1+r\left(1-t_{K}\right)\right)$ the after-tax price of $c_{2}$.
a) Set up the individual budget constraint (using $q$ ) and solve for the optimal $l, c_{1}, c_{2}$ and savings $s$ as a function of $w\left(1-t_{L}\right), q, R, \theta$, and the elasticity of labor supply $e=1 / k$. Derive the indirect utility function $V$. Compute the elasticity of savings with respect to the net-of-tax interest rate $r\left(1-t_{K}\right)$.
b) Assume that the government cannot use lumpsum taxation $(R=0)$ and sets $t_{L}$ and $q$ so as to raise a given amount for public spending $g$ and maximize indirect utility $V$. Set-up the Ramsey static maximization problem for the government. Denote by $\lambda$ the multiplier of the government budget constraint. Assume that $w$ and $r$ are exogenous.

Derive the first order conditions with respect to $t_{L}$ and $q$. Combine these two equations to eliminate $\lambda$ and express $t_{K}$ in terms of $t_{L}, r, e$, and $\theta$. Finally, use the budget constraint to obtain an equation relating $t_{L}, g, w, e$, and $\theta$.
c) Numerical Simulation using the two equations obtained in b): assume that $w=1, r=2$ (a period represents a generation so $r=2$ is reasonable), and $g=1 / 4$.

Obtain the optimal $t_{L}$ and $t_{K}$ in three cases: $(\theta=1, e=1 / 4),(\theta=1 / 5, e=1 / 4)$, and $(\theta=1, e=1)$. Explain intuitively the pattern for the results. Which case is the most realistic? Why is the optimal $t_{K}$ not zero?
d) Find 2 important missing elements (there are many more than 2 ) in the model above and discuss briefly and qualitatively how incorporating these elements might alter the optimal rate of capital income taxation.
3. Consider a model where individuals live for two periods, and supply labor inelastically in period one of their life. Individuals have to save from period one earnings to finance their period 2 consumption (retirement). The before tax interest rate between the two periods is denoted by $r$ (exogenous, uniform and constant over time). Assume that there is an interest income tax at rate $\tau$ in this economy (assume no earnings income tax for simplicity) so that the net-of-tax
interest rate is $\bar{r}=r(1-\tau)$.
Assume that an individual maximizes a utility function of the form $U=\log \left(c_{1}\right)+\theta \log \left(c_{2}\right)$. Earnings in period 1 are equal to $w$.
a) Write down the maximization problem of the individual and find the optimal consumption and saving choices as a function of $\bar{r}, w$, and $\theta$. Explain the effect of an increase in $\bar{r}$ on savings and on retirement wealth.
b) Suppose now that an IRA program is introduced which exempts from taxation interest income up to a given level of savings $s^{*}$. All returns on savings above $s^{*}$ continue to be taxed at rate $\tau$. Solve for the optimal savings decision in this world. Be careful to distinguish different cases as savings are below, equal or above $s^{*}$. What is the effect of the IRA program on savings and on retirement wealth?
c) Are the results from b. consistent with the empirical literature trying to estimate the effects of IRAs on savings and wealth? Discuss briefly the findings of the literature as well as the difficulties that such empirical studies have to confront.

