UC Berkeley
Haas School of Business
Game Theory
(EMBA 296 & EWMBA 211)
Summer 2016

Review, oligopoly, auctions, and signaling

Block 3
Jun 29-30, 2017

LUPI

Many players simultaneously chose an integer between 1 and 99,999. Whoever chooses the lowest unique positive integer (LUPI) wins.

Question What does an equilibrium model of behavior predict in this game?

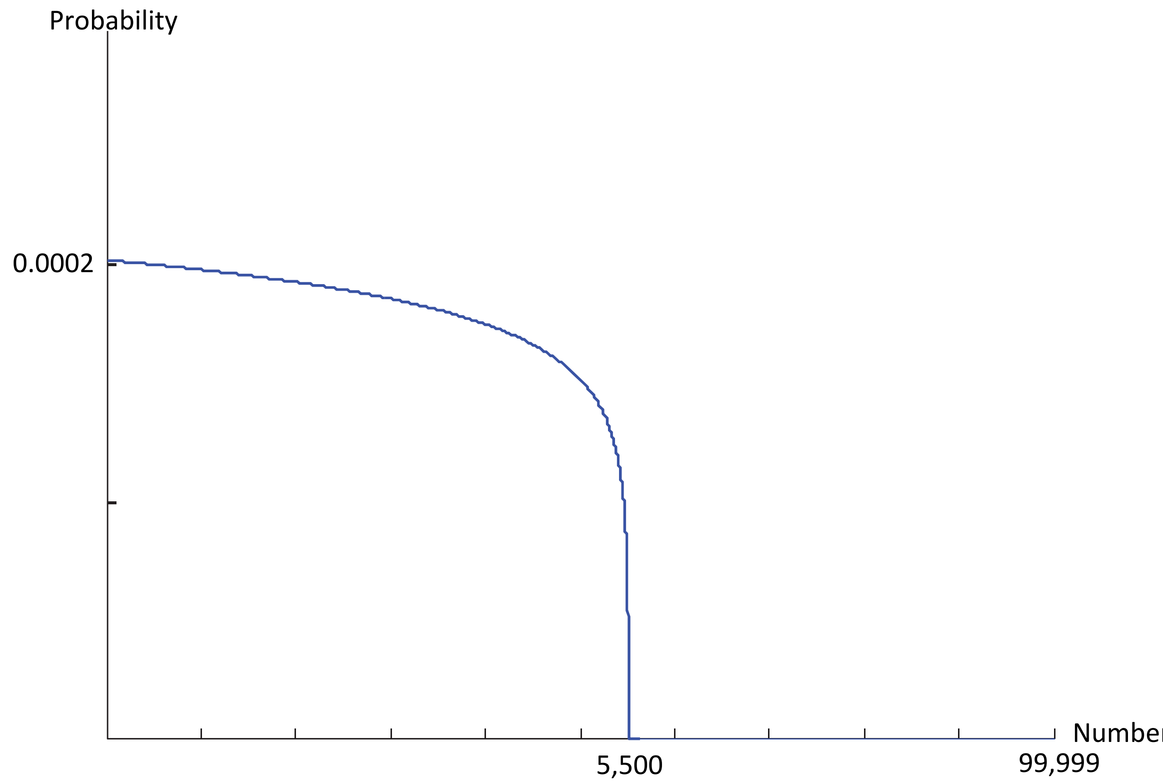
The field version of LUPI, called Limbo, was introduced by the government-owned Swedish gambling monopoly Svenska Spel. Despite its complexity, there is a surprising degree of convergence toward equilibrium.

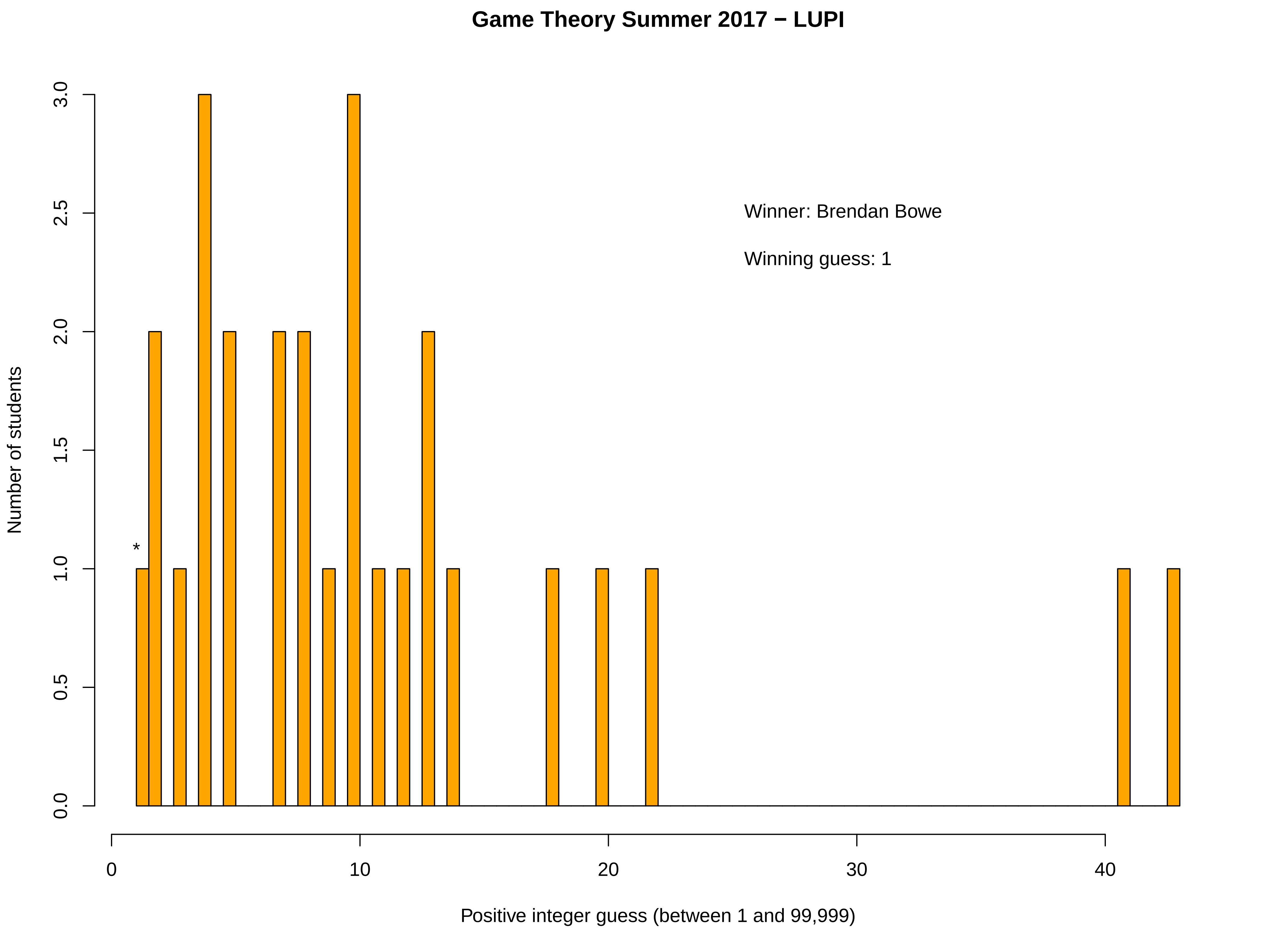
Games with population uncertainty relax the assumption that the exact number of players is common knowledge.

In particular, in a Poisson game (Myerson; 1998, 2000) the number of players N is a random variable that follows a Poisson distribution with mean n so the probability that $N = k$ is given by

$$\frac{e^{-n}n^k}{k!}$$

In the Swedish game the average number of players was $n = 53,783$ and number choices were positive integers up to $99,999$.


Probability
0.0002
Number
5,500
99,999



Game Theory Summer 2017 − LUPI
Winner: Brendan Bowe
Winning guess: 1
*
Number of students
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
10
20
30
40
Positive integer guess (between 1 and 99,999)

Morra

Player 1	Action 1	Guess 1	Player 2	Action 2	Guess 2	Total
Peter Martinez-Fo	1	3	Brendan Bowe	1	2	2
John Illia	2	4	Ivan Jelic	2	4	4
Logan Newell	1	2	Shawn Higbee	2	4	3
Robert Ethier	1	2	Kiran Kumar	2	4	3
Praveen Sampath	1	3	Nicholas Johnsto	2	3	3
Manjunathan Kum	2	3	Mike Della Penna	1	3	3
Leslie Mcmurchie	2	4	ROBOT	2	3	4
Yong Zhang	1	3	Nick Simmons-St	2	4	3
Zack Shalvarjian	2	4	Bud Heath	2	3	4
Adam Speert	1	3	Michael Toomey	1	2	2
OLY RILLERA	1	3	Kota Reichert	1	2	2
Hallie Fox	1	3	Gurjit Thandi	2	3	3
Sara Neff	1	3	David Kwon	2	4	3
Dinesh Kumar	1	3	Mathew Kottoor	2	3	3

A review of the main ideas

We study two (out of four) groups of game theoretic models:

[1] Strategic games – all players simultaneously choose their plan of action once and for all.

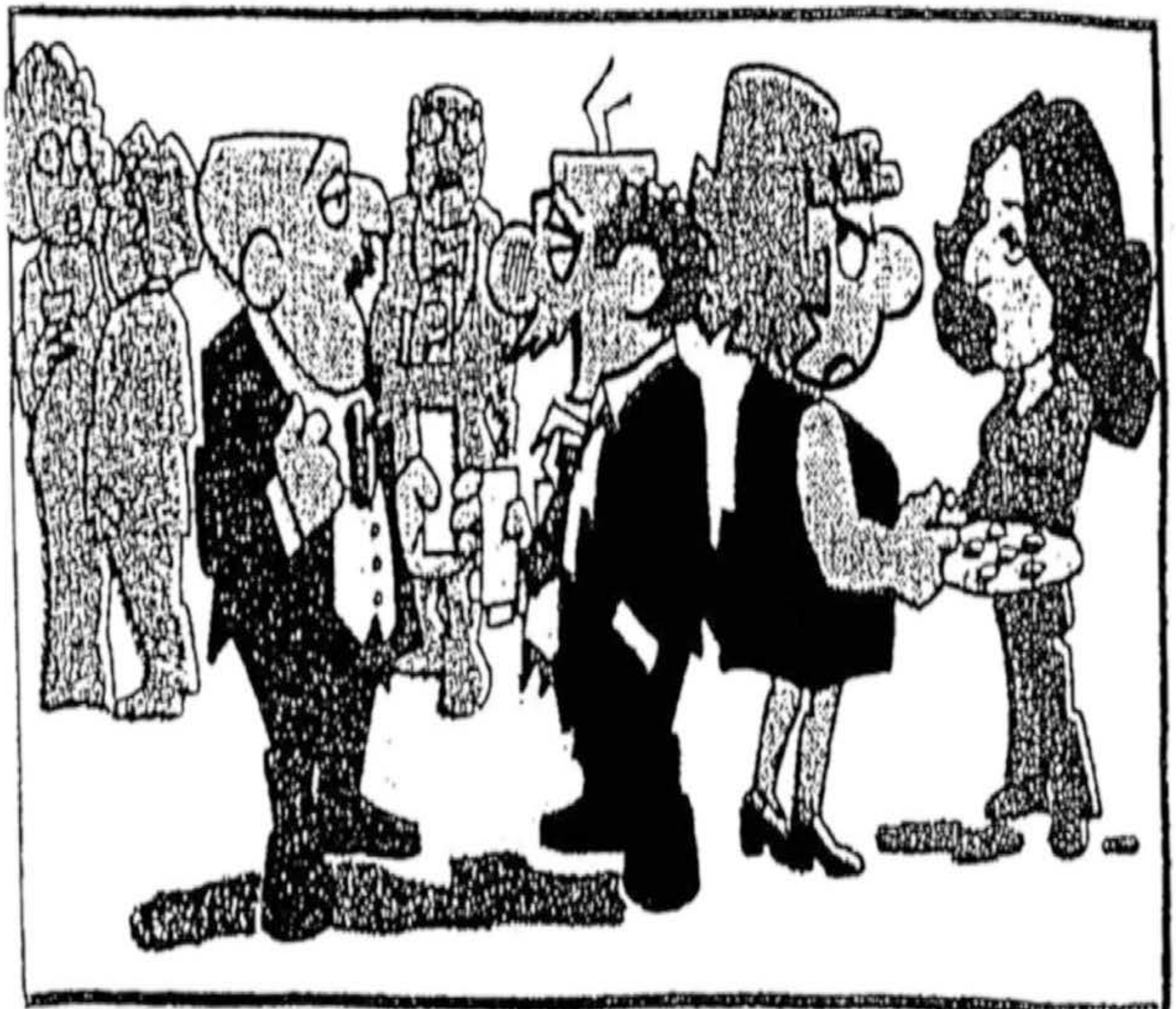
[2] Extensive games (with perfect information) – players choose sequentially (and fully informed about all previous actions).

A solution (equilibrium) is a systematic description of the outcomes that may emerge in a family of games. We study two solution concepts:

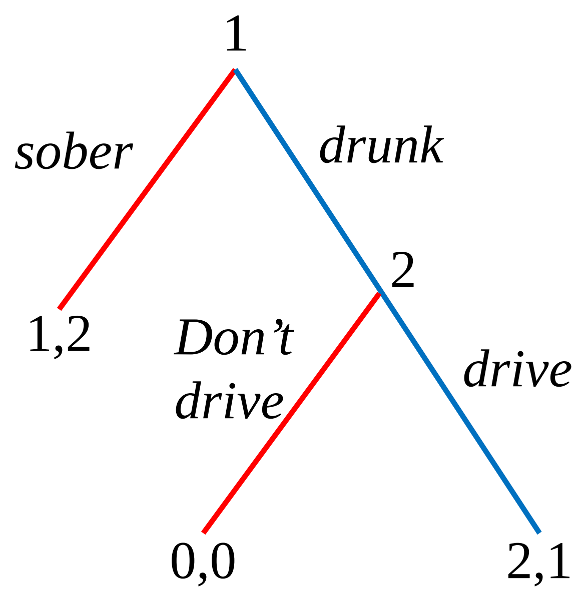
[1] Nash equilibrium – a steady state of the play of a <u>strategic</u> game (no player has a profitable deviation given the actions of the other players).

[1] Subgame equilibrium – a steady state of the play of an <u>extensive</u> game (a Nash equilibrium in every <u>subgame</u> of the extensive game).

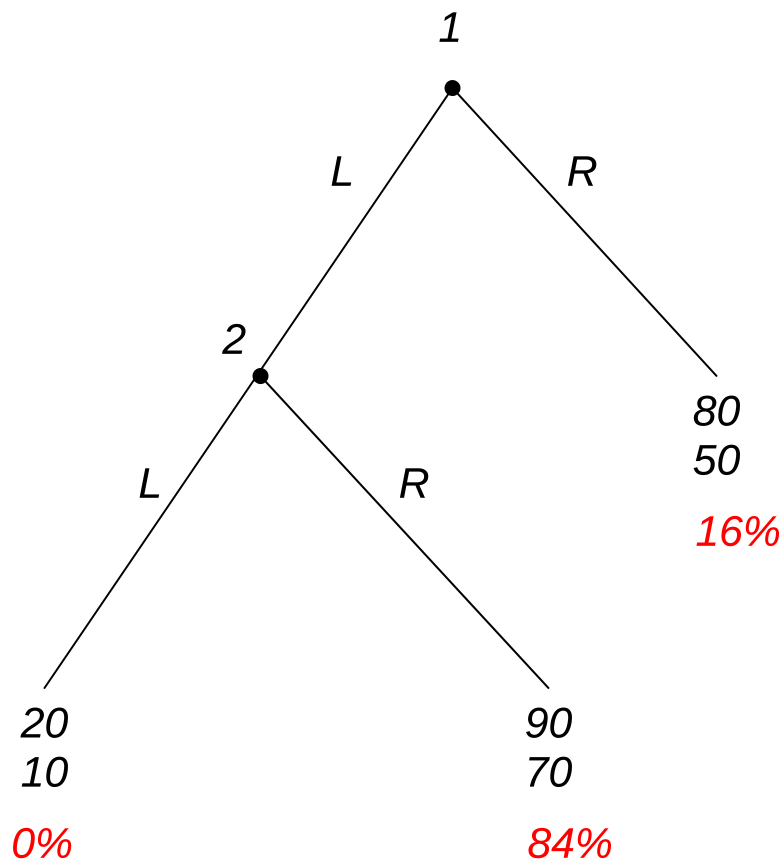
$\Longrightarrow$ Every subgame perfect equilibrium is also a Nash equilibrium.

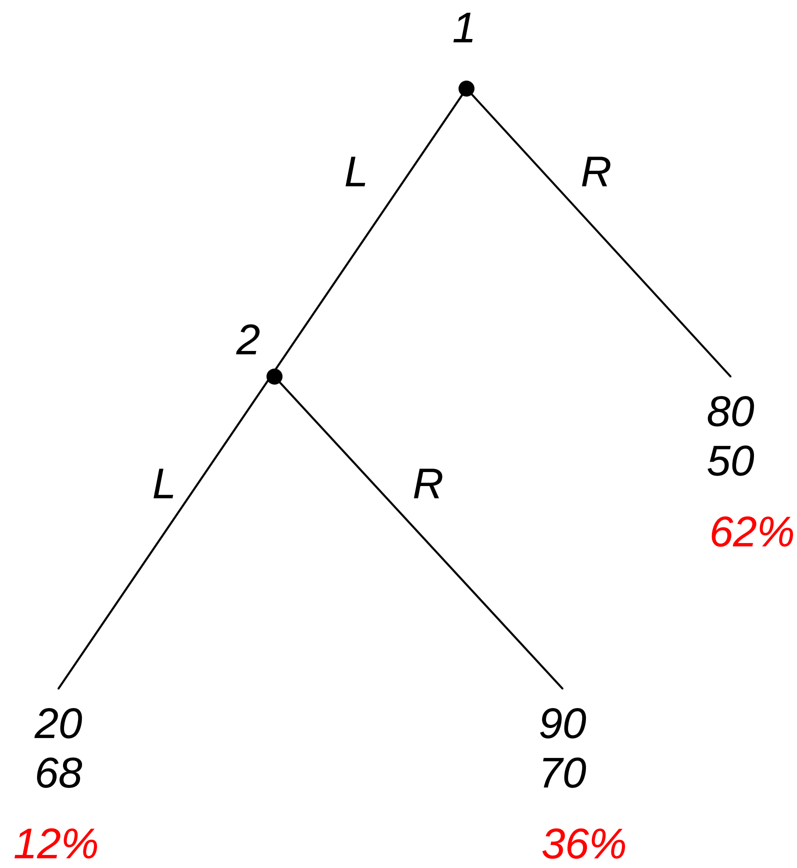


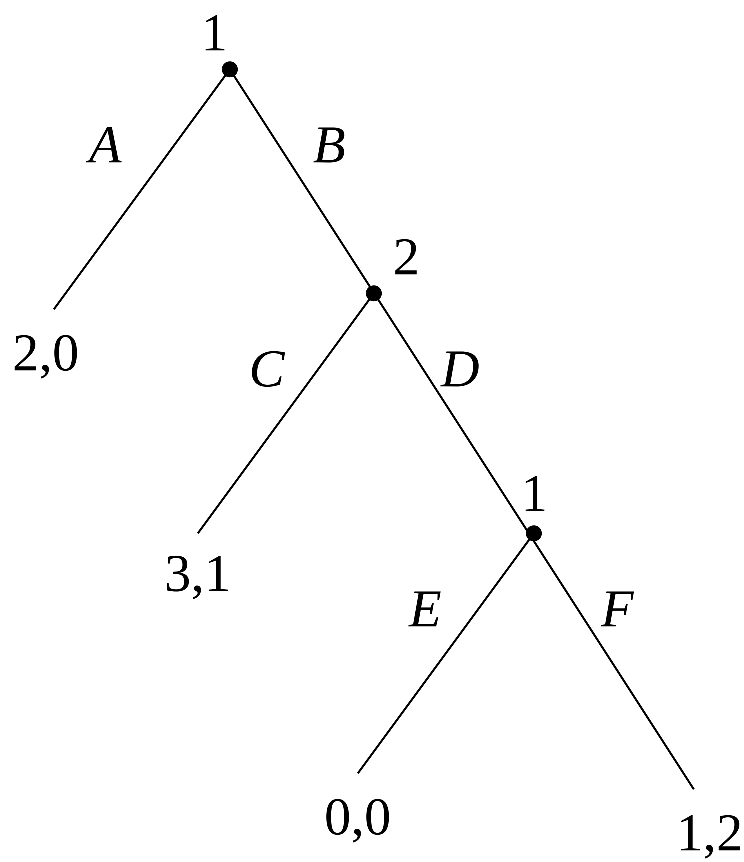
"LORETTA'S DRIVING BECAUSE I'M DRINKING, AND I'M DRINKING BECAUSE SHE'S DRIVING."


1
sober
drunk
1,2
2
Don't
drive
drive
0,0
2,1

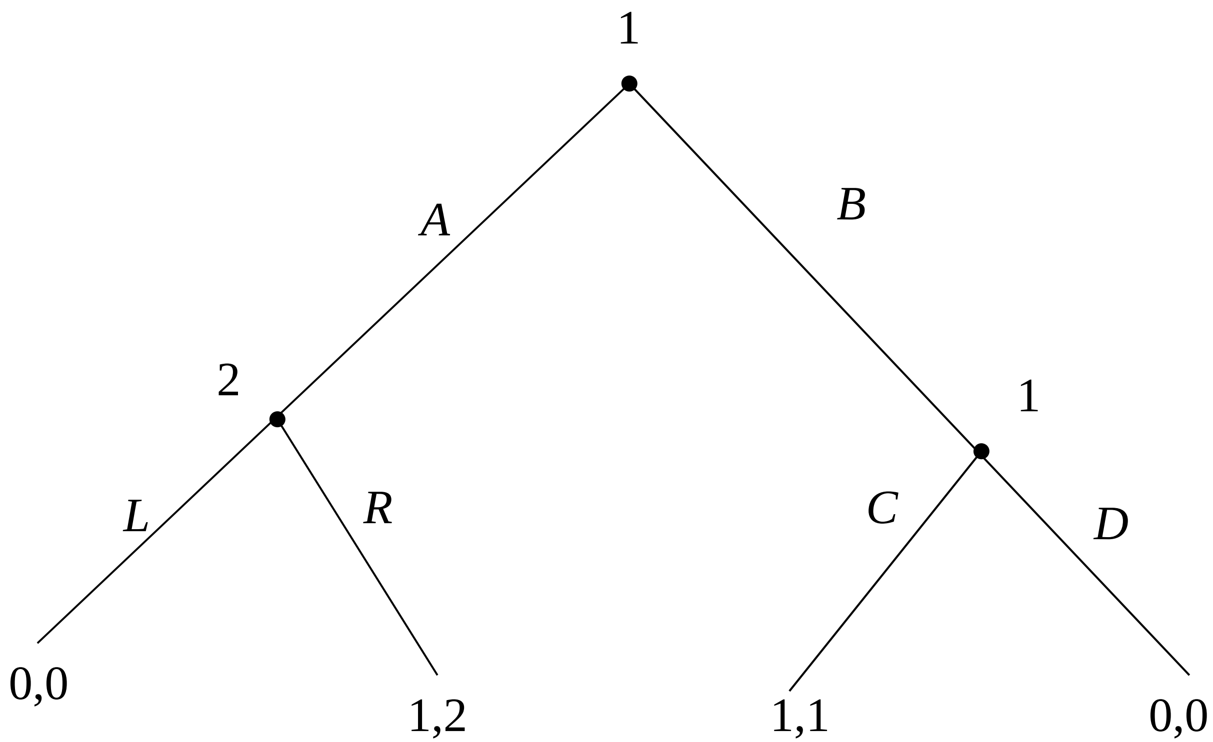
Two entry games in the laboratory

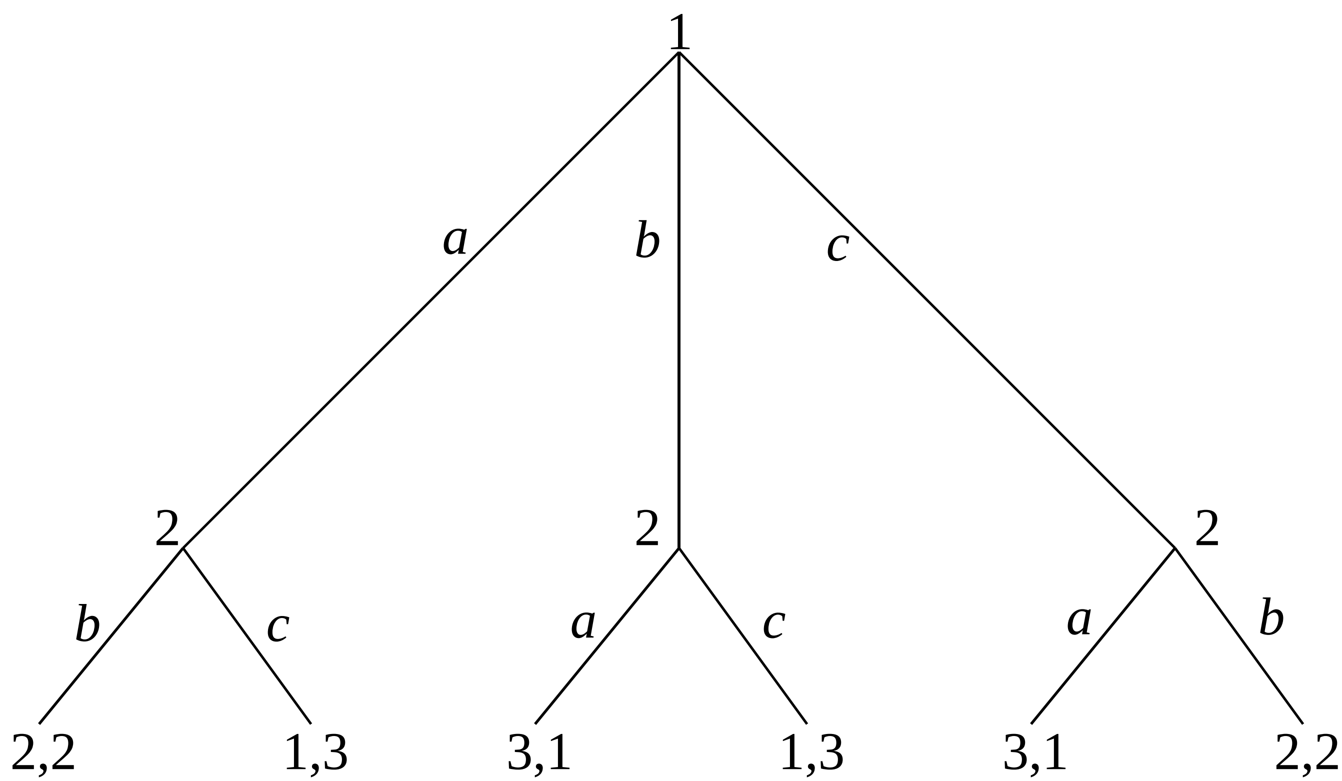



1
L
R
2
L
R
80
50
62%
20
68
12%
90
70
36%

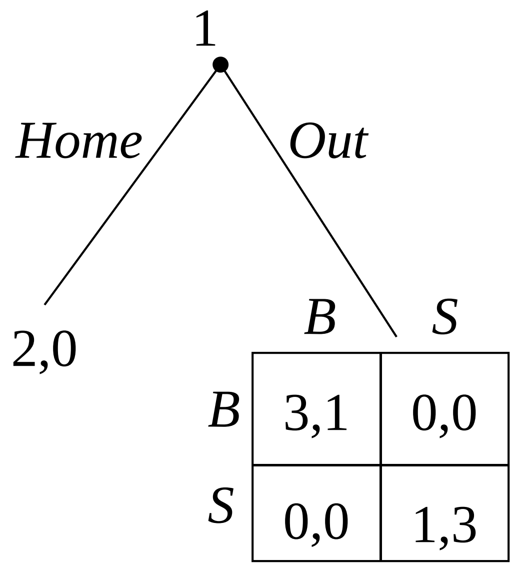

1
A
B
2
2,0
C
D
1
3,1
E
F
0,0
1,2

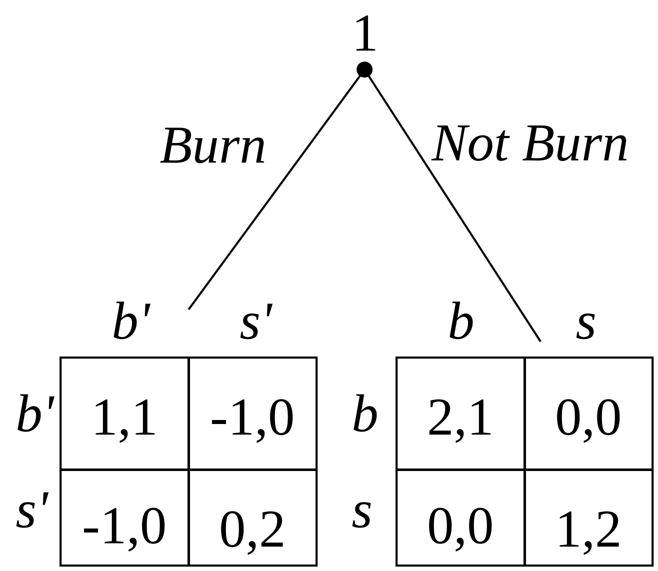
And one more example :-)




1
a
b
c
2
2
2
b
c
a
c
a
b
2,2
1,3
3,1
1,3
3,1
2,2

2/9 (two variants)




1
Burn
Not Burn
b′ s′
b′ 1,1 -1,0
s′ -1,0 0,2
b s
b 2,1 0,0
s 0,0 1,2

	$b'b$	$b's$	$s'b$	$s's$
$Bb'b$	1, 1	1, 1	−1, 0	−1, 0
$Bb's$	1, 1	1, 1	−1, 0	−1, 0
$Bs'b$	−1, 0	−1, 0	0, 2	0, 2
$Bs's$	−1, 0	−1, 0	0, 2	0, 2
$Nb'b$	2, 1	0, 0	2, 1	0, 0
$Nb's$	0, 0	1, 2	0, 0	1, 2
$Ns'b$	2, 1	0, 0	2, 1	0, 0
$Ns's$	0, 0	1, 2	0, 0	1, 2

Oligopolistic competition
(in strategic and extensive forms)

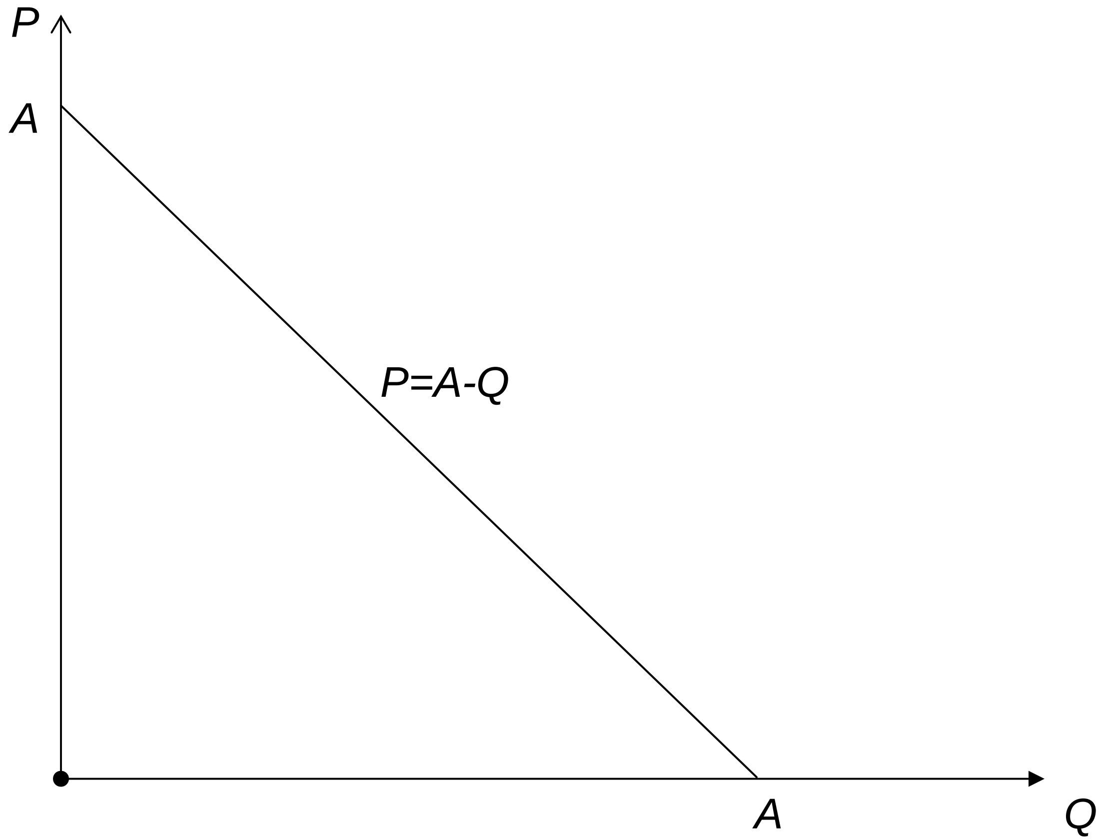
Cournot's oligopoly model (1838)

- A single good is produced by two firms (the industry is a "duopoly").
- The cost for firm $i = 1, 2$ for producing q_i units of the good is given by $c_i q_i$ ("unit cost" is constant equal to $c_i > 0$).
- If the firms' total output is $Q = q_1 + q_2$ then the market price is

  $$P = A - Q$$

  if $A \geq Q$ and zero otherwise (linear inverse demand function). We also assume that $A > c$.

The inverse demand function



To find the Nash equilibria of the Cournot's game, we can use the procedures based on the firms' best response functions.

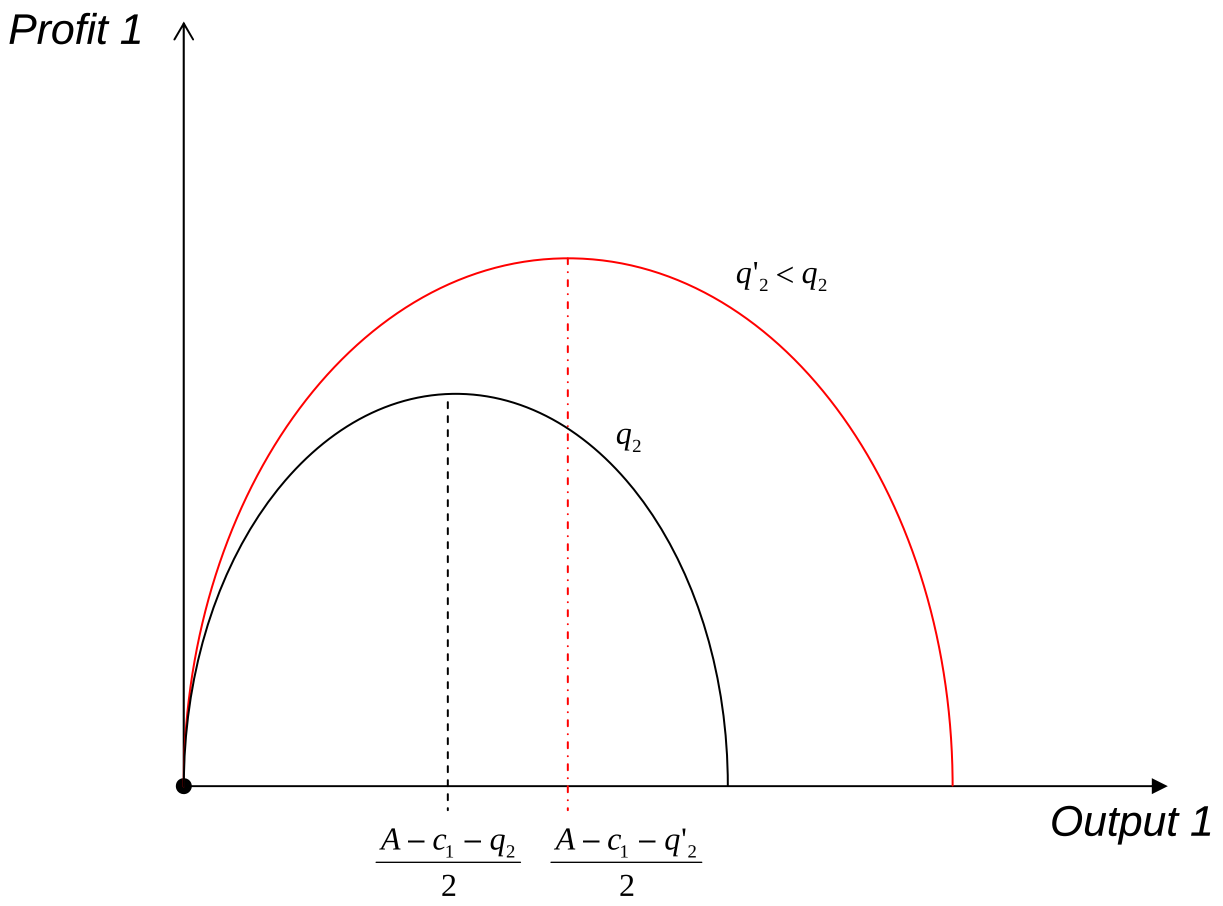
But first we need the firms payoffs (profits):

$$\begin{aligned} \pi_1 &= Pq_1 - c_1 q_1 \\ &= (A - Q)q_1 - c_1 q_1 \\ &= (A - q_1 - q_2)q_1 - c_1 q_1 \\ &= (A - q_1 - q_2 - c_1)q_1 \end{aligned}$$

and similarly,

$$\pi_2 = (A - q_1 - q_2 - c_2)q_2$$

Firm 1's profit as a function of its output (given firm 2's output)



To find firm 1's best response to any given output q_2 of firm 2, we need to study firm 1's profit as a function of its output q_1 for given values of q_2.

Using calculus, we set the derivative of firm 1's profit with respect to q_1 equal to zero and solve for q_1:

$$q_1 = \frac{1}{2}(A - q_2 - c_1).$$

We conclude that the best response of firm 1 to the output q_2 of firm 2 depends on the values of q_2 and c_1.

Because firm 2's cost function is $c_2 \neq c_1$, its best response function is given by

$$q_2 = \frac{1}{2}(A - q_1 - c_2).$$

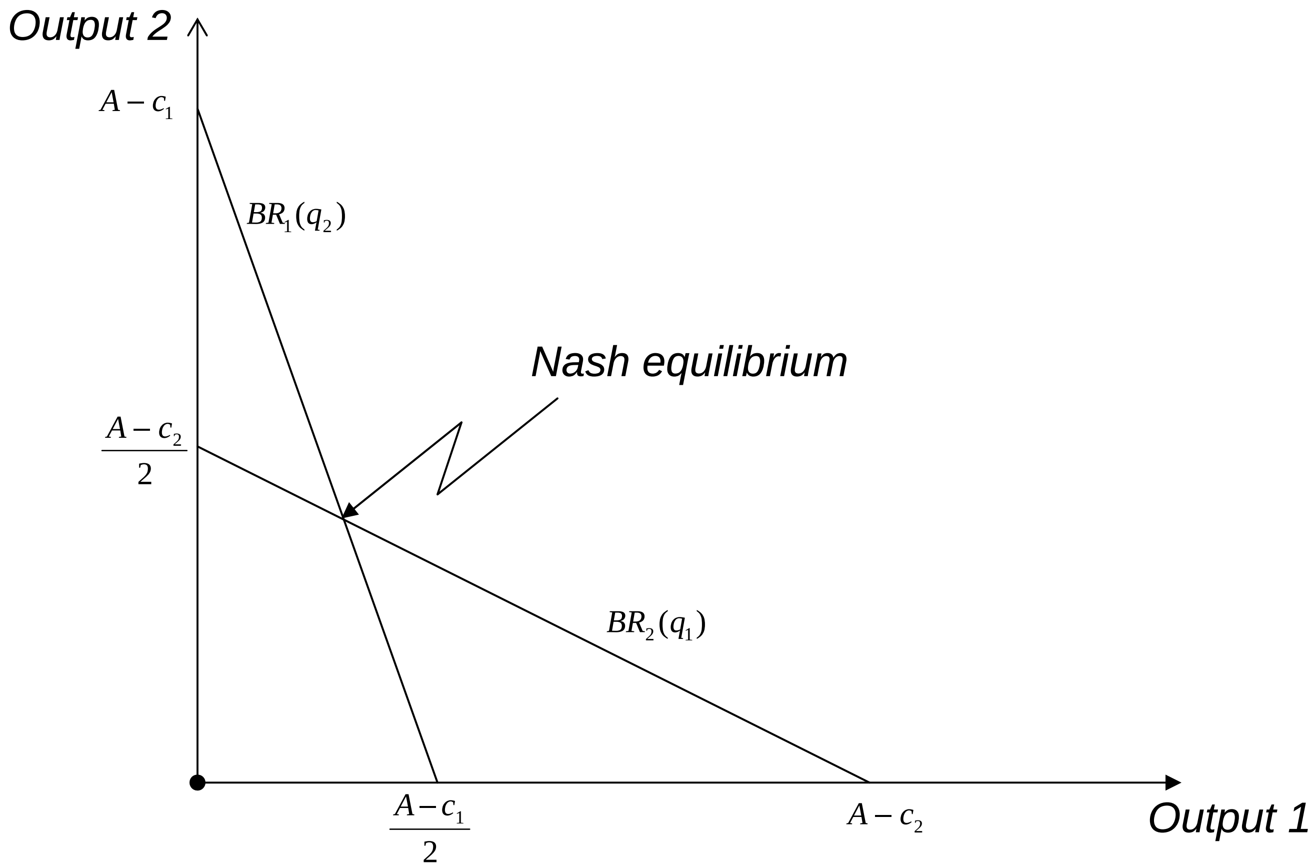
A Nash equilibrium of the Cournot's game is a pair (q_1^*, q_2^*) of outputs such that q_1^* is a best response to q_2^* and q_2^* is a best response to q_1^*.

From the figure below, we see that there is exactly one such pair of outputs

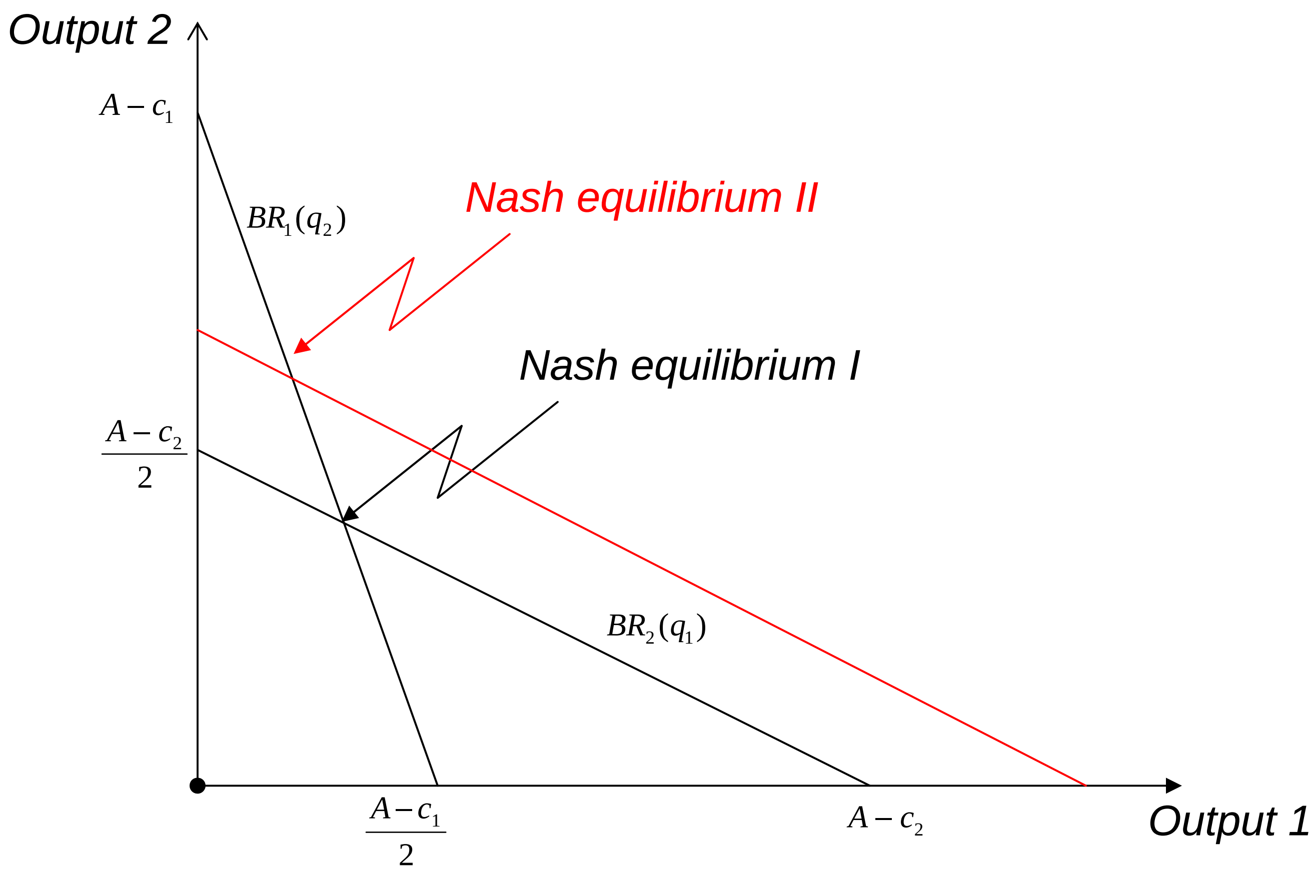
$$q_1^* = \frac{A+c_2-2c_1}{3} \quad \text{and} \quad q_2^* = \frac{A+c_1-2c_2}{3}$$

which is the solution to the two equations above.

The best response functions in the Cournot's duopoly game



Nash equilibrium comparative statics
(a decrease in the cost of firm 2)



A question: what happens when consumers are willing to pay more (*A* increases)?

In summary, this simple Cournot's duopoly game has a unique Nash equilibrium.

Two economically important properties of the Nash equilibrium are (to economic regulatory agencies):

[1] The relation between the firms' equilibrium profits and the profit they could make if they act collusively.

[2] The relation between the equilibrium profits and the number of firms.

[1] Collusive outcomes: in the Cournot's duopoly game, there is a pair of outputs at which *both* firms' profits exceed their levels in a Nash equilibrium.

[2] Competition: The price at the Nash equilibrium if the two firms have the *same* unit cost $c_1 = c_2 = c$ is given by

$$\begin{aligned} P^* &= A - q_1^* - q_2^* \\ &= \frac{1}{3}(A + 2c) \end{aligned}$$

which is above the unit cost c. But as the number of firm increases, the equilibrium price deceases, approaching c (zero profits!).

Stackelberg's duopoly model (1934)

How do the conclusions of the Cournot's duopoly game change when the firms move sequentially? Is a firm better off moving before or after the other firm?

Suppose that $c_1 = c_2 = c$ and that firm 1 moves at the start of the game. We may use backward induction to find the subgame perfect equilibrium.

– First, for *any* output q_1 of firm 1, we find the output q_2 of firm 2 that maximizes its profit. Next, we find the output q_1 of firm 1 that maximizes its profit, *given the strategy* of firm 2.

Firm 2

Since firm 2 moves after firm 1, a strategy of firm 2 is a *function* that associate an output q_2 for firm 2 for each possible output q_1 of firm 1.

We found that under the assumptions of the Cournot's duopoly game Firm 2 has a unique best response to each output q_1 of firm 1, given by

$$q_2 = \frac{1}{2}(A - q_1 - c)$$

(Recall that $c_1 = c_2 = c$).

Firm 1

Firm 1's strategy is the output q_1 the maximizes

$$\pi_1 = (A - q_1 - q_2 - c)q_1 \quad \text{subject to} \quad q_2 = \tfrac{1}{2}(A - q_1 - c)$$

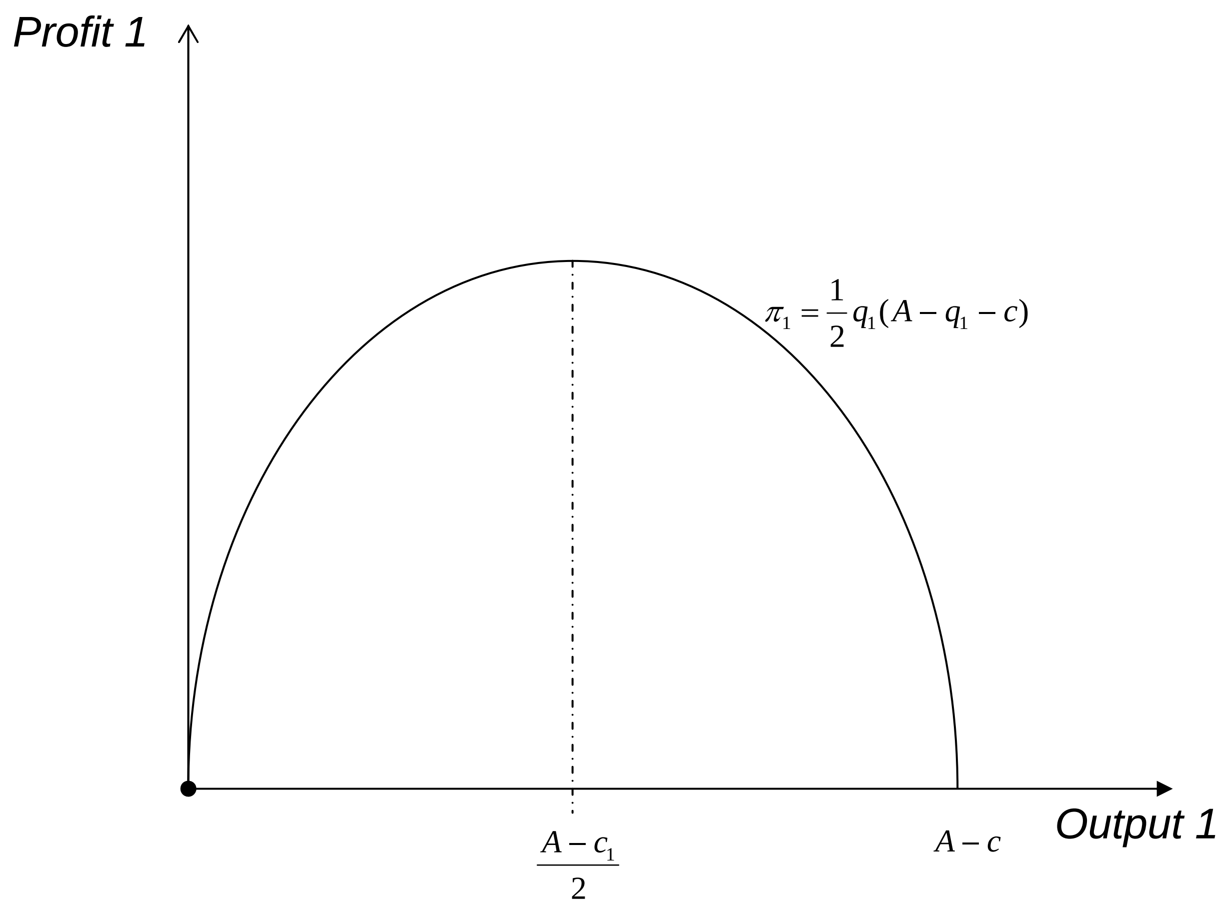
Thus, firm 1 maximizes

$$\pi_1 = (A - q_1 - (\frac{1}{2}(A - q_1 - c)) - c)q_1 = \frac{1}{2}q_1(A - q_1 - c).$$

This function is quadratic in q_1 that is zero when $q_1 = 0$ and when $q_1 = A - c$. Thus its maximizer is

$$q_1^* = \frac{1}{2}(A - c).$$

Firm 1's (first-mover) profit in Stackelberg's duopoly game



We conclude that Stackelberg's duopoly game has a unique subgame perfect equilibrium, in which firm 1's strategy is the output

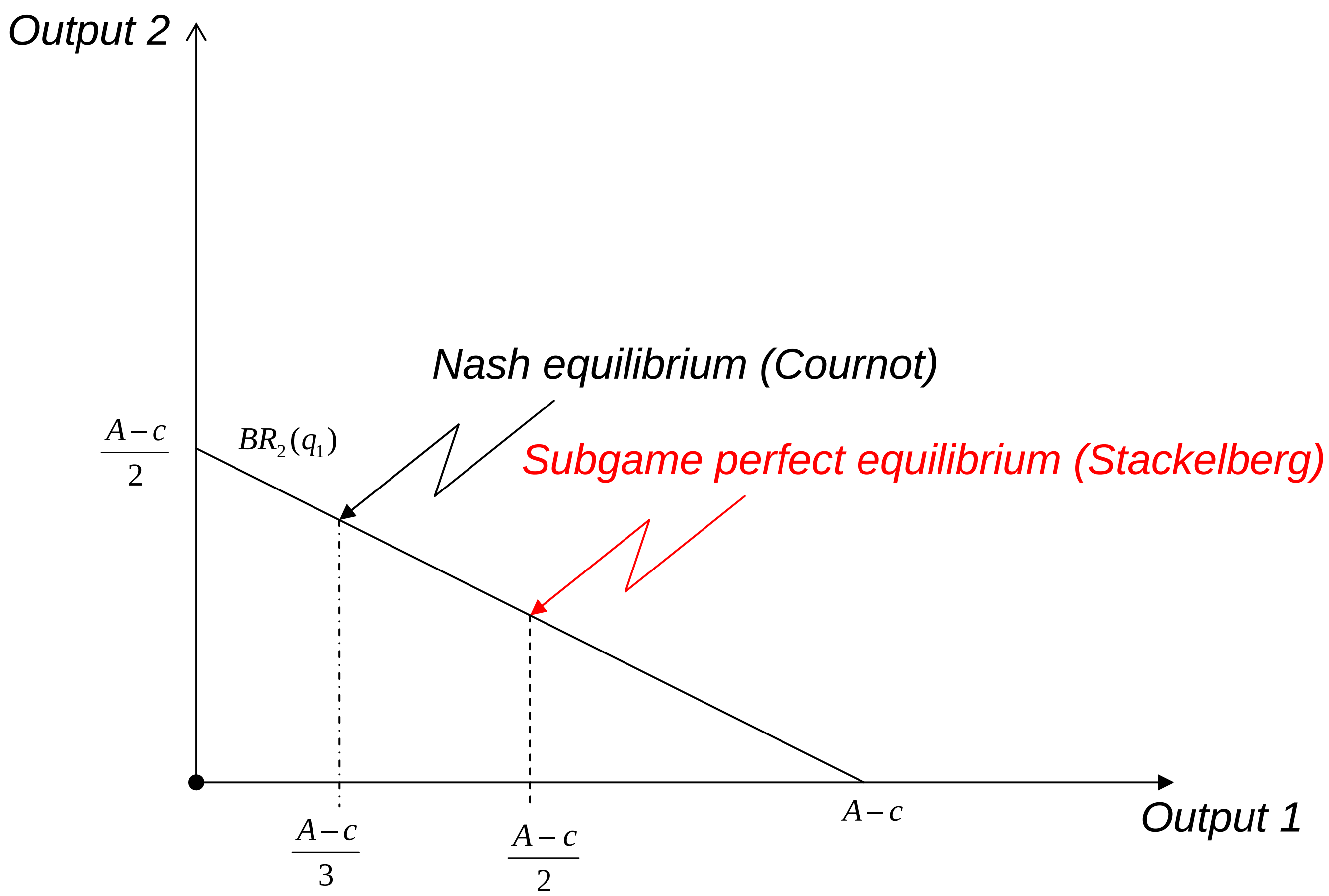
$$q_1^* = \frac{1}{2}(A - c)$$

and firm 2's output is

$$\begin{aligned} q_2^* &= \frac{1}{2}(A - q_1^* - c) \\ &= \frac{1}{2}(A - \frac{1}{2}(A - c) - c) \\ &= \frac{1}{4}(A - c). \end{aligned}$$

By contrast, in the unique Nash equilibrium of the Cournot's duopoly game under the same assumptions ($c_1 = c_2 = c$), each firm produces $\frac{1}{3}(A - c)$.

The subgame perfect equilibrium of Stackelberg's duopoly game



Auctions

Auction design

Two important issues for auction design are:

- Attracting entry
- Preventing collusion

Sealed-bid auction deals better with these issues, but it is more likely to lead to inefficient outcomes.

European 3G mobile telecommunication license auctions

Although the blocks of spectrum sold were very similar across countries, there was an enormous variation in revenues (in USD) per capita:

Austria	100
Belgium	45
Denmark	95
Germany	615
Greece	45
Italy	240
Netherlands	170
Switzerland	20
United Kingdom	650

United Kingdom

- 4 licenses to be auctioned off and 4 incumbents (with advantages in terms of costs and brand).
- To attract entry and deter collusion – an English until 5 bidders remain and then a sealed-bid with reserve price given by lowest bid in the English.
- later a 5th license became available to auction, a straightforward English auction was implemented.

Netherlands

– Followed UK and used (only) an English auction, but they had 5 incumbents and 5 licenses!

– Low participation: strongest potential entrants made deals with incumbents, and weak entrants either stayed out or quit bidding.

Switzerland

– Also followed UK and ran an English auction for 4 licenses. Companies either stayed out or quit bidding.

1. The government permitted last-minute joint-bidding agreements. Demand shrank from 9 to 4 bidders one week before the auction.

2. The reserve price had been set too low. The government tried to change the rules but was opposed by remaining bidders and legally obliged to stick to the original rules.

– Collected 1/30 per capita of UK, and 1/50 of what they had hoped for!

Types of auctions

Sequential / simultaneous

Bids may be called out sequentially or may be submitted simultaneously in sealed envelopes:

- English (or oral) – the seller actively solicits progressively higher bids and the item is soled to the highest bidder.
- Dutch – the seller begins by offering units at a "high" price and reduces it until all units are soled.
- Sealed-bid – all bids are made simultaneously, and the item is sold to the highest bidder.

First-price / second-price

The price paid may be the highest bid or some other price:

- First-price – the bidder who submits the highest bid wins and pay a price equal to her bid.

- Second-prices – the bidder who submits the highest bid wins and pay a price equal to the second highest bid.

Variants: all-pay (lobbying), discriminatory, uniform, Vickrey (William Vickrey, Nobel Laureate 1996), and more.

Private-value / common-value

Bidders can be certain or uncertain about each other's valuation:

- In private-value auctions, valuations differ among bidders, and each bidder is certain of her own valuation and can be certain or uncertain of every other bidder's valuation.

- In common-value auctions, all bidders have the same valuation, but bidders do not know this value precisely and their estimates of it vary.

First-price auction (with perfect information)

To define the game precisely, denote by v_i the value that bidder i attaches to the object. If she obtains the object at price p then her payoff is $v_i - p$.

Assume that bidders' valuations are all different and all positive. Number the bidders 1 through n in such a way that

$$v_1 > v_2 > \cdots > v_n > 0.$$

Each bidder i submits a (sealed) bid b_i. If bidder i obtains the object, she receives a payoff $v_i - b_i$. Otherwise, her payoff is zero.

Tie-breaking – if two or more bidders are in a tie for the highest bid, the winner is the bidder with the highest valuation.

In summary, a first-price sealed-bid auction with perfect information is the following strategic game:

- Players: the n bidders.

- Actions: the set of possible bids b_i of each player i (nonnegative numbers).

- Payoffs: the preferences of player i are given by

$$u_i = \begin{cases} v_i - \overline{b} & \text{if} \quad b_i = \overline{b} \text{ and } v_i > v_j \text{ if } b_j = \overline{b} \\ 0 & \text{if} \quad b_i < \overline{b} \end{cases}$$

where $\overline{b}$ is the highest bid.

The set of Nash equilibria is the set of profiles $(b_1, ..., b_n)$ of bids with the following properties:

$$
\begin{array}{ll}
[1] & v_2 \leq b_1 \leq v_1 \\
[2] & b_j \leq b_1 \text{ \underline{for all} } j \neq 1 \\
[3] & b_j = b_1 \text{ \underline{for some} } j \neq 1
\end{array}
$$

It is easy to verify that all these profiles are Nash equilibria. It is harder to show that there are no other equilibria. We can easily argue, however, that there is no equilibrium in which player 1 does not obtain the object.

$\Longrightarrow$ The first-price sealed-bid auction is socially efficient, but does not necessarily raise the most revenues.

Second-price auction (with perfect information)

A second-price sealed-bid auction with perfect information is the following strategic game:

– Players: the n bidders.

– Actions: the set of possible bids b_i of each player i (nonnegative numbers).

– Payoffs: the preferences of player i are given by

$$u_i = \begin{cases} v_i - \overline{b} & \text{if} \quad b_i > \overline{b} \text{ or } b_i = \overline{b} \text{ and } v_i > v_j \text{ if } b_j = \overline{b} \\ 0 & \text{if} \quad b_i < \overline{b} \end{cases}$$

where $\overline{b}$ is the highest bid submitted by a player other than i.

First note that for any player i the bid $b_i = v_i$ is a (weakly) dominant action (a "truthful" bid), in contrast to the first-price auction.

The second-price auction has many equilibria, but the equilibrium $b_i = v_i$ for all i is distinguished by the fact that every player's action dominates all other actions.

Another equilibrium in which player $j \neq 1$ obtains the good is that in which

[1] $b_1 < v_j$ and $b_j > v_1$
[2] $b_i = 0$ <u>for all</u> $i \neq \{1, j\}$

Common-value auctions and the winner's curse

Suppose we all participate in a sealed-bid auction for a jar of coins. Once you have estimated the amount of money in the jar, what are your bidding strategies in first- and second-price auctions?

The winning bidder is likely to be the bidder with the largest positive error (the largest overestimate).

In this case, the winner has fallen prey to the so-called the winner's curse. Auctions where the winner's curse is significant are oil fields, spectrum auctions, pay per click, and more.

Incomplete and asymmetric information
(an illustration – the market for lemons)

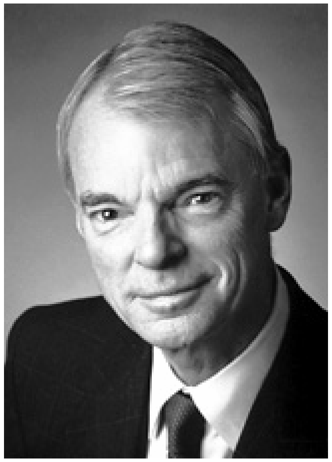
Markets with asymmetric information

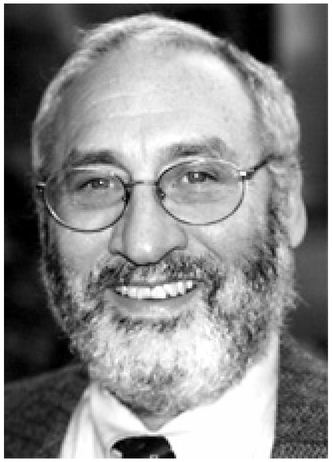
- The traditional theory of markets assumes that market participants have complete information about the underlying economic variables:
  - Buyers and sellers are both perfectly informed about the quality of the goods being sold in the market.
  - If it is not costly to verify quality, then the prices of the goods will simply adjust to reflect the quality difference.

$\Longrightarrow$ This is clearly a drastic simplification!!!

- There are certainly many markets in the real world in which it may be very costly (or even impossible) to gain accurate information:
  - labor markets, financial markets, markets for consumer products, and more.
- If information about quality is costly to obtain, then it is no longer possible that buyers and sellers have the same information.
- The costs of information provide an important source of market friction and can lead to a market breakdown.

Nobel Prize 2001
“for their analyses of markets with asymmetric information”





The Market for Lemons

Example I

- Consider a market with 100 people who want to sell their used car and 100 people who want to buy a used car.
- Everyone knows that 50 of the cars are "plums" and 50 are "lemons."
- Suppose further that

	seller	buyer
lemon	$1000	$1200
plum	$2000	$2400

- If it is easy to verify the quality of the cars there will be no problem in this market.

- Lemons will sell at some price $\$1000 - 1200$ and plums will sell at $\$2000 - 2400$.

- But happens to the market if buyers cannot observe the quality of the car?

– If buyers are risk neutral, then a typical buyer will be willing to pay his expected value of the car

$$\frac{1}{2}1200 + \frac{1}{2}2400 = \$1800.$$

– But for this price only owners of lemons would offer their car for sale, and buyers would therefore (correctly) expect to get a lemon.

– Market failure – no transactions will take place, although there are possible gains from trade!

Example II

- Suppose we can index the quality of a used car by some number q, which is distributed uniformly over $[0, 1]$.

- There is a large number of demanders for used cars who are willing to pay $\frac{3}{2}q$ for a car of quality q.

- There is a large number of sellers who are willing to sell a car of quality q for a price of q.

- If quality is perfectly observable, each used car of quality q would be soled for some price between q and $\frac{3}{2}q$.

- What will be the equilibrium price(s) in this market when quality of any given car cannot be observed?

- The unique equilibrium price is zero, and at this price the demand is zero and supply is zero.

$\Longrightarrow$ The asymmetry of information has destroyed the market for used cars. But the story does not end here!!!

Signaling

- In the used-car market, owners of the good used cars have an incentive to try to convey the fact that they have a good car to the potential purchasers.

- Put differently, they would like choose actions that <u>signal</u> that they are offering a plum rather than a lemon.

- In some case, the presence of a "signal" allows the market to function more effectively than it would otherwise.

Example – educational signaling

- Suppose that a fraction $0 < b < 1$ of workers are *competent* and a fraction $1 - b$ are *incompetent*.

- The competent workers have marginal product of a_2 and the incompetent have marginal product of $a_1 < a_2$.

- For simplicity we assume a competitive labor market and a linear production function

$$L_1 a_1 + L_2 a_2$$

where L_1 and L_2 is the number of incompetent and competent workers, respectively.

- If worker quality is observable, then firm would just offer wages

$$w_1 = a_1 \text{ and } w_2 = a_2$$

  to competent workers, respectively.

- That is, each worker will paid his marginal product and we would have an <u>efficient</u> equilibrium.

- But what if the firm cannot observe the marginal products so it cannot distinguish the two types of workers?

– If worker quality is unobservable, then the "best" the firm can do is to offer the average wage

$$w = (1 - b)a_1 + ba_2.$$

– If both types of workers agree to work at this wage, then there is no problem with adverse selection (more below).

– The incompetent (resp. competent) workers are getting paid more (resp. less) than their marginal product.

- The competent workers would like a way to signal that they are more productive than the others.

- Suppose now that there is some signal that the workers can acquire that will distinguish the two types

- One nice example is education – it is cheaper for the competent workers to acquire education than the incompetent workers.

- To be explicit, suppose that the cost (dollar costs, opportunity costs, costs of the effort, etc.) to acquiring e years of education is

$$c_1 e \text{ and } c_2 e$$

  for incompetent and competent workers, respectively, where $c_1 > c_2$.

- Suppose that workers conjecture that firms will pay a wage $s(e)$ where s is some increasing function of e.

- Although education has no effect on productivity (MBA?), firms may still find it profitable to base wage on education – attract a higher-quality work force.

Market equilibrium

In the educational signaling example, there appear to be several possibilities for equilibrium:

[1] The (representative) firm offers a single contract that attracts both types of workers.

[2] The (representative) firm offers a single contract that attracts only one type of workers.

[3] The (representative) firm offers two contracts, one for each type of workers.

- A <u>separating equilibrium</u> involves each type of worker making a choice that separate himself from the other type.

- In a <u>pooling equilibrium</u>, in contrast, each type of workers makes the same choice, and all getting paid the wage based on their average ability.

  Note that a separating equilibrium is wasteful in a social sense – no social gains from education since it does not change productivity.

Example (cont.)

– Let e_1 and e_2 be the education level actually chosen by the workers. Then, a separating (signaling) equilibrium has to satisfy:

[1] zero-profit conditions

$$\begin{aligned} s(e_1) &= a_1 \\ s(e_2) &= a_2 \end{aligned}$$

[2] self-selection conditions

$$\begin{aligned} s(e_1) - c_1 e_1 &\geq s(e_2) - c_1 e_2 \\ s(e_2) - c_2 e_2 &\geq s(e_1) - c_2 e_1 \end{aligned}$$

– In general, there may by many functions $s(e)$ that satisfy conditions [1] and [2]. One wage profile consistent with separating equilibrium is

$$s(e) = \begin{cases} a_2 & \text{if} \quad e > e^* \\ a_1 & \text{if} \quad e \leq e^* \end{cases}$$

and

$$\frac{a_2 - a_1}{c_2} > e^* > \frac{a_2 - a_1}{c_1}$$

$\Longrightarrow$ Signaling can make things better or worse – each case has to examined on its own merits!

The Sheepskin (diploma) effect

The increase in wages associated with obtaining a higher credential:

– Graduating high school increases earnings by 5 to 6 times as much as does completing a year in high school that does not result in graduation.

– The same discontinuous jump occurs for people who graduate from collage.

– High school graduates produce essentially the same amount of output as non-graduates.