Confronting Theory with Experimental Data and vice versa

European University Institute

May-Jun 2008

Lectures 1-2: Choice under uncertainty

Background

- Decisions under uncertainty enter every realm of economic decision-making.

- Models of choice under uncertainty play a key role in every field of economics.

- Test the empirical validity of particular axioms or to compare the predictive abilities of competing theories.

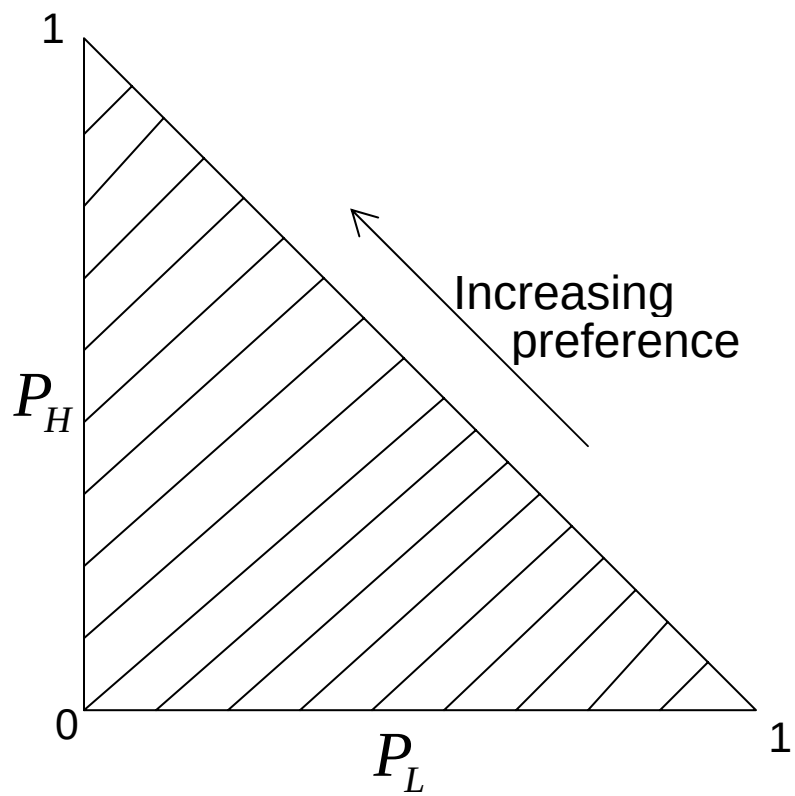
Experiments à la Allais

- Each theory predicts indifference curves with distinctive shapes in the probability triangle.

- By choosing alternatives that theories rank differently, each theory can be tested against the others.

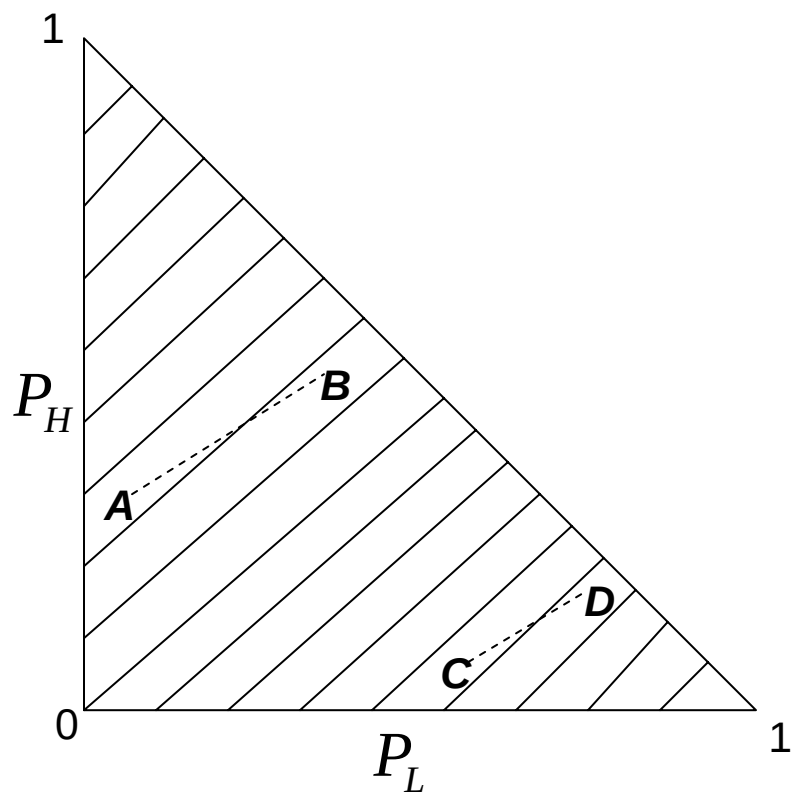
- The criterion typically used to evaluate a theory is the fraction of choices it predicts correctly.

The Marschak-Machina probability triangle



H, *M*, and *L* are three degenerate gambles with certain outcomes *H*>*M*>*L*

A violation of Expected Utility Theory (EUT)



EUT requires that indifference lines are parallel so one must choose either ***A*** and ***C***, or ***B*** and ***D***.

Contributions

Results have generated the most impressive dialogue between observation and theorizing:

- Violations of EUT raise criticisms about the status of the Savage axioms as the touchstone of rationality.

- These criticisms have generated the development of various alternatives to EUT, such as Prospect Theory.

Limitations

Choice scenarios narrowly tailored to reveal *anomalies* limits the usefulness of data for other purposes:

- Subjects face *extreme* rather than *typical* decision problems designed to encourage violations of specific axioms.

- Small data sets force experimenters to pool data and to ignore individual heterogeneity.

Research questions

Consistency

- Is behavior under uncertainty consistent with the utility maximization model?

Structure

- Is behavior consistent with a utility function with some special structural properties?

Recoverability

- Can the underlying utility function be recovered from observed choices?

Heterogeneity

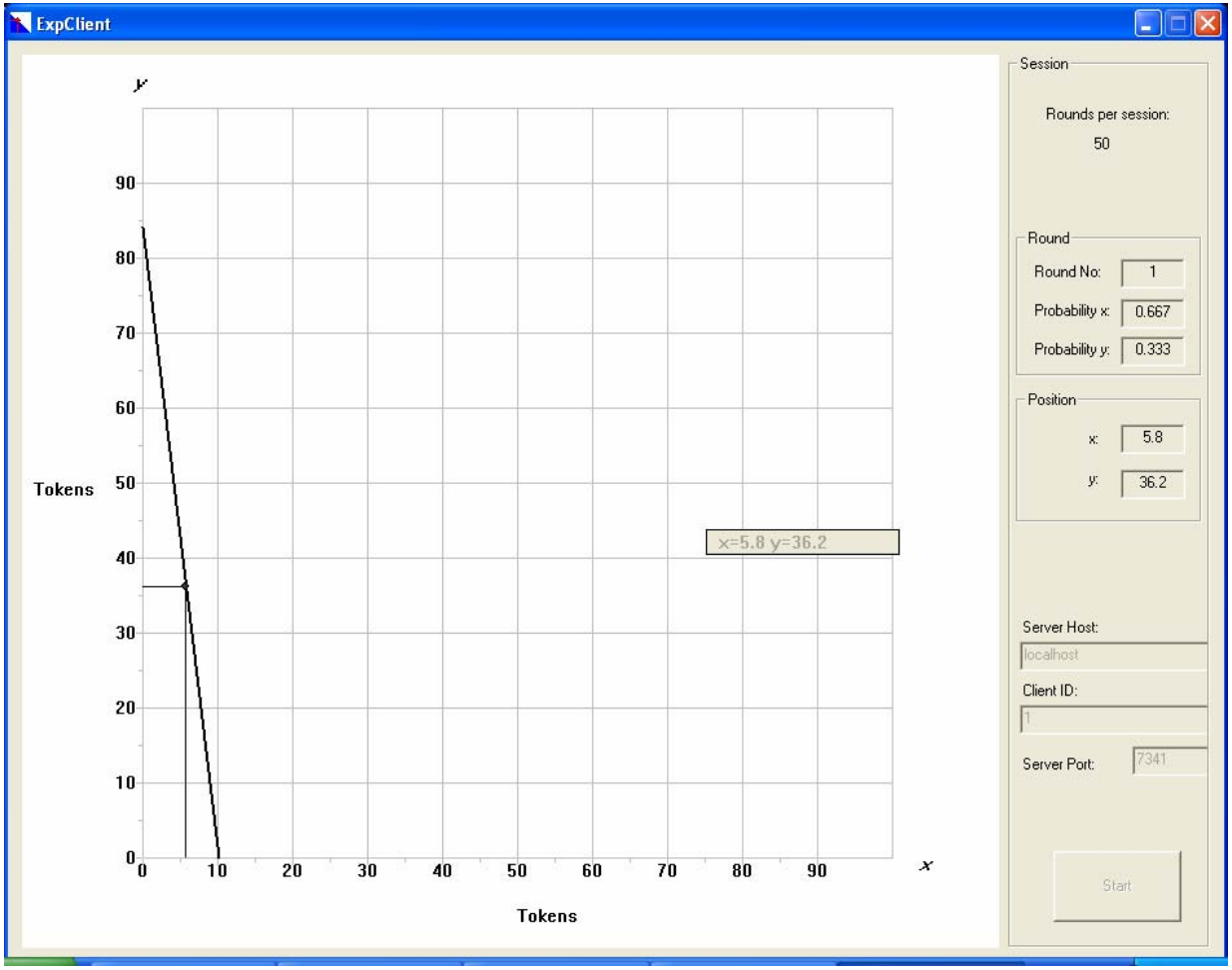
- To what degree do preferences differ across individuals?

A new experimental design

An experimental design that has a couple of fundamental innovations over previous work:

- A selection of a bundle of contingent commodities from a budget set (a portfolio choice problem).
- A graphical experimental interface that allows for the collection of a rich individual-level data set.

The computer program dialog window



- The choice of a portfolio subject to a budget constraint provides more information about preferences than a binary choice.

- A large menu of decision problems that are representative, in the statistical sense and in the economic sense.

- A rich dataset that provides the opportunity to interpret the behavior at the level of the individual subject.

Rationality

Let $\{(p^i, x^i)\}_{i=1}^{50}$ be some observed individual data (p^i denotes the i-th observation of the price vector and x^i denotes the associated portfolio).

A utility function $u(x)$ *rationalizes* the observed behavior if it achieves the maximum on the budget set at the chosen portfolio

$$u(x^i) \geq u(x) \text{ for all } x \text{ s.t. } p^i \cdot x^i \geq p^i \cdot x.$$

Revealed preference

A portfolio x^i is *directly revealed preferred* to a portfolio x^j if $p^i \cdot x^i \geq p^i \cdot x^j$, and x^i is *strictly directly revealed preferred* to x^j if the inequality is strict.

The relation *indirectly revealed preferred* is the transitive closure of the directly revealed preferred relation.

Generalized Axiom of Revealed Preference (GARP) *If x^i is indirectly revealed preferred to x^j, then x^j is not strictly directly revealed preferred (i.e. $p^j \cdot x^j \leq p^j \cdot x^i$) to x^i.*

GARP is tied to utility representation through a theorem, which was first proved by Afriat (1967).

Afriat's Theorem *The following conditions are equivalent:*

- *The data satisfy GARP.*
- *There exists a non-satiated utility function that rationalizes the data.*
- *There exists a concave, monotonic, continuous, non-satiated utility function that rationalizes the data.*

Goodness-of-fit

- Verifying GARP is conceptually straightforward but it can be difficult in practice.

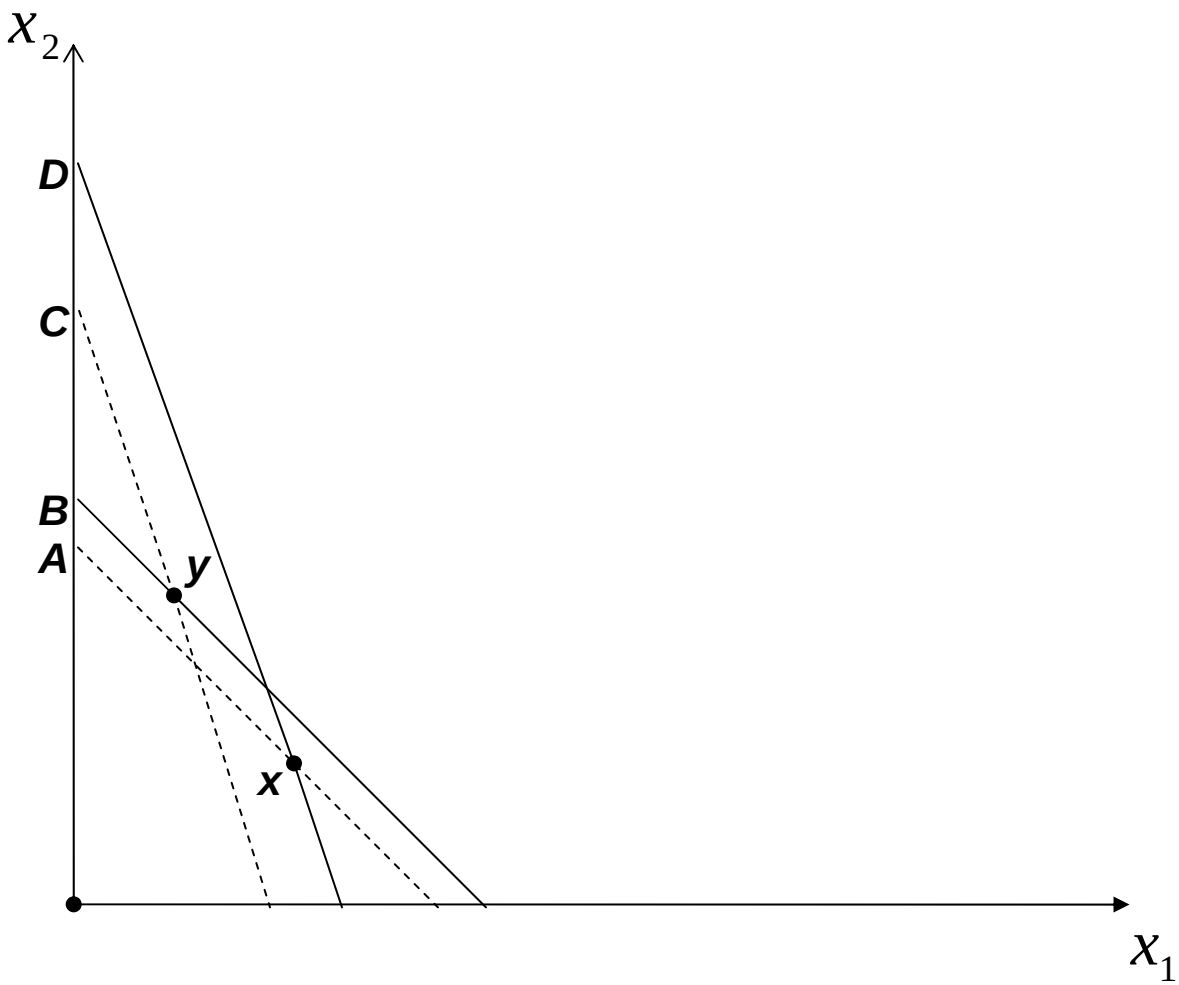
- Since GARP offers an exact test, it is necessary to measure the extent of GARP violations.

- Measures of GARP violations based on three indices: Afriat (1972), Varian (1991), and Houtman and Maks (1985).

Afriat's critical cost efficiency index (CCEI) *The amount by which each budget constraint must be relaxed in order to remove all violations of GARP.*

The CCEI is bounded between zero and one. The closer it is to one, the smaller the perturbation required to remove all violations and thus the closer the data are to satisfying GARP.

The construction of the CCEI for a simple violation of GARP



The agent is ‘wasting' as much as *A/B<C/D* of his income by making inefficient choices.

A benchmark level of consistency

A random sample of hypothetical subjects who implement the power utility function

$$u(x) = \frac{x^{1-\rho}}{1-\rho},$$

commonly employed in the empirical analysis of choice under uncertainty, with error.

The likelihood of error is assumed to be a decreasing function of the utility cost of an error.

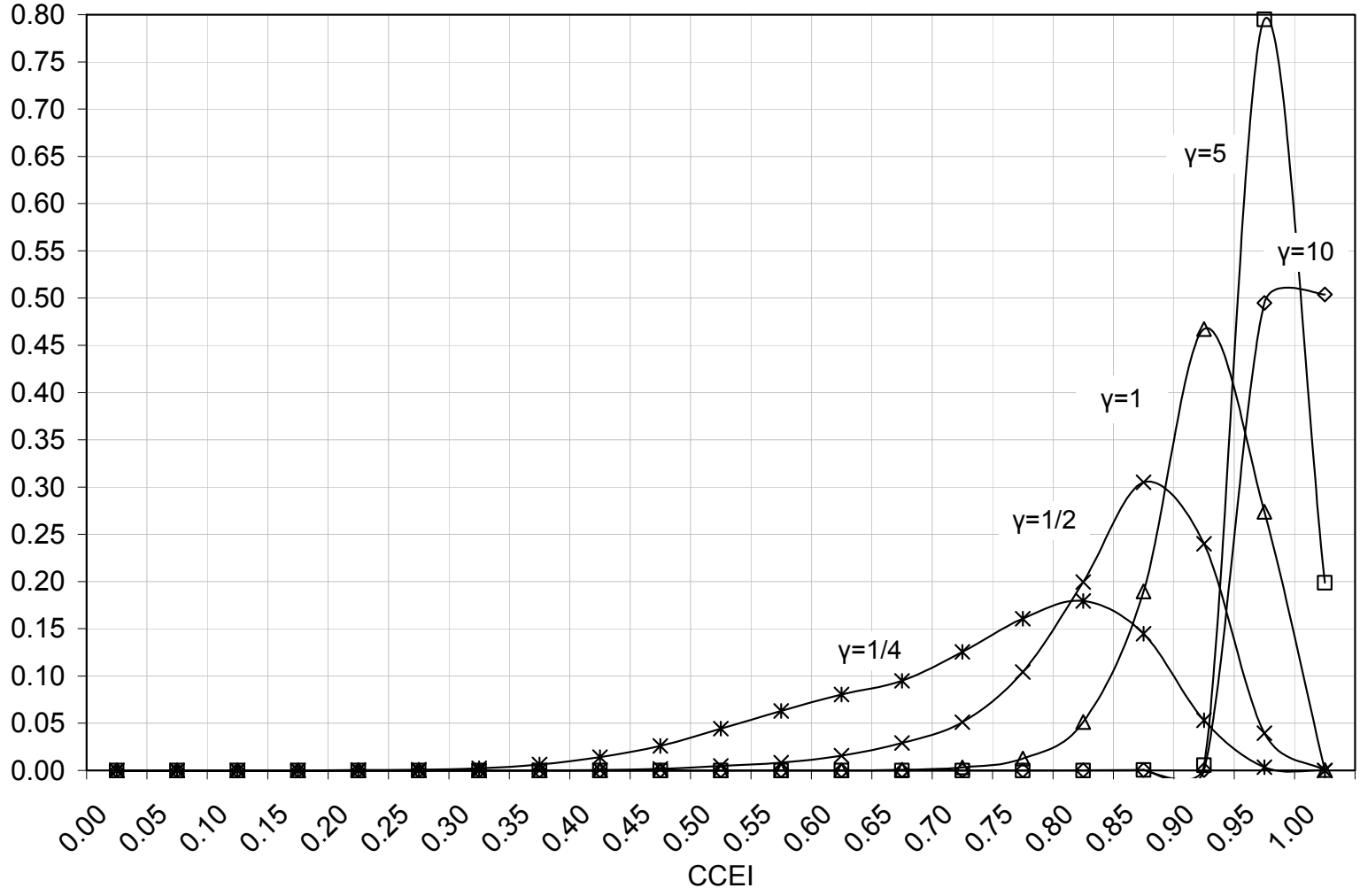
More precisely, we assume an idiosyncratic preference shock that has a logistic distribution

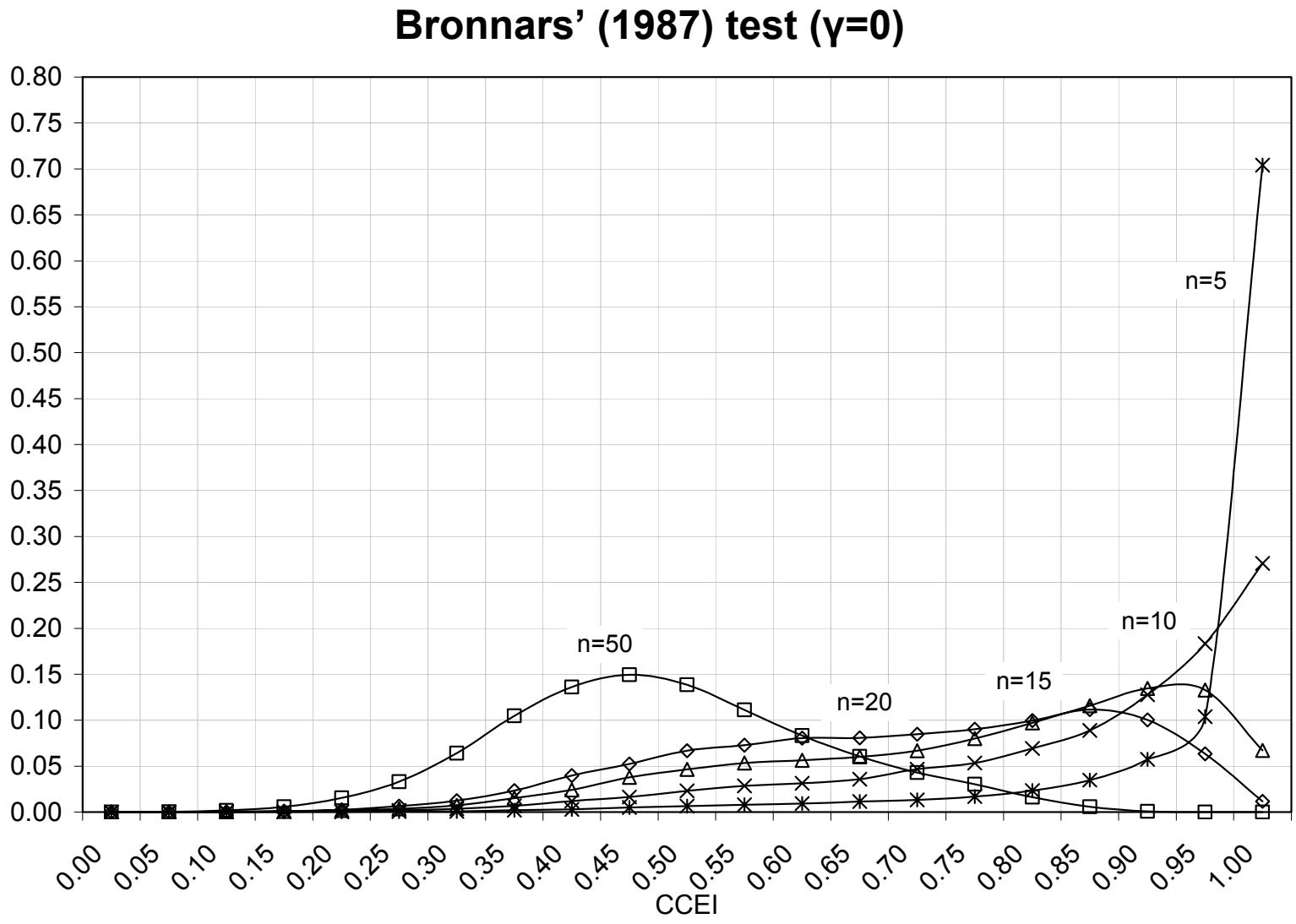
$$\mathsf{Pr}(x^*) = \frac{e^{\gamma \cdot u(x^*)}}{\int\limits_{x:p\cdot x=1} e^{\gamma \cdot u(x)}},$$

where the precision parameter γ reflects sensitivity to differences in utility.

If utility maximization is not the correct model, is our experiment sufficiently powerful to detect it?

The distributions of GARP violations – ρ=1/2 and different γ




Bronnars' (1987) test (γ=0)
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
n=5
n=10
n=15
n=20
n=50
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
CCEI

Recoverability

- GARP imposes on the data the complete set of conditions implied by utility-maximization.

- Revealed preference relations in the data thus contain the information that is necessary for recovering preferences.

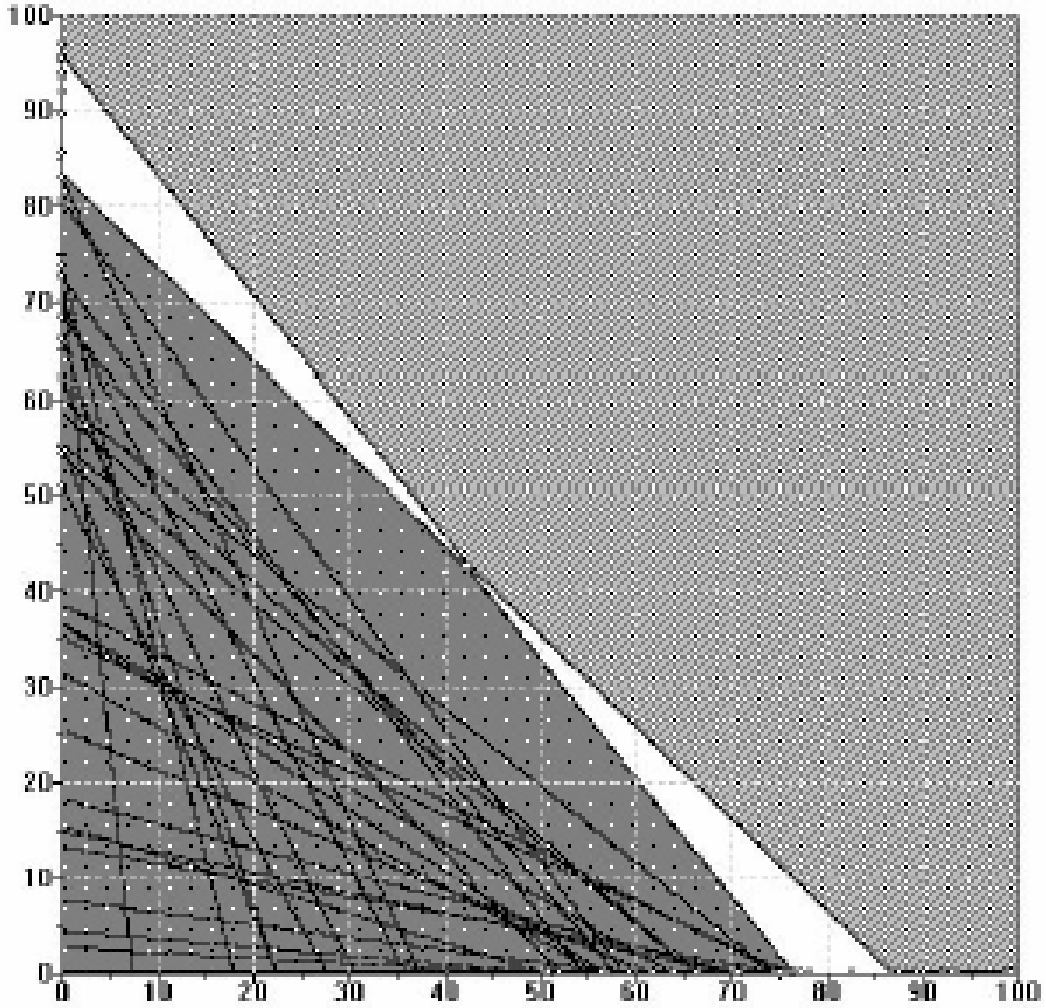
- Varian's (1982) algorithm serves as a partial solution to this so-called *recoverability problem*.

Let $S(x^0)$ be the set of prices at which x^0 could be chosen and be consistent with the observed data and let

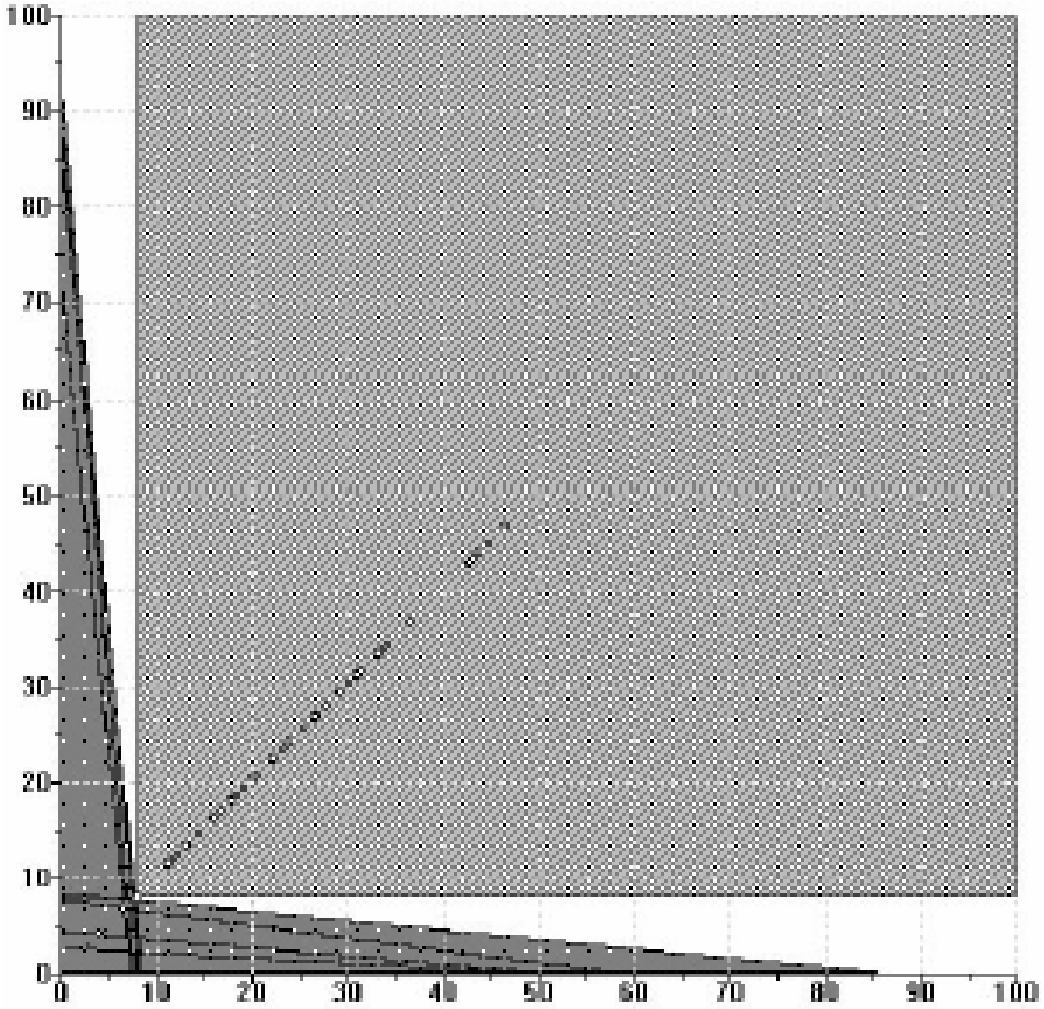
$$\begin{aligned} RW(x^0) &= \{x : x^0 R x \ \forall p \in S(x^0)\} \\ RP(x^0) &= \{x : x R x^0 \ \forall p \in S(x^0)\}. \end{aligned}$$

$RP(x^0)$ and $RW(x^0)^C$ form the tightest inner and outer bounds on the set of allocations preferred to x^0.

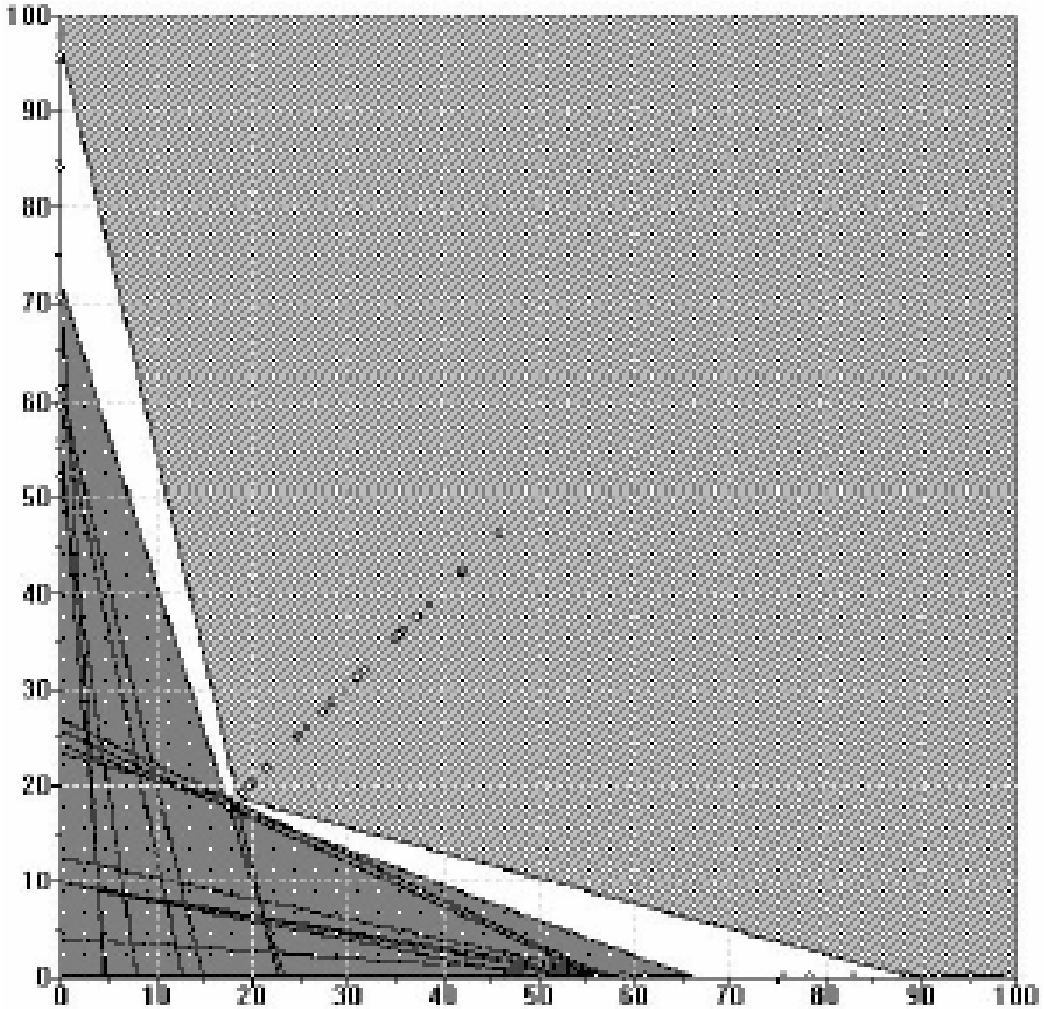
Risk neutrality



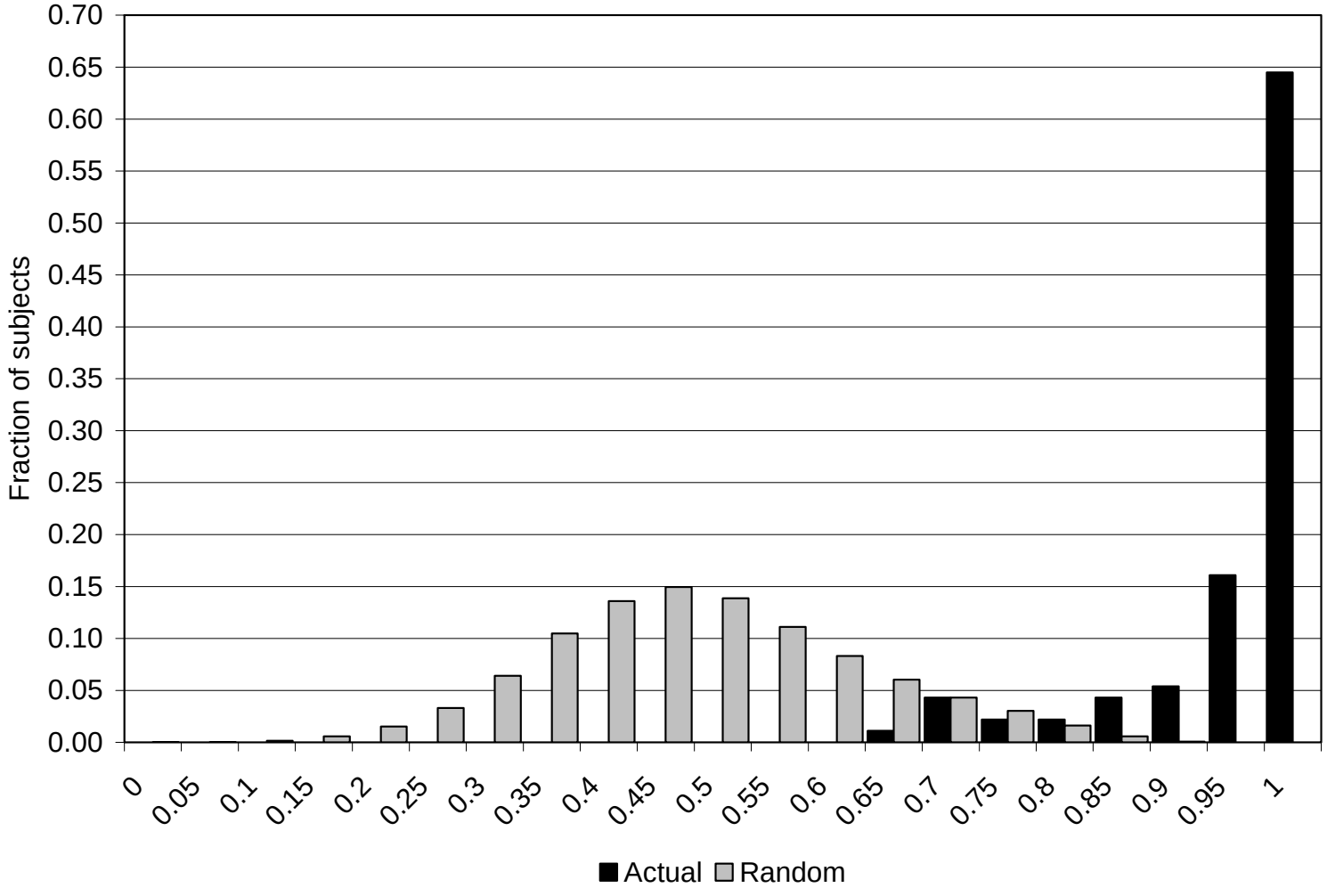
Infinite risk aversion



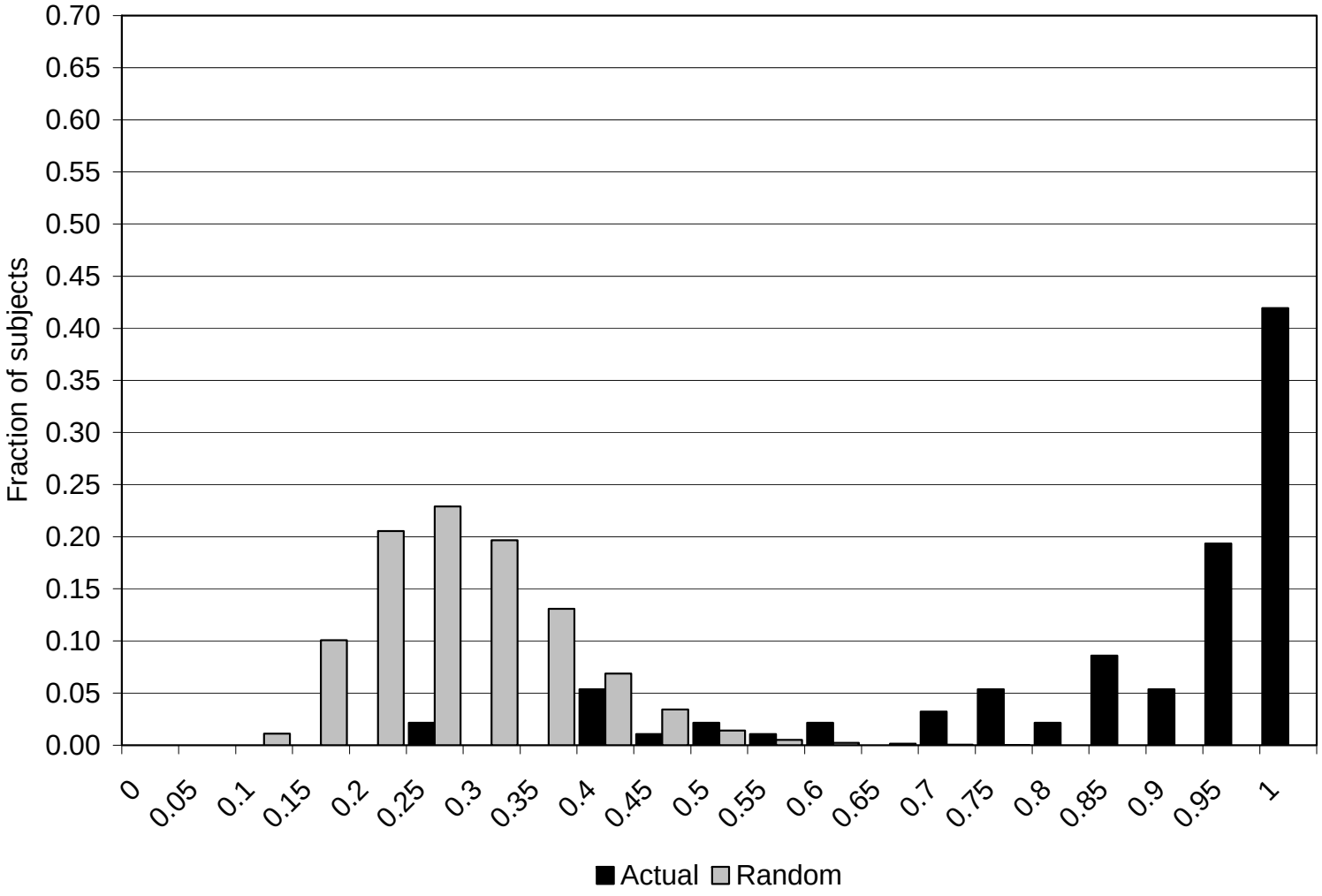
Loss / disappointment aversion



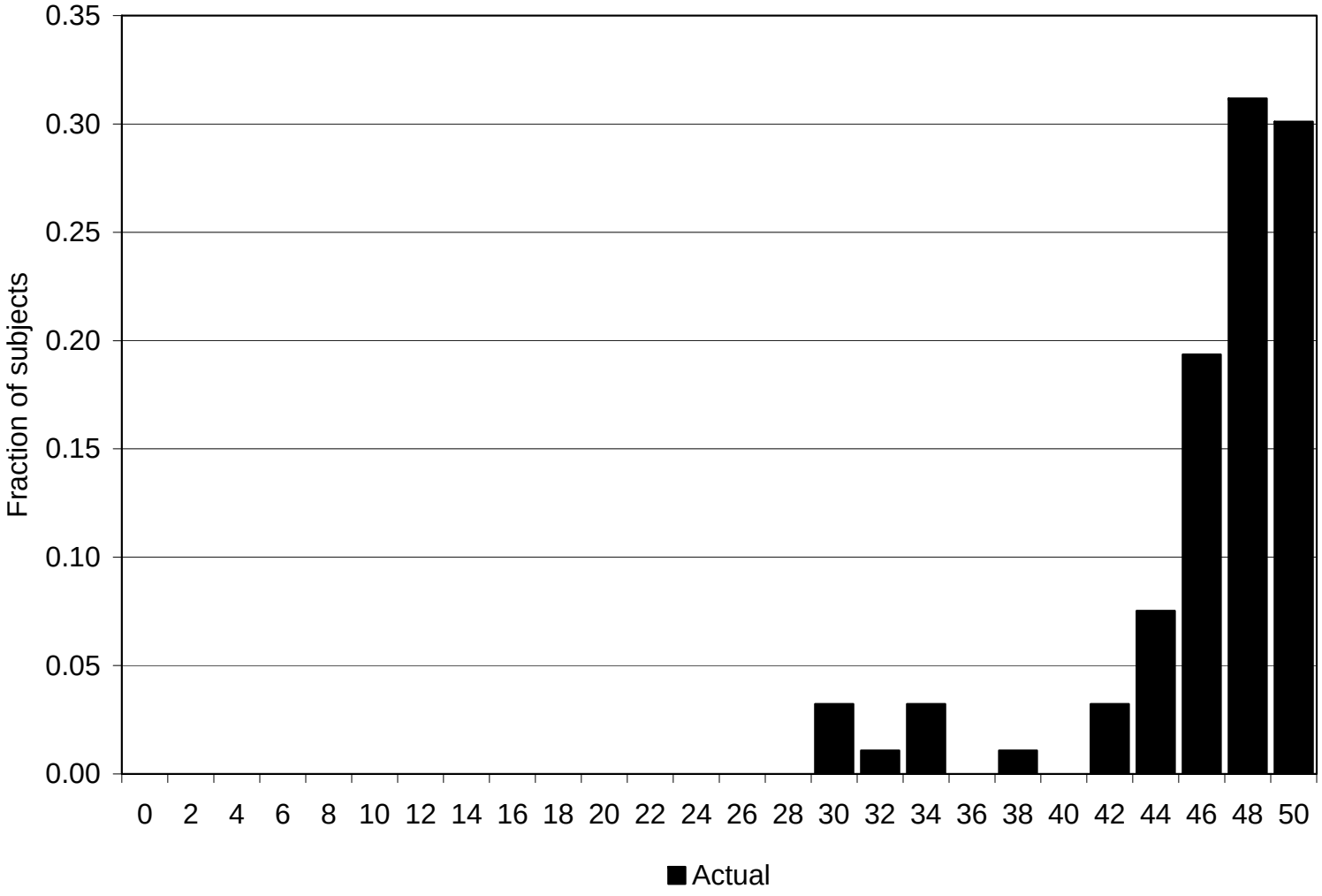
The distributions of GARP violations - Afriat (1972) CCEI



The distributions of GARP violations - Varian (1991)



The distribution of GARP violations - Houtman and Maks (1985)



Granularity

- A measure of the size of the components, or descriptions of components, that make up a system (Wikipedia).

- There is no taxonomy that allows us to classify all subjects unambiguously.

- A review of the full data set reveals striking regularities *within* and marked heterogeneity *across* subjects.

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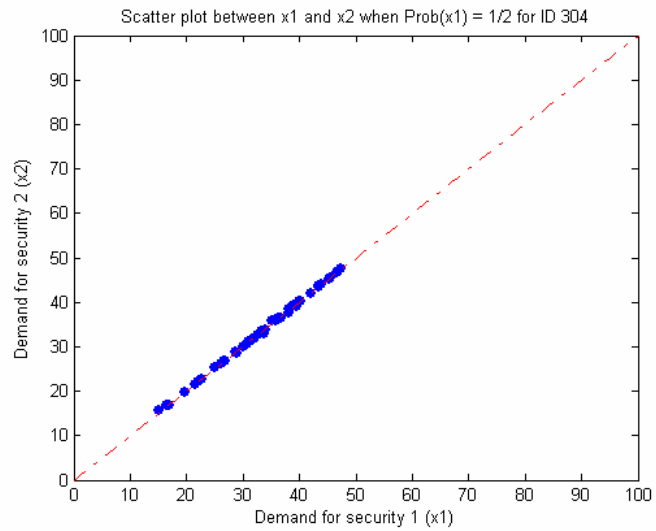

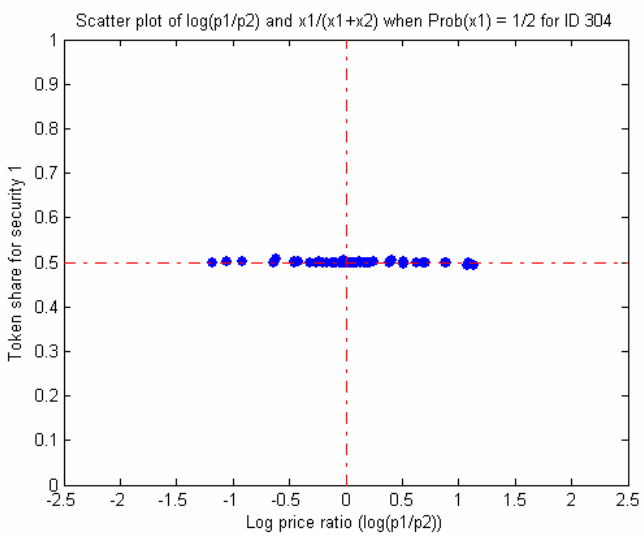

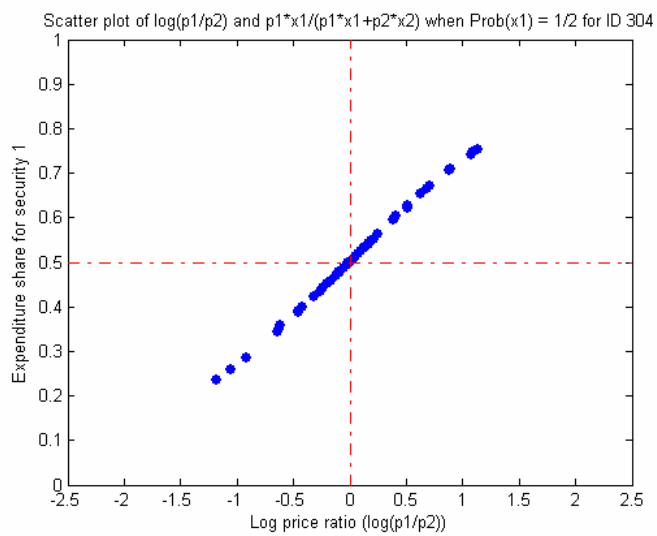

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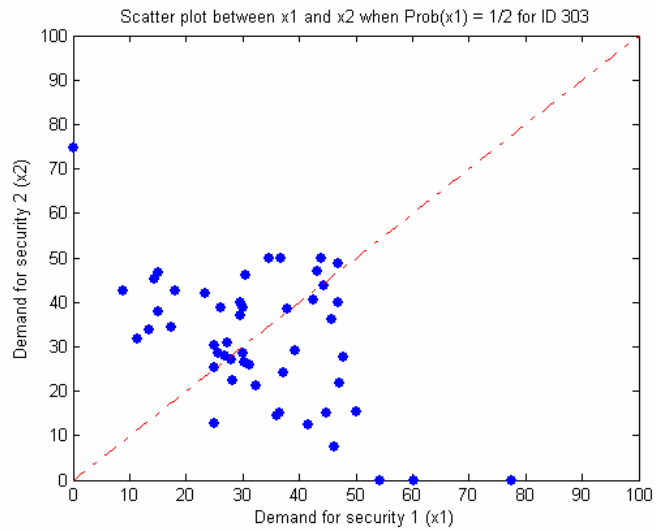

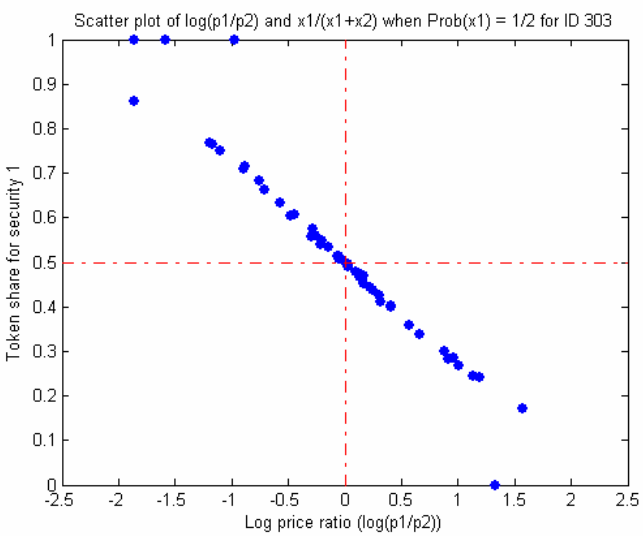

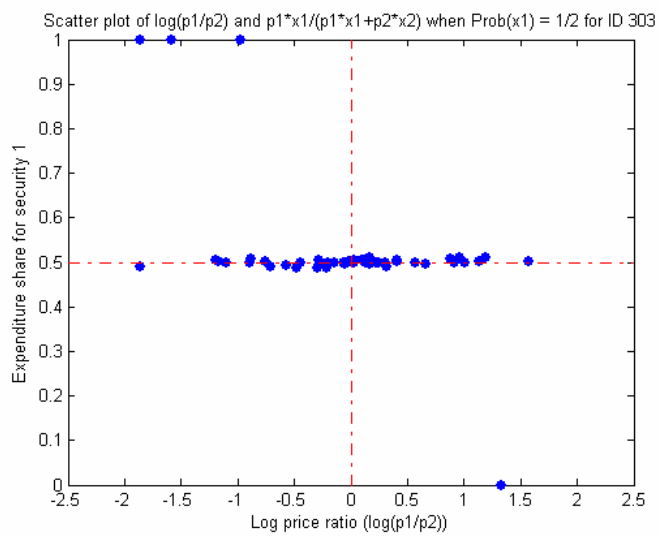

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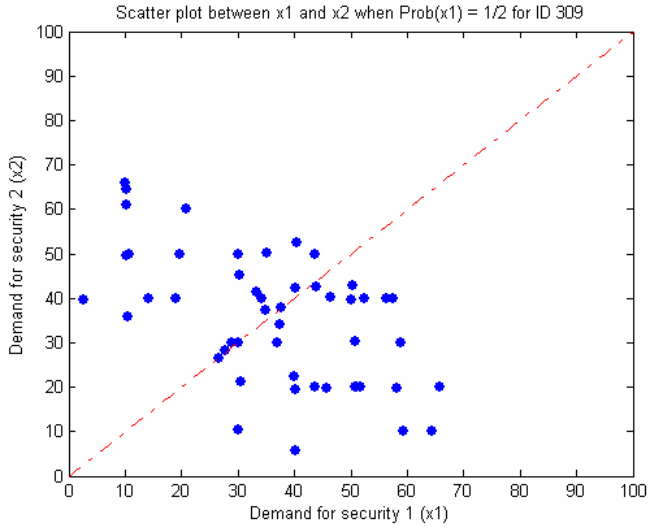

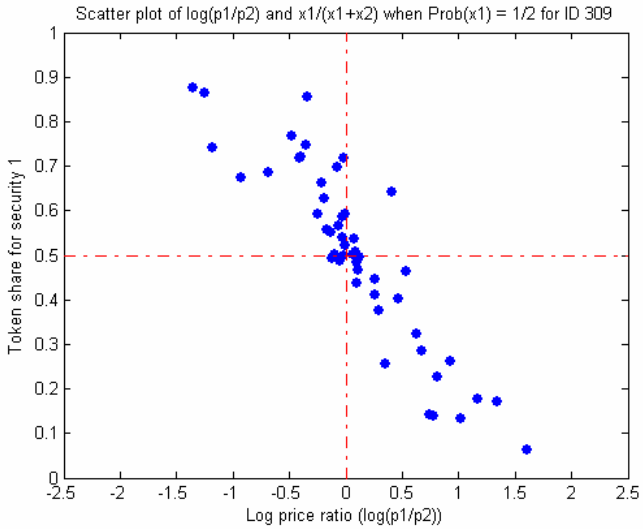

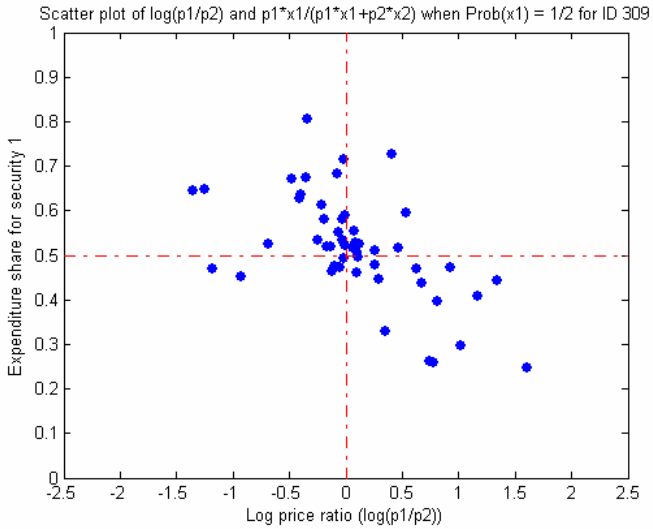

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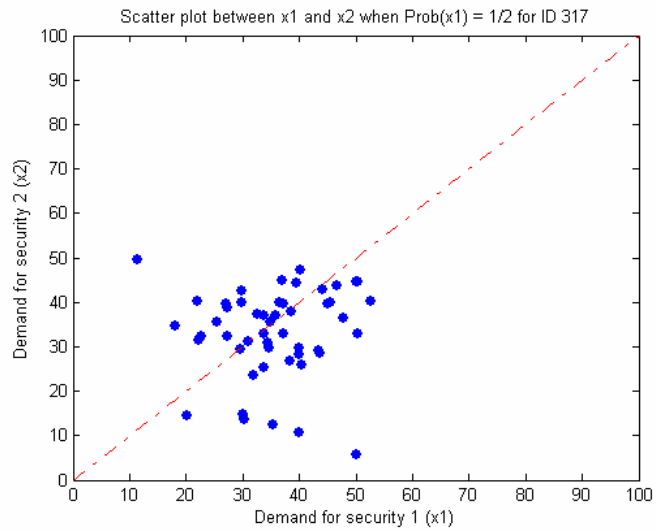

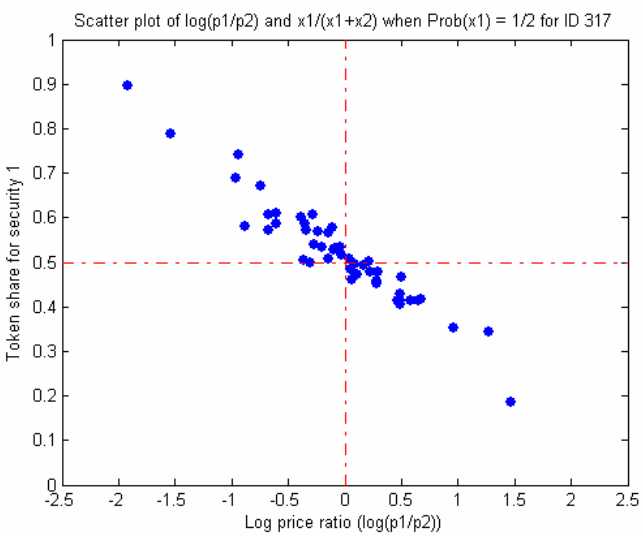

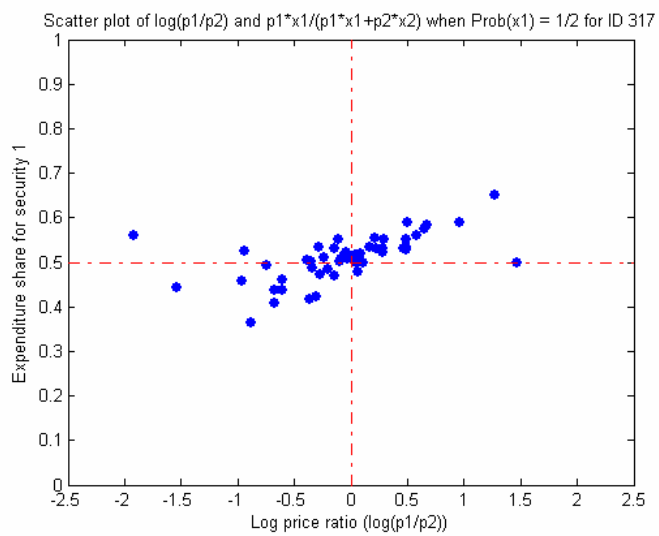

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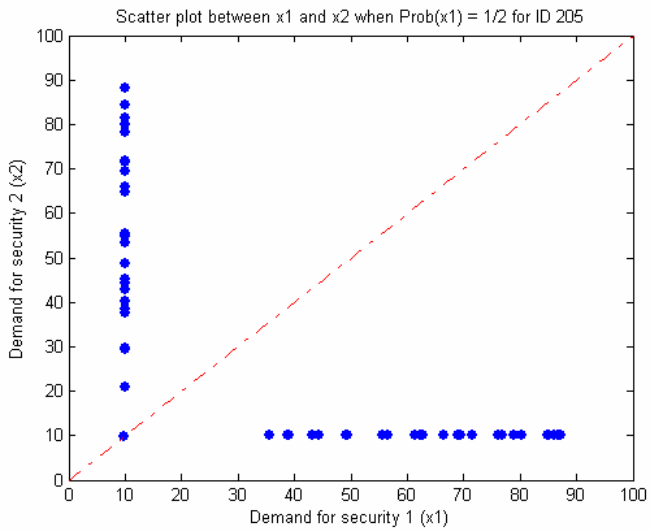

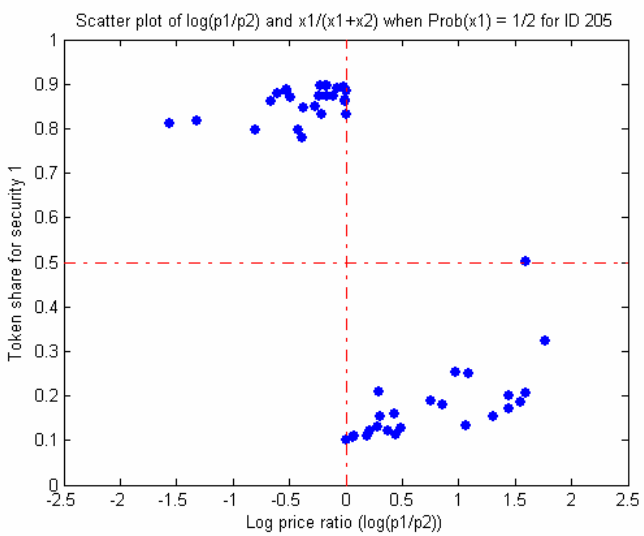

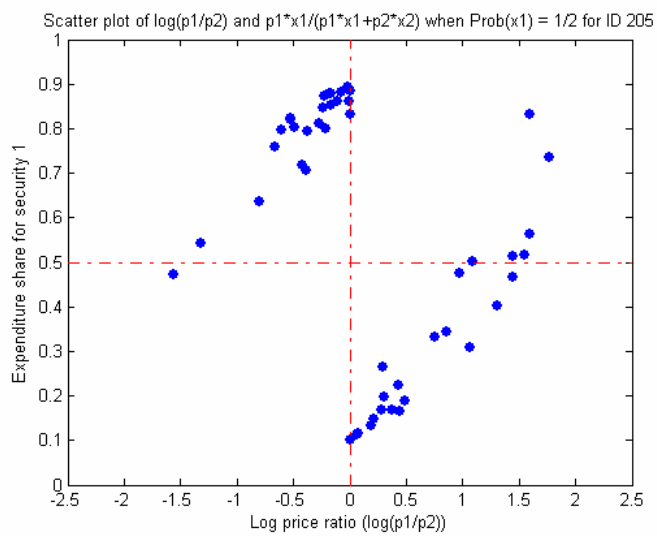

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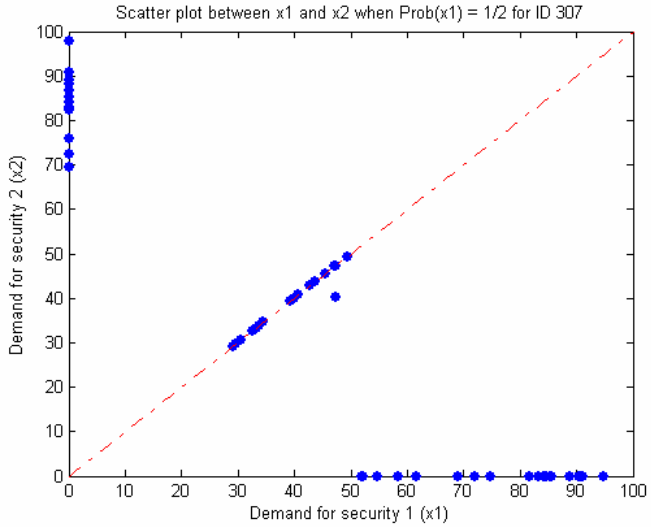

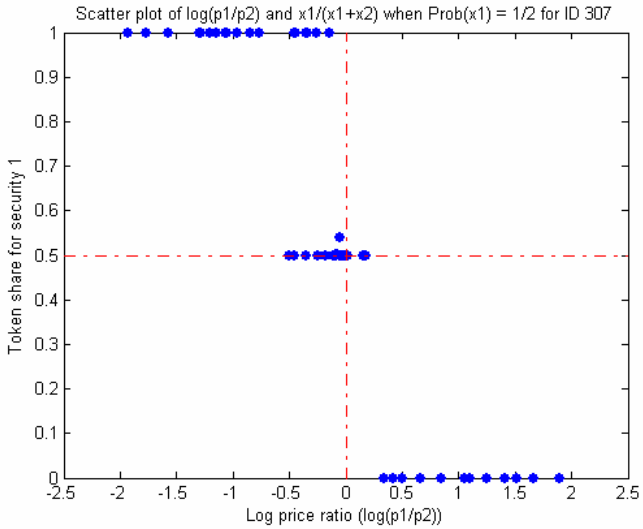

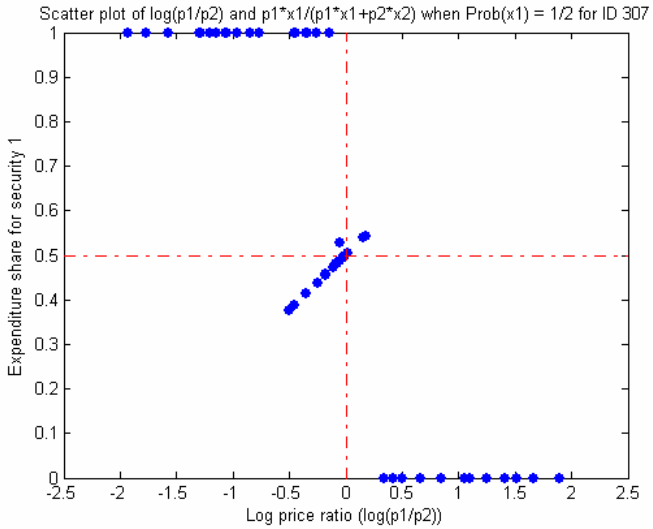

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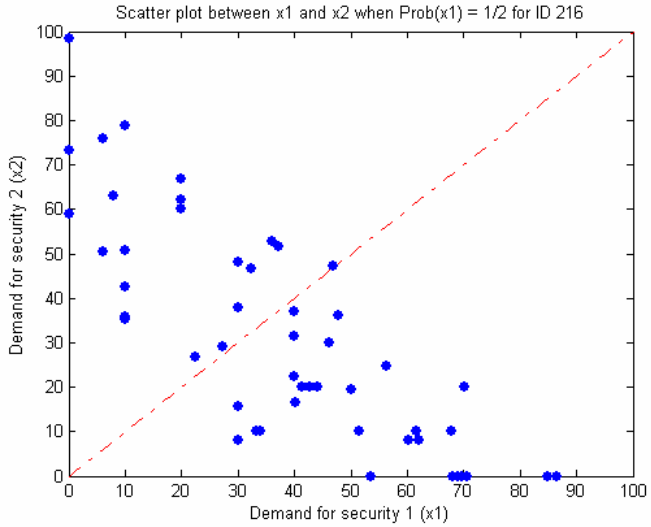

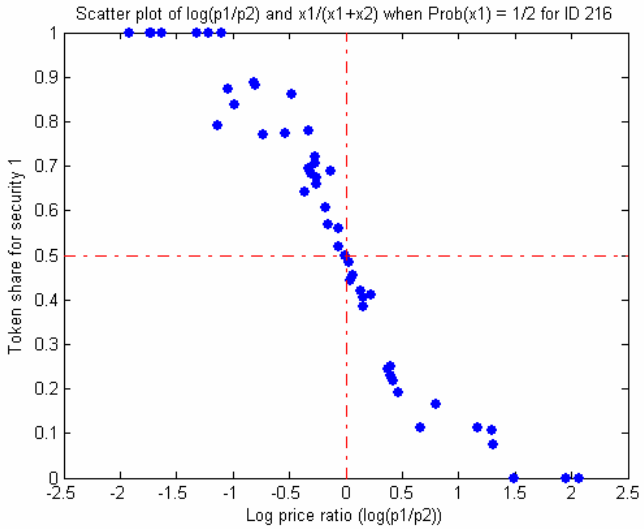

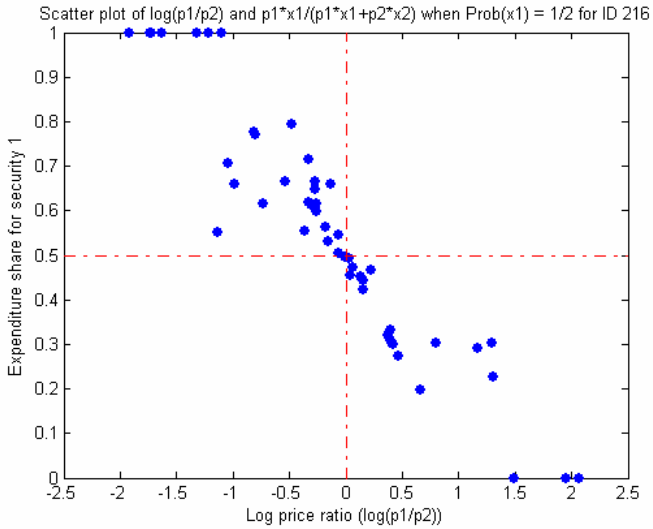

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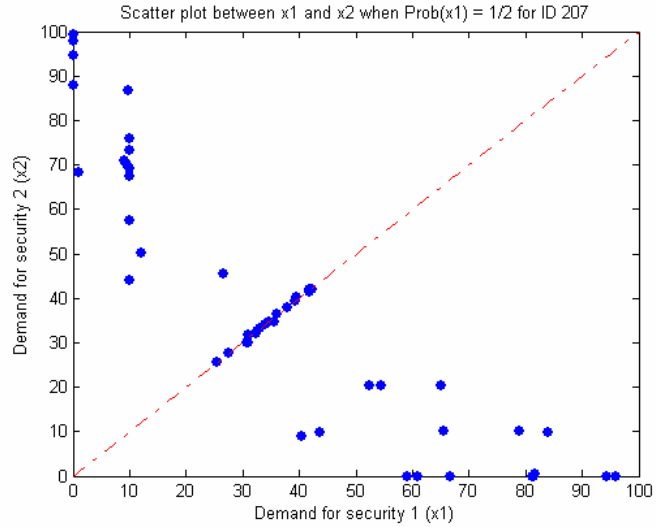

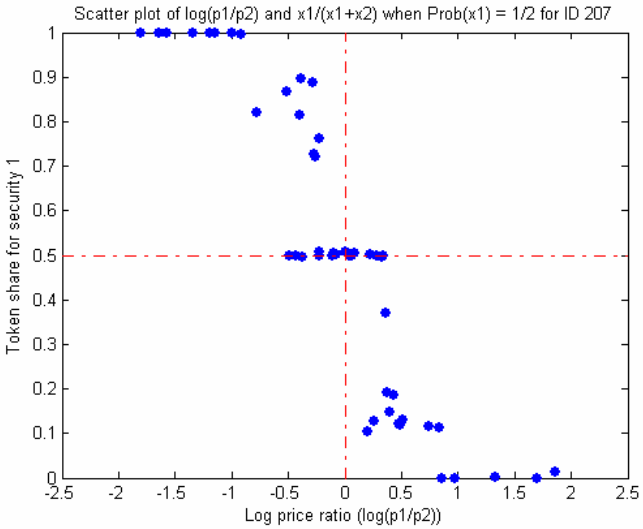

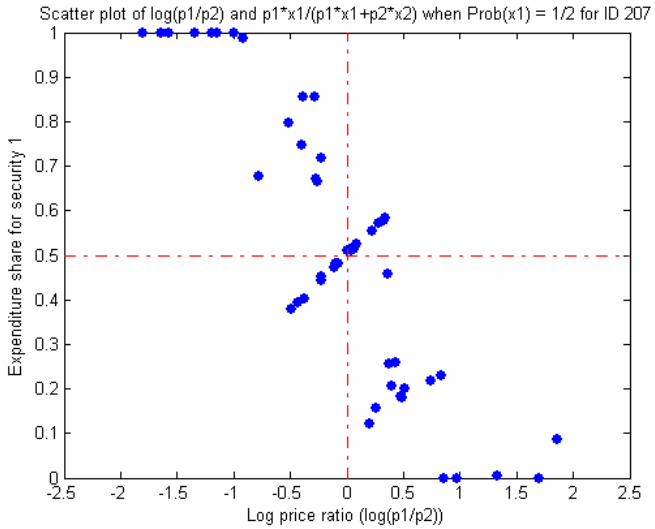

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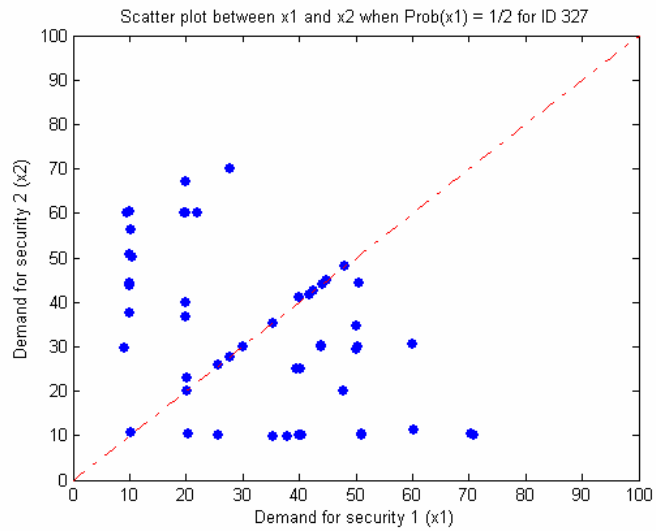

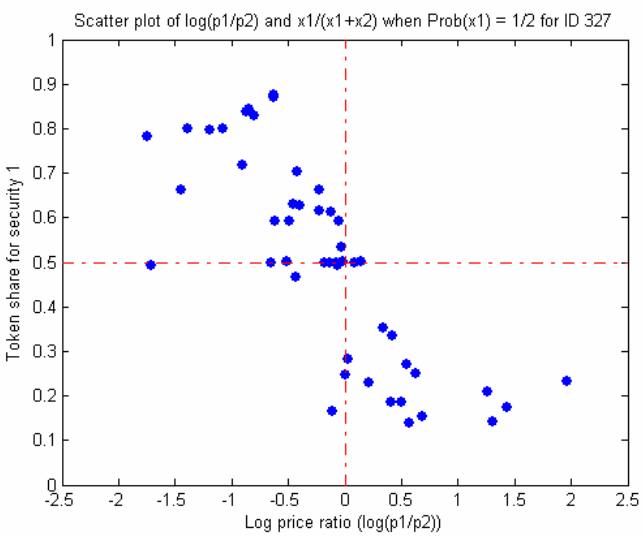

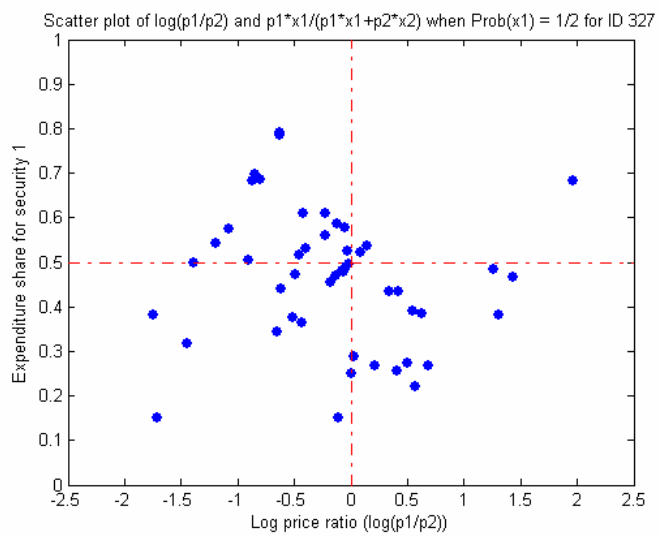

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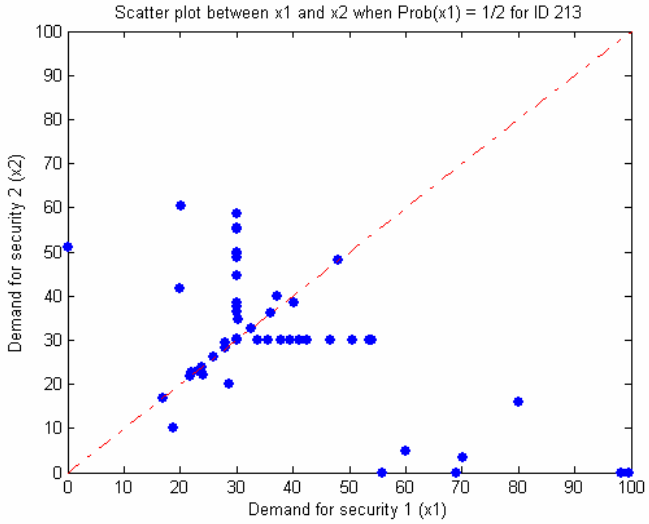

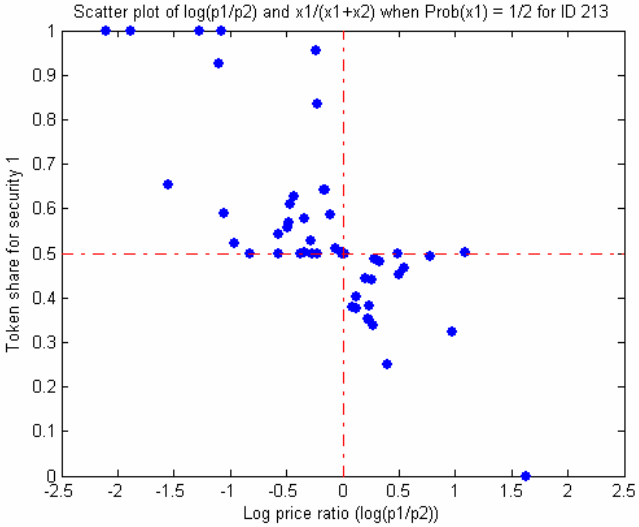

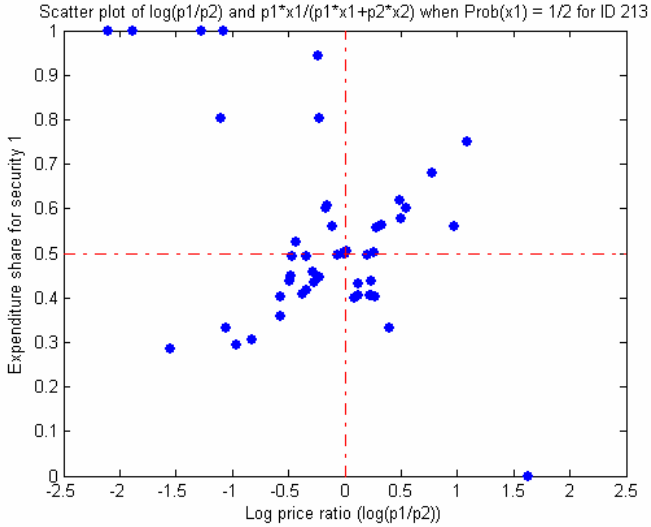

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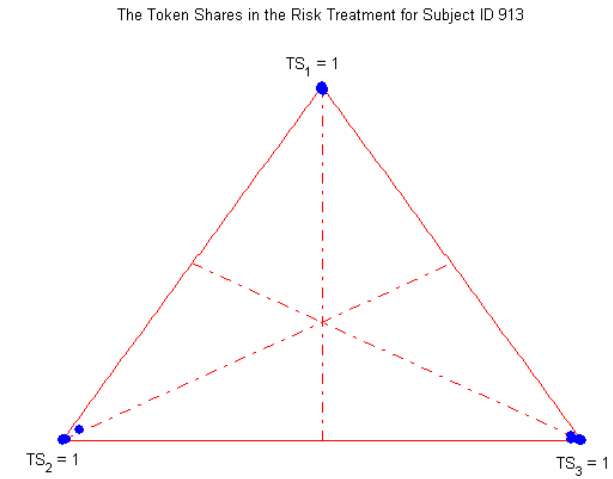
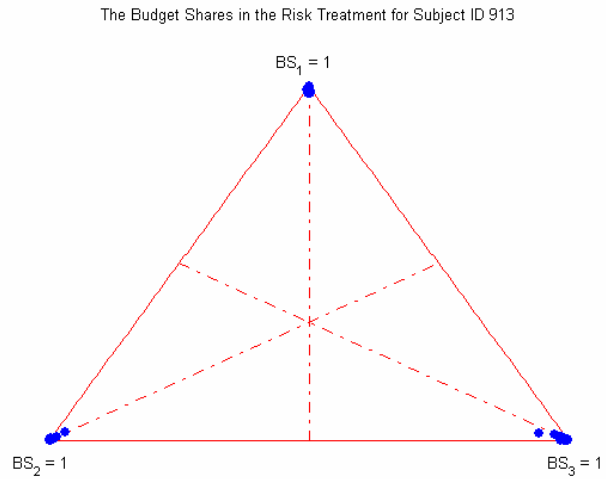
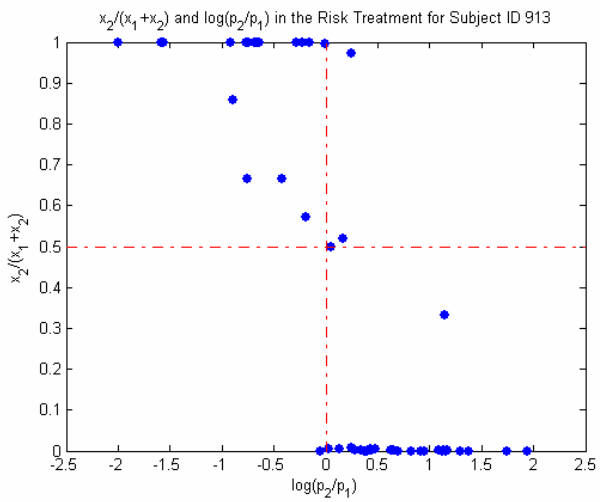
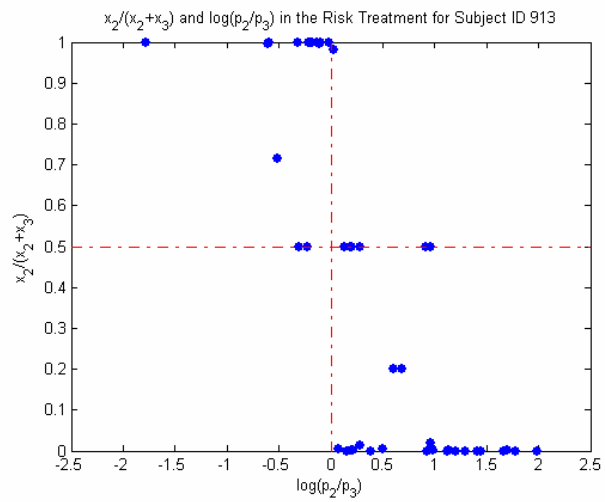
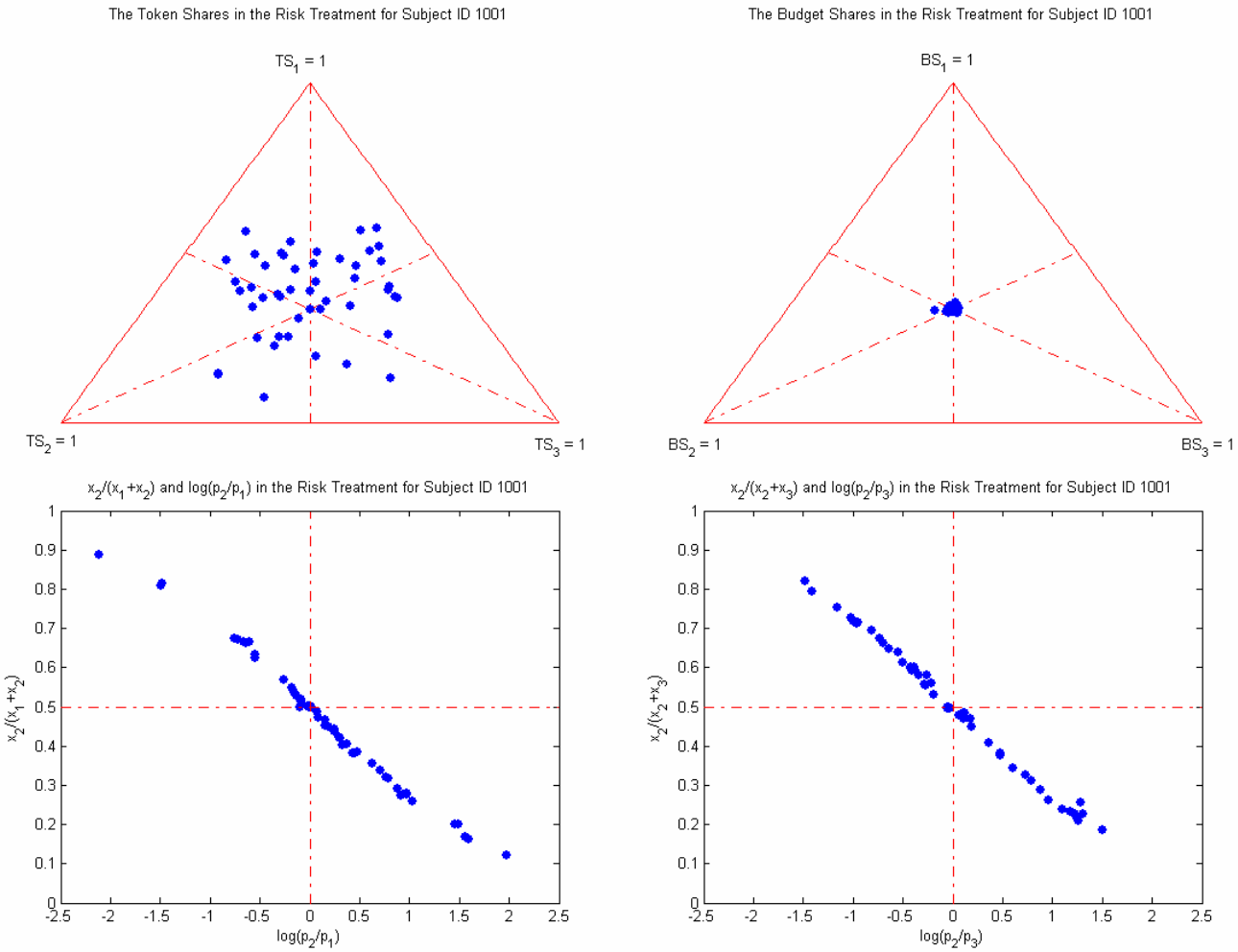

The Token Shares in the Risk Treatment for Subject ID 913
TS1 = 1
TS2 = 1
TS3 = 1


The Budget Shares in the Risk Treatment for Subject ID 913
BS1 = 1
BS2 = 1
BS3 = 1

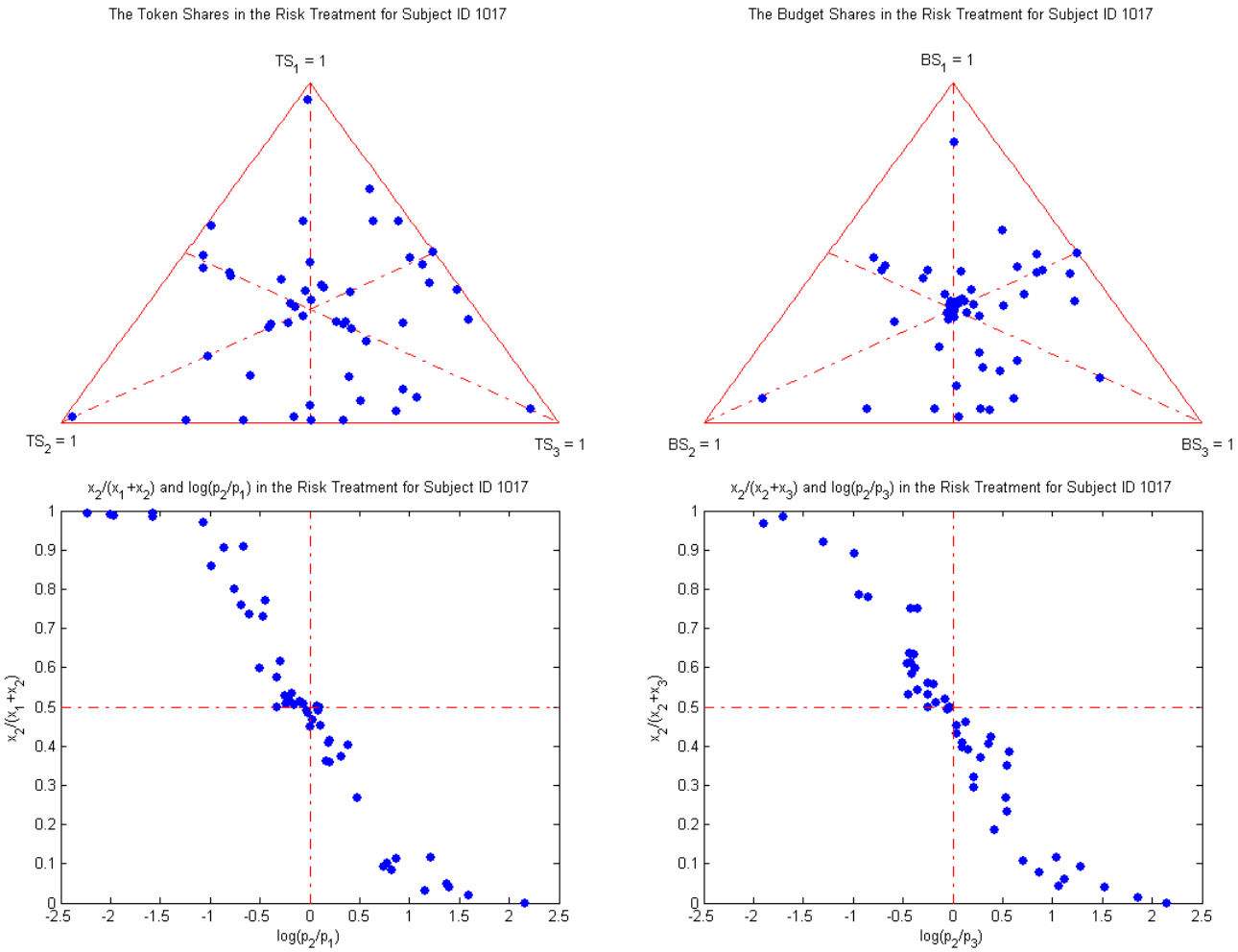

x2/(x1+x2) and log(p2/p1) in the Risk Treatment for Subject ID 913
x2/(x1+x2)
log(p2/p1)


x2/(x2+x3) and log(p2/p3) in the Risk Treatment for Subject ID 913
x2/(x2+x3)
log(p2/p3)

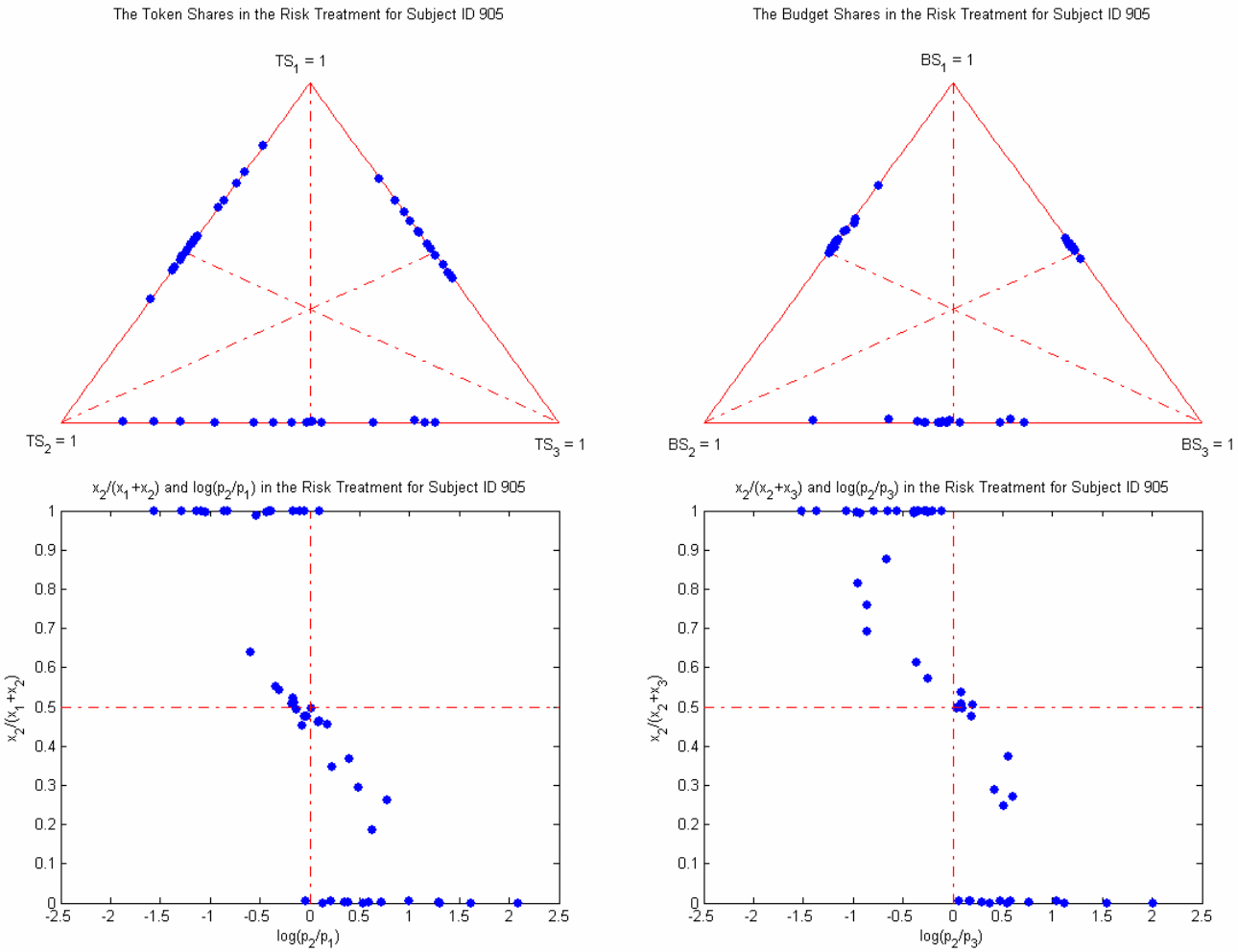
ID 1001 (1.000)


The Token Shares in the Risk Treatment for Subject ID 1001
TS1 = 1
TS2 = 1
TS3 = 1
The Budget Shares in the Risk Treatment for Subject ID 1001
BS1 = 1
BS2 = 1
BS3 = 1
x2/(x1+x2) and log(p2/p1) in the Risk Treatment for Subject ID 1001
x2/(x1+x2)
log(p2/p1)
x2/(x2+x3) and log(p2/p3) in the Risk Treatment for Subject ID 1001
x2/(x2+x3)
log(p2/p3)

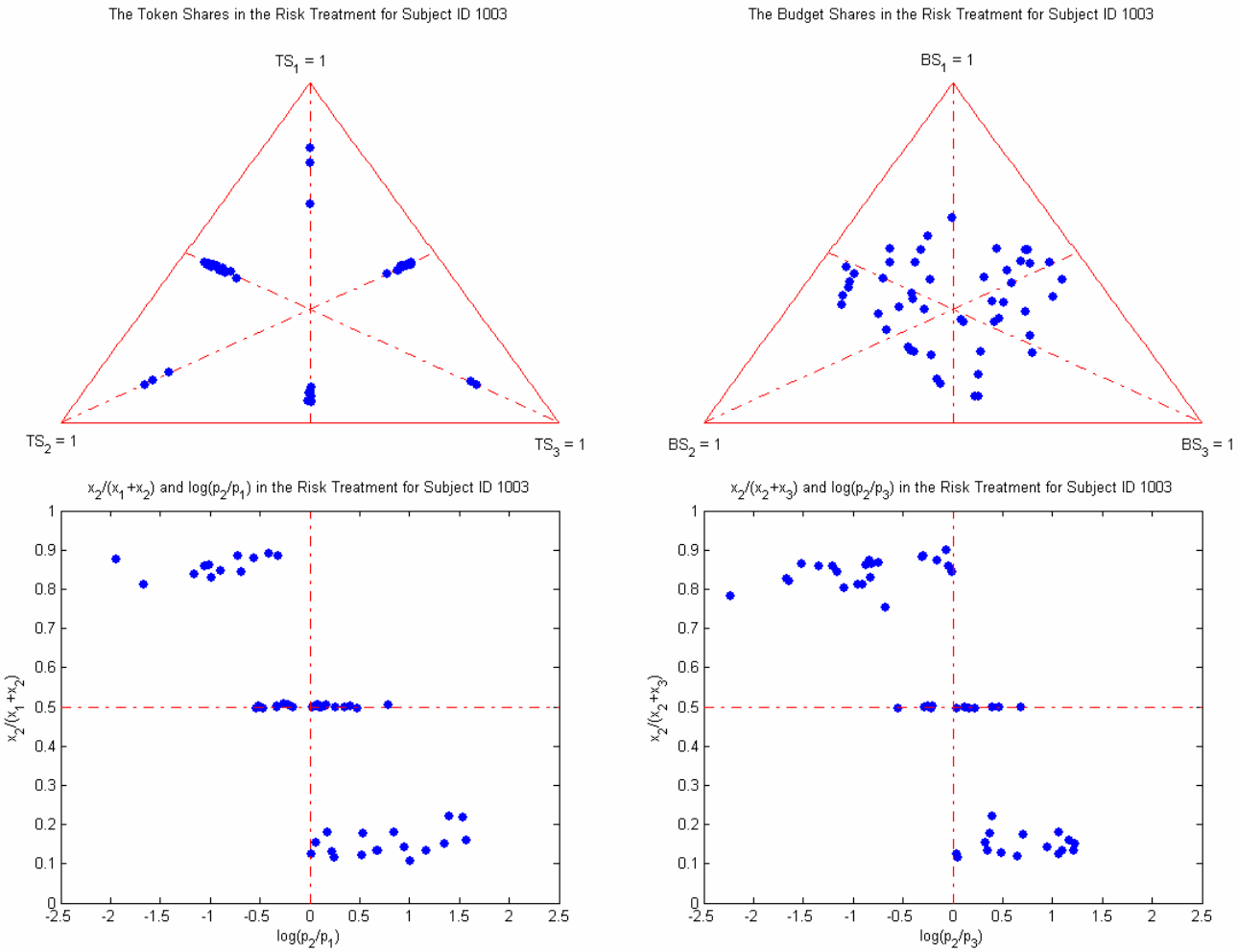
ID 1017 (1.000)


The Token Shares in the Risk Treatment for Subject ID 1017
TS1 = 1
TS2 = 1
TS3 = 1
The Budget Shares in the Risk Treatment for Subject ID 1017
BS1 = 1
BS2 = 1
BS3 = 1
x2/(x1+x2) and log(p2/p1) in the Risk Treatment for Subject ID 1017
x2/(x1+x2)
log(p2/p1)
x2/(x2+x3) and log(p2/p3) in the Risk Treatment for Subject ID 1017
x2/(x2+x3)
log(p2/p3)

ID 905 (1.000)


The Token Shares in the Risk Treatment for Subject ID 905
TS1 = 1
TS2 = 1
TS3 = 1
The Budget Shares in the Risk Treatment for Subject ID 905
BS1 = 1
BS2 = 1
BS3 = 1
x2/(x1+x2) and log(p2/p1) in the Risk Treatment for Subject ID 905
x2/(x1+x2)
log(p2/p1)
x2/(x2+x3) and log(p2/p3) in the Risk Treatment for Subject ID 905
x2/(x2+x3)
log(p2/p3)

ID 1003 (1.000)


The Token Shares in the Risk Treatment for Subject ID 1003
TS1 = 1
TS2 = 1
TS3 = 1
The Budget Shares in the Risk Treatment for Subject ID 1003
BS1 = 1
BS2 = 1
BS3 = 1
x2/(x1+x2) and log(p2/p1) in the Risk Treatment for Subject ID 1003
x2/(x1+x2)
log(p2/p1)
x2/(x2+x3) and log(p2/p3) in the Risk Treatment for Subject ID 1003
x2/(x2+x3)
log(p2/p3)

Risk aversion

A "low-tech" approach of estimating an individual-level power utility function directly from the data:

$$u(x) = \frac{x^{1-\rho}}{(1-\rho)}.$$

ρ is the Arrow-Pratt measure of relative risk aversion. The aversion to risk increases as ρ increases.

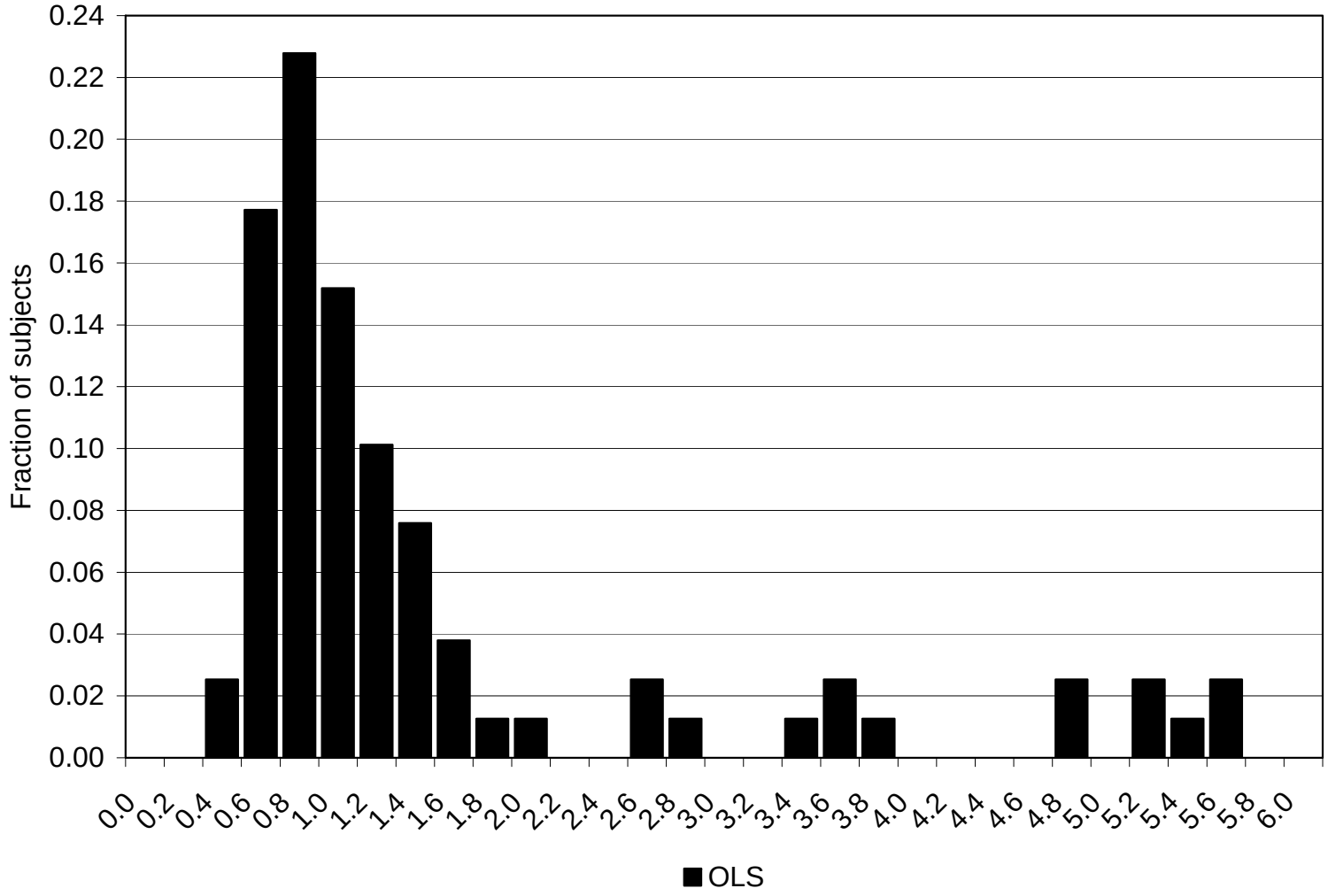
This generates the following individual-level econometric specification for each subject n:

$$\log\left(\frac{x^i_{2n}}{x^i_{1n}}\right) = \alpha_n + \beta_n \log\left(\frac{p^i_{1n}}{p^i_{2n}}\right) + \epsilon^i_n$$

where $\epsilon^i_n \sim N(0, \sigma^2_n)$.

We generate estimates of $\hat{\rho}_n = 1/\hat{\beta}_n$ which allows us to test for heterogeneity of risk preferences.

The distribution of the individual Arrow-Pratt measures (OLS)



Loss/disappointment aversion

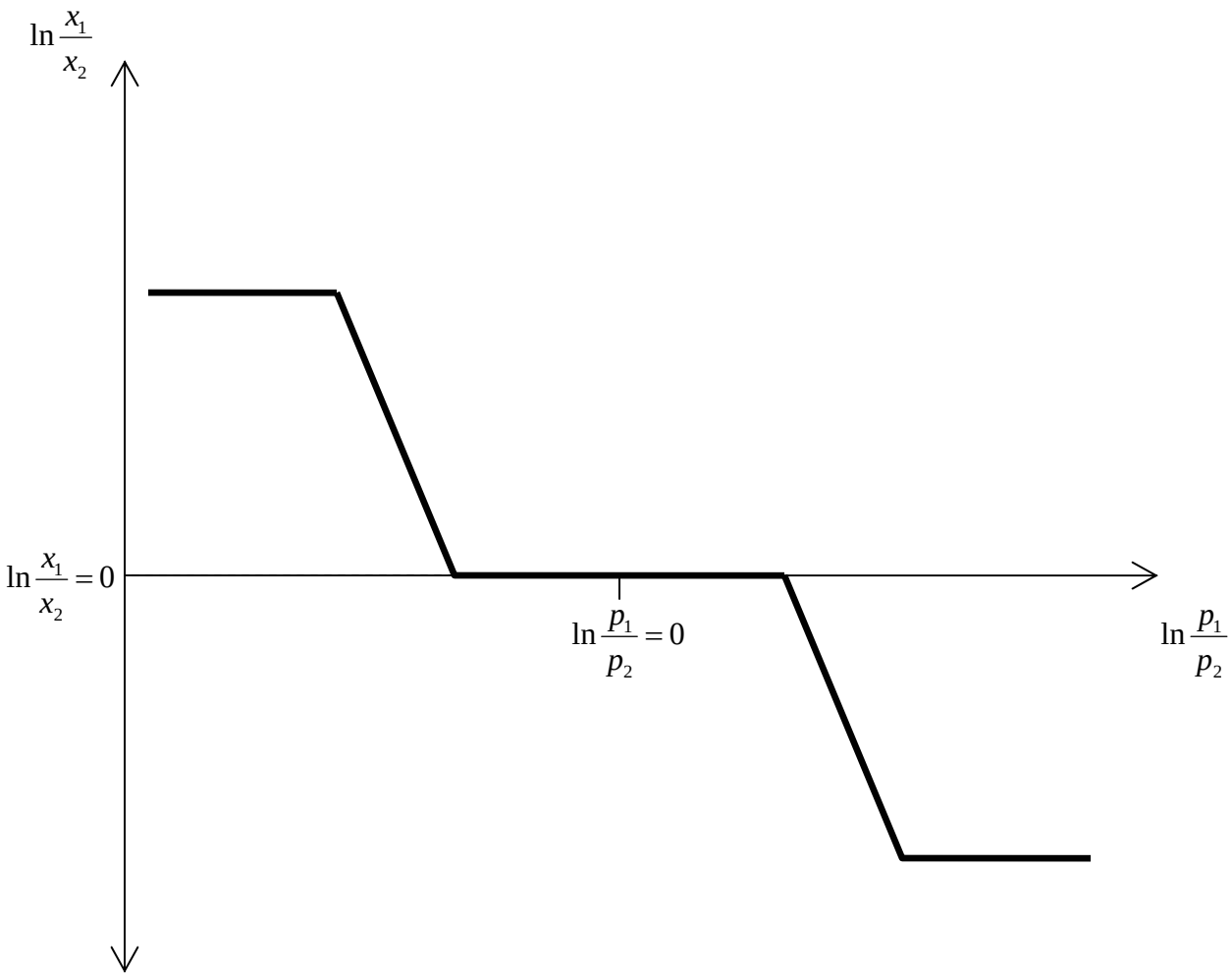
The theory of Gul (1991) implies that the utility function over portfolios takes the form

$$\min\left\{\alpha u\left(x_1\right)+u\left(x_2\right), u\left(x_1\right)+\alpha u\left(x_2\right)\right\},$$

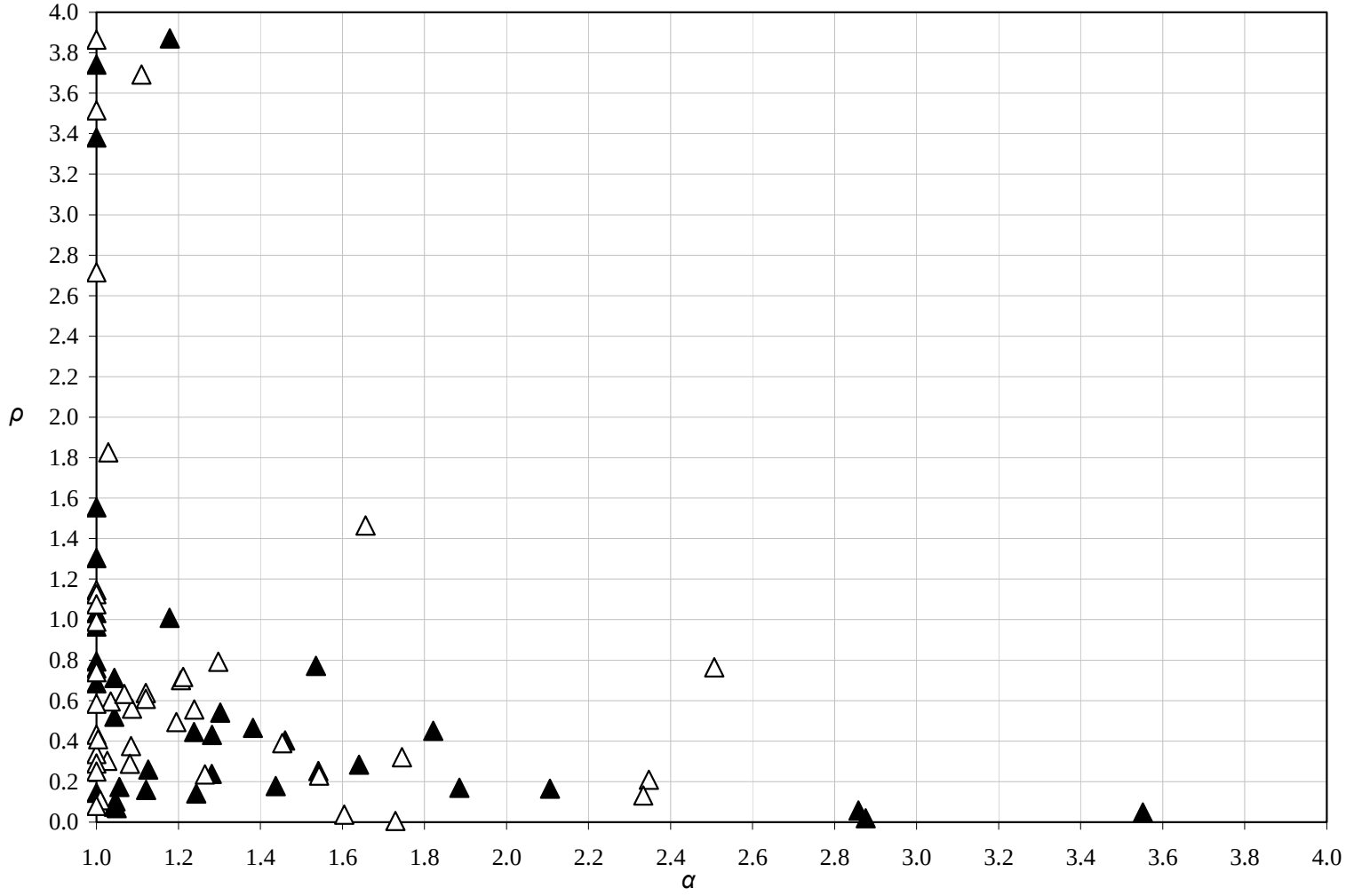
where $\alpha \geq 1$ measures loss/disappointment aversion and $u(\cdot)$ is the utility of consumption in each state.

If $\alpha > 1$ there is a kink at the point where $x_1 = x_2$ and if $\alpha = 1$ we have the standard EUT representation.

An illustration of the derived demand



Scatterplot of the estimated CRRA parameters



Type-mixture model (TMM)

A *unified* account of both procedural rationality and substantive rationality.

- Allow EU maximization to play the role of the underlying preference ordering.
- Account for subjects' underlying preferences and their choice of decision rules.

Ingredients

- The "true" underlying preferences are represented by a power utility function.

- A discrete choice among the fixed set of prototypical heuristics, D, S and $B(\omega)$.

- The probability of choosing each particular heuristic is a function of the budget set.

- Subjects could make mistakes when trying to maximize EU by employing heuristic S.

- In contrast, when following heuristic D or $B(\omega)$ subjects' hands do not tremble.

- A subject may prefer to choose heuristic $B(\omega)$ or D instead of the noisy version of heuristic S.

Specification

The underlying preferences of each subject are assumed to be represented by

$$u(x) = \frac{x^{1-\rho}}{(1-\rho)}$$

(power utility function as long as consumption in each state meets the secure level ω).

Let $\varphi(p)$ be the portfolio which gives the subject the maximum (expected) utility achievable at given prices p.

The *ex ante* expected payoff from attempting to maximize EU by employing heuristic S is given by

$$U_S(p) = \mathbb{E}[\pi u\left(\tilde{\varphi}_1(p)\right) + (1-\pi) u\left(\tilde{\varphi}_2(p)\right)]$$

$\tilde{\varphi}(p)$ is a random portfolio s.t. $p \cdot \tilde{\varphi}(p) = 1$ for every $p = (p_1, p_2)$, and $p_1[\tilde{\varphi}_1(p) - \varphi_1(p)] = \varepsilon$ and $\epsilon_n^i \sim N(0, \sigma_n^2)$.

When following heuristic D or B subjects' hands do not tremble. We therefore write

$$U_D(p) = u(1/(p_1 + p_2))$$

and

$$U_B(p) = \max\{\pi u(0) + (1-\pi)u(1/p_2), \pi u(1/p_1) + (1-\pi)u(0)\}$$

Estimation

The probability of choosing heuristic $k = D, S, B(\omega)$ is given by a standard logistic discrete choice model:

$$\text{Pr}(\text{heuristic } \tau \,|p; \beta, \rho, \sigma) = \frac{e^{\beta U_\tau}}{\sum\limits_{k=D,S,B} e^{\beta U_k}}$$

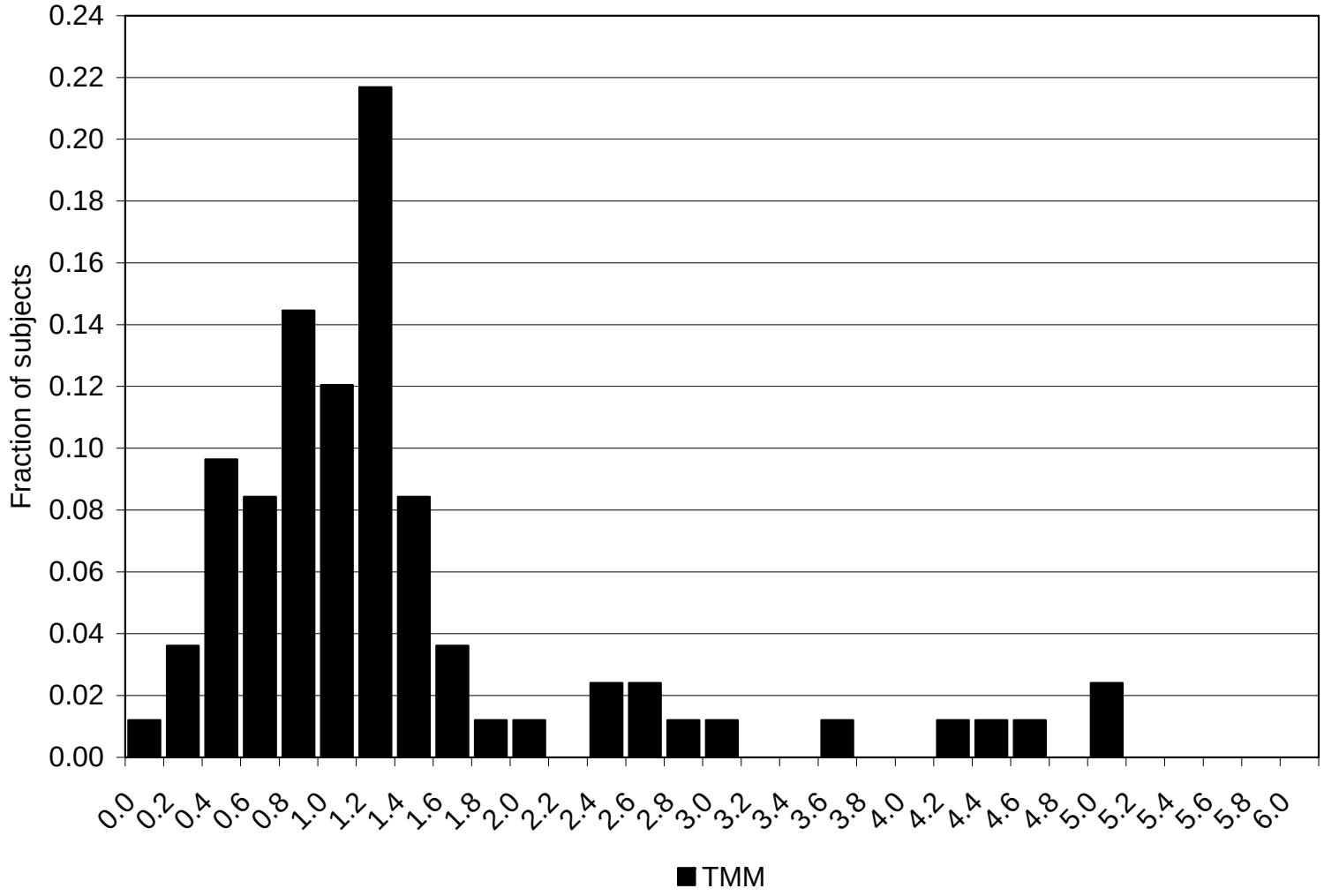
where U_D, U_S and U_B is the payoff specification for heuristic D, S and $B(\omega)$, respectively.

- The $\hat{\beta}$ estimates are significantly positive, implying that the TMM has some predictive power.

- Most subjects exhibit moderate to high levels of risk preferences around $\hat{\rho} = 0.8$.

- There is a strong correlation between the estimated $\hat{\rho}$ parameters from "low-tech" OLS and TMM estimations.

The distribution of the individual Arrow-Pratt measures (TMM)



Goodness-of-fit

- Compare the choice probabilities predicted by the TMM and empirical choice probabilities.

- Nadaraya-Watson nonparametric estimator with a Gaussian kernel function.

- The empirical data are supportive of the TMM model (fits best in the symmetric treatment).

Ambiguity

- The distinction between settings with risk and ambiguity dates back to at least the work of Knight (1921).

- Ellsberg (1961) countered the reduction of subjective uncertainty to risk with several thought experiments.

- A large theoretical literature (axioms over preferences) has developed models to accommodate this behavior.

- But what matters most is the implications of the models for choice behavior.

Consider the following two-color Ellsberg-type urns (Halevy, 2007):

I. 5 red balls and 5 black balls

II. an unknown number of red and black

III. a bag containing 11 tickets with the numbers 0-10; the number written on the drawn ticket determines the number of red balls

IV. a bag containing 2 tickets with the numbers 0 and 10; the number written on the drawn ticket determines the number of red balls.

- A clever experiment to verify the connection between the reduction of objective compound lotteries and attitudes to ambiguity.

- Four different urns are used to elicit choices in the presence of risk, ambiguity, and two degrees of compound uncertainty.

- Different models generate different predictions about how the reservation values (BDM) for these four urns will be ordered.

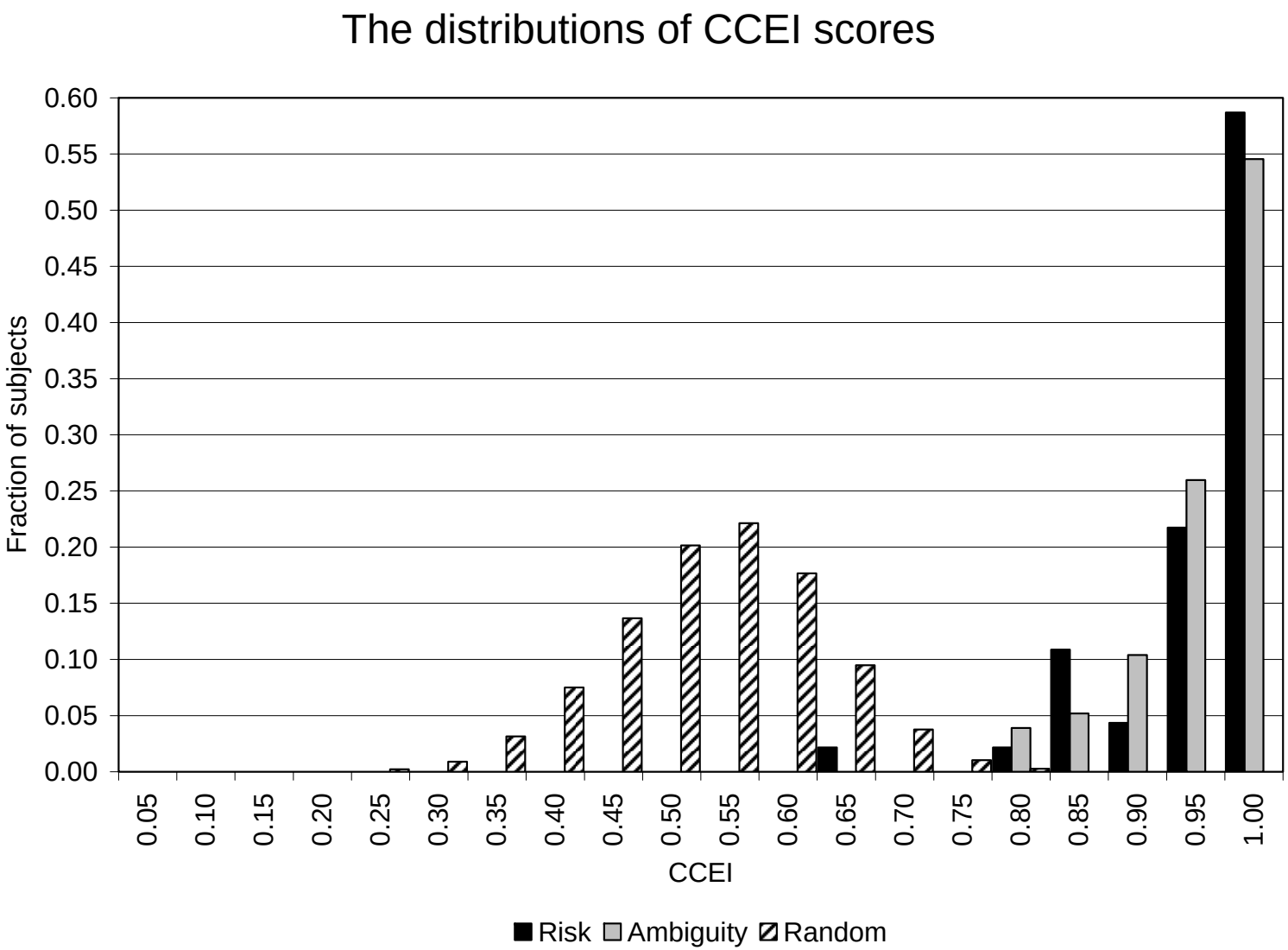
- The experiment can therefore classify each subject according to which model predicts his ordering of reservation values.

- Now, consider three states of nature and corresponding Arrow security (pays one dollar in one state and nothing in the other states).

- One state has an objectively known probability, whereas the probabilities of the other states are ambiguous.

- The presence of ambiguity could cause not just a departure from EU, but a more fundamental departure from rationality.

- Our analysis suggests otherwise – choices under ambiguity are at least as rationalizable as choices under risk.

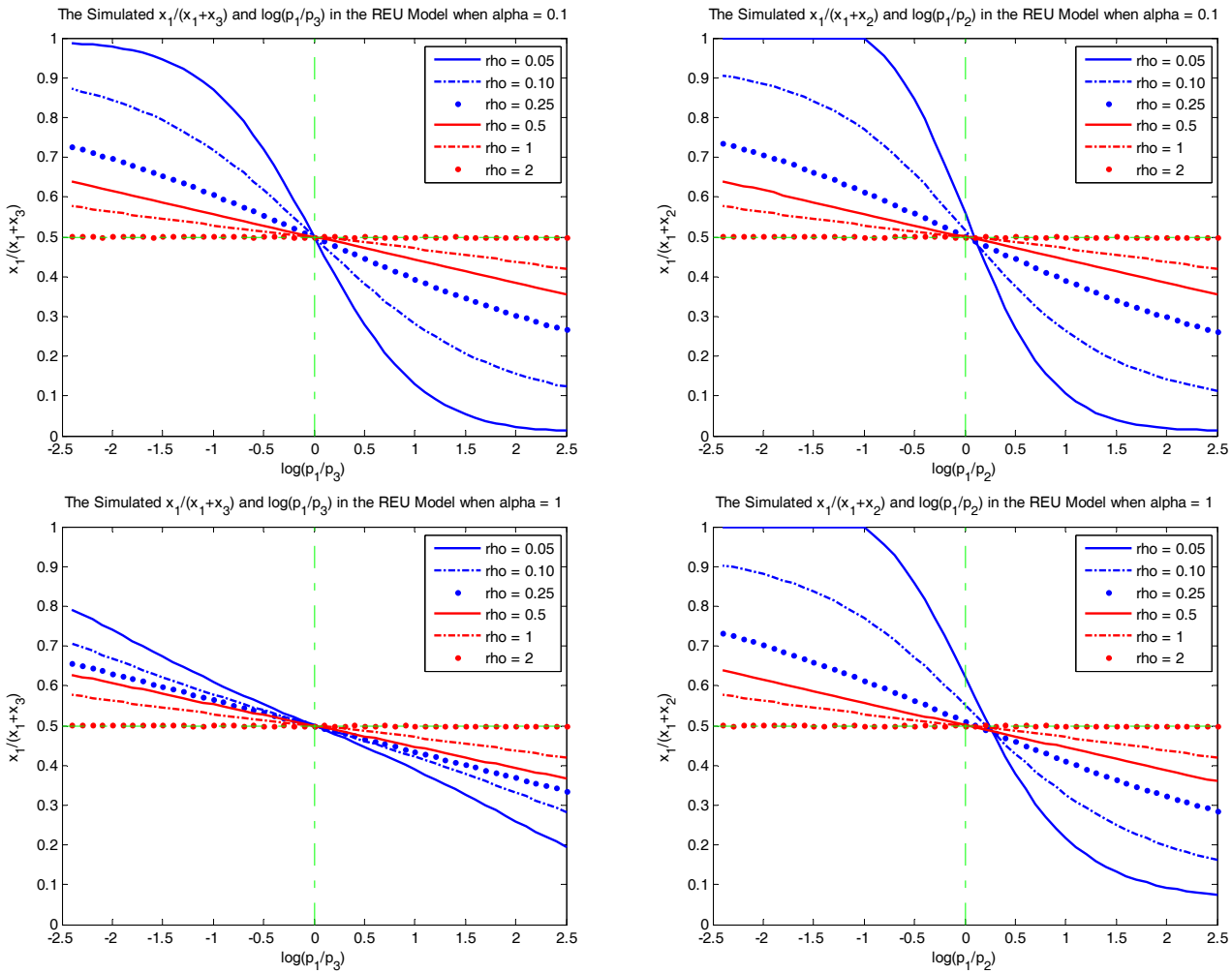

The distributions of CCEI scores
Fraction of subjects
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
CCEI
Risk
Ambiguity
Random

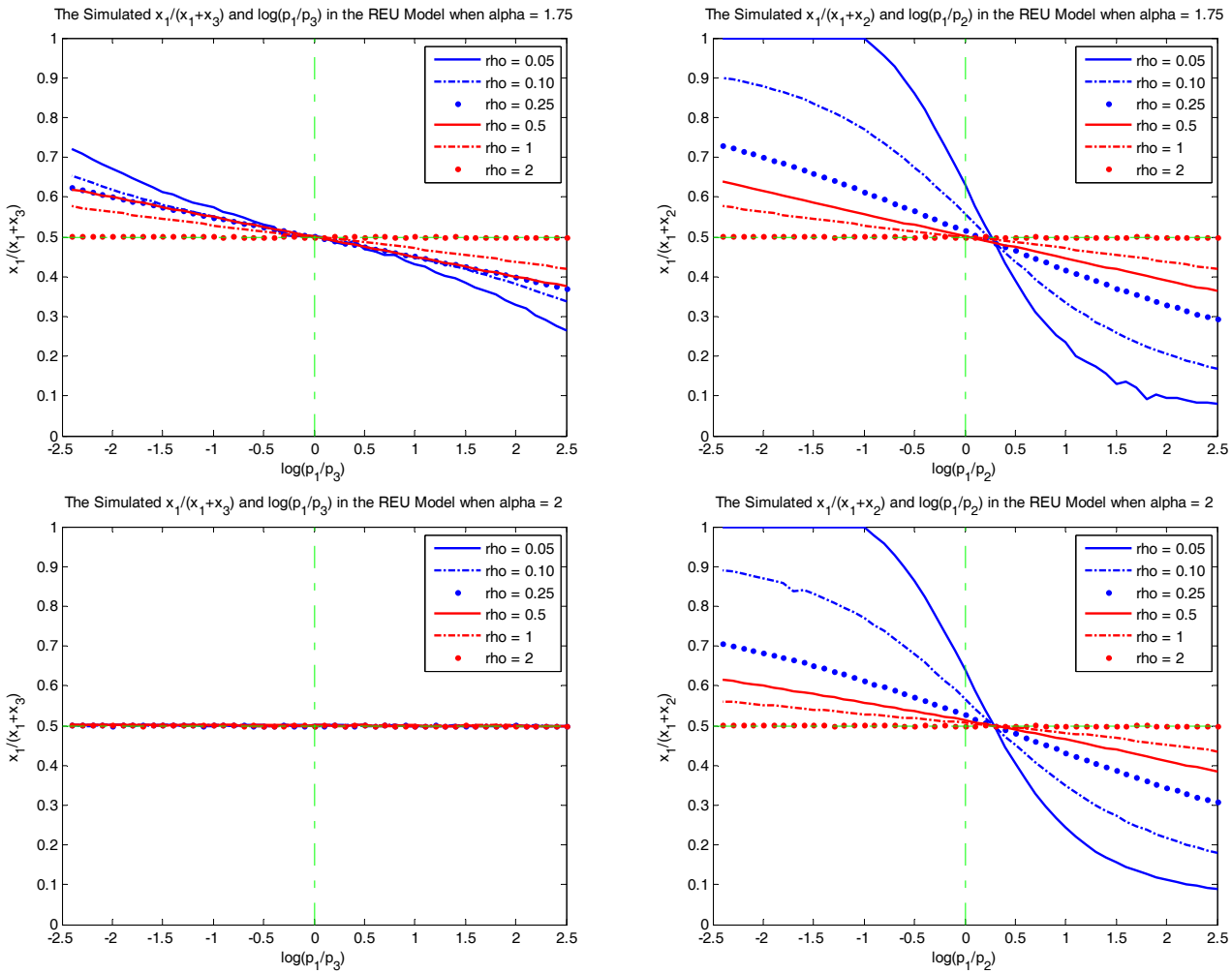
Structure, recoverability and extrapolation

The conventional parametric approach:

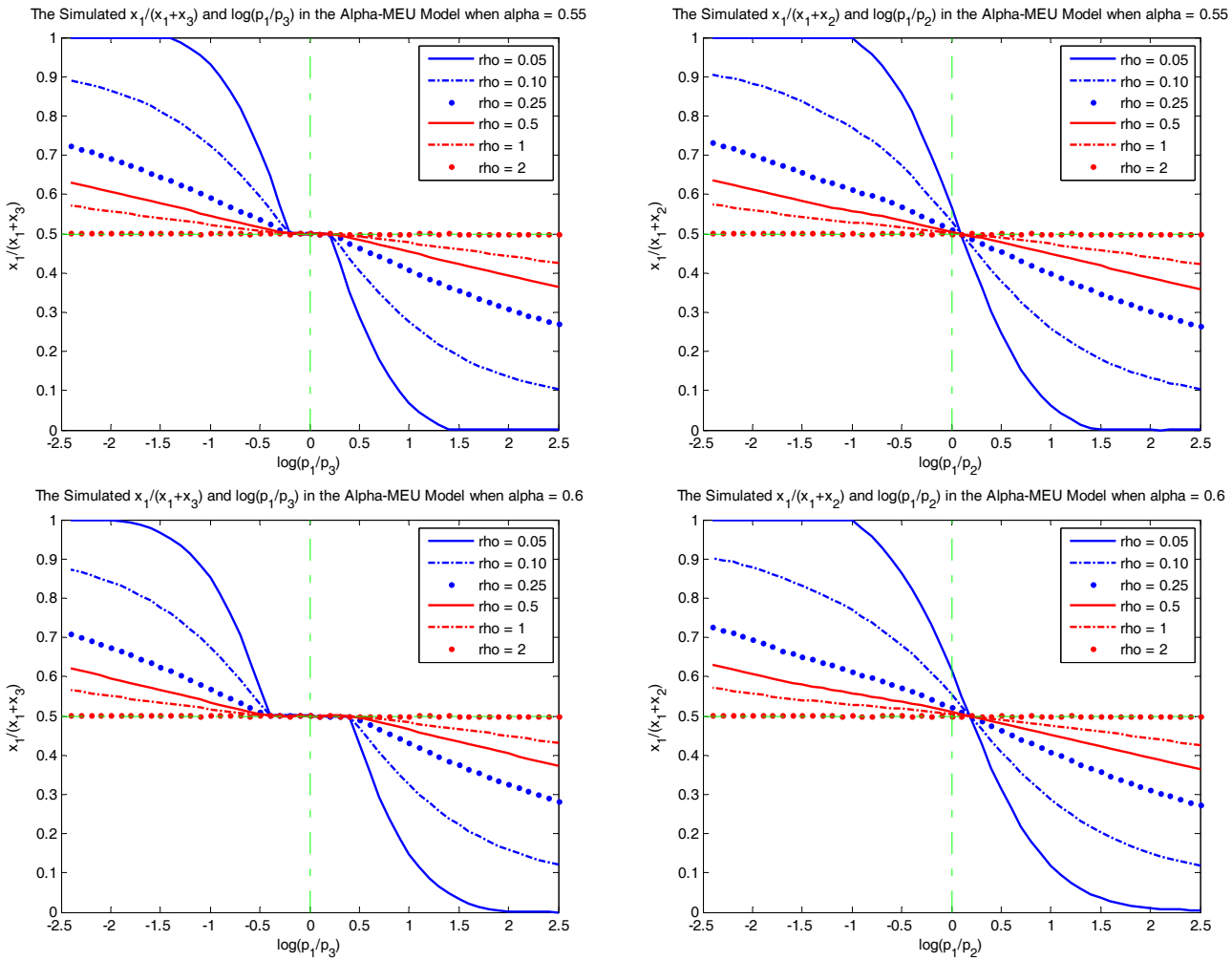
- Choose a parametric form for the underlying utility function and fit the associated demand function to the data.
- Test to see if they conform to the special restrictions imposed by hypotheses concerning functional structure.
- Construct an estimate of the underlying utility function and forecast demand behavior in new situations.

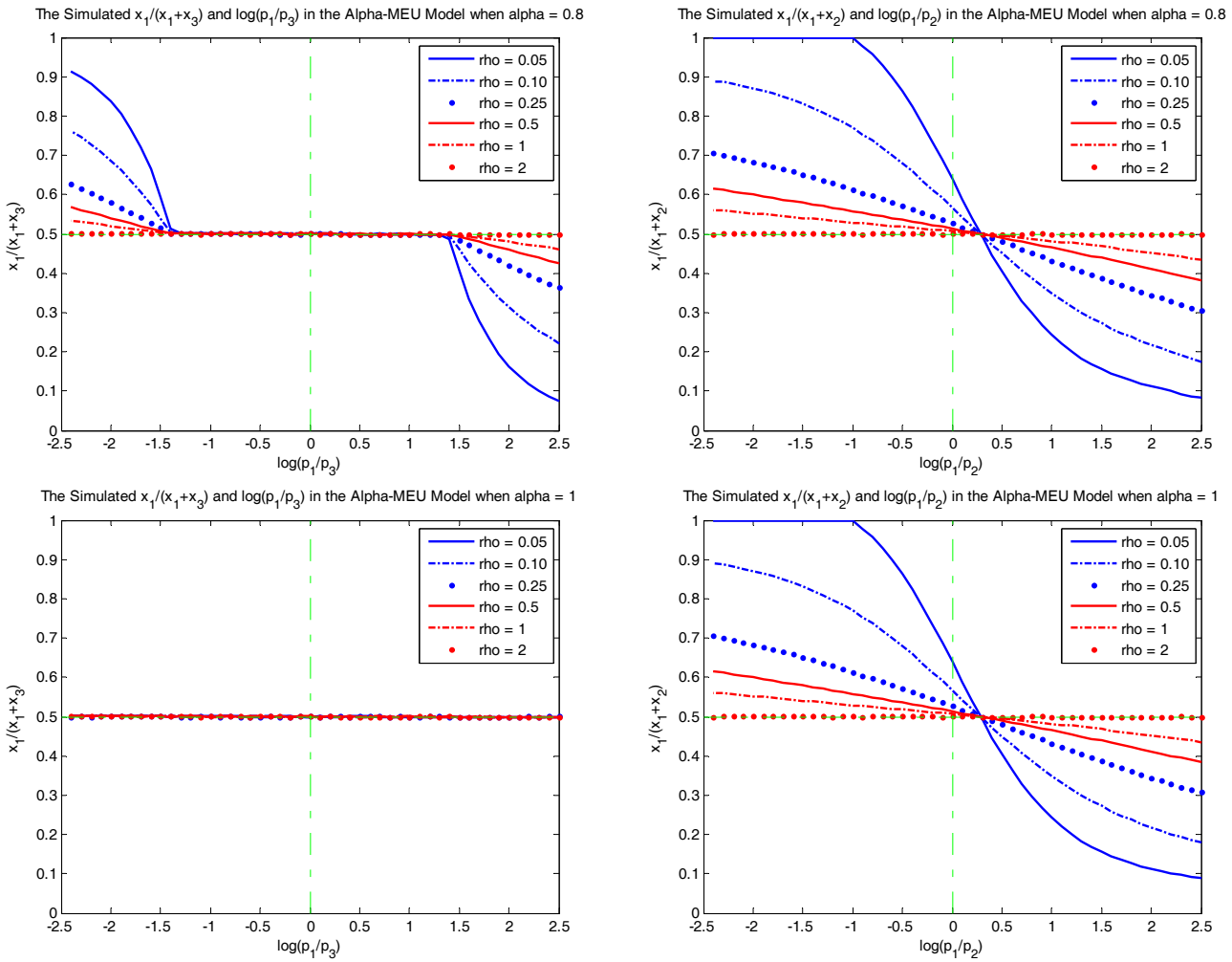
Recursive Expected Utility (REU)




The Simulated x1/(x1+x3) and log(p1/p3) in the REU Model when alpha = 1.75
The Simulated x1/(x1+x2) and log(p1/p2) in the REU Model when alpha = 1.75
The Simulated x1/(x1+x3) and log(p1/p3) in the REU Model when alpha = 2
The Simulated x1/(x1+x2) and log(p1/p2) in the REU Model when alpha = 2
x1/(x1+x3)
x1/(x1+x2)
log(p1/p3)
log(p1/p2)
rho = 0.05
rho = 0.10
rho = 0.25
rho = 0.5
rho = 1
rho = 2

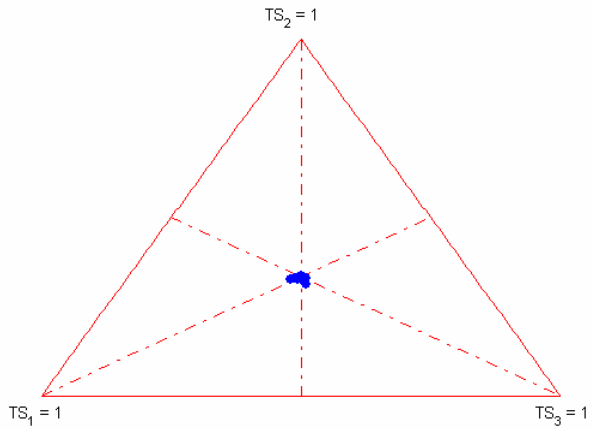
α-Maxmin Expected Utility (α-MEU)



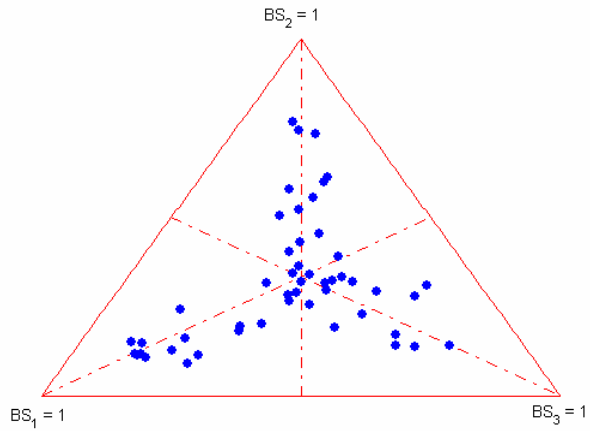

The Simulated x1/(x1+x3) and log(p1/p3) in the Alpha-MEU Model when alpha = 0.8
The Simulated x1/(x1+x2) and log(p1/p2) in the Alpha-MEU Model when alpha = 0.8
The Simulated x1/(x1+x3) and log(p1/p3) in the Alpha-MEU Model when alpha = 1
The Simulated x1/(x1+x2) and log(p1/p2) in the Alpha-MEU Model when alpha = 1
x1/(x1+x3)
x1/(x1+x2)
log(p1/p3)
log(p1/p2)
rho = 0.05
rho = 0.10
rho = 0.25
rho = 0.5
rho = 1
rho = 2

Individual-level data

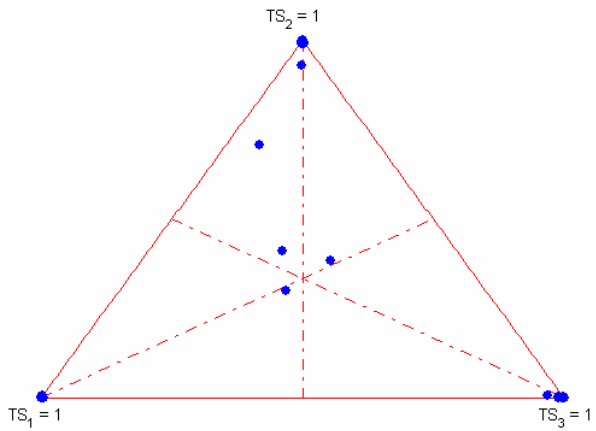
The Token Shares in the Ambiguity Treatment for Subject ID 129



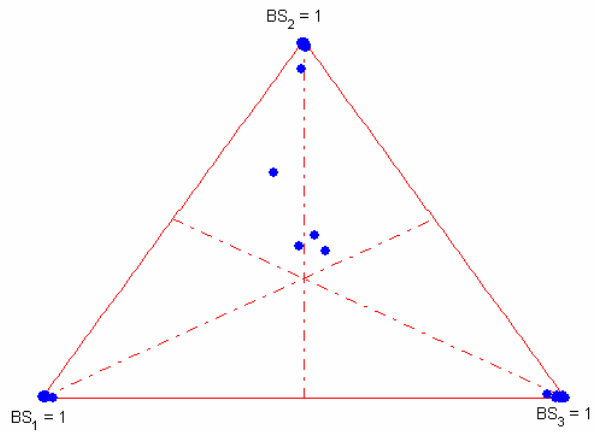
The Budget Shares in the Ambiguity Treatment for Subject ID 129



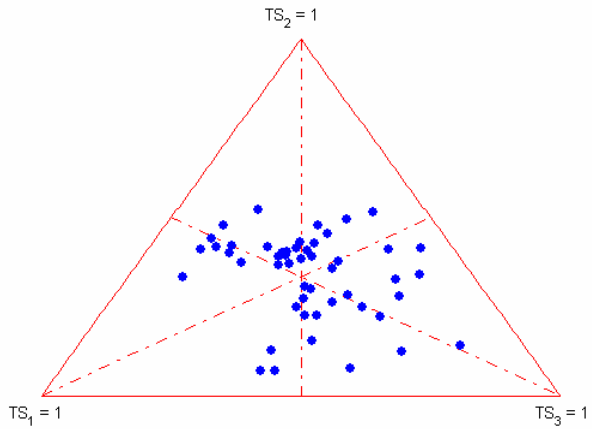
The Token Shares in the Ambiguity Treatment for Subject ID 314



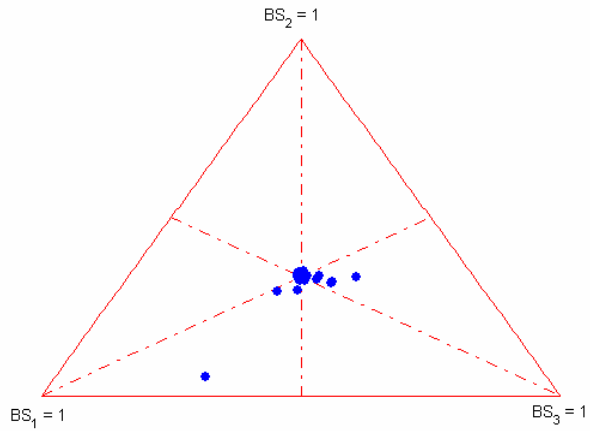
The Budget Shares in the Ambiguity Treatment for Subject ID 314



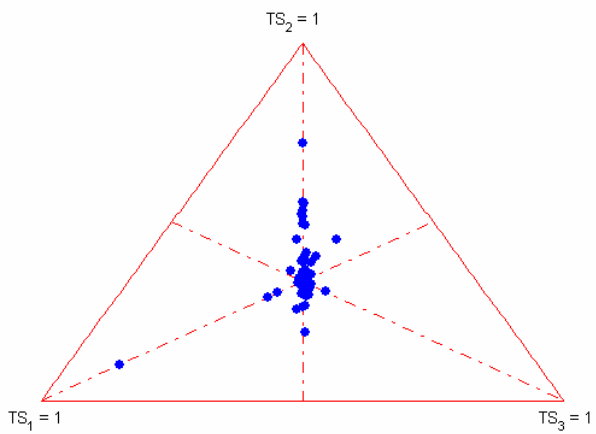
The Token Shares in the Ambiguity Treatment for Subject ID 339



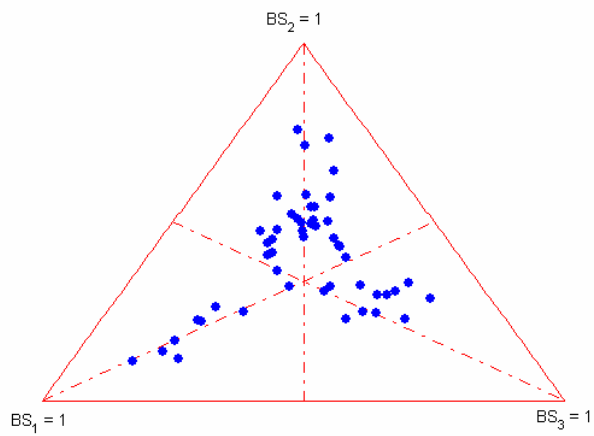
The Budget Shares in the Ambiguity Treatment for Subject ID 339



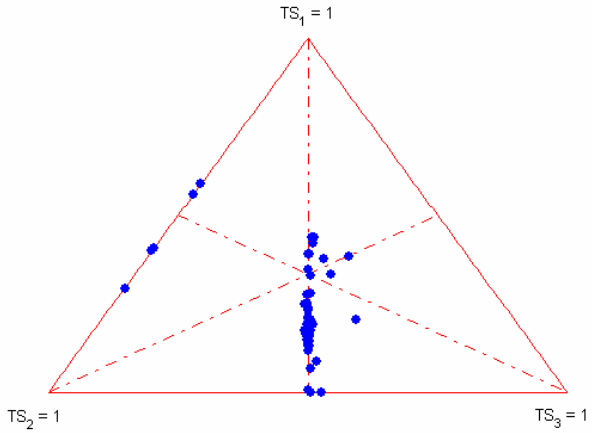
The Token Shares in the Ambiguity Treatment for Subject ID 322



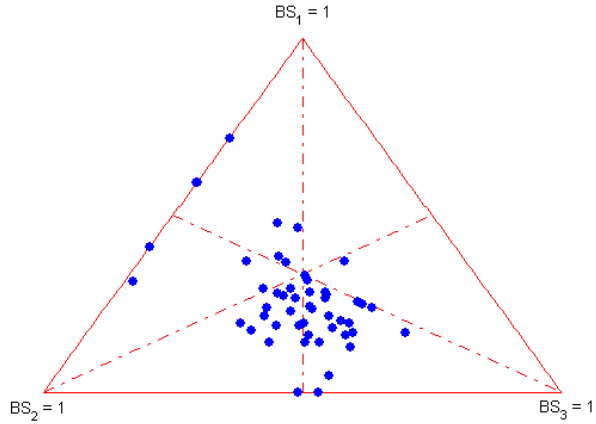
The Budget Shares in the Ambiguity Treatment for Subject ID 322



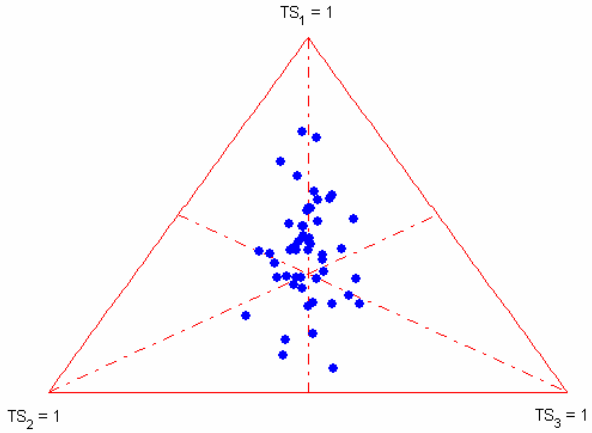
The Token Shares in the Ambiguity Treatment for Subject ID 130



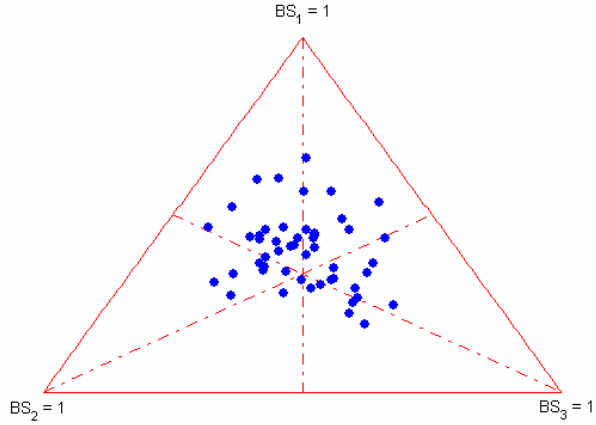
The Budget Shares in the Ambiguity Treatment for Subject ID 130



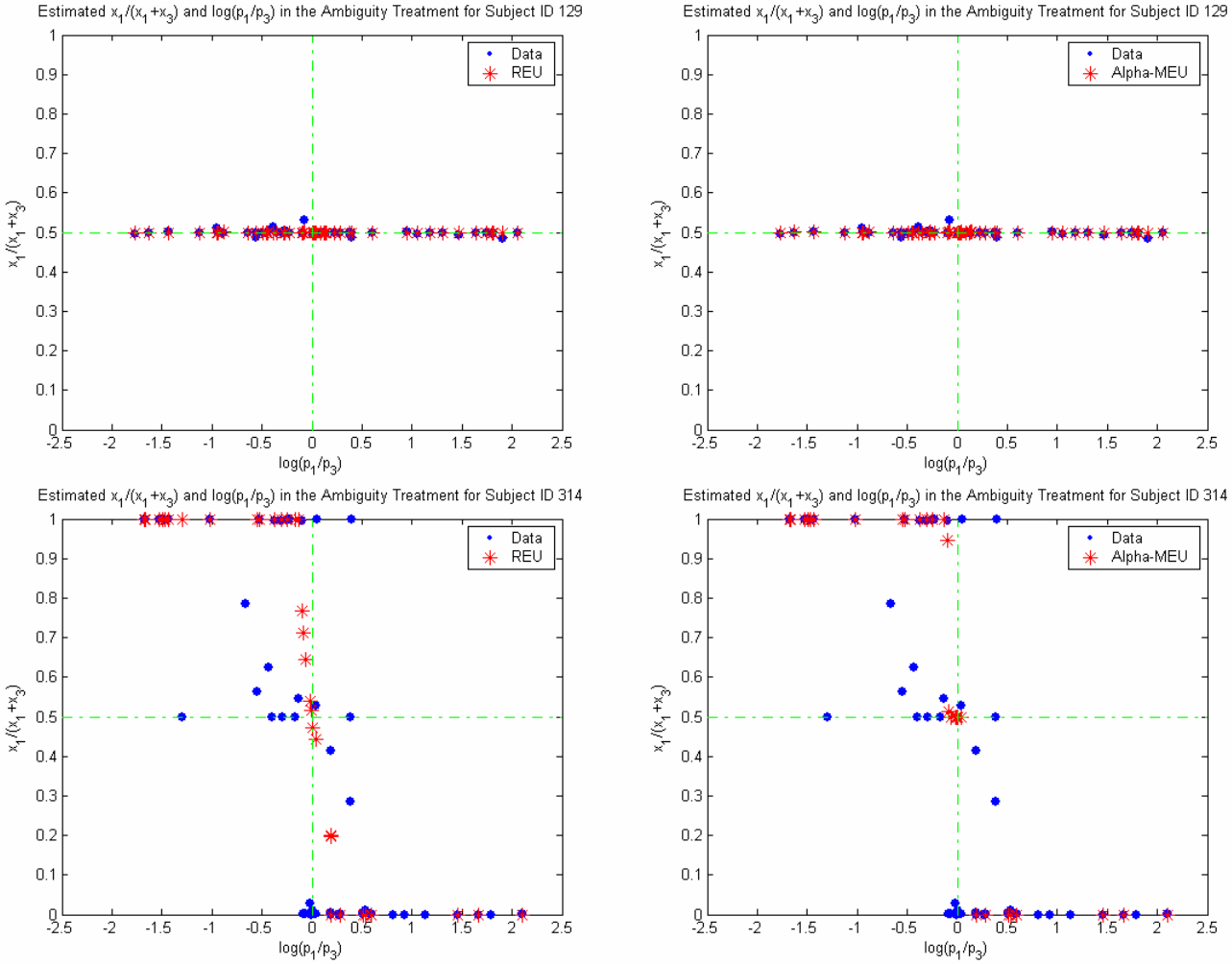
The Token Shares in the Ambiguity Treatment for Subject ID 407

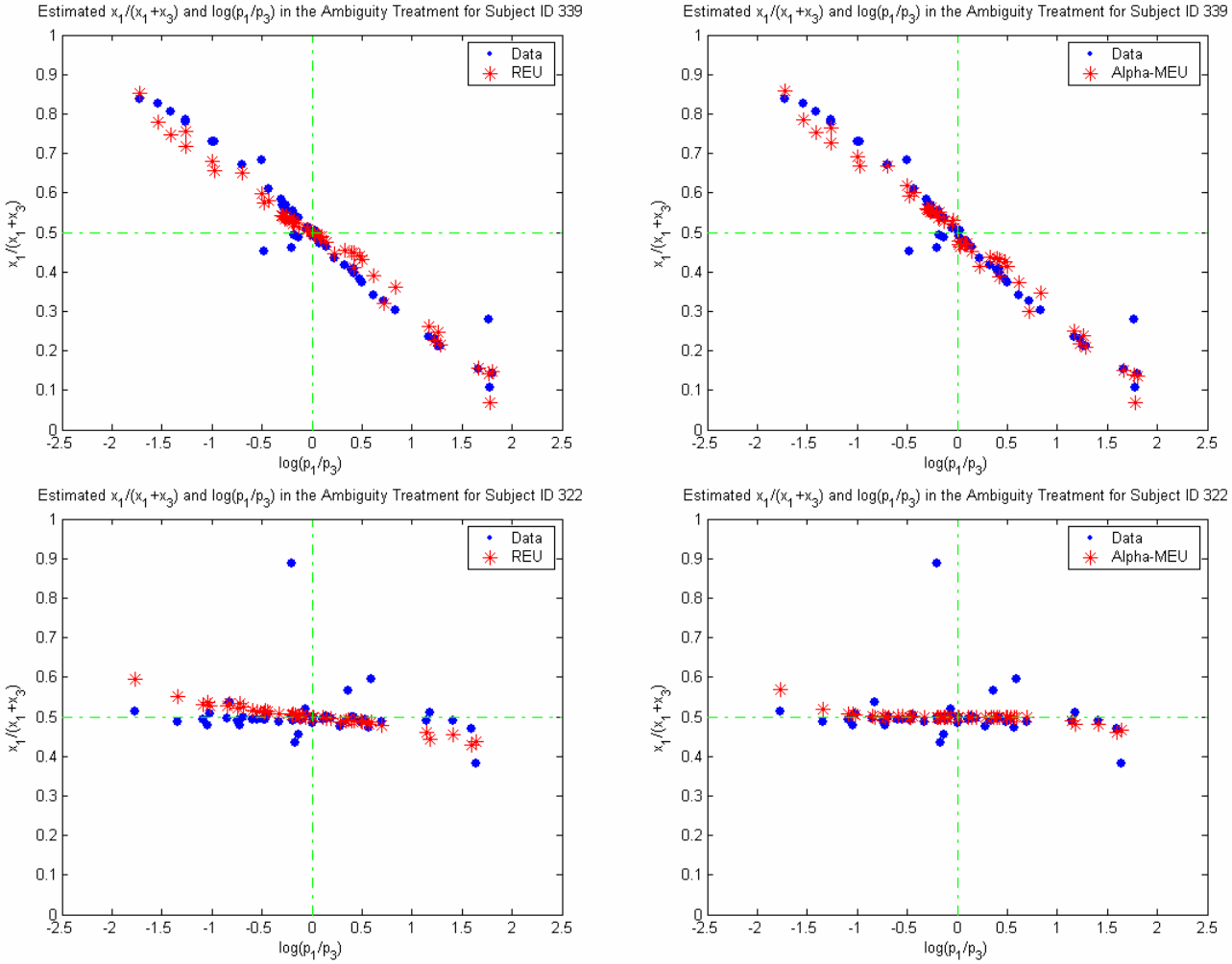


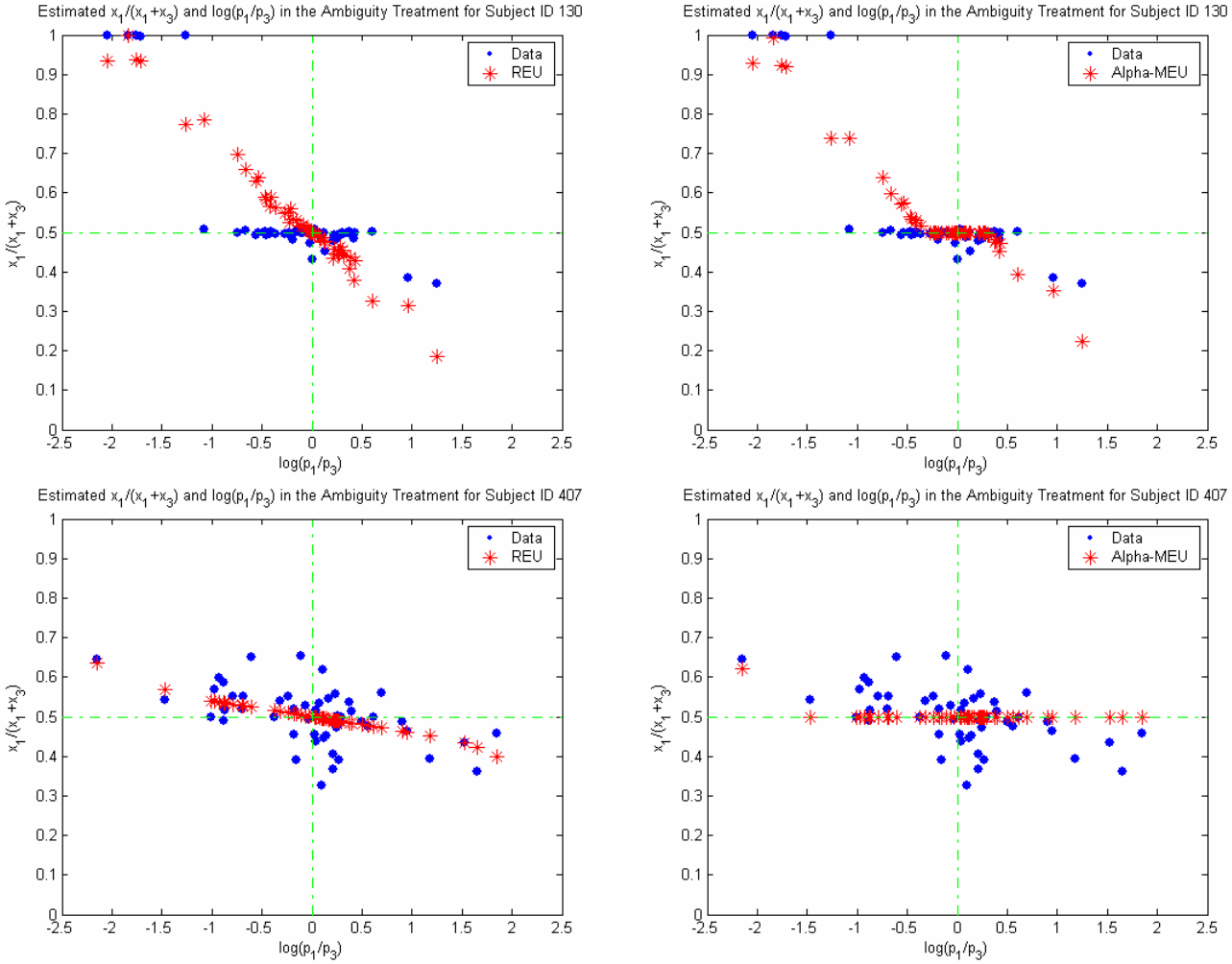
The Budget Shares in the Ambiguity Treatment for Subject ID 407



Estimation results




Estimated x1/(x1+x3) and log(p1/p3) in the Ambiguity Treatment for Subject ID 339
Data
REU
Estimated x1/(x1+x3) and log(p1/p3) in the Ambiguity Treatment for Subject ID 339
Data
Alpha-MEU
Estimated x1/(x1+x3) and log(p1/p3) in the Ambiguity Treatment for Subject ID 322
Data
REU
Estimated x1/(x1+x3) and log(p1/p3) in the Ambiguity Treatment for Subject ID 322
Data
Alpha-MEU
x1/(x1+x3)
log(p1/p3)


Estimated $x_1/(x_1+x_3)$ and $\log(p_1/p_3)$ in the Ambiguity Treatment for Subject ID 130
Data
REU
Estimated $x_1/(x_1+x_3)$ and $\log(p_1/p_3)$ in the Ambiguity Treatment for Subject ID 130
Data
Alpha-MEU
Estimated $x_1/(x_1+x_3)$ and $\log(p_1/p_3)$ in the Ambiguity Treatment for Subject ID 407
Data
REU
Estimated $x_1/(x_1+x_3)$ and $\log(p_1/p_3)$ in the Ambiguity Treatment for Subject ID 407
Data
Alpha-MEU
$x_1/(x_1+x_3)$
$\log(p_1/p_3)$

Takeaways

- There is much that can be learned about a theory from the data, quite apart from any notion of "testing" the theory.

- Develop tools, both methodological and analytical, for providing a more comprehensive analysis of individual choice.

- Avoid imposing theoretical preconceptions on the data and instead to recover preferences from the ground up.

- Behavior in more complex settings will require richer experimental data as well as new theoretical and analytical techniques.