Economics 201A
Problem Set 1
Due 10 September 2019

1. Consider the following classical condition:

   Definition 1. A choice rule $\mathcal{C}$ satisfies **path independence** if, for all $A, B \in 2^X \setminus \emptyset$,

   $$\mathcal{C}(A \cup B) = \mathcal{C}\big(\mathcal{C}(A) \cup \mathcal{C}(B)\big)$$

   (a) Prove the following claim: If $\mathcal{C}$ is nonempty and rationalizable, then $\mathcal{C}$ satisfies path independence.

   (b) Provide an example of a nonempty choice rule that satisfies path independence, but is not rationalizable.

2. The following axiom is another condition proposed by Sen:

   Axiom 2 (Sen's γ)**.** If $x \in C(A) \cap C(B)$, then $x \in C(A \cup B)$.

   Let X be finite and $\mathcal{C}$ be a nonempty choice rule. Prove the following claims:

   (a) If $\mathcal{C}$ is rationalizable, then $\mathcal{C}$ satisfies Sen's γ.

   (b) If $\mathcal{C}$ satisfies Sen's α and Sen's γ, then there exists a (not necessarily transitive) binary relation $\succsim$ on X such that

   $$C(A) = \{x \in A : x \succsim y, \text{ for all } y \in A\}$$

3. Let $\succsim$ be a binary relation on X. The set $Z \subseteq X$ is **$\succsim$-dense** if, for all $x, y \in X$, there exists a $z \in Z$ such that $x \succ z \succ y$. Prove that if $\succsim$ is complete and transitive and there exists a countable $\succsim$-dense set, then there exists a utility representation $u : X \to \mathbb{R}$ of $\succsim$. (Hint: The proof is very similar to the proof of a utility representation for a weak order on a countable set.)