University of California, Berkeley
Economics 201B
Spring 2009 Final Exam–May 21, 2009

Instructions: You have three hours to do this exam. The exam is out of a total of 300 points; allocate your time accordingly. **Please write your solutions to Parts I, II and III in separate bluebooks.**

Part I

1. (100 points) Define or state and *briefly* discuss the importance of each of the following within or for economic theory:
   (a) Kakutani's Fixed Point Theorem
   (b) First Welfare Theorem in the Arrow-Debreu Economy
   (c) Second Welfare Theorem in an exchange economy
   (d) Transversality Theorem

Part II

2. (75 points)Theorem 4 from Lecture 13 is a theorem asserting that core allocations in exchange economies satisfy a perturbation of the definition of Walrasian quasiequilibrium.
   (a) State the theorem.
   (b) Give the first two bullets in the proof of the theorem.

Part III

3. (125 points) Consider the function $z : \Delta^0 \times \mathbf{R} \to \mathbf{R}^2$ defined by

$$z((p_1, p_2), \alpha) = \left(\log p_1 + \frac{2}{9p_1} - p_1, \frac{(p_1)^2 - p_1 \log p_1 - \frac{2}{9}}{p_2}\right) + \left(\alpha, -\frac{p_1 \alpha}{p_2}\right)$$

Here, Δ^0 is the normalized price simplex $\{(p_1, p_2) \in \mathbf{R}^2_{++} : p_1 + p_2 = 1\}$, and $\log t$ denotes the natural logarithm of t, so that $\frac{d}{dt} \log t = \frac{1}{t}$. In answering the following questions, it may be useful to you to know that $\lim_{t \to 0, t>0} t \log t = 0$.

(a) Show that, for every $\alpha \in \mathbf{R}$ and for any $\varepsilon > 0$, there exists an exchange economy with two agents whose excess demand function is $z((p_1, p_2), \alpha)$ whenever $p_1 \in [\varepsilon, 1-\varepsilon]$.

(b) Verify that if $\alpha < \frac{7}{9}$, then $z(\cdot, \alpha)$ satisfies the conditions of the Debreu-Gale-Kuhn-Nikaido Lemma, and conclude that there exists p such that $z(p, \alpha) = 0$.

(c) Define $\hat{z} : \mathbf{R}_{++} \times \left(-\infty, \frac{7}{9}\right) \to \mathbf{R}$ by $\hat{z}(\hat{p}, \alpha)$ is the first component of $z\left(\left(\frac{\hat{p}}{\hat{p}+1}, \frac{1}{\hat{p}+1}\right), \alpha\right)$. Let A denote the set of all $\alpha \in \left(-\infty, \frac{7}{9}\right)$ such that the economy with excess demand $z(\cdot, \alpha)$ is not regular. Using the Transversality Theorem, show that A is a set of Lebesgue measure zero.

(d) Using the Index Theorem, show that if $\alpha < \frac{7}{9}$ and $\alpha \notin A$, then the economy with excess demand $z(\cdot, \alpha)$ has an odd number of equilibria.

(e) Using the Implicit Function Theorem, prove directly from the definition that if $\alpha < \frac{7}{9}$ and $\alpha \notin A$, then the equilibrium correspondence $E(\alpha) = \{\hat{p} \in \mathbf{R}_{++} : \hat{z}(\hat{p}, \alpha) = 0\}$ is lower hemicontinuous at α.