

Measuring Benchmark Damages in Antitrust Litigation

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Abstract

We compare the two dominant approaches to estimation of benchmark damages in antitrust litigation, the forecasting approach and the dummy variable approach. We give conditions under which the two approaches are equivalent and present the results of a small simulation study.

I. Introduction

The quantitative evaluation of monetary damages from alleged antitrust violations occupies a central place in antitrust litigation. The two most common approaches to evaluating damages involve the use of *yardsticks* and *benchmarks*.¹ In a typical yardstick approach, one compares prices during the period in which the antitrust violation is believed to have had an effect (the “impact period”) to prices in other markets that are deemed to be reasonably comparable to the market at issue. In contrast, the benchmark approach evaluates prices only in the market at issue, comparing prices in the impact period to available prices before and/or after the alleged period of impact (the “control period”).

In this paper, we offer a detailed evaluation of the benchmark approach to damages. We have found the benchmark approach to be the most commonly used damages methodology. To focus the analysis, we assume that the antitrust violation at issue involves price fixing. We also assume that the appropriate legal remedy involves *overcharges* rather than *lost profits*.² Our particular focus is a comparison of the forecasting and dummy variable approaches, which we define in Section

¹Alternative approaches involve variations on the yardstick approach, such as a comparison of rates of return and/or profit margins across industries.

²For a broad discussion of these alternative measures, see Hovenkamp (2005, Section 17.5(a)).

III. Our analysis underscores that these competing approaches to computing benchmark damages estimates typically yield similar estimates, despite seemingly different implementation schemes. Econometrically, our analysis is quite simple. The results we discuss are applications of standard results on the algebra of regressions. Practically, our analysis is highly relevant. Many millions of dollars change hands annually on the basis of benchmark damages estimates.

We are not the first to consider the advantages and disadvantages of each of these methodologies.³ However, we believe our results comparing the forecasting and dummy variable approaches, while straightforward, are perhaps new and certainly underappreciated. In order to focus on the central methodological issues, we begin in Section II by describing the basic regression framework. In Section III, we discuss alternative versions of the dummy variable approach, offering in the process a suggestion as to how to compare the various methodologies. We also describe the forecasting approach and compare it to the dummy variable approach. Section IV presents three propositions that directly compare the dummy variable and forecasting approaches. The propositions tend to support the use of the dummy variable approach over the forecasting approach. However, there are particular advantages associated with the forecasting approach, and these are discussed in Section V. In Section VI, we return to the dummy variable approach, discussing some important model specification issues. In Section VII, we offer an example that illustrates the differences between the various approaches. Section VIII concludes.

II. The Basic Model

Let Y_t denote a measure of the outcome of an alleged conspiracy (e.g., price), X_t a vector of exogenous covariates not causally affected by the conspiracy (e.g., demand and cost variables), and D_t a dummy variable indicating the period of the alleged conspiracy, i.e., the impact or conspiracy period.

We assume that there is data both before and during the alleged conspiracy period. Let T_0 denote the beginning of the alleged conspiracy, so that $t = 1, 2, \dots, T_0 - 1$ corresponds to the pre-conspiracy

³See, for example, Fisher (1980), Rubinfeld and Steiner (1983), Rubinfeld (1985), Rubinfeld (2008), and Higgins and Johnson (2003). See especially White, Marshall and Kennedy (2006); the authors strongly prefer the forecasting approach and are highly critical of the dummy-variable approach.

control period and $t = T_0, T_0 + 1, \dots, T$ to the conspiracy period. We focus on the basic model

$$Y_t = \alpha + \beta'X_t + \theta D_t + \gamma'D_tX_t + \varepsilon_t \quad (1)$$

where ε_t is a mean zero residual that is independent of X_t , D_t , and D_tX_t .⁴ This relatively general specification takes into account the possibility that the alleged conspiracy will affect price (the outcome) directly, as given by given by θD_t (e.g., through an increase in price at each point in time in the damage period). However, it also takes into account the possibility that the effect of the conspiracy will be felt through one or more of the covariate demand and cost variables, as given by the term $\gamma'D_tX_t$. This allows the effect of X_t on Y_t to differ between the control period and the impact period. This can be a desirable feature, for example, in an industry and time period where excess profits are being dissipated over time by the process of market entry.

We assume that the covariates are not causally related to the conspiracy. When the covariates *are* caused by the conspiracy, neither the forecasting approach nor the basic dummy variable approach is appropriate if applied using the model in Equation (1). When using the forecasting approach, one bases the forecast on covariates; this approach will be tainted if the covariates are caused by the conspiracy. For the dummy variable approach, the covariates are endogenous, which would require a different econometric model, such as instrumental variables.⁵ Note, however, that assuming no *causal* relationship between the covariates and the conspiracy does not rule out the possibility that the covariates are *correlated* with the conspiracy. Indeed, we focus on the case where the covariates have different levels during pre-conspiracy period than during the conspiracy period.⁶

We focus on what is to be done when the model in (1) is appropriate and there are sufficient data to apply either approach. For simplicity, we assume that the period in which there are antitrust damages and the conspiracy period are identical. Allowing for the two to be different would add some complexity to the specification, but would not change any of the fundamental points to be

⁴That is, we assume that Equation (1) is the data generating process.

⁵The forecasting approach will also require a more complex model that correctly specifies the values of the taint-free covariates in the impact period.

⁶The case where the covariates have equal average levels between the pre-conspiracy period and the conspiracy period is discussed in Higgins and Johnson (2003); see their assumption 4.

made in the paper.

The model in Equation (1) is identically equivalent to the following model of counterfactual outcomes:

$$Y_t(1) = \alpha + (\beta' + \gamma')X_t + \theta + u_t \quad (2)$$

$$Y_t(0) = \alpha + \beta'X_t + v_t \quad (3)$$

where $Y_t(1)$ is the outcome during the conspiracy period, and $Y_t(0)$ is the outcome that would occur were there no conspiracy. Under this formulation, $Y_t = D_t Y_t(1) + (1 - D_t) Y_t(0)$ and $\varepsilon_t = D_t u_t + (1 - D_t) v_t$. The formulation in Equations (2) and (3) is useful for understanding some of the conceptual points we raise, below.

III. The Dummy Variable and Forecasting Approaches

One standard approach to the evaluation of overcharges estimates a regression model for the entire period for which data are available, and evaluates damages by looking at the statistical significance and magnitude of the coefficient on a dummy variable that distinguishes the impact period from the control period. When using this “dummy variable” approach, a secondary issue arises. Should one evaluate damages by assuming a constant price differential through the impact period (as suggested by the coefficient on the dummy variable) or should one allow for non-constant price effects of the alleged conspiracy?

When the time period or periods in which the alleged antitrust behavior affected prices is sufficiently long and the necessary data are available, a second standard approach to the evaluation of overcharges is a two step procedure. First, one estimates a regression model that “explains” prices using only data for the control period in which the market was unimpeded. Second, the regression model is used to predict but-for prices in the impact period.⁷ This approach is conventionally referred to simply as a “forecasting” (or “before-after”) approach.

To apply the dummy variable approach, we estimate Equation (1) for the entire time period.

⁷There must be sufficiently variability to allow one to appropriately account for non-collusive variables that might have affected price in the impact period.

Continuing to assume that the impact of the covariates on price is unaffected by the conspiracy, θ measures the “average” effect of the conspiracy on price per unit of time. More generally, the impact of the covariates on price may be correlated with the conspiracy, although not directly caused by it. In this more general scenario, the difference between prices with and without the conspiracy per unit of time is given by $\mathbb{E}[Y_t(1) - Y_t(0)|X_t] = \theta + \gamma'X_t$. If Q_t represents output, and all covariates are deemed to be associated with the alleged conspiracy, then aggregate overcharges are a sales-weighted sum of the difference in prices with and without the conspiracy, or

$$\widehat{OC}_1 = \sum_t D_t Q_t \left\{ \widehat{Y}_t(1) - \widehat{Y}_t(0) \right\} \equiv \widehat{\theta} \sum_t D_t Q_t + \widehat{\gamma}' \sum_t D_t Q_t X_t \quad (4)$$

where $\widehat{\theta}$ and $\widehat{\gamma}$ are regression estimates of the parameters and $\widehat{Y}_t(1) = \widehat{\alpha} + (\widehat{\beta}' + \widehat{\gamma}')X_t + \widehat{\theta}$ and $\widehat{Y}_t(0) = \widehat{\alpha} + \widehat{\beta}'X_t$.

For some purposes, it may be desirable to impose the restriction that $\gamma = 0$ (i.e., the effect of the covariates on price is the same in the impact period and the control period). In this case, we would obtain a different estimate of damages, given by

$$\widehat{OC}_2 = \widehat{\theta}_S \sum_t D_t Q_t \quad (5)$$

where $\widehat{\theta}_S$ is estimated from the “short” regression that omits the interaction between covariates and the conspiracy period dummy. Our main focus is on \widehat{OC}_1 , but we discuss \widehat{OC}_2 in Section VI.

The forecasting approach estimates the following model:

$$Y_t = \alpha + X_t \beta + e_t \quad (6)$$

for the control period $t = 1, 2, \dots, T_0$. Overcharges are then based on the difference between the predicted outcome (e.g., price) and the actual outcome, during the conspiracy period. If \widehat{Y}_t^f is the out-of-sample (damage period) price forecast, then aggregate overcharges using the forecasting

method are given by:

$$\widehat{FC} = \sum_t D_t(Y_t - \widehat{Y}_t^f)Q_t \quad (7)$$

As a general rule in this context, the forecasting approach and the dummy variable approach will generate different overcharge estimates. This is because price gaps are weighted by sales here (cf., Salkever 1976). In the next section, we discuss the issues involved in choosing one approach rather than the other.

IV. When Do the Approaches Differ?

Equation (1) was introduced as a description of the true relationship between the outcome, the conspiracy period, and the covariates. A related interpretation of Equation (1) is as an in-sample decomposition of prices into predicted and unexplained components. Specifically, we have

$$Y_t = \widehat{\alpha} + \widehat{\beta}'X_t + \widehat{\theta}D_t + \widehat{\gamma}'D_tX_t + \widehat{\varepsilon}_t \quad (8)$$

where $\widehat{\varepsilon}_t$ is a fitted price residual which has zero correlation with the regression prediction, in the sample at hand, by construction.

We can use this decomposition to connect the forecasting and the dummy variable approaches. We describe three important results. All proofs are contained in the appendix.

Proposition 1: When quantity varies over the conspiracy period, the forecasting and dummy variable approaches will differ, depending on whether or not quantity is correlated with the fitted residual during the conspiracy period. Formally,

$$\widehat{FC} = \widehat{OC}_1 + \sum_t D_tQ_t\widehat{\varepsilon}_t \quad (9)$$

Proposition 2: When quantity is constant over the conspiracy period, the forecasting and dummy variable approaches yield numerically identical overcharge estimates.

Proposition 3: Assuming that quantity is population uncorrelated with omitted factors during the conspiracy period, the dummy variable and forecasting approaches share a probability limit. However, the forecasting approach produces a more variable estimate of damages than the dummy variable approach.

Proposition 1 establishes that the difference between the forecasting and dummy variable approaches hinges on whether the quantity of sales would affect price in the regression model. Classical demand theory would suggest that when price is unexpectedly high (i.e., when ε_t is high) that quantity is likely to be low. Hence, one presumption is that the forecasting estimate of overcharges will negatively biased relative to the dummy variable estimate of overcharges. However, note that Equation (1) is not typically interpreted as an inverse demand equation, but rather a reduced form model for price. Consequently, there may be no economic basis for the assumption that quantity and unexplained price deviations are negatively related. Moreover, there is little to be gained by making such a presumption, since it is straightforward to estimate damages using both approaches, and to assess the difference.

Proposition 2 follows from the formal result in Proposition 1. If quantity is constant, then the difference between the two approaches is proportional to $\sum_t D_t \hat{\varepsilon}_t$, which is zero by construction. Proposition 3 also follows from the formal result in Proposition 1. Unless quantity is constant over the conspiracy period, the term $\sum_t D_t Q_t \hat{\varepsilon}_t$ will add noise to the forecasting estimate that is not present with the dummy variable approach.

V. Why Forecast?

Given that the forecasting approach may under some circumstances generate biased and inefficient damage estimates, when should the approach be used? In this section, we discuss the potential advantages of the forecasting approach.

Advocates of the forecasting approach sometimes use sophisticated model selection procedures to choose the regression model. One motivation for this approach is that the model selection process is based purely on data *prior* to the conspiracy period and will therefore not be corrupted by any effects that the conspiracy might have had on the covariates in the conspiracy period. There is an important benefit associated with this approach, but there is a further drawback. The benefit is that an appropriate model searching methodology minimizes the scope for “overfitting” and “cherry-picking”.⁸ If data *during* the conspiracy period are used to choose the regression model,

⁸This is a benefit emphasized by White et al. (2006).

then there is a risk that the model will produce a biased damages estimate, which is inappropriate. For example, it is always possible to use an in-sample model selection procedure to produce a damages estimate of zero, just by adding a sufficient number of irrelevant covariates so that the model fully explains prices in the conspiracy period (“overfitting”).

Choosing the model that is most beneficial to a particular position (“cherrypicking”) can also be a problem. If an expert knows the damages estimate that is beneficial to the client, there is a risk that in-sample model selection could be tantamount to choosing the model that generates a damages estimate that is most preferred. A forecasting approach that is based on an appropriate model selection methodology serves as a good disciplining device.⁹

The drawback of using data prior to the conspiracy period to select the model is that it may be too disciplining. In particular, the use of only pre-conspiracy data prevents the expert from selecting a model using all of his or her knowledge of the economics of the problem. Particularly in dynamic markets, the relationship between covariates and prices may be so rapidly evolving that the pre-conspiracy period will not be an especially good guide to model selection for the conspiracy period. In such a setting, prior knowledge may be of great value and the expert may want to use such knowledge. Suppose, for example, that the market at issue is a highly innovative one in which new technologies are developed on average every two years, and also that the rate of innovation is growing over time. Suppose also that the conspiracy period is four years long. Then, the forecasting approach is likely to underestimate the extent to which innovation would likely have occurred in the but-for world during the conspiracy period.

Weighing these considerations, some would conclude that the model selection procedure associated with forecasting is on balance desirable, especially when damages do not involve dynamic markets. Our analysis suggests, however, that one should consider combining the model selection procedure commonly used in the forecasting approach with the dummy variable approach. To be specific, the model could be chosen based on data prior to the conspiracy period. However, the model could then be estimated using the dummy variable approach and the complete data set

⁹A second advantage of the forecasting approach is more technical. If model selection is based purely on the pre-conspiracy period, and one believes that observations before the conspiracy are (approximately) independent of observations during the conspiracy, then the model selection process does not affect the standard errors for the damages estimate.

that includes the conspiracy period. Propositions 1 through 3 suggest that the dummy variable approach would then work at least as well as the forecasting approach.

VI. Saving Degrees of Freedom in the Dummy Variable Model

An important consideration in the dummy variable model is whether overcharges can be estimated with greater precision by imposing the restriction $\gamma = 0$. Imposing this restriction could in principle either increase or decrease the variability of the overcharges estimate. To see why, observe that the conditional variances of the overcharge estimates including and then excluding $D_t X_t$ are given by

$$V[\widehat{OC}_1] = \left(\sum_t D_t Q_t \right)^2 V[\widehat{\theta}] + \left(\sum_t D_t Q_t X_t \right)' V[\widehat{\gamma}] \left(\sum_t D_t Q_t X_t \right) \quad (10)$$

$$+ \left(\sum_t D_t Q_t X_t \right)' C[\widehat{\gamma}, \widehat{\theta}] \left(\sum_t D_t Q_t \right) + \left(\sum_t D_t Q_t \right) C[\widehat{\theta}, \widehat{\gamma}] \left(\sum_t D_t Q_t X_t \right)$$

$$V[\widehat{OC}_2] = \left(\sum_t D_t Q_t \right)^2 V[\widehat{\theta}_S] \quad (11)$$

where $V[\widehat{\theta}]$ and $V[\widehat{\gamma}]$ represent the variance of the estimator of the dummy variable coefficient and the variance of the estimator of the change in the slope coefficients, respectively. Furthermore, $C[\widehat{\gamma}, \widehat{\theta}]$ is the $k \times 1$ vector of covariances between the slope coefficient estimators and the dummy variable coefficient estimator.

These equations clarify that it is not possible to determine *a priori* whether imposing the restriction that $\gamma = 0$ will improve efficiency. To see why, consider the three terms in Equation (10). The first term, which relates to $V[\widehat{\theta}]$, can be either larger or smaller than $V[\widehat{\theta}_S]$.¹⁰ The second term in Equation (10), summarizing the variability in the estimate of the *change* in the effect of the covariates on price, is strictly positive and typically will be large. The reason is that a precise estimate of the change in the effect of the covariates on price requires sufficient variation in the covariates *both* before and during the alleged conspiracy. Often, covariates that are suspected to have a substantial effect on prices are notably different during the alleged conspiracy period, and

¹⁰Under some restrictions, such as the Higgings-Johnson restrictions, we have that $V[\widehat{\theta}_S] < V[\widehat{\theta}]$.

there is insufficient variation in the key covariates prior to the alleged conspiracy to obtain a good estimate. The third and final term in Equation (10), pertaining to the covariance between the change in the level of price and the change in the effect of covariates, $C[\widehat{\gamma}, \widehat{\theta}]$, can consist of terms which are all positive, all negative, or a mixture of signs.

With this background, we can now motivate the conclusion that the variance associated with the first approach to overcharges can either be larger or smaller than the variance associated with the second approach. Even if $V[\widehat{\theta}]$ is larger than $V[\widehat{\theta}_S]$, the third and final term in Equation (10), pertaining to covariances, can be negative and large enough in magnitude to offset both the second term in Equation (10) and the difference between $V[\widehat{\theta}]$ and $V[\widehat{\theta}_S]$.¹¹

While theory leaves us with an indeterminate solution, we have found in nearly all the simulations we have examined that the variances associated with the two approaches are quite similar. The most important difference between the two approaches is in their bias, not in their variance. The first approach to estimating overcharges, using the longer specification that allows for an estimate of γ , is more robust in terms of bias because it is consistent even when the effect of covariates on price is different between the pre- and post-conspiracy periods, i.e., when $\gamma \neq 0$. The second approach, using the shorter specification when $\gamma = 0$, is more fragile because it is not consistent with $\gamma \neq 0$. Consequently, our general recommendation is to use the first approach to estimating overcharges, unless one has substantial confidence that the effect of covariates on price is unchanged.

VII. An Example

In this section, we present the results of a simulation study intended to demonstrate the practical relevance of the issues discussed above. We set $T = 100$, with the alleged conspiracy period beginning in period 66. The data generating process is given by equation (1), where the outcome is price, P_t , with ε_t being a sequence of independent and identically distributed (*iid*) normal variables with mean zero and variance ten. We also (arbitrarily) choose the following model parameters:

¹¹To the best of our knowledge, there is no parametric restriction that guarantees an improvement in precision from imposing the restriction $\gamma = 0$. For example, even in data generating processes where $\gamma = 0$, it can still be more efficient to allow for a change in the effect of the covariates on price. Because of this, we are not aware of any statistical test that would clearly point to whether it was more appropriate to include or exclude the interaction term from the regression, from the point of view of minimizing the variability of the overcharge estimate.

$\alpha = 10$, $\beta = 2$ (there is one covariate), $\theta = 4$, and γ is equal to either 0 or 1. In summary, the model for price is given by:

$$P_t = 10 + 2X_t + 4D_t + \gamma D_t X_t + \varepsilon_t \quad (12)$$

The covariate X_t is generated according to

$$X_t = \phi_{1X} + \phi_{2X}t + \phi_{3X}X_{t-1} + e_{tX} \quad (13)$$

where $X_0 = 0$, $\phi_{1X} = 1$, $\phi_{2X} = -0.015$, $\phi_{3X} = 0.25$, e_{tX} is distributed *iid* normal with mean 0 and variance 10, and ε_t and e_{tX} are mutually independent. This specification allows for trend and persistence in the covariate.

The AR(1) specification in Equation (13) does not allow for the possibility that X_t is correlated with the dummy in the conspiracy period. However, because of the trend that is incorporated, in some samples there will be a notable (spurious) correlation. This mimics real world settings in which these methods are used. It will often appear that one or several covariates move differently before the alleged conspiracy period than during, but these apparent differences will potentially be consistent with a complex time series process underlying one or more covariates and with spurious correlation between the covariates and the outcome variable during the alleged conspiracy period.

We consider two basic specifications of the quantity of sales. The first specification simply holds quantity constant at 150. The second AR(1) specification arises from the model

$$Q_t = \phi_{1Q} + \phi_{2Q}t + \phi_{3Q}Q_{t-1} + \nu P_t + e_{tQ} \quad (14)$$

where ϕ_{1Q} is set to 150, ϕ_{2Q} is set to -0.05, ϕ_{3Q} is set to 0.25, e_{tQ} is distributed *iid* standard normal, and ε_t , e_{tX} , and e_{tQ} are mutually independent. The coefficient ν is set to either 0 or -1.

The results of this simulation are given in Table 1, which presents estimates of the bias and standard deviation of the overcharges methodologies discussed, using 240,000 replications of data sets of size $T = 100$. While the table contains the figures for the standard deviation of these estimators, we focus on the simulation estimates of bias. In all four models, and many others we

have examined, differences in standard deviation among the methods are generally quite minor, as compared with differences in bias.

Model 1 holds quantity constant and generates the covariate X_t according to Equation (13) and price P_t according to Equation (12), with $\gamma = 0$. This corresponds to a setting in which the effect of the covariate on price is the same before and during the alleged conspiracy. Model 2 is identical to the first, but sets $\gamma = 1$. This implies an increase in the partial correlation between the covariate and price during the conspiracy period, as compared to before. Model 3 allows quantity to vary according to Equation (13), but there is no partial correlation between quantity and price. Model 3 continues to use the parameterization $\gamma = 1$. Model 4 is identical to the third, but additionally allows quantity to vary with price by setting the coefficient on price, ν , equal to -1.

TABLE 1: BIAS AND STANDARD DEVIATION OF OVERCHARGE METHODOLOGIES

Description of Data Generating Process	OC ₁	OC ₁	FC
<u>Model 1</u> Q_t constant; $\gamma = 0$	5.4 (2,669.8)	6.8 (2,556.0)	5.4 (2,669.8)
<u>Model 2</u> Q_t constant; $\gamma = 1$	5.4 (2,669.8)	1,131.9 (2,588.2)	5.4 (2,669.8)
<u>Model 3</u> Q_t an AR(1) unrelated to prices; $\nu = 0, \gamma = 1$	7.0 (3,411.8)	1,442.0 (3,308.2)	7.0 (3,411.9)
<u>Model 4</u> Q_t an AR(1) related to prices; $\nu = -1, \gamma = 1$	-17.8 (2,972.5)	1,352.3 (2,863.1)	-344.0 (2,973.6)

Note: Table presents simulation estimates of bias and standard deviation (parentheses) three approaches to estimating overcharges. Estimates based on 240,000 simulated data sets, each with 100 observations. See text for details.

Model 1 corresponds to a setting of constant quantity and no change in the effect of covariates on price. Because quantity is constant in this model, Proposition 2 asserts that the first dummy variable approach (\widehat{OC}_1) and the forecasting approach (\widehat{FC}) will lead to identical estimates. This is borne out in the simulations. In each of the 240,000 replications, \widehat{OC}_1 and \widehat{FC} are identical. The sample estimates of bias, presented in the first row of the table, are thus also identical. The estimated bias of \$5.40 is statistically insignificant (results available on request), and economically insignificant compared to the true damages amount for this model of \$14,000.

Because the effect of covariates on price is constant in this model, the discussion in Section VI indicates that there should be no important difference between the two varieties of the dummy variable approach: the first approach (\widehat{OC}_1) allows the effect of covariates on price to change during the alleged conspiracy and the second approach (\widehat{OC}_2) correctly imposes that the assumption that the effect of covariates on price remain fixed (i.e., that $\gamma = 0$). Consistent with our expectation, the two dummy variable approaches lead to highly similar estimated biases (\$6.80 as compared to \$5.40).

Model 2 modifies the data generating process to allow the effect of covariates on price to change during the alleged conspiracy period. In this setting, since quantity is constant as in the first model, the first dummy variable approach and the forecasting approach are identical. However, because the effect of the covariates changes during the alleged conspiracy period, the second dummy variable approach (which imposes the constraint that $\gamma = 0$) is inferior to the first approach, which allows γ to be different from zero. Table 1 shows that the simulation estimate of the bias of the second dummy variable approach is quite large, estimated at \$1,131.90, or just over 8 percent of the true parameter.

Model 3 allows quantity to vary according to Equation (13), as discussed, but does not allow any predictable relationship between quantity and price. This model retains the assumption that the effect of the covariate on prices differs before and during the alleged conspiracy period. Because quantity and price have a zero partial correlation, both the first dummy variable approach and the forecasting approach generate consistent estimates (Proposition 3). Interestingly, while the forecasting approach is more variable than the first dummy variable method, the difference is trivial, at least for this example.

The simulation estimates of bias are very similar, but not identical. (The estimated sample bias differs in the second decimal place, but the differences are masked by rounding.) However, because the effect of the covariates changes over time, the second dummy variable approach is not consistent. The simulation estimate of bias is \$1,442.00, which is above 8 percent of the true parameter.

Finally, Model 4 allows for quantity and price to be related, with $\nu = -1$. In this data generating process, this implies a negative partial correlation between the reduced form residuals and quantity. From equation (9), we know that this will lead the forecasting method to exhibit negative bias, and the simulation results bear this out. While the first dummy variable approach

continues to perform well in this setting, the forecasting method has a bias nearly as large as the second dummy variable approach. The estimated bias of the forecasting method, $-\$344.00$, represents over 2 percent of the true parameter. In this setting, because the effect of the covariate changes over time, the second dummy variable model performs the least well of the three methods considered, with an estimated bias of $\$1,355.40$, or over 9 percent of the true parameter.

VIII. Conclusion

In this paper, we have reviewed two major approaches to the estimation of overcharges: the dummy variable approach and the forecasting approach. The dummy variable approach is based on a regression model that explains price before and during the alleged conspiracy period. There are two leading variants of the dummy variable approach, corresponding to whether the effects of covariates are allowed to differ before and during the alleged conspiracy period, or are instead imposed to be the same throughout. We consider both of these variants. The forecasting approach formulates a model for price before the alleged conspiracy period and then compares price forecasts with actual prices. For both the dummy variable approach and the forecasting approach, a quantity-weighted difference in prices is used to estimate overcharges.

We show that the first dummy variable approach, in which the effects of covariates are allowed to differ over time, is numerically equivalent to the forecasting approach when quantity is constant. When quantity varies over time, but not in a manner related to unobserved determinants of price, then both the forecasting and the first dummy variable approaches generate consistent estimates of overcharges, but the forecasting approach is less efficient (has more variability) than the first dummy variable approach. This raises a question as to the merits of the forecasting approach.

We also show that there is some justification for the second dummy variable approach. When the effects of covariates on price are indeed constant over time, then the second dummy variable approach can have less variability than the first dummy variable approach. However, this is not guaranteed. Moreover, in simulation results, we do not find important differences in the variability of the two dummy variable approaches. On the other hand, if the restriction that the effects of covariates are constant is, in fact, false, then the second dummy variable approach can be biased.

Overall, our discussion points to an important role for the first dummy variable approach. The primary drawback of the first dummy variable approach is the possibility that analysts will “overfit” the regression model, including a great number of covariates that do not belong in the regression model. This can lead to imprecise overcharges estimates, and perhaps even spurious overcharges estimates if inappropriate covariates are included. To ameliorate these problems, we suggest that further consideration be given to the use of a model selection procedure (such as that currently used in the forecasting approach) in conjunction with the first dummy variable approach.

Appendix

Proof of Proposition 1: We begin by noting that $\widehat{Y}_t^f = \widehat{\alpha} + \widehat{\beta}X_t$, where $\widehat{\alpha}$ and $\widehat{\beta}$ are identical to the dummy variable regression estimates. Then, using Equation (8), we have

$$\widehat{FC} = \sum_t D_t Q_t \left(\widehat{\theta} D_t + \widehat{\gamma}' D_t X_t + \widehat{\varepsilon}_t \right) = \widehat{\theta} \sum_t D_t Q_t + \widehat{\gamma}' \sum_t D_t X_t Q_t + \sum_t D_t Q_t \widehat{\varepsilon}_t \quad (\text{A.1})$$

$$= \widehat{OC}_1 + \sum_t D_t Q_t \widehat{\varepsilon}_t \quad (\text{A.2})$$

□

Proof of Proposition 2: Applying Proposition 1, note that when quantity is constant, say $Q_t = \overline{Q}$, we have $\sum_t D_t Q_t \widehat{\varepsilon}_t = \overline{Q} \sum_t D_t \widehat{\varepsilon}_t = 0$ by the orthogonality of fitted residuals and covariates. □

Proof of Proposition 3: Applying Proposition 1, the dummy variable approach and the forecasting approach share a probability limit whenever

$$\sum_t D_t Q_t \widehat{\varepsilon}_t = \sum_t D_t Q_t \varepsilon_t - (\widehat{\alpha} - \alpha) - \left(\widehat{\beta} - \beta \right)' X_t - \left(\widehat{\theta} - \theta \right) - (\widehat{\gamma} - \gamma)' X_t \quad (\text{A.3})$$

converges in probability to zero. Since each of $\widehat{\alpha}$, $\widehat{\beta}$, $\widehat{\theta}$, and $\widehat{\gamma}$ is consistent, this condition holds whenever $\sum_t D_t Q_t \varepsilon_t$ converges in probability to zero. Note then that a sufficient condition is $0 = E[D_t Q_t \varepsilon_t] = E[Q_t \varepsilon_t | D_t = 1] / P(D_t = 1)$, which holds whenever Q_t and ε_t are uncorrelated in the population during the alleged conspiracy (since ε_t is mean zero). Finally, note that the variance of $\sum_t D_t Q_t \widehat{\varepsilon}_t$ will be strictly positive unless Q_t fails to predict prices in each sample. □

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