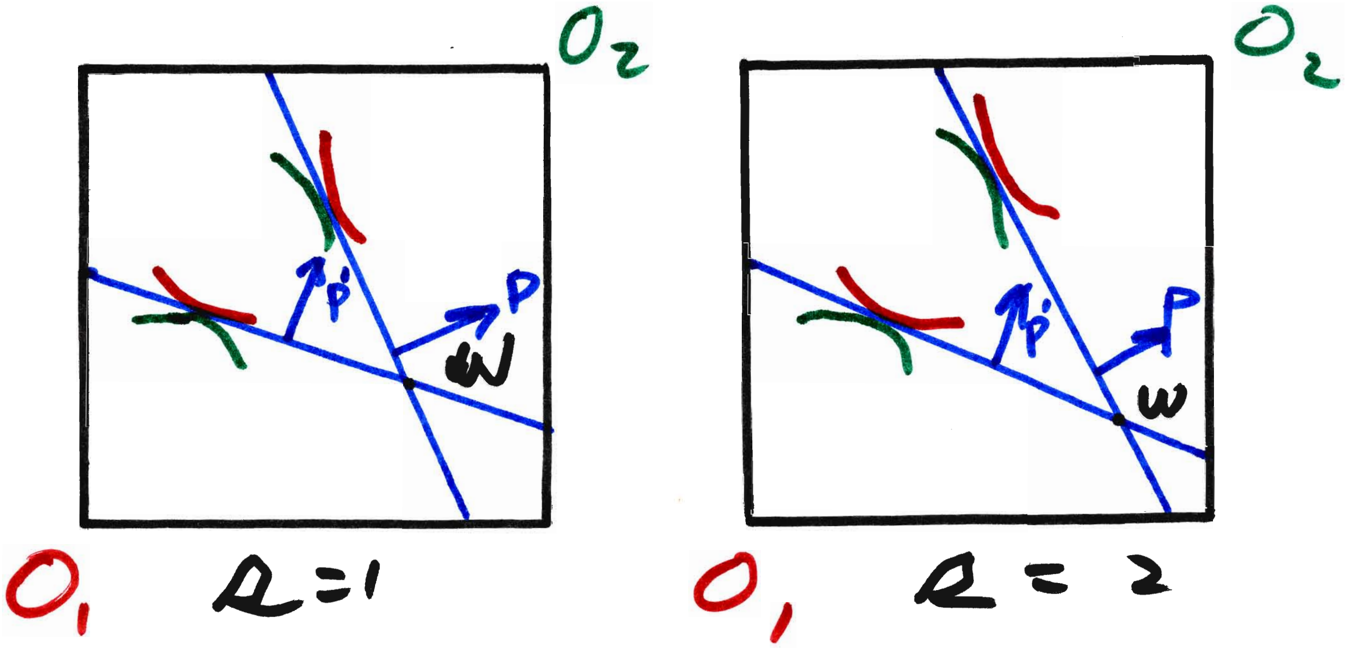
Economics 201B–Second Half

Lecture 14, 4/29/10

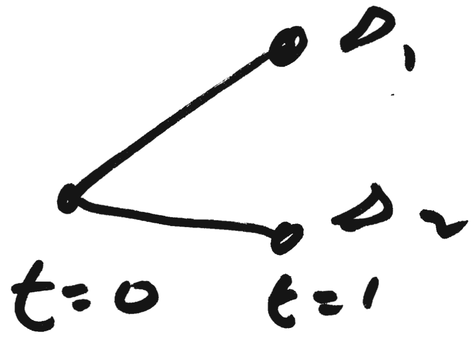
GEI Model (Continued)

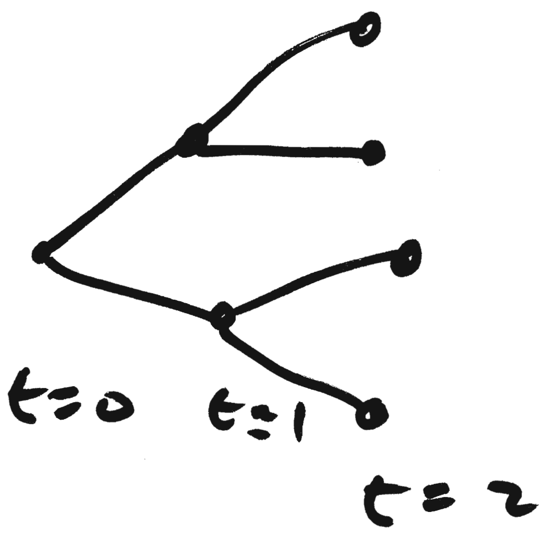
- Saw in Lecture 13 that Equilibrium need not be Pareto Optimal.
- Model V(b):
  - Either 2 periods, $L > 1$, securities pay off in goods; or more than 2 periods with $L = 1$. Two situations work much the same way.
  - *Natural Conjecture*: In the GEI model, Walrasian Equilibrium is constrained Pareto Optimal, i.e. there is no allocation achievable by the given markets that Pareto dominates the Equilibrium allocation. Note that the conjecture is ambiguous, since the allocations achievable by the given markets depend on the prices.
    - Under any reasonable interpretation, the conjecture is *false*. Hart Example (19.F.2 in MWG, but with different spin).
      - $I = 2$, $L = 2$, $S = 2$. There are no securities, so you can't trade consumption across states.
      - States $s = 1$ and $s = 2$ have identical Edgeworth boxes.
      - Each Edgeworth box has three Equilibrium prices; let p and p' be two of them.
      - Within each Edgeworth box, agent 1 prefers p to p', agent 2 prefers p' to p

Hart Example



Model V(b)





Securities pay in goods

Securities traded at $t=0$

Securities pay off in Numeraire good, or $L=1$

Securities traded at $t=0$, $t=1$

· Fix utility functions v_1, v_2 realizing the preferences of agents 1,2 in the common Edgeworth box.

· $x_{\ell s i}$ denotes the consumption of good ℓ by agent i in state s. Let

$$\begin{aligned} u_1(x_{111}, x_{211}, x_{121}, x_{221}) &= 100 v_1(x_{111}, x_{211}) + v_1(x_{121}, x_{221}) \\ u_2(x_{112}, x_{212}, x_{122}, x_{222}) &= v_2(x_{112}, x_{212}) + 100 v_2(x_{122}, x_{222}) \end{aligned}$$

Agent 1 cares much more about state 1, agent 2 cares much more about state 2.

· There are nine equilibria, including the following 4:

$s=1$	$s=2$
p	p
p	p'
p'	p
p'	p'

(p, p') gives 1's preferred outcome in state 1, which 1 cares about most, and gives 2's preferred outcome in state 2, which 2 cares about most.

· Let $100 \to \infty$. Then eventually, (p, p') Pareto dominates (p', p), and clearly (p, p') is achievable using the given markets, so (p', p) cannot be "constrained Pareto Optimal" in any reasonable sense of the term.

– Equilibrium may not exist ("Hart Points"):

∗ Two versions:

· 2 periods, 2 securities pay in goods at time 1.

· 3 periods, states form a binary tree (2 nodes at time 1, 4 nodes at time 2)

∗ It's customary in these models to have consumption at period 0 in the 2-period case, and at periods 0 and 1 in the 2-period case. We will assume for simplicity that consumption occurs only in the final period.

* Focus for now on 2 periods, securities pay in goods.

* Suppose securities payoffs are given by

$$S_1 \quad \text{pays} \quad \begin{cases} \left(\frac{2}{3}, \frac{4}{3}\right) & \text{in state 1} \\ \\ \left(\frac{4}{3}, \frac{2}{3}\right) & \text{in state 2} \end{cases}$$

$$S_2 \quad \text{pays} \quad \begin{cases} (1,1) & \text{in state 1} \\ (1,1) & \text{in state 2} \end{cases}$$

* If the spot prices turn out to be $\left(\frac{1}{2}, \frac{1}{2}\right)$ in both states, then securities have the following payoffs:

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

  which is singular. You can't use the securities to move income between states 1 and 2! Prices at which the rank of the securities payoff matrix drops are called Hart points.

* If the spot prices turn out to be $\left(\frac{1-\varepsilon}{2}, \frac{1+\varepsilon}{2}\right)$ in both states, then securities have the following payoffs:

$$\begin{pmatrix} 1+\frac{\varepsilon}{3} & 1 \\ 1-\frac{\varepsilon}{3} & 1 \end{pmatrix}$$

  which is *not* singular. You *can* move income between states 1 and 2 using the securities, but you need to hold a *large* position, very long in one and very short in the other.

* With one good and three periods, the securities have prices at period 1; at Hart points, the rank of the securities return matrix drops.

        - Consequently, in either case, the budget set (in goods) is *not* uhc or closed graph in the spot prices. Moreover, demand for securities is not bounded below. Equilibrium need not exist.
        - Radner: Put an exogenous lower bound on short sales of securities. Then demand is bounded below and uhc, and equilibrium exists. However, equilibrium need not be Pareto Optimal if either
            - the short sale constraint is binding; or
            - the equilibrium price is a Hart point.
        - Duffie-Shafer: Except for a set of endowments and securities payoffs of measure zero, Walrasian equilibrium exists. Idea:
            - Change demand at Hart points to make it uhc.
            - Use a different fixed point argument to find a "pseudoequilibrium."
            - If pseudoequilibrium price is not a Hart point, then it's an equilibrium.
            - Generically, pseudoequilibrium doesn't occur at a Hart point.

- What about first Welfare Theorem?
    - Hart Example suggests there's an open set of parameters in which equilibrium is constrained optimal, and an open set where it is not constrained optimal. It's much worse than that.
        - Consider the 3 period, $L = 1$ model, binary tree, 1 security.

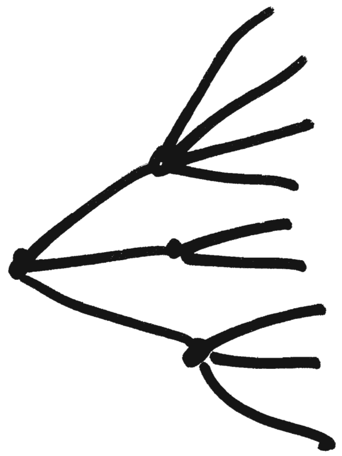
      * Changing the period 0 plans changes the period 1 equilibrium securities prices, so changes the constrained feasible set.
      * Agents don't take this into account at Walrasian Equilibrium. Hence

        Theorem 1 (Geanakoplos-Polemarchakis) *In Model V(b), let J be the maximum number of branches leading from any node in the event tree. If the number of securities is less than J, then generically* all *Walrasian Equilibria are constrained suboptimal.*

- Model V(a): Dynamic Completeness
  - Multiple Periods, 1 good, Event Tree.

    Theorem 2 (Magill-Shafer) *Let J be the maximum number of branches leading from any node in the event tree. If the number of securities is greater than or equal to J, then generically in the securities payoffs and endowments, every Walrasian Equilibrium produces dynamically complete markets, and thus the equilibrium allocations are Pareto Optimal.*

Model V(a)



J = 4

t=0 1 2

No intermediate consumption, need J securities

With intermediate consumption, need J+1 securities