Economics 201A–Fall 2005–Second Half
The Second Welfare Theorem with Nonconvex Preferences

This handout is based on Anderson, "The Second Welfare Theorem with Nonconvex Preferences," *Econometrica* 56(1988), 361-382. As the diagram on page 2 shows, the second welfare theorem may fail if preferences are nonconvex. Specifically, it gives an economy with two goods and two agents, and a Pareto optimum x^* so that so that the utility levels of x^* cannot be approximated by an Walrasian equilibrium with transfers; moreover, if p^* is the price which locally supports x^*, and T is the income transfer which makes x affordable with respect to the prices p^*, there is a unique Walrasian equilibrium with transfers (y^*, q^*, T); y^* is much more favorable to agent I and much less favorable to agent II than x^* is.

Theorem 3.3 of the paper shows that this is, in a sense, the worst that can happen under standard assumptions on preferences.[1] Specifically, given a Pareto optimum x^*, there is a Walrasian quasiequilibrium with transfers (y^*, p^*, T) such that all but L people are indifferent between x^* and y^*, where L is the number of goods. Those L people are treated quite harshly (they get zero consumption). One could be less harsh and give these L people carefully chosen consumption bundles in the convex hull of their quasidemand sets, *but one would then have to forbid them from trading*, a prohibition that would in practice be difficult to enforce.

[1] See the paper for the precise assumptions needed on preferences.

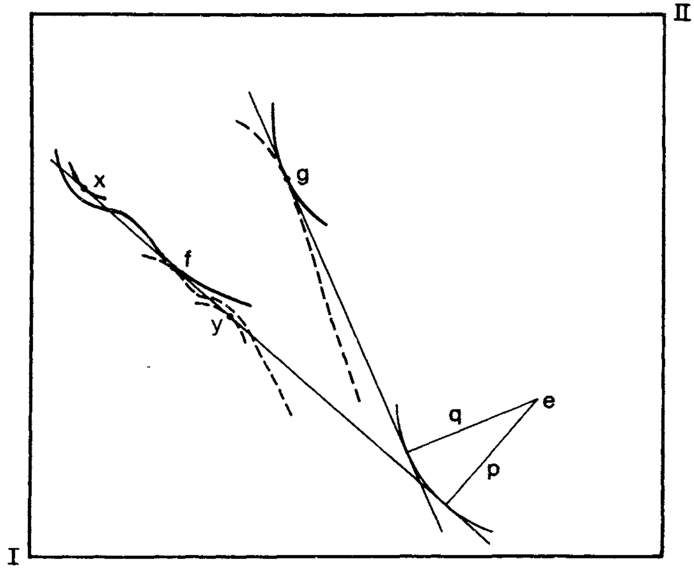

FIGURE 1.

there were an approximate Walrasian allocation g, it would have the property that $f(a) \not\succ_a g(a)$ for all a (observe that $f(a)$ is in the budget set). However, as in the convex case, allowing the government to dictate f as the initial allocation destroys the interpretation of the Second Welfare Theorem as a story of decentralized allocation.

In Theorem 3.3, we show that the government can achieve the utility levels desired for all but k agents, where k is the dimension of the commodity space. In other words, the pathology illustrated in Figure 1 disappears (at least for most agents) provided that the number of agents is large relative to the number of commodities. The proof is elementary, relying primarily on the Shapley-Folkman Theorem. We focus on a particular choice of decentralizing price $\bar{p}$; this price is used by Mas-Colell in the proof of his theorem, and is closely related to the so-called gap-minimizing price studied in Anderson (1987); essentially, $\bar{p}$ is the price which minimizes the measure by which support fails in Mas-Colell's Theorem. Given any Pareto optimum f, there is an income transfer t and a quasiequilibrium $\tilde{f}$ with respect to t such that all but k agents are indifferent between f and $\tilde{f}$. If preferences are monotone and a mild assumption on the distribution of goods at f is satisfied, then we may show that $\bar{p}$ is strictly positive, and hence $\tilde{f}$ is a Walrasian equilibrium with respect to t. As an alternative, we can achieve an approximate equilibrium (i.e., total excess demand is bounded, independent of the number of agents) $\hat{f}$ such that all agents are indifferent between f and $\hat{f}$. It is worth emphasizing that Theorem 3.3 is a universal theorem, applying to all exchange economics, rather than a generic theorem. However, there is no guarantee that $\tilde{f}(a)$ is close to $f(a)$ for *any a*. A