Econ 230B

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Problem Set 1 Solution

1. (4.5 pts)

a) (0.5 pts) e is the elasticity of income with respect to the net-of-tax rate $1-\tau$. There are no income effects, so this elasticity is both compensated and uncompensated.

Total tax $T = \tau \sum_i z_i = \tau(1-\tau)^e \sum_i z_i^0$.

FOC in τ gives $\tau^* = 1/(1+e)$.

b) (1pt)

$$\hat{e} = \frac{(1/n)\sum_i \log(z_{i2}) - (1/n)\sum_i \log(z_{i1})}{\log(1-\tau_2) - \log(1-\tau_1)}$$

obtained by OLS regression $\log(z_{it}) = \alpha + e \log(1-\tau_t) + \epsilon_{it}$

c) (0.5 pt) Assuming that incomes are multiplied by $e^g > 1$ because of growth from year 1 and year 2, previous $\hat{e}$ is biased upward. To get consistent estimate of e, need to subtract the growth rate from the numerator:

$$\hat{e} = \frac{(1/n)\sum_i \log(z_{i2}) - (1/n)\sum_i \log(z_{i1}) - g}{\log(1-\tau_2) - \log(1-\tau_1)}$$

d) (1 pt)

$$\hat{g} = (1/n)\sum_i \log(z_{i1}) - (1/n)\sum_i \log(z_{i0})$$

obtained by OLS regression $\log(z_{it}) = \alpha + g\, t + \epsilon_{it}$

Using all three years, DD estimate:

$$\hat{e}_R = \frac{[(1/n)\sum_i \log(z_{i2}) - (1/n)\sum_i \log(z_{i1})] - [(1/n)\sum_i \log(z_{i1}) - (1/n)\sum_i \log(z_{i0})]}{\log(1-\tau_2) - \log(1-\tau_1)}$$

Obtained with OLS regression: $\log(z_{it}) = \alpha + \beta\, t + e \log(1-\tau_t) + \epsilon_{it}$

e) (0. 75 pt) Total tax $T = \tau \sum_i z_i = \tau \sum_i (1-\tau)^{e_i} z_i^0$.

FOC: $\sum_i z_i - \tau \sum_i e_i (1-\tau)^{e_i - 1} z_i^0$

implies $\sum_i z_i = [\tau/(1-\tau)] \sum_i e_i (1-\tau)^{e_i} z_i^0$

that is, $\sum_i z_i = [\tau/(1-\tau)] \sum_i e_i z_i$

Let us note $\bar{e} = \sum_i e_i z_i / \sum_i z_i$ the average elasticity weighted by incomes (high incomes have a disproportionate effect on total elasticity), we have:

$\tau/(1-\tau) = 1/\bar{e}$, that is, $\tau = 1/(1+\bar{e})$.

f) (0.75 pt) Total tax $T = \tau \sum_i z_i = \tau \sum_i (1-\tau)^{e_i} z_i^0(R)$.

FOC: $\sum_i z_i - [\tau/(1-\tau)] e \sum_i z_i + \tau \sum_i (1-\tau)^{e_i} (z_i^0)'(R) \partial R/\partial \tau = 0$

but last term is zero because at the optimum, R is maximized and thus $\partial R/\partial \tau = 0$. Therefore, the FOC is the same as in a) and $\tau = 1/(1+e)$ as in a).

2. (3 pts) Many different possible reforms.

3. (2.5 pts)

a) (1 pt) Let us denote $L = 1 - l$ leisure. We have $u = u(c, L)$.

Individual maximization:

Max $u(wl - T(w), 1 - l)$.

First order condition $u_c w - u_L = 0$ implies that labor supply $l(w, T(w))$ depends on w and $T(w)$

Planner's program:

Max $\int u(wl(w) - T(w), 1 - l(w)) f(w) dw$ st $\int T(w) f(w) dw \geq 0$

Lagrangian $L = u(wl(w) - T(w), 1 - l(w)) f(w) + \lambda T(w) f(w)$

FOC with respect to $T(w)$: $\partial L/\partial T(w) = 0$

implies $u_c(-1 + w\partial l/\partial T(w)) - u_L \partial l/\partial T(w) + \lambda = 0$

implies $u_c = \lambda$ (using individual FOC $u_c w - u_L = 0$)

Interpretation: The marginal utility of consumption is equalized over all individuals.

b) (1 pt) $du/dw = u_c[l - T'(w) + w dl/dw] - u_L dl/dw = u_c[l - T'(w)]$ (using individual FOC $u_c w - u_L = 0$)

To get $T'(w)$, let us differentiate wrt w the government FOC $u_c = \lambda$ which takes the form $a'(wl - T(w)) = \lambda$ with separable utility.

We obtain $a''(wl - T(w)) \cdot [l - T'(w) + w dl/dw] = 0$

which implies $l - T'(w) = -w dl/dw]$

So $du/dw = -w u_c dl/dw$

So we have indeed $du/dw < 0$ iff $dl/dw > 0$

c) (0.5 pts) $a'(wl - T(w)) = \lambda$ so consumption is constant across individuals but high skilled workers work more, and thus utility is decreasing in skills.

This is a FB model of income taxation. It generates complete redistribution and requires the high skilled individuals to work harder (because they produce a lot).