Economics 201b
Spring 2010
Problem Set 3
Due Thursday April 8

Unless otherwise specified, all equilibria are understood to mean without transfers.

1. Consider a Robinson Crusoe economy with a linear technology $f(x_1) = \alpha x_1$ and linear preferences: $U(x_1, x_2) = x_1 + \beta x_2$ where $0 < \alpha, \beta < \infty$. Let the endowment be $\omega = (0, L)$. Give a complete case-by-case analytic characterization of all equilibria. For each case, draw a picture.

2. In an Arrow-Debreu economy with strongly monotone preferences, consider a 4-tuple $(p, x, y, T)$ where $T$ is an income transfer and $x_i \in D_i(p, y, T)$ for all $I$ consumers. Suppose all but one market clear - without loss of generality, assume the first $L-1$ markets clear, then show explicitly that, in fact, all markets clear. When writing your solution use the standard notation (e.g. $l$, $i$, $x_i$, $y_j$, $\theta_{ij}$) for Arrow-Debreu economies found in the notes for lecture 4. Do not assume it is a pure exchange economy.

3. Consider a two-consumer, one-firm Arrow-Debreu economy. The technology of the firm is $Y = \{(y_1, y_2) | y_1 \leq 0, y_2 = e \log(1 - y_1)\}$. The endowments are $\omega_1 = (e, 0)$ and $\omega_2 = (e^2, 0)$, and the utilities are $U_1(x_{11}, x_{21}) = \frac{\log(x_{11})}{e} + x_{21} - \frac{1}{e}$ and $U_2(x_{12}, x_{22}) = \log(x_{12}) + x_{22} - 2$. Let $\theta_1$ and $\theta_2$ be the two agent's shares of the firm's profit.

   (a) Give an analytic characterization of all equilibria. Show your work in detail; in particular find a *simple, clean* expression for equilibrium labor.

   (b) Suppose that the agents bargain for their shares $\theta_i$ of the firm's profits. What is the Nash bargaining solution for the shares? Explain. (Recall, the Nash Bargaining solution is the split of shares - ($\theta_1^*$, $\theta_2^*$) - that solves the following maximization problem

   $$\underset{\theta_1, \theta_2}{\operatorname{argmax}} \quad (U_1^{\theta_1} - \underline{U_1})(U_2^{\theta_2} - \underline{U_2})$$

   where $U_i^{\theta_i}$ is agent $i$'s equilibrium utility when the shares are $(\theta_1, \theta_2)$, and $\underline{U_i}$ is agent $i$'s utility when there is no access to the firm's technology.)

4. Consider a two-person, two-good exchange economy with the following nonconvex preferences: $U_i(x_{1i}, x_{2i}) = \max\{x_{1i}, x_{2i}\}$ for $i = 1, 2$. Suppose the social endowment is $\bar{\omega} = (1, \gamma)$ with $\gamma > 0$.

   (a) Give a careful analytic characterization of all *exact* Pareto Optimal allocations. Answer will depend on $\gamma$. Draw pictures demonstrating the different possibilities.

   (b) Are there any values of $\gamma$ for which the Second Welfare Theorem fails? Prove your answer.