UC Berkeley
Haas School of Business
Game Theory
(EMBA 296 & EWMBA 211)
Summer 2019

Review, oligopoly, signaling, and more...

Block 3

June 29, 2019

Game plan

Before I leave you in Steve's capable hands...

- Review
- Oligopolistic competition
- Signaling
- The tragedy of the commons
- Evolutionary stability

A review of the main ideas

We study two (out of four) groups of game theoretic models:

[1] Strategic games – all players simultaneously choose their plan of action once and for all.

[2] Extensive games (with perfect information) – players choose sequentially (and fully informed about all previous actions).

A solution (equilibrium) is a systematic description of the outcomes that may emerge in a family of games. We study two solution concepts:

[1] Nash equilibrium – a steady state of the play of a <u>strategic</u> game (no player has a profitable deviation given the actions of the other players).

[1] Subgame equilibrium – a steady state of the play of an <u>extensive</u> game (a Nash equilibrium in every <u>subgame</u> of the extensive game).

$\Longrightarrow$ Every subgame perfect equilibrium is also a Nash equilibrium.

Oligopolistic competition
(in strategic and extensive forms)

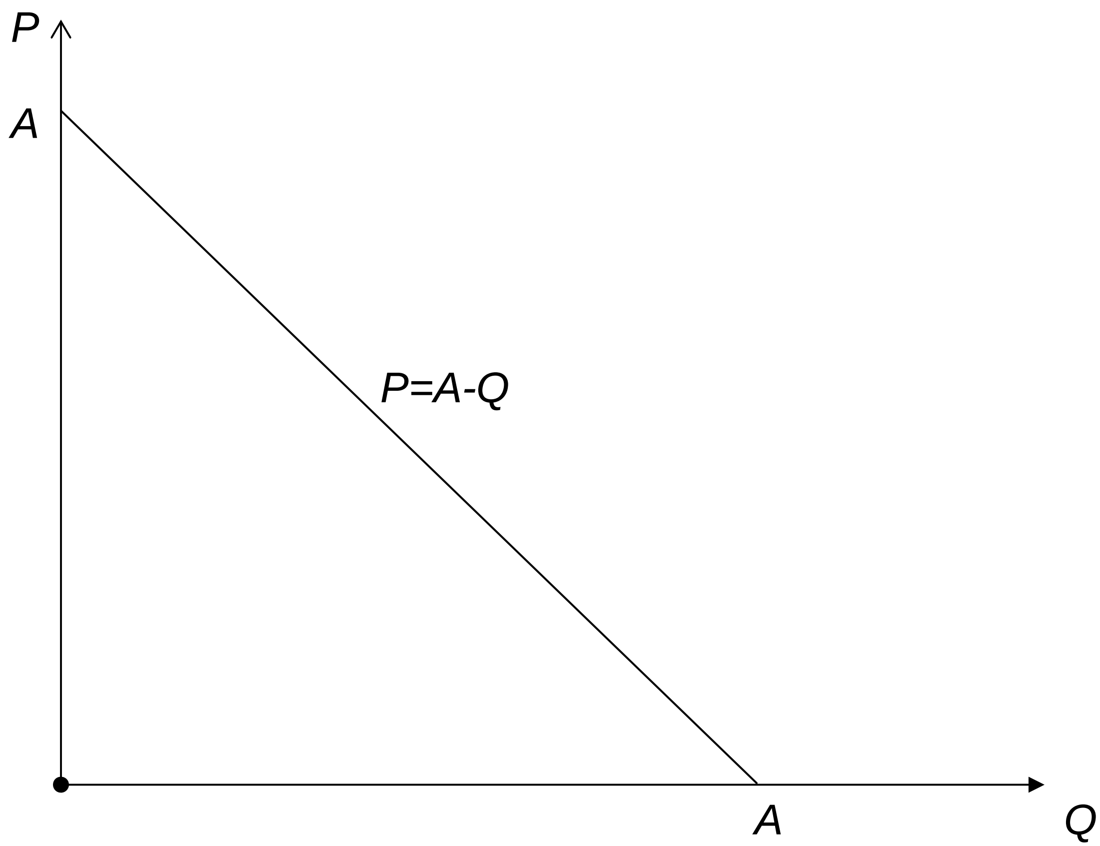
Cournot's oligopoly model (1838)

- A single good is produced by two firms (the industry is a "duopoly").
- The cost for firm $i = 1, 2$ for producing q_i units of the good is given by $c_i q_i$ ("unit cost" is constant equal to $c_i > 0$).
- If the firms' total output is $Q = q_1 + q_2$ then the market price is

$$P = A - Q$$

  if $A \geq Q$ and zero otherwise (linear inverse demand function). We also assume that $A > c$.

The inverse demand function



To find the Nash equilibria of the Cournot's game, we can use the procedures based on the firms' best response functions.

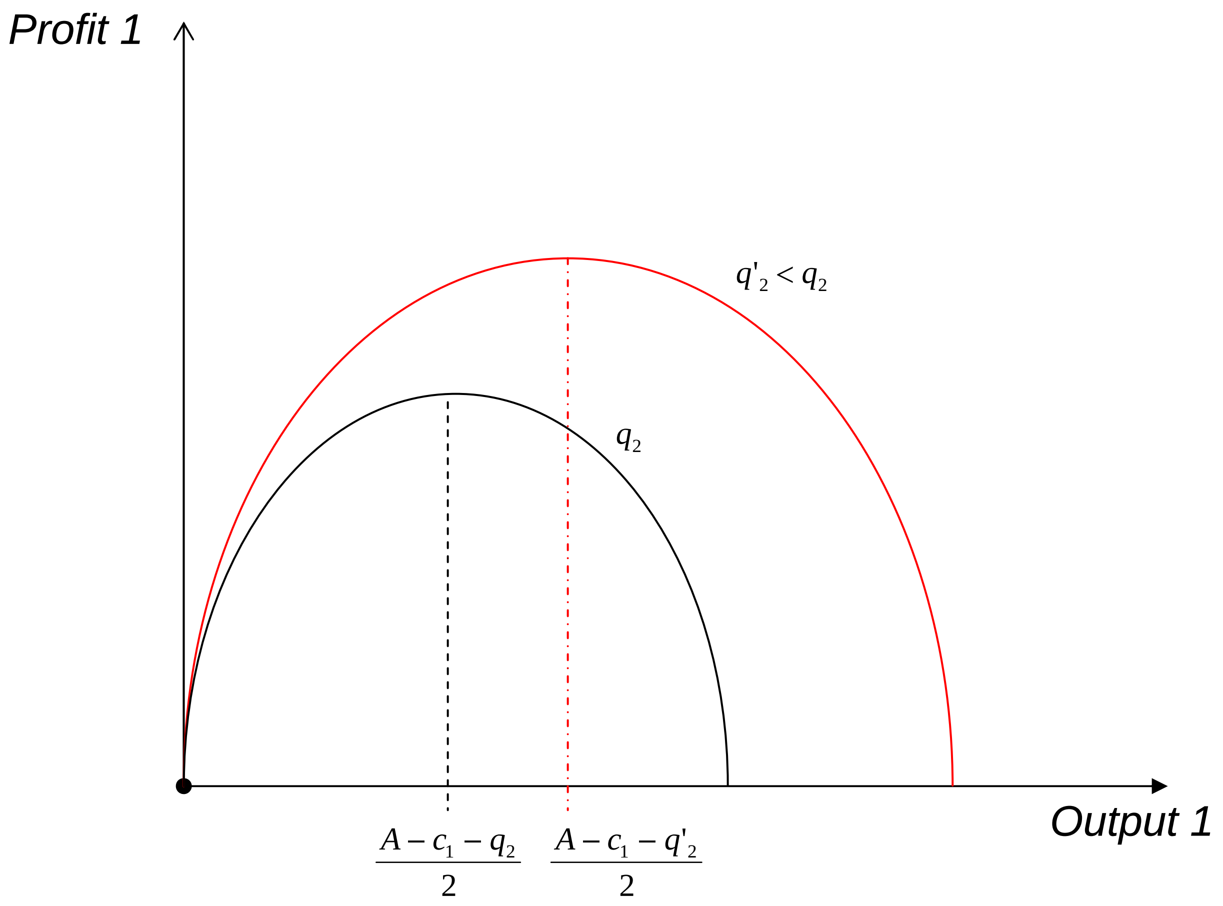
But first we need the firms payoffs (profits):

$$\begin{aligned}\pi_1 &= Pq_1 - c_1q_1 \\ &= (A-Q)q_1 - c_1q_1 \\ &= (A-q_1-q_2)q_1 - c_1q_1 \\ &= (A-q_1-q_2-c_1)q_1\end{aligned}$$

and similarly,

$$\pi_2 = (A-q_1-q_2-c_2)q_2$$

Firm 1's profit as a function of its output
(given firm 2's output)



To find firm 1's best response to any given output q_2 of firm 2, we need to study firm 1's profit as a function of its output q_1 for given values of q_2.

Using calculus, we set the derivative of firm 1's profit with respect to q_1 equal to zero and solve for q_1:

$$q_1 = \frac{1}{2}(A - q_2 - c_1).$$

We conclude that the best response of firm 1 to the output q_2 of firm 2 depends on the values of q_2 and c_1.

Because firm 2's cost function is $c_2 \neq c_1$, its best response function is given by

$$q_2 = \frac{1}{2}(A - q_1 - c_2).$$

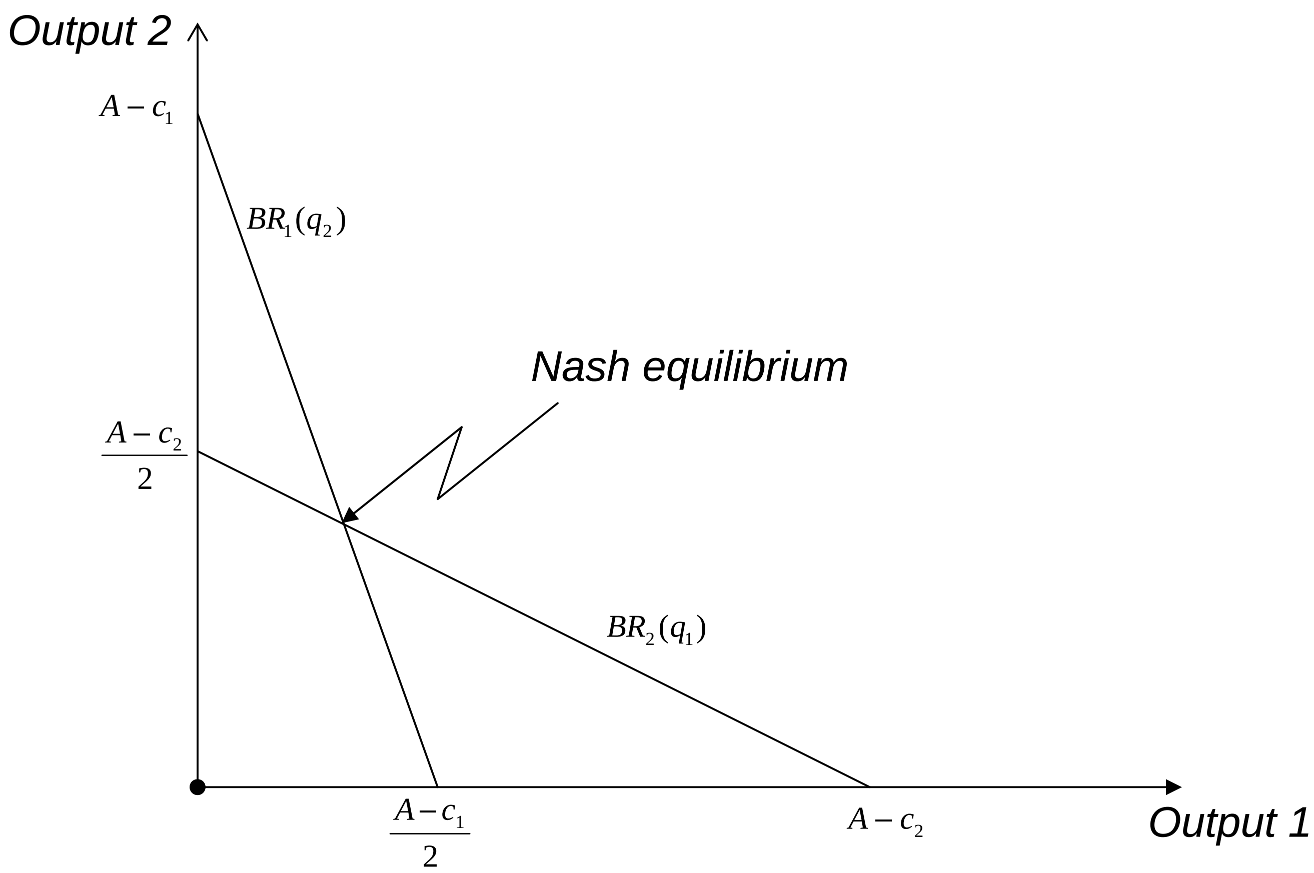
A Nash equilibrium of the Cournot's game is a pair (q_1^*, q_2^*) of outputs such that q_1^* is a best response to q_2^* and q_2^* is a best response to q_1^*.

From the figure below, we see that there is exactly one such pair of outputs

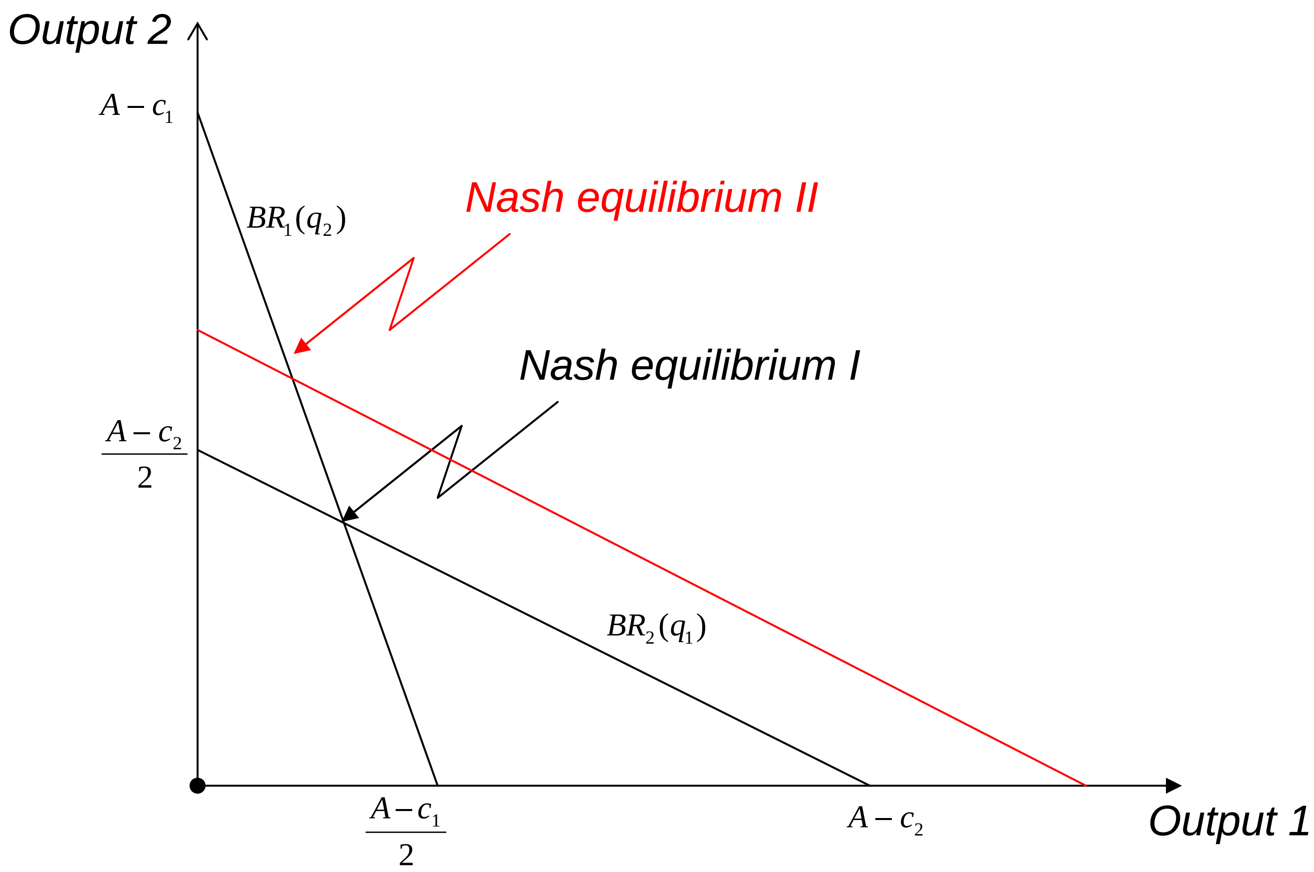
$$q_1^* = \frac{A+c_2-2c_1}{3} \quad \text{and} \quad q_2^* = \frac{A+c_1-2c_2}{3}$$

which is the solution to the two equations above.

The best response functions in the Cournot's duopoly game



Nash equilibrium comparative statics
(a decrease in the cost of firm 2)



A question: what happens when consumers are willing to pay more (*A* increases)?

In summary, this simple Cournot's duopoly game has a unique Nash equilibrium.

Two economically important properties of the Nash equilibrium are (to economic regulatory agencies):

[1] The relation between the firms' equilibrium profits and the profit they could make if they act collusively.

[2] The relation between the equilibrium profits and the number of firms.

[1] Collusive outcomes: in the Cournot's duopoly game, there is a pair of outputs at which *both* firms' profits exceed their levels in a Nash equilibrium.

[2] Competition: The price at the Nash equilibrium if the two firms have the *same* unit cost $c_1 = c_2 = c$ is given by

$$\begin{aligned} P^* &= A - q_1^* - q_2^* \\ &= \frac{1}{3}(A + 2c) \end{aligned}$$

which is above the unit cost c. But as the number of firm increases, the equilibrium price deceases, approaching c (zero profits!).

Stackelberg's duopoly model (1934)

How do the conclusions of the Cournot's duopoly game change when the firms move sequentially? Is a firm better off moving before or after the other firm?

Suppose that $c_1 = c_2 = c$ and that firm 1 moves at the start of the game. We may use backward induction to find the subgame perfect equilibrium.

- First, for *any* output q_1 of firm 1, we find the output q_2 of firm 2 that maximizes its profit. Next, we find the output q_1 of firm 1 that maximizes its profit, *given the strategy* of firm 2.

Firm 2

Since firm 2 moves after firm 1, a strategy of firm 2 is a *function* that associate an output q_2 for firm 2 for each possible output q_1 of firm 1.

We found that under the assumptions of the Cournot's duopoly game Firm 2 has a unique best response to each output q_1 of firm 1, given by

$$q_2 = \frac{1}{2}(A - q_1 - c)$$

(Recall that $c_1 = c_2 = c$).

Firm 1

Firm 1's strategy is the output q_1 the maximizes

$$\pi_1 = (A - q_1 - q_2 - c)q_1 \quad \text{subject to} \quad q_2 = \tfrac{1}{2}(A - q_1 - c)$$

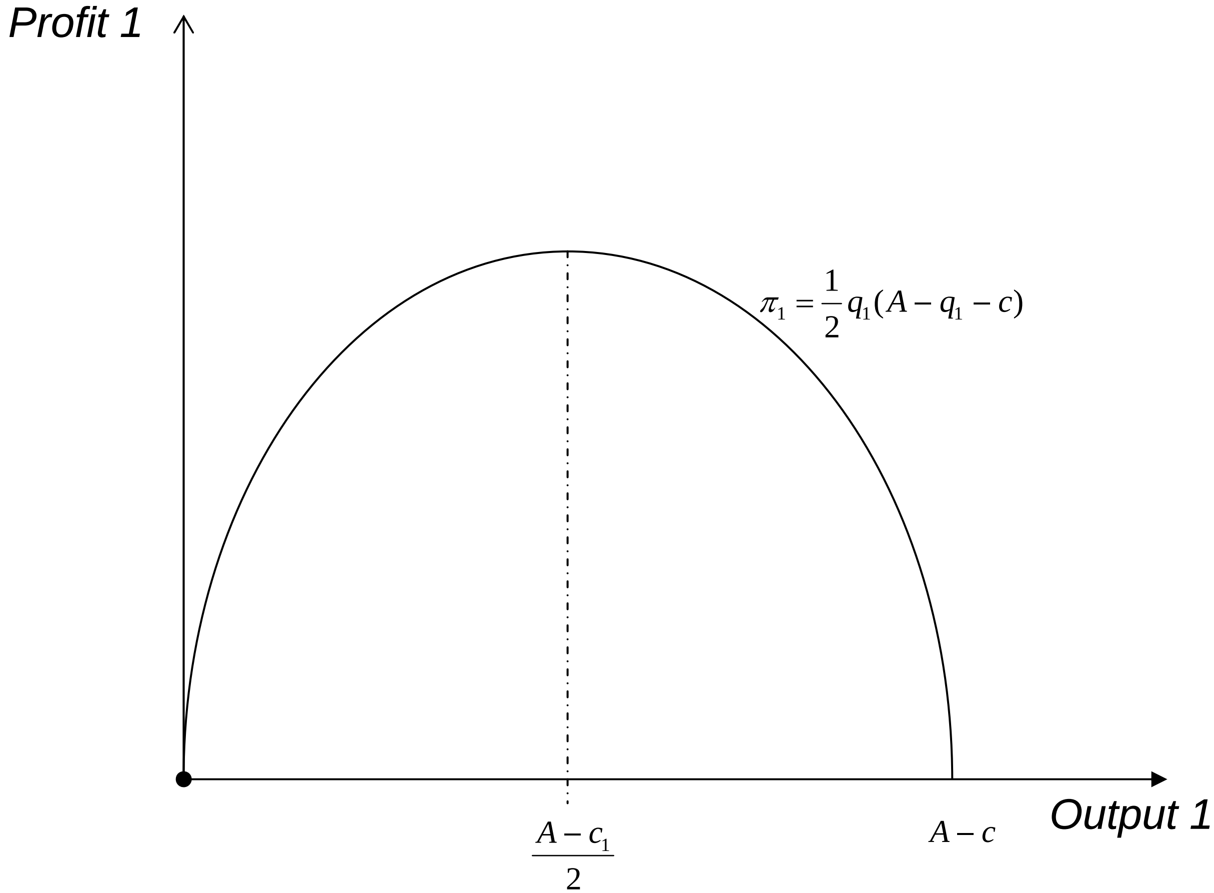
Thus, firm 1 maximizes

$$\pi_1 = (A - q_1 - (\frac{1}{2}(A - q_1 - c)) - c)q_1 = \frac{1}{2}q_1(A - q_1 - c).$$

This function is quadratic in q_1 that is zero when $q_1 = 0$ and when $q_1 = A - c$. Thus its maximizer is

$$q_1^* = \frac{1}{2}(A - c).$$

Firm 1's (first-mover) profit in Stackelberg's duopoly game



We conclude that Stackelberg's duopoly game has a unique subgame perfect equilibrium, in which firm 1's strategy is the output

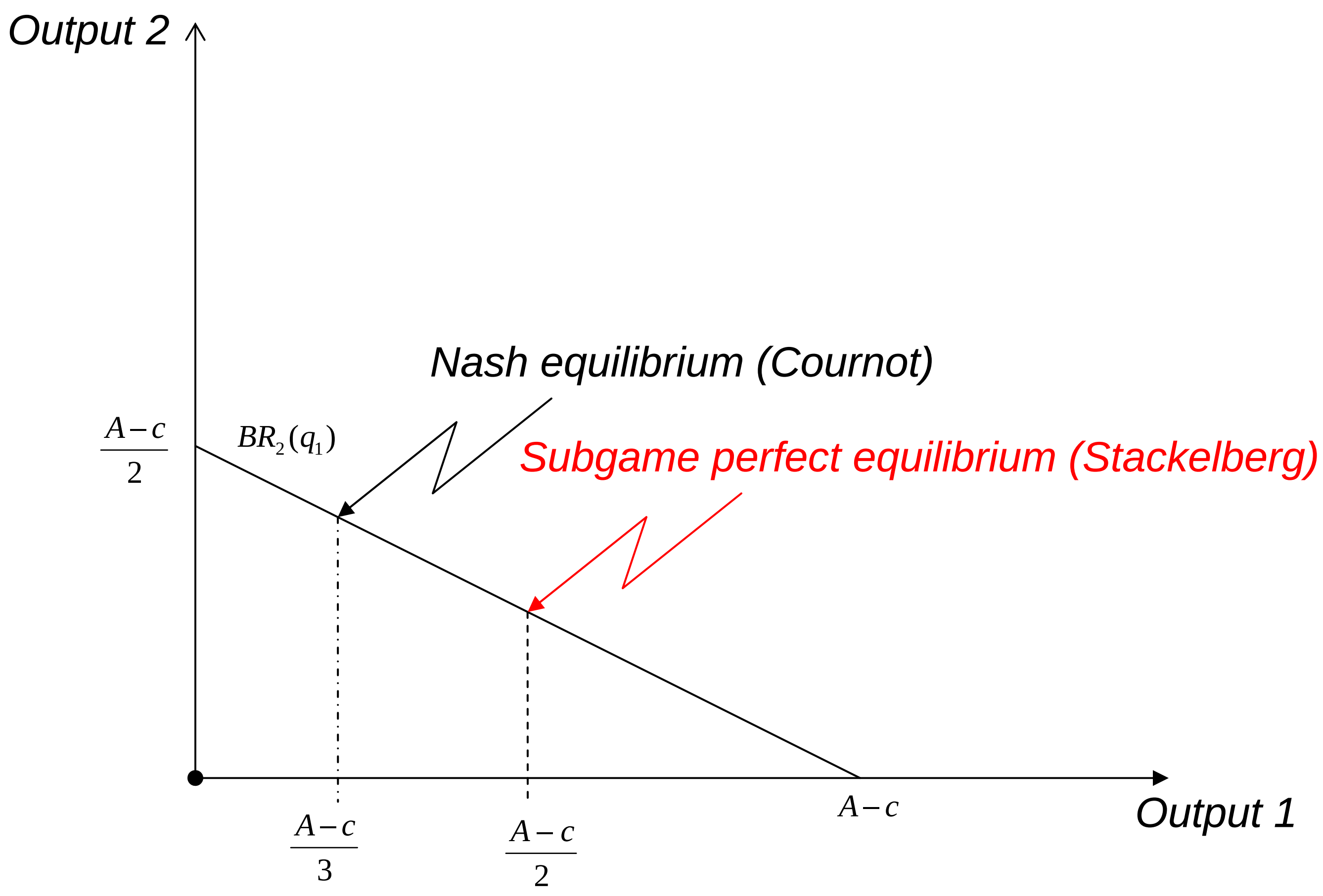
$$q_1^* = \frac{1}{2}(A - c)$$

and firm 2's output is

$$\begin{aligned} q_2^* &= \frac{1}{2}(A - q_1^* - c) \\ &= \frac{1}{2}(A - \frac{1}{2}(A - c) - c) \\ &= \frac{1}{4}(A - c). \end{aligned}$$

By contrast, in the unique Nash equilibrium of the Cournot's duopoly game under the same assumptions ($c_1 = c_2 = c$), each firm produces $\frac{1}{3}(A - c)$.

The subgame perfect equilibrium of Stackelberg's duopoly game



Incomplete and asymmetric information: the market for lemons
(if time permits)

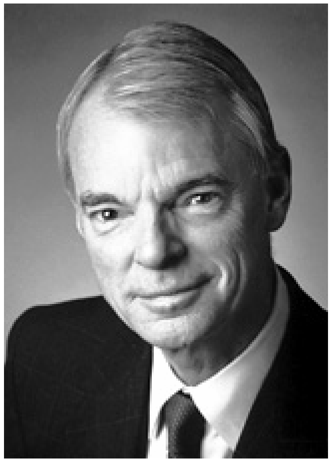
Markets with asymmetric information

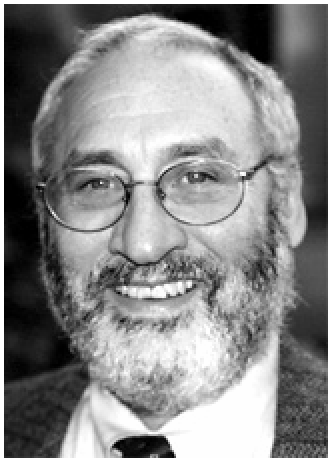
- The traditional theory of markets assumes that market participants have complete information about the underlying economic variables:
  - Buyers and sellers are both perfectly informed about the quality of the goods being sold in the market.
  - If it is not costly to verify quality, then the prices of the goods will simply adjust to reflect the quality difference.

$\Longrightarrow$ This is clearly a drastic simplification!!!

- There are certainly many markets in the real world in which it may be very costly (or even impossible) to gain accurate information:
    - labor markets, financial markets, markets for consumer products, and more.
- If information about quality is costly to obtain, then it is no longer possible that buyers and sellers have the same information.
- The costs of information provide an important source of market friction and can lead to a market breakdown.

Nobel Prize 2001
“for their analyses of markets with asymmetric information”





The Market for Lemons

Example I

- Consider a market with 100 people who want to sell their used car and 100 people who want to buy a used car.

- Everyone knows that 50 of the cars are “plums” and 50 are “lemons.”

- Suppose further that

	seller	buyer
lemon	\$1000	\$1200
plum	\$2000	\$2400

– If it is easy to verify the quality of the cars there will be no problem in this market.

– Lemons will sell at some price $\$1000 - 1200$ and plums will sell at $\$2000 - 2400$.

– But happens to the market if buyers cannot observe the quality of the car?

– If buyers are risk neutral, then a typical buyer will be willing to pay his expected value of the car

$$\frac{1}{2}1200 + \frac{1}{2}2400 = \$1800.$$

– But for this price only owners of lemons would offer their car for sale, and buyers would therefore (correctly) expect to get a lemon.

– Market failure – no transactions will take place, although there are possible gains from trade!

Example II

- Suppose we can index the quality of a used car by some number q, which is distributed uniformly over $[0, 1]$.
- There is a large number of demanders for used cars who are willing to pay $\frac{3}{2}q$ for a car of quality q.
- There is a large number of sellers who are willing to sell a car of quality q for a price of q.

– If quality is perfectly observable, each used car of quality q would be soled for some price between q and $\frac{3}{2}q$.

– What will be the equilibrium price(s) in this market when quality of any given car cannot be observed?

– The unique equilibrium price is zero, and at this price the demand is zero and supply is zero.

$\Longrightarrow$ The asymmetry of information has destroyed the market for used cars. But the story does not end here!!!

Signaling

- In the used-car market, owners of the good used cars have an incentive to try to convey the fact that they have a good car to the potential purchasers.

- Put differently, they would like choose actions that signal that they are offering a plum rather than a lemon.

- In some case, the presence of a "signal" allows the market to function more effectively than it would otherwise.

Example – educational signaling

- Suppose that a fraction $0 < b < 1$ of workers are *competent* and a fraction $1 - b$ are *incompetent*.

- The competent workers have marginal product of a_2 and the incompetent have marginal product of $a_1 < a_2$.

- For simplicity we assume a <u>competitive</u> labor market and a linear production function

$$L_1 a_1 + L_2 a_2$$

where L_1 and L_2 is the number of incompetent and competent workers, respectively.

– If worker quality is observable, then firm would just offer wages

$$w_1 = a_1 \text{ and } w_2 = a_2$$

to competent workers, respectively.

– That is, each worker will paid his marginal product and we would have an <u>efficient</u> equilibrium.

– But what if the firm cannot observe the marginal products so it cannot distinguish the two types of workers?

– If worker quality is unobservable, then the “best” the firm can do is to offer the average wage

$$w = (1 - b)a_1 + ba_2.$$

– If both types of workers agree to work at this wage, then there is no problem with adverse selection (more below).

– The incompetent (resp. competent) workers are getting paid more (resp. less) than their marginal product.

- The competent workers would like a way to signal that they are more productive than the others.
- Suppose now that there is some signal that the workers can acquire that will distinguish the two types
- One nice example is education – it is cheaper for the competent workers to acquire education than the incompetent workers.

– To be explicit, suppose that the cost (dollar costs, opportunity costs, costs of the effort, etc.) to acquiring e years of education is

$$c_1 e \text{ and } c_2 e$$

for incompetent and competent workers, respectively, where $c_1 > c_2$.

– Suppose that workers conjecture that firms will pay a wage $s(e)$ where s is some increasing function of e.

– Although education has no effect on productivity (MBA?), firms may still find it profitable to base wage on education – attract a higher-quality work force.

Market equilibrium

In the educational signaling example, there appear to be several possibilities for equilibrium:

[1] The (representative) firm offers a single contract that attracts both types of workers.

[2] The (representative) firm offers a single contract that attracts only one type of workers.

[3] The (representative) firm offers two contracts, one for each type of workers.

- A separating equilibrium involves each type of worker making a choice that separate himself from the other type.

- In a pooling equilibrium, in contrast, each type of workers makes the same choice, and all getting paid the wage based on their average ability.

  Note that a separating equilibrium is wasteful in a social sense – no social gains from education since it does not change productivity.

Example (cont.)

– Let e_1 and e_2 be the education level actually chosen by the workers. Then, a separating (signaling) equilibrium has to satisfy:

[1] zero-profit conditions

$$\begin{aligned} s(e_1) &= a_1 \\ s(e_2) &= a_2 \end{aligned}$$

[2] self-selection conditions

$$\begin{aligned} s(e_1) - c_1 e_1 &\geq s(e_2) - c_1 e_2 \\ s(e_2) - c_2 e_2 &\geq s(e_1) - c_2 e_1 \end{aligned}$$

– In general, there may by many functions $s(e)$ that satisfy conditions [1] and [2]. One wage profile consistent with separating equilibrium is

$$s(e) = \begin{cases} a_2 & \text{if} \quad e > e^* \\ a_1 & \text{if} \quad e \leq e^* \end{cases}$$

and

$$\frac{a_2 - a_1}{c_2} > e^* > \frac{a_2 - a_1}{c_1}$$

$\Longrightarrow$ Signaling can make things better or worse – each case has to examined on its own merits!

The Sheepskin (diploma) effect

The increase in wages associated with obtaining a higher credential:

- Graduating high school increases earnings by 5 to 6 times as much as does completing a year in high school that does not result in graduation.
- The same discontinuous jump occurs for people who graduate from collage.
- High school graduates produce essentially the same amount of output as non-graduates.

The tragedy of the commons

William Forster Lloyd (1833)

- Cattle herders sharing a common parcel of land (the commons) on which they are each entitled to let their cows graze. If a herder put more than his allotted number of cattle on the common, overgrazing could result.

- Each additional animal has a positive effect for its herder, but the cost of the extra animal is shared by all other herders, causing a so-called "free-rider" problem. Today's commons include fish stocks, rivers, oceans, and the atmosphere.

Garrett Hardin (1968)

- This social dilemma was populated by Hardin in his article "The Tragedy of the Commons," published in the journal *Science*. The essay derived its title from Lloyd (1833) on the over-grazing of common land.

- Hardin concluded that "...the commons, if justifiable at all, is justifiable only under conditions of low-population density. As the human population has increased, the commons has had to be abandoned in one aspect after another."

- "The only way we can preserve and nurture other and more precious freedoms is by relinquishing the freedom to breed, and that very soon. "Freedom is the recognition of necessity" – and it is the role of education to reveal to all the necessity of abandoning the freedom to breed. Only so, can we put an end to this aspect of the tragedy of the commons."

"Freedom to breed will bring ruin to all."

Let's put some game theoretic analysis (rigorous sense) behind this story:

- There are n players, each choosing how much to produce in a production activity that 'consumes' some of the clean air that surrounds our planet.

- There is K amount of clean air, and any consumption of clean air comes out of this common resource. Each player $i = 1, ..., n$ chooses his consumption of clean air for production $k_i \geq 0$ and the amount of clean air left is therefore

$$K - \sum_{i=1}^{n} k_i.$$

– The benefit of consuming an amount $k_i \geq 0$ of clean air gives player i a benefit equal to $\ln(k_i)$. Each player also enjoys consuming the reminder of the clean air, giving each a benefit

$$\ln\left(K - \sum_{i=1}^{n} k_i\right).$$

– Hence, the value for each player i from the action profile (outcome) $k = (k_1, ..., k_n)$ is give by

$$v_i(k_i, k_{-i}) = \ln(k_i) + \ln\left(K - \sum_{j=1}^{n} k_j\right).$$

– To get player i's best-response function, we write down the first-order condition of his payoff function:

$$\frac{\partial v_i(k_i, k_{-i})}{\partial k_i} = \frac{1}{k_i} - \frac{1}{K - \sum_{j=1}^{n} k_j} = 0$$

and thus

$$BR_i(k_{-i}) = \frac{K - \sum_{j \neq i} k_j}{2}.$$

The two-player Tragedy of the Commons

– To find the Nash equilibrium, there are n equations with n unknown that need to be solved. We first solve the equilibrium for two players. Letting $k_i(k_j)$ be the best response of player i, we have two best-response functions:

$$k_1(k_2) = \frac{K - k_2}{2} \quad \text{and} \quad k_2(k_1) = \frac{K - k_1}{2}.$$

– If we solve the two best-response functions simultaneously, we find the unique (pure-strategy) Nash equilibrium

$$k_1^{NE} = k_2^{NE} = \frac{K}{3}.$$

Can this two-player society do better? More specifically, is consuming $\frac{K}{3}$ clean air for each player too much (or too little)?

- The 'right way' to answer this question is using the Pareto principle (Vilfredo Pareto, 1848-1923) – can we find another action profile $k = (k_1, k_2)$ that will make both players better off than in the Nash equilibrium?

- To this end, the function we seek to maximize is the social welfare function w given by

$$w(v_1, v_2) = v_1 + v_2 = \sum_{i=1}^{2} \ln(k_i) + 2\ln\left(K - \sum_{i=1}^{2} k_i\right).$$

– The first-order conditions for this problem are

$$\frac{\partial w(k_1, k_2)}{\partial k_1} = \frac{1}{k_1} - \frac{2}{K - k_1 - k_2} = 0$$

and

$$\frac{\partial w(k_1, k_2)}{\partial k_2} = \frac{1}{k_2} - \frac{2}{K - k_1 - k_2} = 0.$$

– Solving these two equations simultaneously result the unique Pareto optimal outcome

$$k_1^{PO} = k_2^{PO} = \frac{K}{4}.$$

The n-player Tragedy of the Commons

– In the n-player Tragedy of the Commons, the best response of each player $i = 1, ..., n$, $k_i(k_{-i})$, is given by

$$BR_i(k_{-i}) = \frac{K - \sum_{j \neq i} k_j}{2}.$$

– We consider a symmetric Nash equilibrium where each player i chooses the same level of consumption of clean air k^* (it is subtle to show that there cannot be asymmetric Nash equilibria).

– Because the best response must hold for each player i and they all choose the same level k^{SNE} then in the symmetric Nash equilibrium all best-response functions reduce to

$$k^{SNE} = \frac{K - \sum_{j \neq i} k^{SNE}}{2} = \frac{K - (n-1)k^{SNE}}{2}$$

or

$$k^{SNE} = \frac{K}{n+1}.$$

Hence, the sum of clean air consumed by the firms is $\frac{n}{n+1}K$, which increases with n as Hardin conjectured.

What is the socially optimal outcome with n players? And how does society size affect this outcome?

– With n players, the social welfare function w given by

$$\begin{aligned} w(v_1, ..., v_n) &= \sum_{i=1}^{n} v_i \\ &= \sum_{i=1}^{n} \ln(k_i) + n \ln\left(K - \sum_{i=1}^{n} k_i\right). \end{aligned}$$

And the n first-order conditions for the problem of maximizing this function are

$$\frac{\partial w(k_1, ..., k_n)}{\partial k_i} = \frac{1}{k_i} - \frac{n}{K - \sum_{j=1}^{n} k_j} = 0$$

for $i = 1, ..., n$.

– Just as for the analysis of the Nash equilibrium with n players, the solution here is also symmetric. Therefore, the Pareto optimal consumption of each player k^{PO} can be found using the following equation:

$$\frac{1}{k^{PO}} - \frac{n}{K - nk^{PO}} = 0$$

or

$$k^{PO} = \frac{K}{2n}$$

and thus the Pareto optimal consumption of air is equal $\frac{K}{2}$, for any society size n. for $i = 1, ..., n$.

Finally, we show there is no asymmetric equilibrium.

- To this end, assume there are two players, i and j, choosing two different $k_i \neq k_j$ in equilibrium.

- Because we assume a Nash equilibrium the best-response functions of i and j must hold simultaneously, that is

$$k_i = \frac{K - \bar{k} - k_j}{2} \quad \text{and} \quad k_j = \frac{K - \bar{k} - k_i}{2}$$

  where $\bar{k}$ be the sum of equilibrium choices of all other players except i and j.

– However, if we solve the best-response functions of players i and j simultaneously, we find that

$$k_i = k_j = \frac{K - \bar{k}}{3}$$

contracting the assumption we started with that $k_i \neq k_j$.

Evolutionary game theory

Evolutionary stability

A single population of players. Players interact with each other pair-wise and randomly matched.

Players are assigned modes of behavior (mutation). Utility measures each player's ability to survive.

ε of players consists of mutants taking action a while others take action a^*.

Evolutionary stable strategy (ESS)

Consider a two-player payoff symmetric game

$$G = \langle \{1, 2\}, (A, A), (u_1, u_2) \rangle$$

where

$$u_1(a_1, a_2) = u_2(a_2, a_1)$$

(players exchanging a_1 and a_2).

$a^* \in A$ is ESS if and only if for any $a \in A$, $a \neq a^*$ and $\varepsilon > 0$ sufficiently small

$$(1-\varepsilon)u(a^*, a^*) + \varepsilon u(a^*, a) > (1-\varepsilon)u(a, a^*) + \varepsilon u(a, a)$$

which is satisfied if and only if for any $a \neq a^*$ either

$$u(a^*, a^*) > u(a, a^*)$$

or

$$u(a^*, a^*) = u(a, a^*) \text{ and } u(a^*, a) > u(a, a)$$

Three results on ESS

[1] If a^* is an ESS then (a^*, a^*) is a NE.

Suppose not. Then, there exists a strategy $a \in A$ such that

$$u(a, a^*) > u(a^*, a^*).$$

But, for ε small enough

$$(1 - \varepsilon)u(a^*, a^*) + \varepsilon u(a^*, a) < (1 - \varepsilon)u(a, a^*) + \varepsilon u(a, a)$$

and thus a^* is not an ESS.

[2] If (a^*, a^*) is a strict NE ($u(a^*, a^*) > u(a, a^*)$ for all $a \in A$) then a^* is an ESS.

Suppose a^* is not an ESS. Then either

$$u(a^*, a^*) \leq u(a, a^*)$$

or

$$u(a^*, a^*) = u(a, a^*) \text{ and } u(a^*, a) \leq u(a, a).$$

so (a^*, a^*) can be a NE but not a strict NE.

[3] The two-player two-action game

	a	a'
a	w, w	x, y
a'	y, x	z, z

has a strategy which is ESS.

If $w > y$ or $z > x$ then (a, a) or (a', a') are strict NE, and thus a or a' are ESS.

If $w < y$ and $z < x$ then there is a unique symmetric mixed strategy NE (α^*, α^*) where

$$\alpha^*(a) = (z - x)/(w - y + z - x)$$

and $u(\alpha^*, \alpha) > u(\alpha, \alpha)$ for any $\alpha \neq \alpha^*$.