

**Confronting Theory with Experimental Data and vice versa**

**European University Institute**

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**Lectures 7-8: Social networks**

## Introduction

- Stock exchanges are assumed to be good approximations to the theoretical ideal of *frictionless* markets.
- Apart from centralized exchanges (NYSE), most financial transactions take place in networks.
- Networks are incomplete - one or more intermediaries link the initial seller and final buyer.

- Intermediation is costly, so incompleteness constitutes a potentially important market imperfection.
- The impact of network architecture on the efficiency and dynamics of financial networks.
- The results are applicable to any model of exchange, whether the commodities are real or financial.

## **A prototype model**

- A connected graph in which the nodes represent traders and the edges represent the possibility of trade.
- Agents bargain over terms of trade and exchange an indivisible asset according to a standard protocol.
- Costly intermediation (frictions), network architecture (incompleteness) and market breakdown (market freezes).

- A finite set of agents  $N$  and  $k < N$  identical and indivisible units of an asset.
- Each agent  $i \in N$  can hold at most one unit at any date  $t$  (capacity constraint).
- The initial distribution of assets is denoted by  $e = (e_1, \dots, e_N)$ , where  $e_i \in \{0, 1\}$ .

## The network

- A *directed* graph  $(N, E)$ , where  $N$  is the set of nodes and the set of edges is

$$E = \cup_{i=1}^N \{(i, j) : j \in N_i\} \subset N \times N.$$

- The set of agents with whom agent  $i$  can trade (neighbors) is denoted by

$$N_i = \{j : (i, j) \in E\}.$$

## Payoffs

- Agent  $i$  receives a flow utility of

$$u_i = (1 - \delta)\bar{u}_i \geq 0$$

from holding an asset, where  $\bar{u}_i \geq 0$  is the present value of holding the asset forever.

- The non-zero flow utilities are generic, so WLOG we can order the agents such that

$$u_1 > u_2 > \dots > u_{N'} > 0 \text{ and } u_i = 0 \text{ for } i = N' + 1, \dots, N.$$

- The first  $N \geq N' > k$  agents are called *investors* and the last  $N - N'$  agents are called *intermediaries*.
- The unique PE allocation is  $x^* = \arg \max \{u \cdot x : x \in X\}$  where  $X$  is the set of all attainable allocations.
- At  $x^*$  the sum of flow utilities - and hence the present value of the stream of flow utilities - is maximized in each period.

## Trading protocol

- Let  $B(x)$  and  $S(x)$  denote the set of buyers and sellers when the initial allocation is  $x$ .
- At each date  $t$ , with initial allocation  $x_{t-1}$ , an agent  $i \in S(x_{t-1})$  is chosen at random.
- Seller  $i$  makes a take-it-or-leave-it offer  $(p)$  to one of the buyers  $j \in B(x_{t-1}) \cap N_i$ .

- An extensive-form game of perfect information. *Markov perfect equilibrium* (MPE) as the solution concept.

- The Markov equilibrium strategy of agent  $i$  is denoted by  $f_i$ , where

$$f_i : X \rightarrow B(x) \times \mathbb{R}_+ \text{ if } i \in S(x)$$

and

$$f_i : X \times S(x) \times \mathbb{R}_+ \rightarrow \{A, R\} \text{ if } i \in B(x).$$

## Asymptotic analysis of equilibrium

**Result 1** *The Pareto-efficient allocation  $x^*$  is an absorbing state in any equilibrium.*

- Let  $\{\Phi_t\}$  denote the random path of equilibrium allocations and  $v(x)$  be the equilibrium payoffs.
- It follows from feasibility that, for any attainable allocation  $x$ ,

$$v(x) \cdot x \leq \mathbb{E} \left[ \sum_{t=0}^{\infty} \delta^t u \cdot \Phi_t \mid \Phi_0 = x \right] \leq \sum_{t=0}^{\infty} \delta^t u \cdot x^* = \bar{u} \cdot x^*$$

and the inequality is strict if  $\Phi_t \neq x^*$  for some  $t$  with positive probability.

- Now, starting at the allocation  $x^*$ , any  $i \in S(x^*)$  can achieve an equilibrium payoff of  $v_i(x^*) \geq \bar{u}_i$  (hold the asset forever).
- This implies that  $v_i(x^*) = \bar{u}_i$  for all  $i \in S(x^*)$ . It then follows that

$$\mathbf{P} [\Phi_t = x^*, \forall t | \Phi_0 = x^*] = 1$$

as required.

**Result 2** *In any equilibrium, any absorbing set is a singleton (there are no limit cycles).*

- Call  $A \subset X$  an *absorbing set* if it is a minimal set with the property that

$$\mathbf{P}[\Phi_{t+1} \in A | \Phi_t \in A] = 1$$

(once  $A$  is entered, each element of  $A$  is reached infinitely often with probability one).

- If  $A$  is not a singleton then more than  $k$  agents hold units of the asset as  $\Phi$  cycles through the elements of  $A$ .

- Let  $S(A) = \cup_{x \in A} S(x)$  and index the elements of  $S(A)$  by  $i_1, i_2, \dots, i_m$  so that  $u_{i_r} > u_{i_{r+1}}$  for  $r = 1, \dots, m - 1$  and  $m > k$ .
- Eventually, we must reach an allocation  $x^k$  where the agents  $i_1, \dots, i_k$  all hold the asset and will never give it up.
- Then  $x^k$  is an absorbing state, contradicting the definition that  $A$  is not a singleton.

**Result 3** *In any equilibrium, the process  $\Phi = \{\Phi_t\}$  must reach an absorbing state with probability one.*

- By contradiction. Let  $x^1, \dots, x^m$  denote the absorbing states and suppose that  $\Phi$  does not reach an absorbing state with probability one.
- If  $\Phi$  reaches one of them with positive probability then it reaches one with probability one (Markov assumption).
- This implies that  $\Phi$  reaches  $\{x^1, \dots, x^m\}$  with probability zero. Thus,  $X \setminus \{x^1, \dots, x^m\}$  contains an absorbing state, contradicting the definition of  $x^1, \dots, x^m$ .

## Accessibility

The efficient asset holders are said to be *accessible* if

$$\forall i = 1, \dots, k, \exists j \in N' \text{ such that } i \in N_j$$

(the intermediaries form a non-empty, connected network and every investor is directly connected to at least one intermediary).

The assumption is very strong but it provides some understanding of the role for intermediaries in helping markets to achieve efficiency.

**Result 4** *When the investors are accessible, there exists  $0 < \delta_0 < 1$  such that the PE allocation  $x^*$  is the only absorbing state of any equilibrium with  $\delta_0 < \delta < 1$ .*

- The existence of the intermediaries ensures that there is a path from any investor to any other investor.
- A benchmark by which we can judge markets that do not satisfy the (restrictive) assumptions needed for the result.

## Future research

- Characterization of the frictionless case and an analysis of the critical parameter values at which market freezes occur.
- At what level do market frictions cause a market freeze? And how does the network architecture affect that level?
- Laboratory implementation of the theoretical model to test the usefulness of the theory for interpreting empirical data.

## Putting the network under the microscope

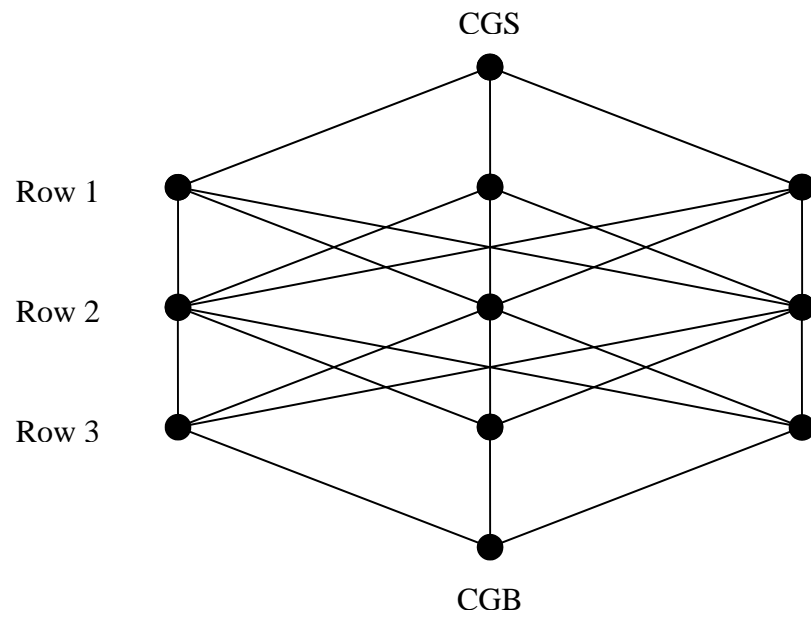
- The strengths of data from the real world are its relevance and availability.
- In real-world settings we observe behavior but not preferences, technologies, or private information.
- Laboratory data are especially useful for comparing network structures and market institutions.

## Experimental design

- The trading mechanism is closer to the well known auction paradigm rather than to the bargaining paradigm.
- The game is played in normal form – it can be played repeatedly in a short amount of time, generating a large data set.
- The platform is sufficiently flexible to allow a variety of network architectures, pricing rules, and payoff functions.

- Subjects are arranged in the rectangular array with  $m$  rows and  $n$  columns.
- In addition to the human traders, there are two computer-generated traders, called CGS and CGB.
- Trades are restricted to adjacent rows but, subject to these constraints, any pattern of trading links is allowed.

# The 3x3 network



- The CGS has one unit of an asset which he is willing to sell for zero and the CGS is willing to buy the asset for 100.
- Each trader simultaneously chooses a *bid* and an *ask* (the asset has no value to the traders).
- After all subjects entered a bid and ask price, the feasible trades are executed according to the trading protocol.

ExpClient

Server Host:  Server Port:

Client ID:

Round:  View Results...

Period in round:

You are a type-  participant

Your Tokens Endowment:

Bid:

Ask:

```
graph TD; A((A)) --> C((C)); A((A)) --> D((D)); B((B)) --> C((C)); B((B)) --> D((D)); C((C)) --> E((E)); C((C)) --> F((F)); D((D)) --> E((E)); D((D)) --> F((F));
```

CA: 0 CA: 0

CB: 100 CB: 100

Please make your decisions

ExpClient

Server Host:  Server Port:

Client ID:

Round:

Period in round:

You are a type-  participant

Your Tokens Endowment:

Bid:   
Ask:

Node	CA	B	P	A	B	CB	A
A	0	10		20			
B	0	20	10	30			
C		30		40			
D		40		50			
E		50				100	60
F		60				100	70

End of period  
Your total earnings in this period are 100 tokens  
Please press OK and wait for the next period

## Payoffs

- The traders were endowed with 100 tokens each. The payoff is defined by the formula

$$\text{payoff} = 10 + \max \{0, \text{trading profit}\}$$

- A compromise (which leaves us dissatisfied) between two conflicting objectives:
  - Rewarding on the basis of (positive or negative) trading profits, just like market traders.
  - Removing the incentive for passivity by subtracting most of the endowment from earnings.

- A payoff function that deducted more trading losses from subjects' earnings:

$$\text{payoff} = 50 + \max \{-40, \text{trading profit}\} .$$

- Other things being equal, under this payoff function, payoffs are higher, but the incentive to trade may be smaller (a significant risk of loss from trading).

## Treatments

Exp.	Networks	# of obs.
<b>Baseline</b>	$2 \times 3$	6/36/270
	$3 \times 3$	6/54/180
<b>Auction</b>	$1 \times 3$	5/15/150
<b>competition</b>	$2 \times 2$	9/36/270
	$2 \times 3$	6/36/180
<b>Bid-price</b>	$2 \times 3$	6/36/180
	$3 \times 2$	6/36/180
<b>Asymmetric</b>	$2 \times 2$	8/32/240
<b>Loss</b>	$3 \times 2$	6/36/180

$a/b/c$  – # of networks, # of subjects, and # of obs. per row.

## The trading game

- A Bertrand pricing model – a unique NE in which all bid and ask prices are equal to 100 and trading profits are zero (except in the top row).
- The incompleteness of the networks requires intermediate trades, and the strategic form of the game does not allow for recontracting,
- Trades may not be completed given the strategic uncertainty about the possibility of reselling the asset.

## Prices

**Convergence** *In the auction treatment, convergence to the equilibrium price occurs in the first few periods and the bids remain at that level throughout the game. In the baseline treatment, convergence is slower, but prices very rapidly reach the neighborhood of the competitive price.*

**Intermediation** *The rates of convergence of the average winning bids and asks to the equilibrium value are not significantly different in the corresponding rows of the three networks in the baseline and auction treatments despite the greater amount of intermediation.*

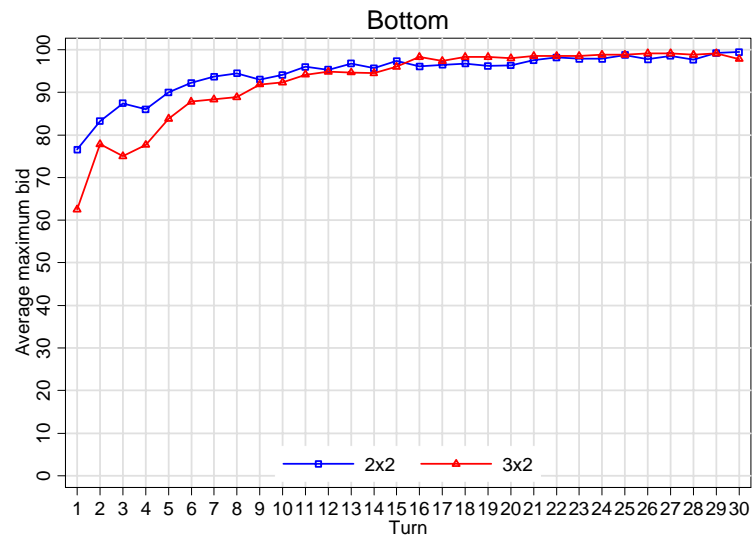
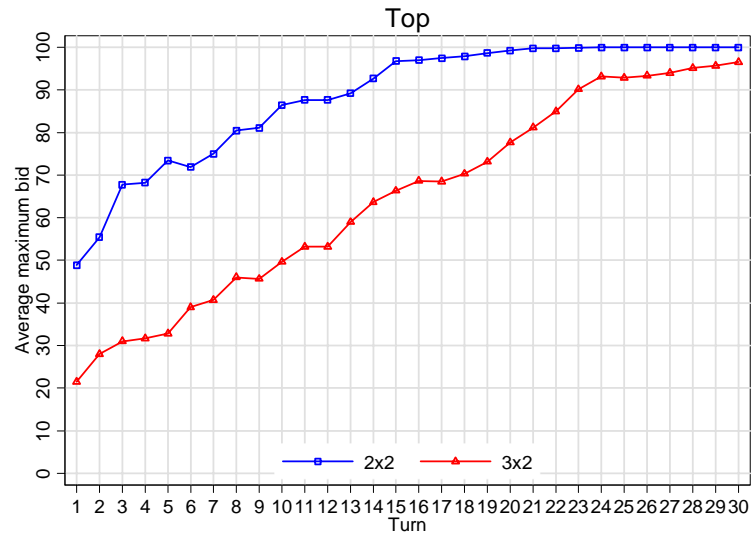
Average wining bids

Network	Row	1	2	3
1 × 3	1	95.9	100	100
2 × 3	2	90.0	99.9	100
3 × 3	3	97.2	100	100
2 × 3	1	77.4	99.9	100
3 × 3	2	88.2	99.9	100

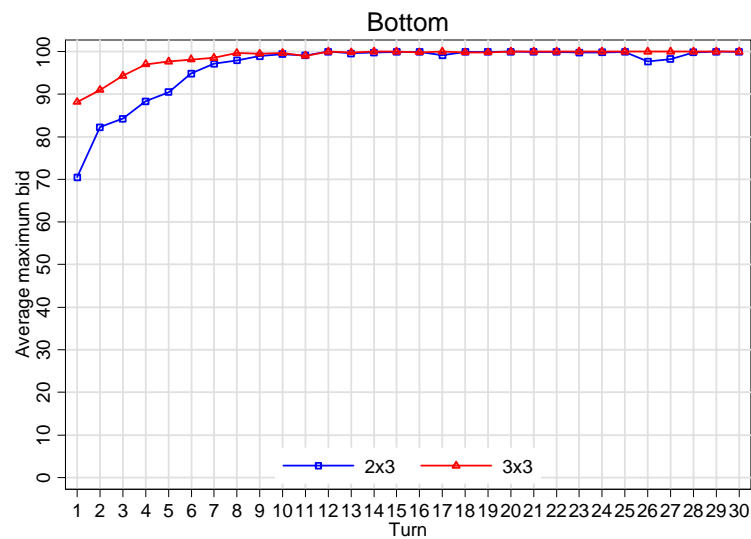
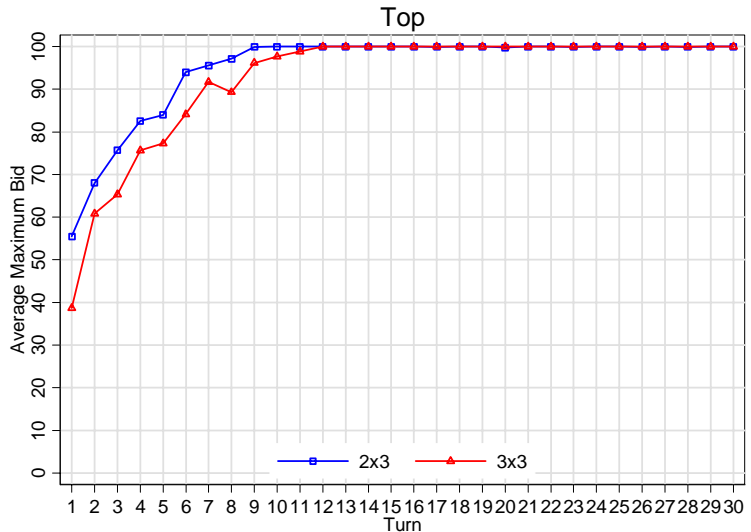
Average wining asks

Network	Row	1	2	3
2 × 3	1	70.7	87.8	94.3
3 × 3	2	80.1	90.8	80.3

# The effect of intermediation 2x2 (blue) vs. 3x2 (red)



# 2x3 (blue) vs. 3x3 (red)

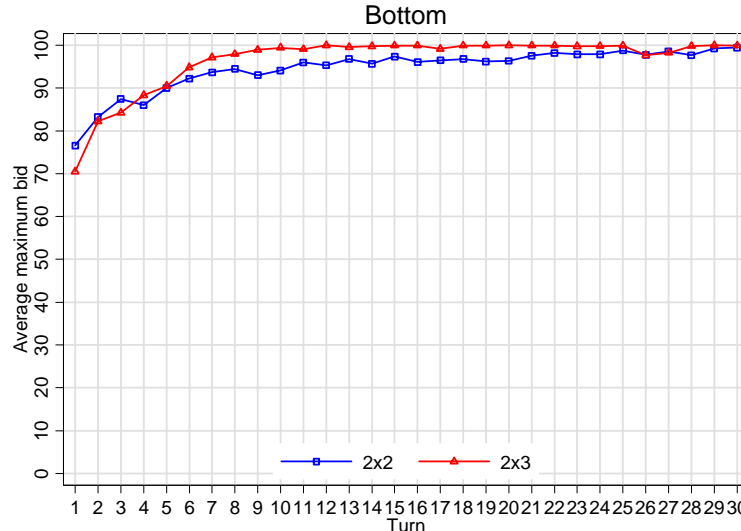
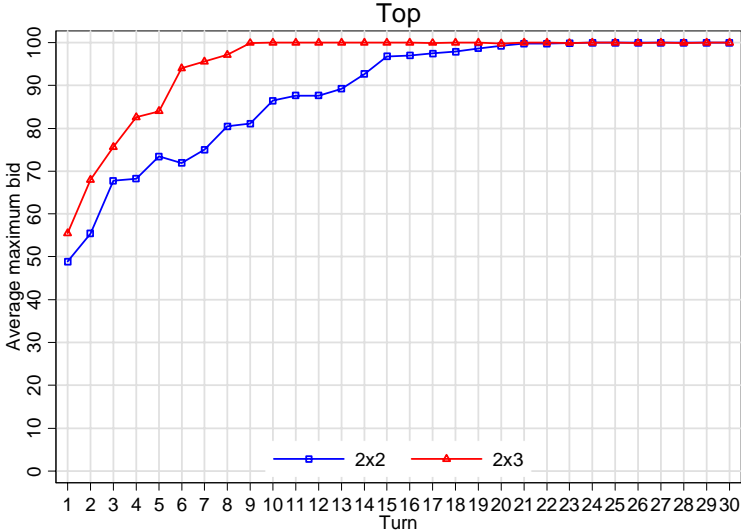


## Sensitivity

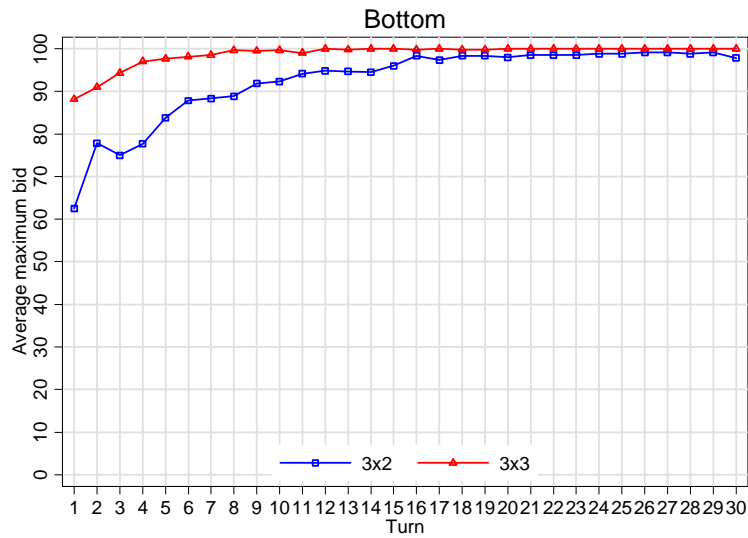
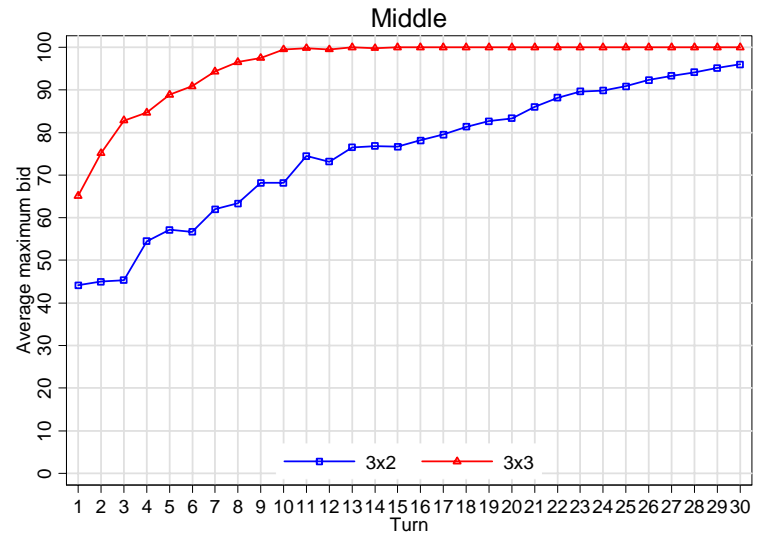
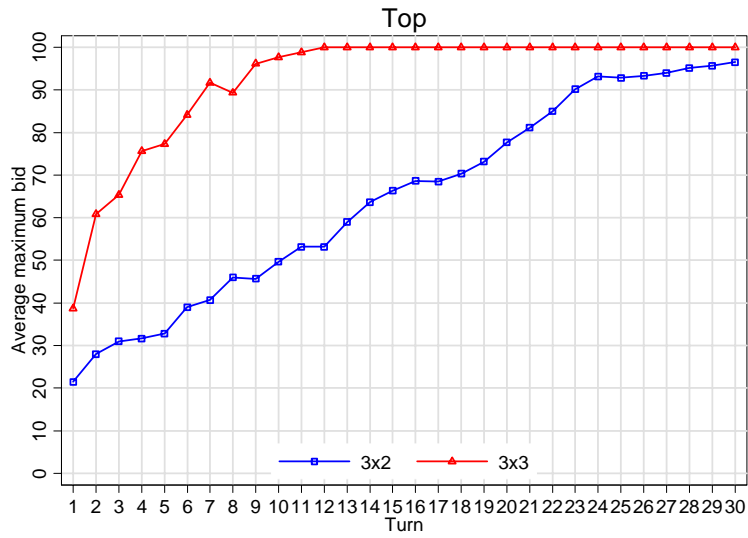
**Competition** *Comparing networks with low ( $n = 2$ ) and high ( $n = 3$ ) competition, the rates of convergence of the average winning bids and asks are not significantly different in the  $2 \times 3$  and  $2 \times 2$  networks. In the  $3 \times 3$  and  $3 \times 2$  networks, the differences are larger.*

**Pricing rule** *In the  $2 \times 3$  network, there are no significant differences between the rates of convergence to equilibrium bid prices in the bid-price and baseline treatments. In the  $3 \times 2$  network, the rates of convergence in the competition treatment are slightly higher than in the bid-price treatment.*

# The effect of competition 2x2 (blue) vs. 2x3 (red)

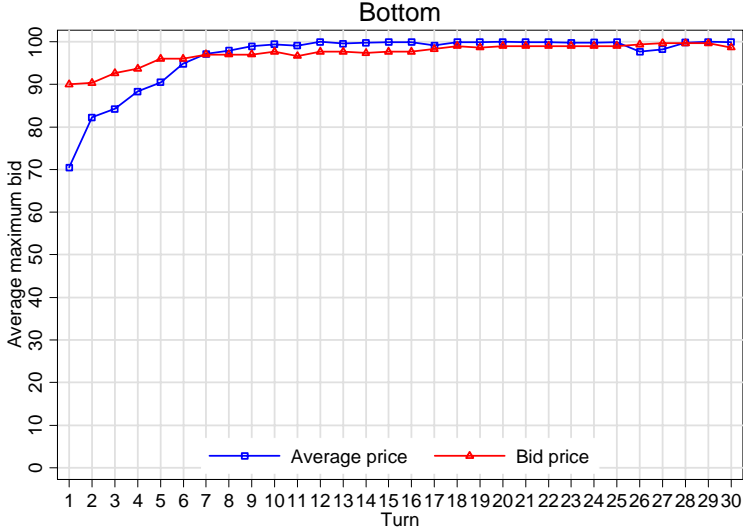
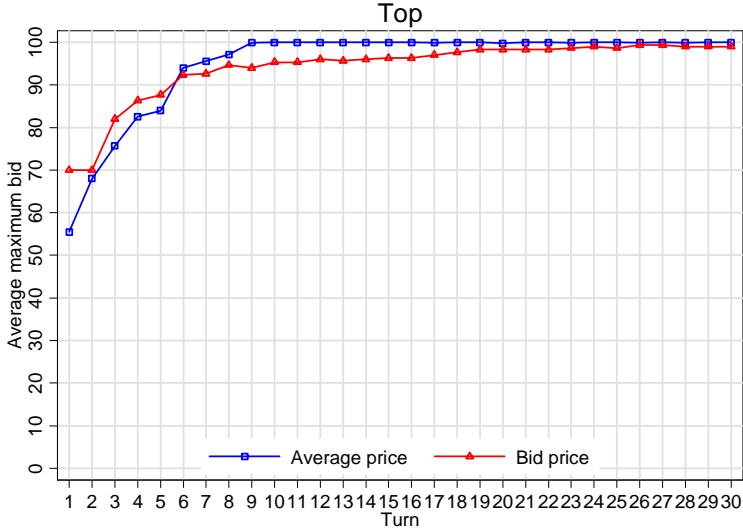


# 3x2 (blue) vs. 3x3 (red)



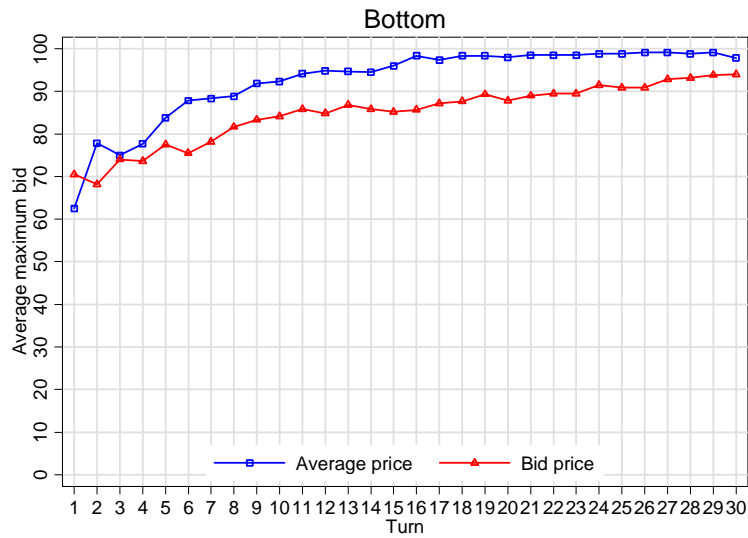
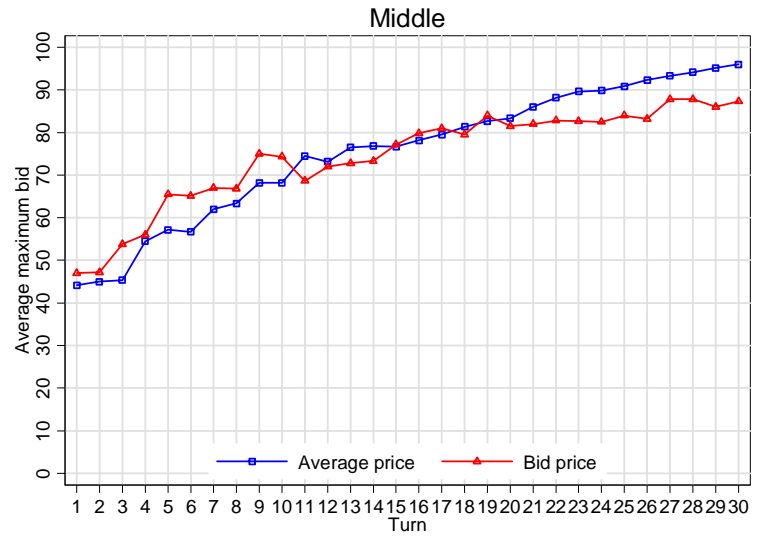
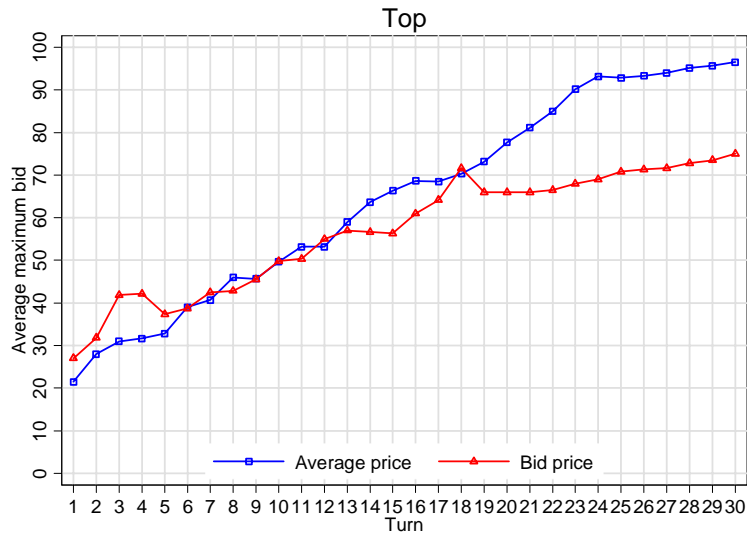
# The effect of price setting

## 2x3 network average price (blue) vs. bid price (red)



# The effect of price setting

## 3x2 network average price (blue) vs. bid price (red)

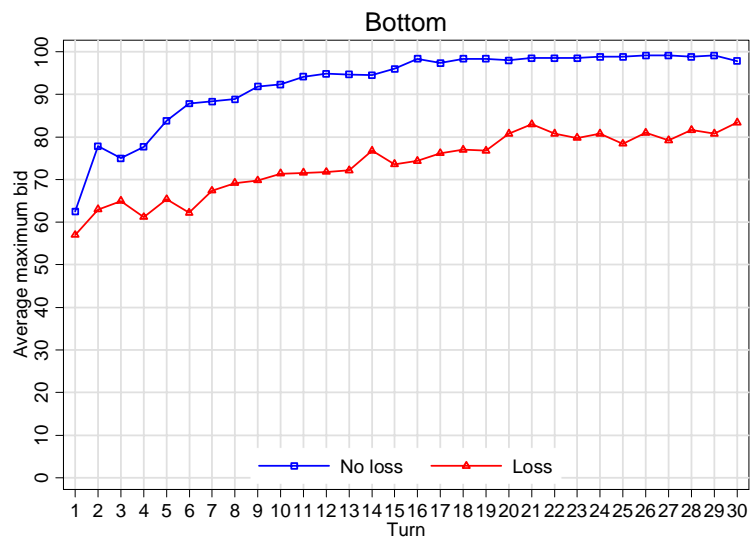
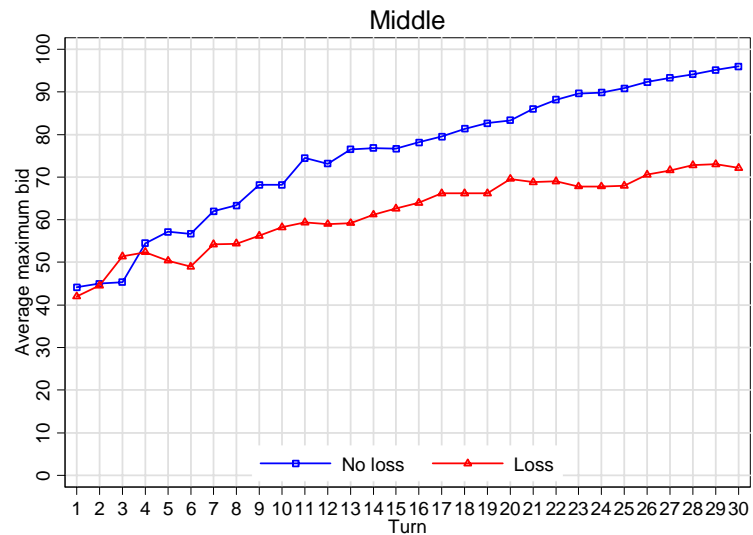
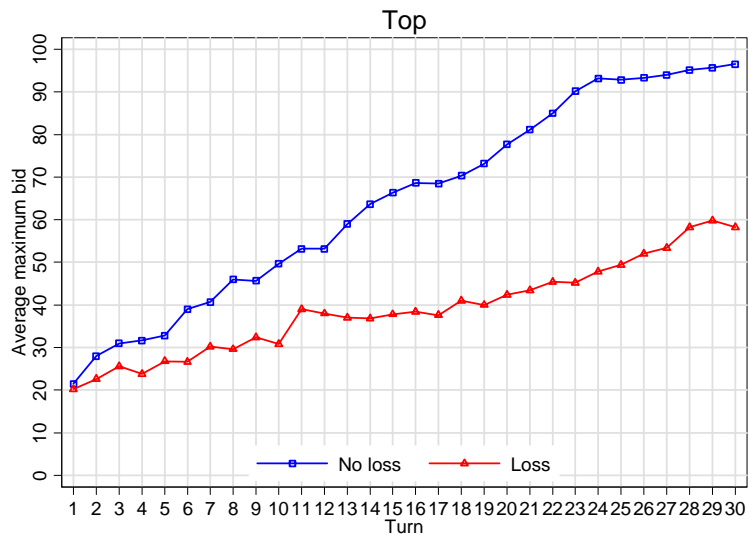


**Payoffs** *In all three rows of the  $3 \times 2$  network, the rates of convergence in the loss treatment are much slower than in the competition treatment and in some cases fails to converge.*

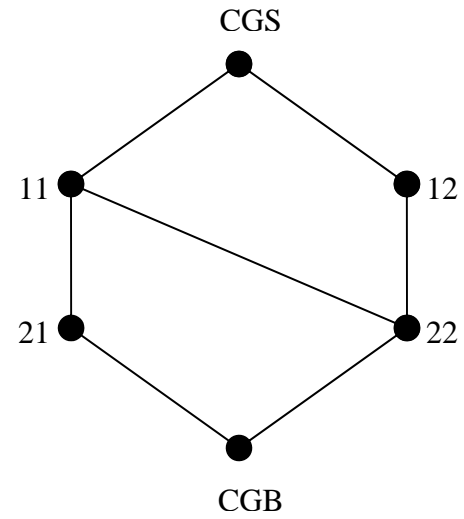
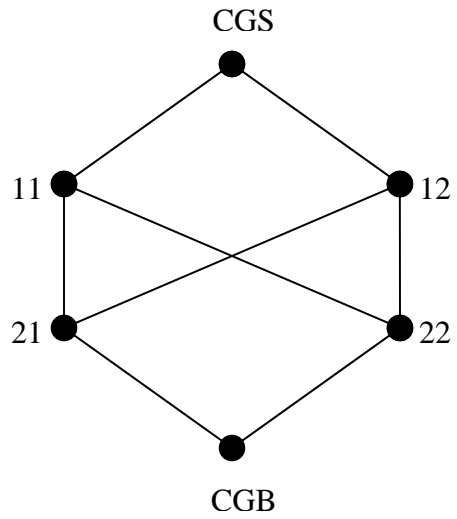
**Asymmetry** *The pricing behavior in a given row in the asymmetric  $2 \times 2$  network is very symmetric. The effect of the network asymmetry is revealed by generally lower prices and slower convergence compare to the symmetric  $2 \times 2$  network.*

# The effect of losses

## 3x2 network no loss (blue) vs. loss (red)

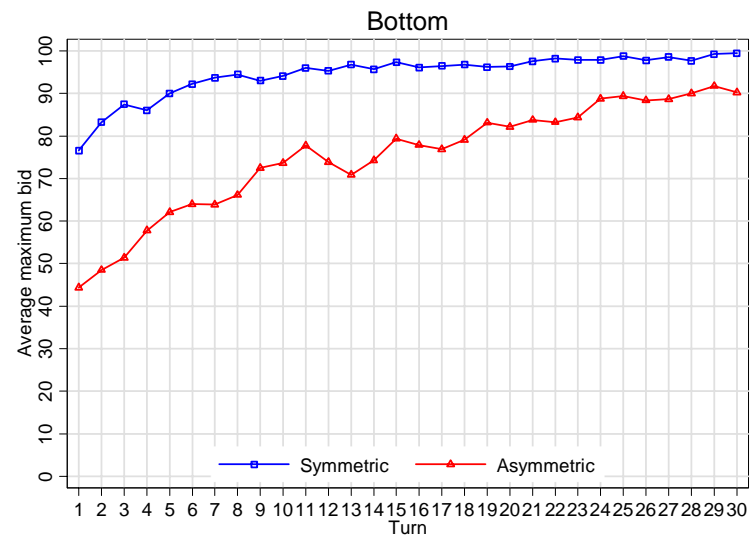
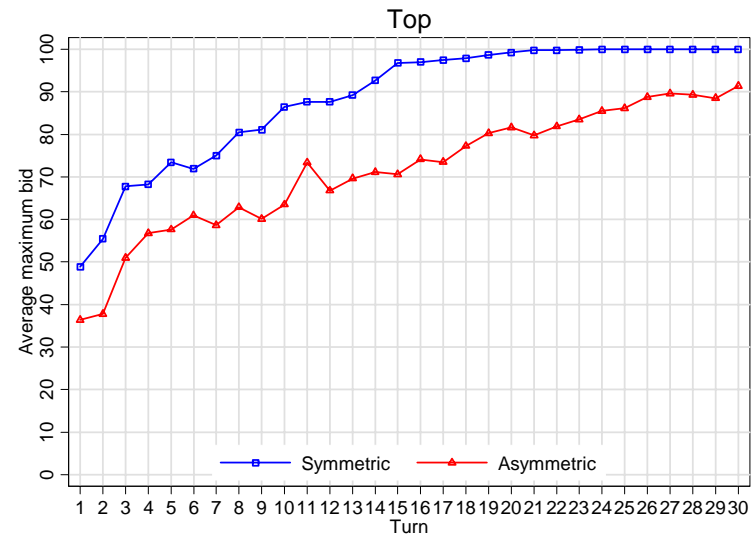


## The symmetric (right) and asymmetric (left) 2x2 networks



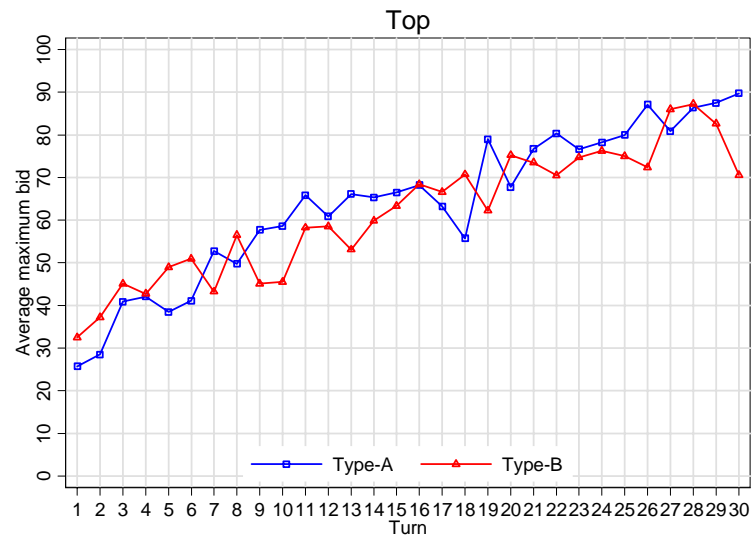
# The effect of asymmetry

## 2x2 network symmetric (blue) vs. asymmetric (red)



# The effect of asymmetry

## 2x2 network two links (blue) vs. one link (red)



## Efficiency

**Efficiency** *On the whole, trade is approximately efficient in the sense that, it tends to be lower in the early trading periods and rises as subjects become more confident about the behavior of other agents and as the prices bid and asked convergence to competitive equilibrium prices.*

## Conclusion

- Competitive prices can account for the pricing behavior observed in the laboratory in variety of networks and trading protocols.
- Significant differences can be identified in the pricing behavior different networks, and different trading protocols lead to different dynamics.
- There are many other questions that can be addressed using this design or variations thereof.