UC Berkeley
Haas School of Business
Game Theory
(EMBA 296 & EWMBA 211)
Summer 2016

Risk preferences

Block 3
Jul 7-9, 2016

The fundamental tradeoffs in life

People's attitudes towards risk, time and other people enter every realm of (financial) decision-making:

$$\begin{array}{rcl} \text{risk} & \Longleftrightarrow & \text{return} \\ \text{today} & \Longleftrightarrow & \text{tomorrow} \\ \text{self} & \Longleftrightarrow & \text{others} \end{array}$$

Risk, time and social preferences are thus important inputs into any broader measure of welfare and enter virtually every field of economics.

The fundamental questions concerning preferences

I Consistency

– Is behavior consistent with a model of utility maximization (Homo Economicus)?

II Structure

– What are the structural properties of the underlying utility function?

III Recoverability

– Can underlying preferences be recovered from observed choices?

IV Linkages

– What are the linkages between preferences in various environments?

The touchstones of (financial) decision-making

Rational choice ‘simply’ requires consistent preferences over all possible alternatives, and choices that correspond to the most preferred alternative from the feasible set.

goals

preferences ↗ ↖ constraints

Insofar as preferences are rational, then the techniques of economic analysis may be brought to bear on modeling the decisions governed by these preferences.

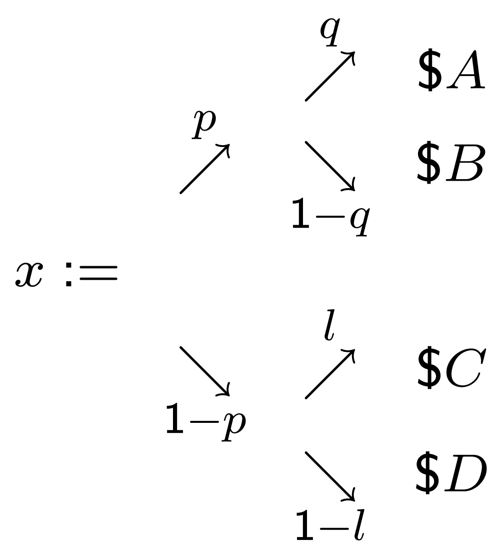
Life is full of lotteries :-(

$$x := \begin{array}{ll} \overset{p}{\nearrow} & \$A \\ & \\ \underset{1-p}{\searrow} & \$B \end{array} \qquad\qquad y := \begin{array}{ll} \overset{p}{\nearrow} & \$A \\ \xrightarrow{q} & \$B \\ \underset{1-p-q}{\searrow} & \$C \end{array}$$

A risky lottery (left) and an ambiguous lottery (right)

$$x := \begin{array}{c} 1/2 \\ \nearrow \\ \\ \searrow \\ 1/2 \end{array} \quad \begin{array}{c} \$A \\ \\ \\ \\ \$B \end{array} \qquad\qquad y := \begin{array}{c} ? \\ \nearrow \\ \\ \searrow \\ 1-? \end{array} \quad \begin{array}{c} \$A \\ \\ \\ \\ \$B \end{array}$$

A compounded lottery



The reduction of a compounded lottery

$$
x := \begin{array}{cccc|l}
 & & \overset{q}{\nearrow} & \$A & pq \\
 & \overset{p}{\nearrow} & \underset{1-q}{\searrow} & \$B & p(1-q) \\
 & & & & \\
 & & \overset{l}{\nearrow} & \$C & (1-p)l \\
 & \underset{1-p}{\searrow} & \underset{1-l}{\searrow} & \$D & (1-p)(1-l)
\end{array}
$$

Consumer's preferences

Formally, we represent the consumer's preferences by a binary relation $\succsim$ defined on the set of lotteries.

For any pair of lotteries x and y, if the consumer says that x is at least as good as y, we write

$$x \succsim y$$

and say that x is (weakly) preferred to y.

Foundations of Economic Analysis (1947)

Paul A. Samuelson (1915-2009) – the first American Nobel laureate in economics and the foremost (academic) economist of the 20th century (and the uncle of Larry Summers…).

The basic assumptions about preferences

The theory begins with three assumptions about preferences. These assumptions are so fundamental that we can refer to them as "axioms" of decision theory.

[1] Completeness

$$x \succsim y \text{ or } y \succsim x$$

for any pair of bundles x and y.

[2] Transitivity

$$\text{if } x \succsim y \text{ and } y \succsim z \text{ then } x \succsim z$$

for any three bundles x, y and z.

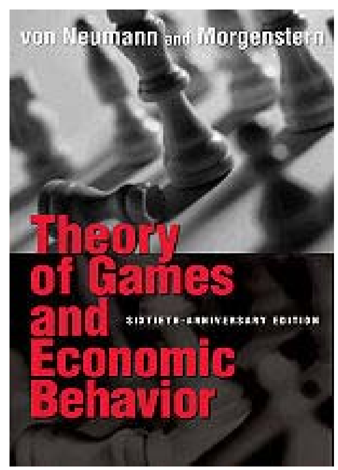
Together, completeness and transitivity constitute the formal definition of *rationality* as the term is used in economics. Rational economic agents are ones who

- have the ability to make choices [1], and

- whose choices display a logical consistency [2].

The third axiom about preferences toward risk is *independence*, due to von Neumann and Morgenstern.

The paternity of decision theory and game theory (1944)



[3] The third axiom is called independence:

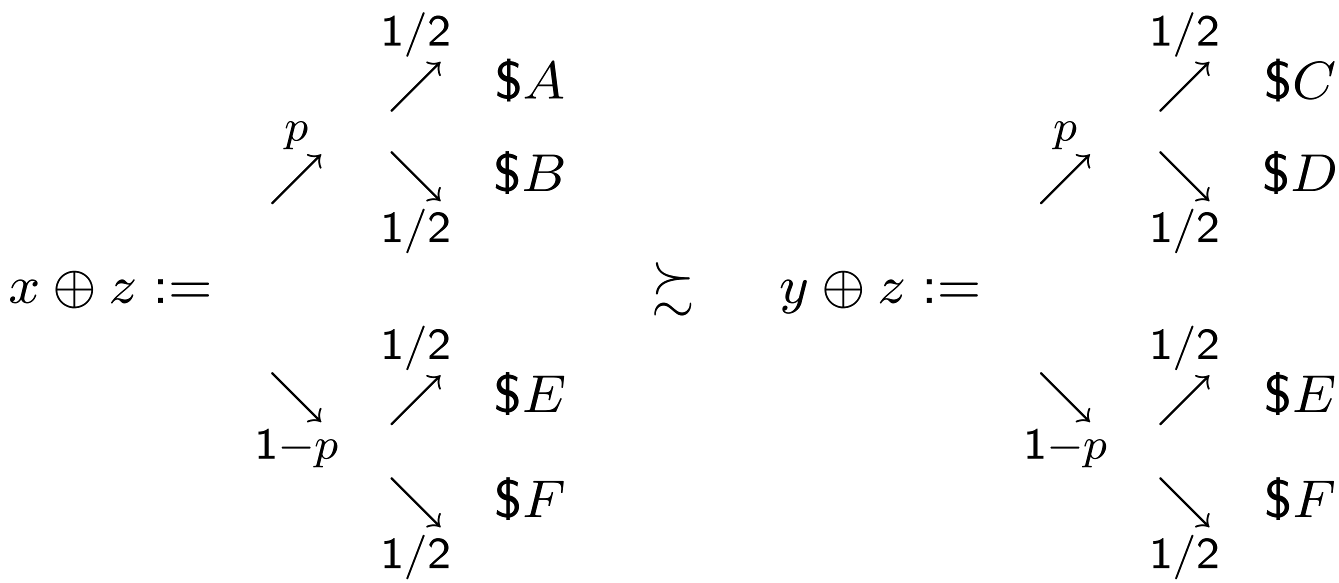
Independence

For any lotteries x, y, z and $0 < p < 1$

$$\text{if } x \succ y \quad \text{then } px + (1-p)z \succ py + (1-p)z.$$

Independence

$$x := \begin{array}{c} 1/2 \\ \nearrow \\ \\ \searrow \\ 1/2 \end{array} \begin{array}{c} \$A \\ \\ \\ \$B \end{array} \quad \succsim \quad y := \begin{array}{c} 1/2 \\ \nearrow \\ \\ \searrow \\ 1/2 \end{array} \begin{array}{c} \$C \\ \\ \\ \$D \end{array}$$



x ⊕ z :=
p
1−p
1/2
1/2
1/2
1/2
$A
$B
$E
$F
≿
y ⊕ z :=
p
1−p
1/2
1/2
1/2
1/2
$C
$D
$E
$F

von Neumann and Morgenstern Expected Utility Theory (EUT)

Allais (1953) I

– Choose between the two gambles:

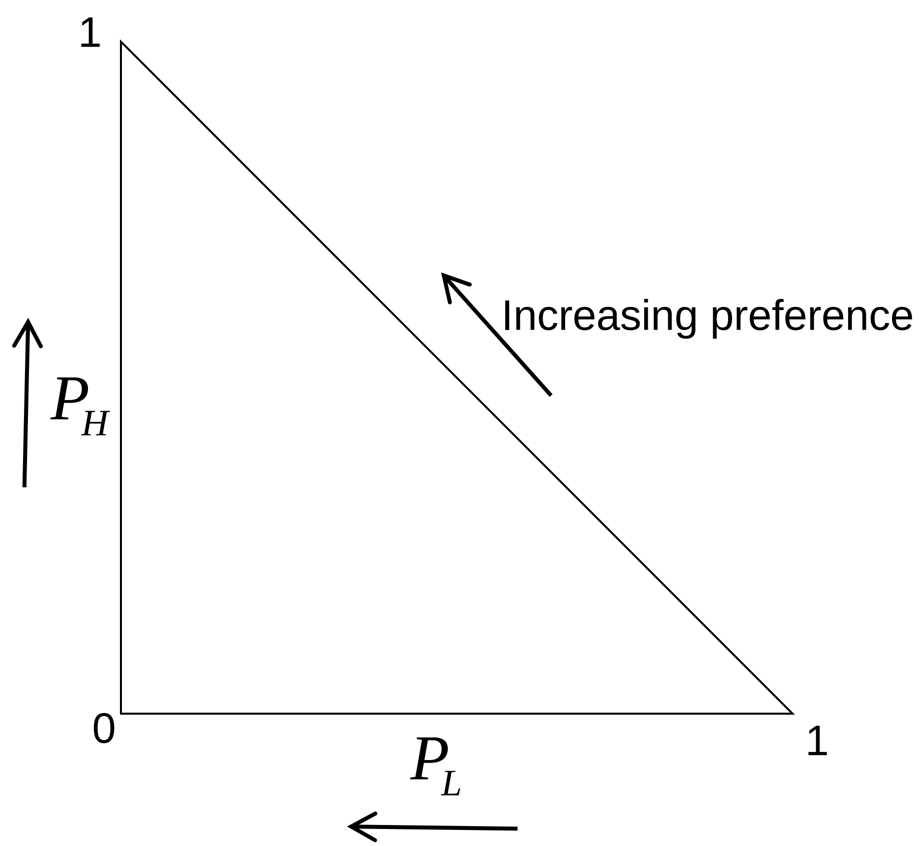
$$x := \begin{cases} \xrightarrow{.33} \$25,000 \\ \xrightarrow{.66} \$24,000 \\ \xrightarrow{.01} \$0 \end{cases} \qquad y := \xrightarrow{1} \$24,000$$

Allais (1953) II

– Choose between the two gambles:

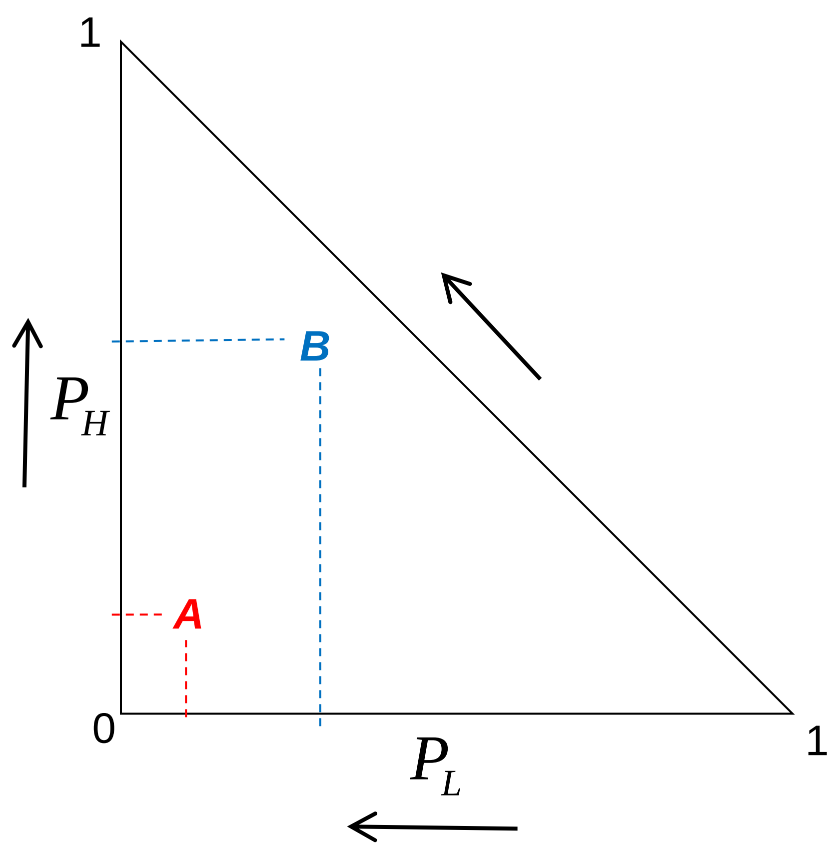
$$z := \begin{array}{l} \overset{.33}{\nearrow} \ \$25,000 \\ \\ \underset{.67}{\searrow} \ \$0 \end{array} \qquad w := \begin{array}{l} \overset{.34}{\nearrow} \ \$24,000 \\ \\ \underset{.66}{\searrow} \ \$0 \end{array}$$

The (Marschak-Machina) probability triangle



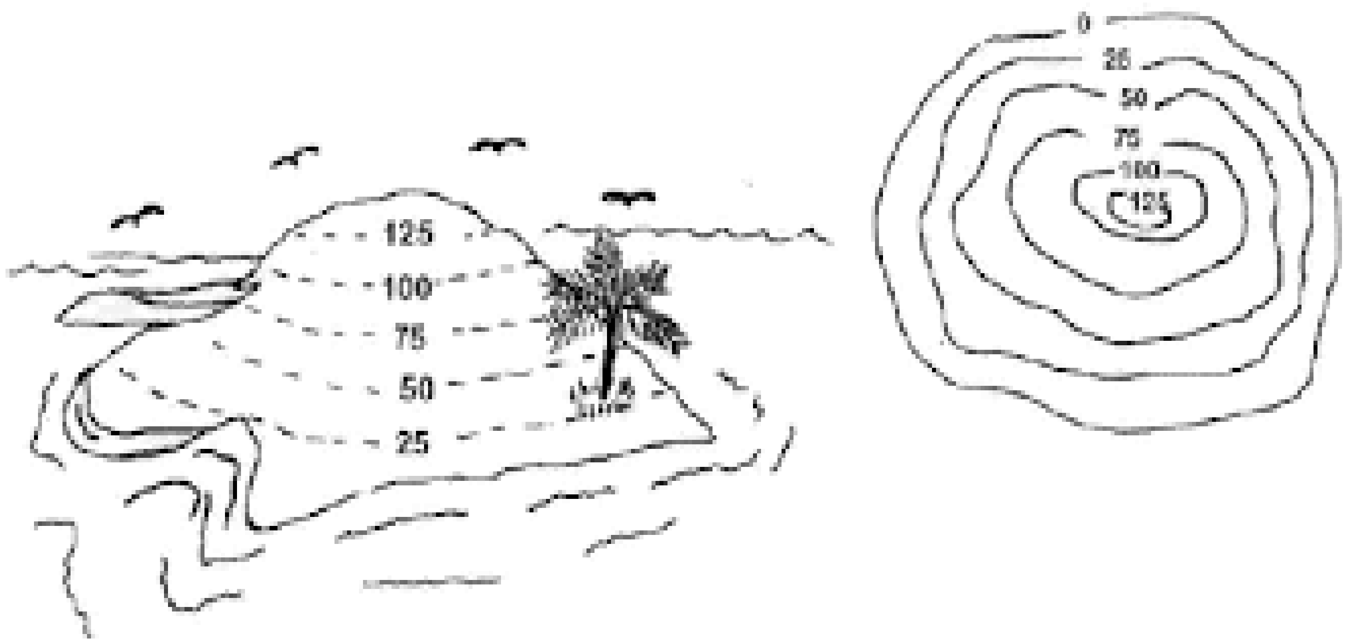
Consider three monetary payouts H, *M*, and *L* where *H>M>L*

Risk profiling

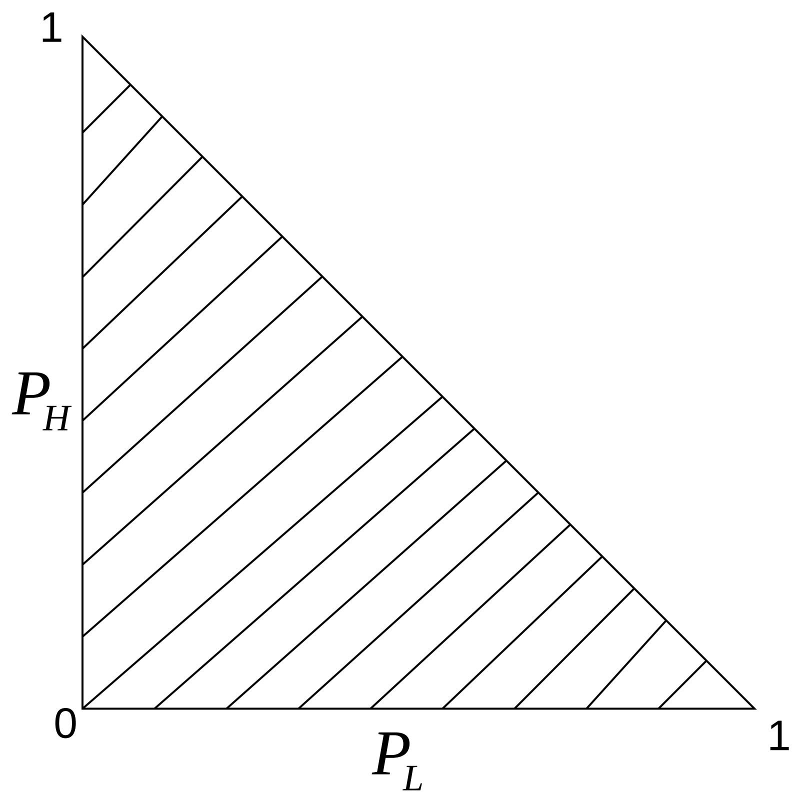


A “complete” risk profiling requires knowing all possible comparisons like between *A* and *B*.

A topographic map

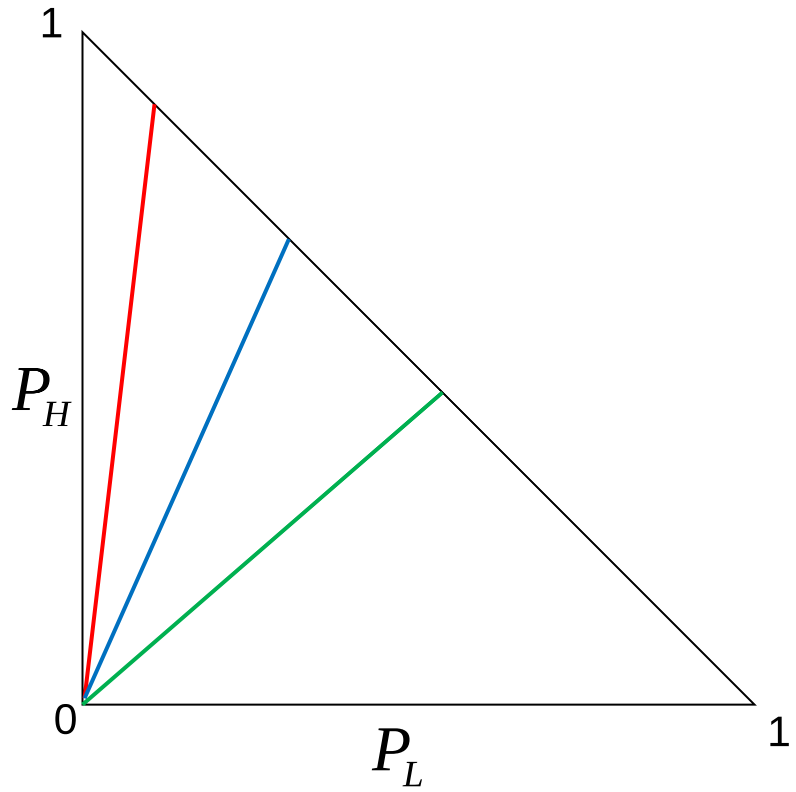


An indifference map of a loss-neutral (expected utility) individual



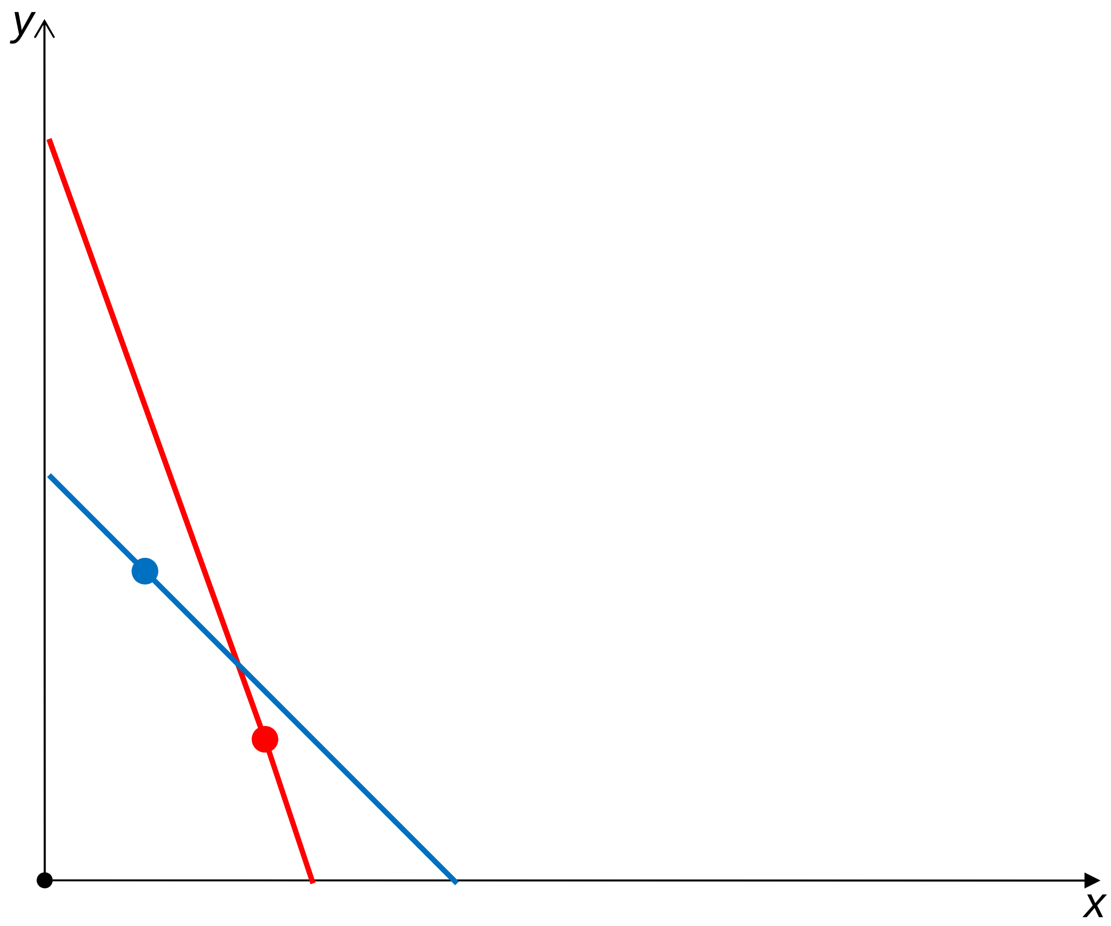
Expected Utility Theory (EUT) requires that indifference lines are parallel

Loss neutral and more risk tolerant

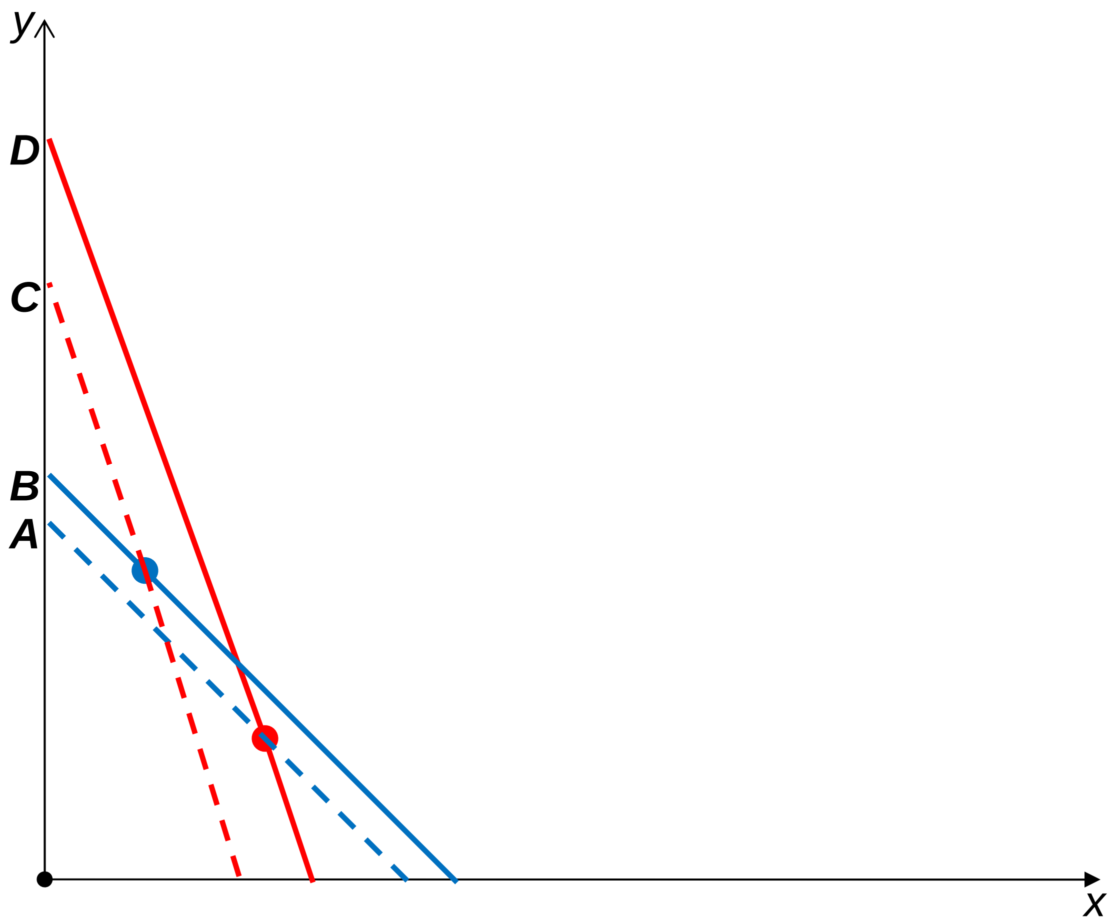


Mr. Green is more risk tolerant than Mr. Blue who is more risk tolerant than Mr. Red. The gentlemen are loss neutral.

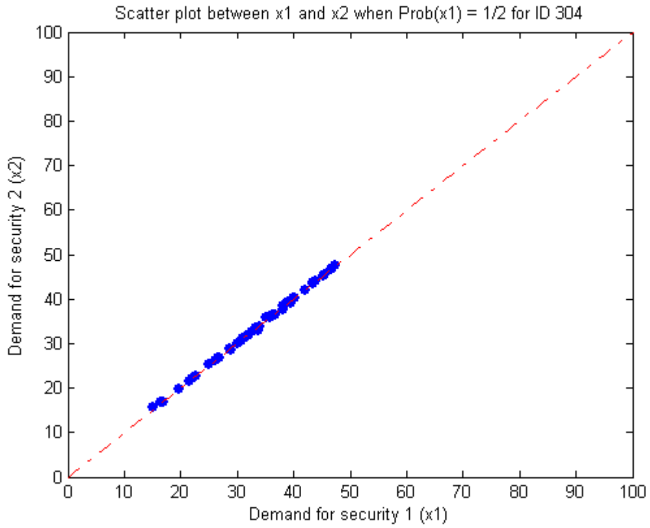
The construction of a Homo Economicus score (CCEI) 1

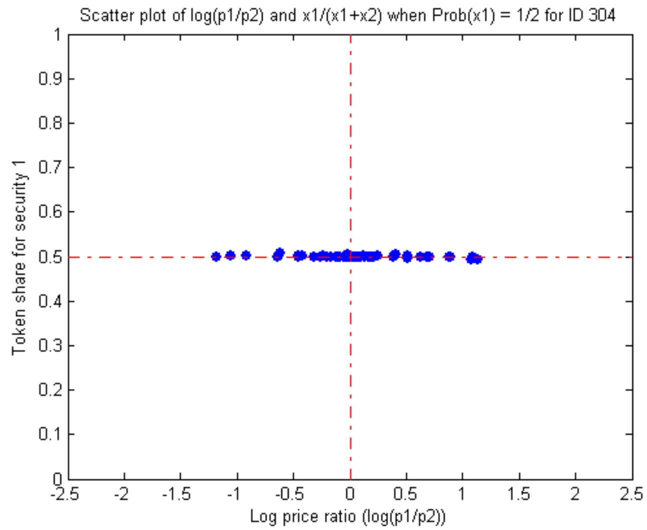


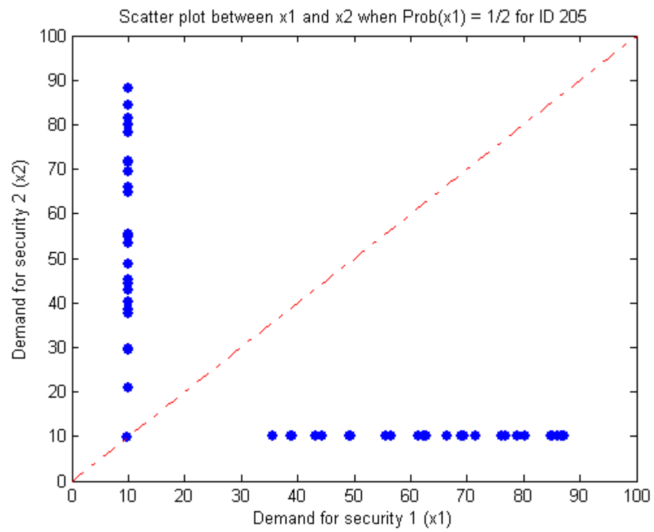
The construction of a Homo Economicus score (CCEI) 2



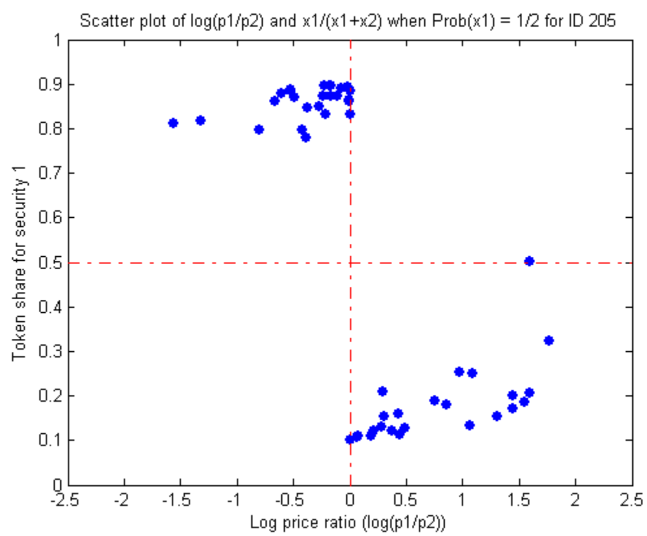
Individual-level data



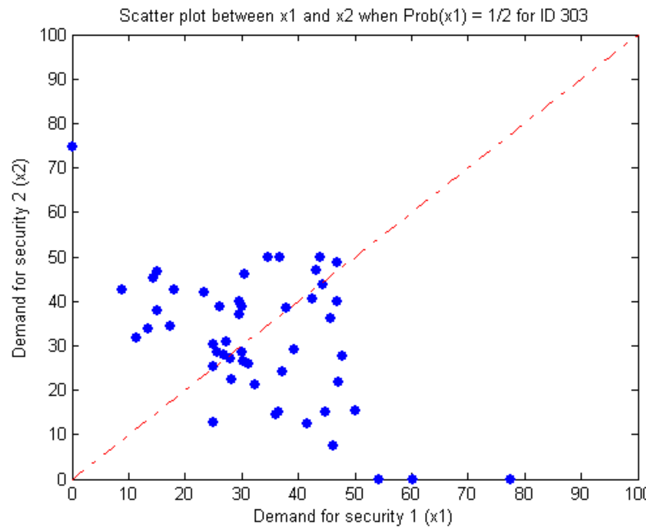


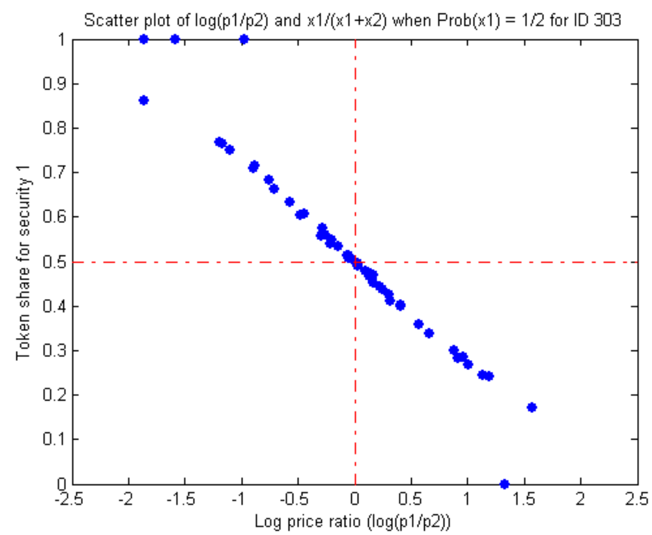


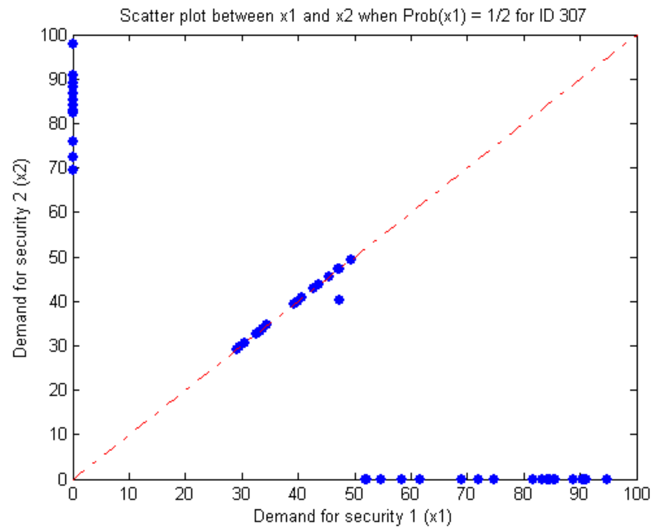
Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 205
Demand for security 2 (x2)
Demand for security 1 (x1)

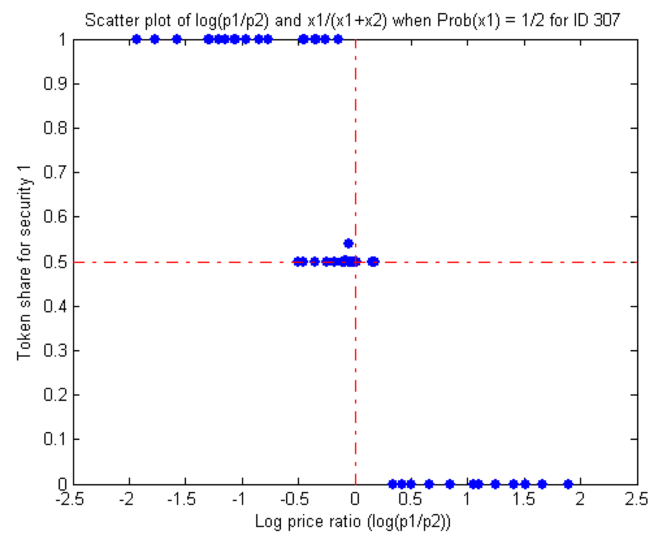


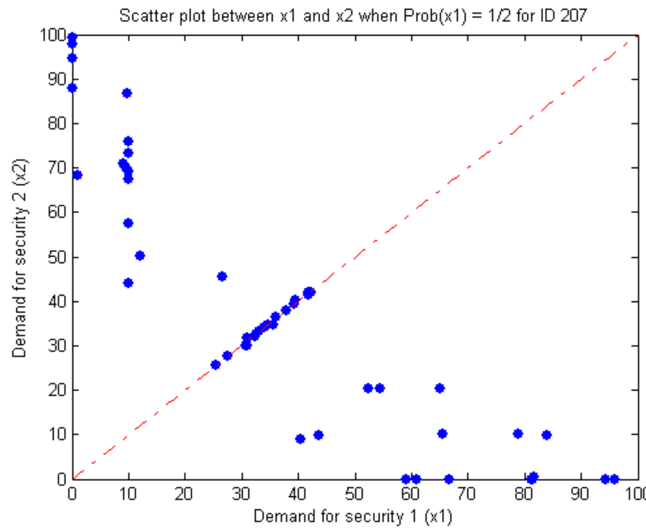
Scatter plot of log(p1/p2) and x1/(x1+x2) when Prob(x1) = 1/2 for ID 205
Token share for security 1
Log price ratio (log(p1/p2))

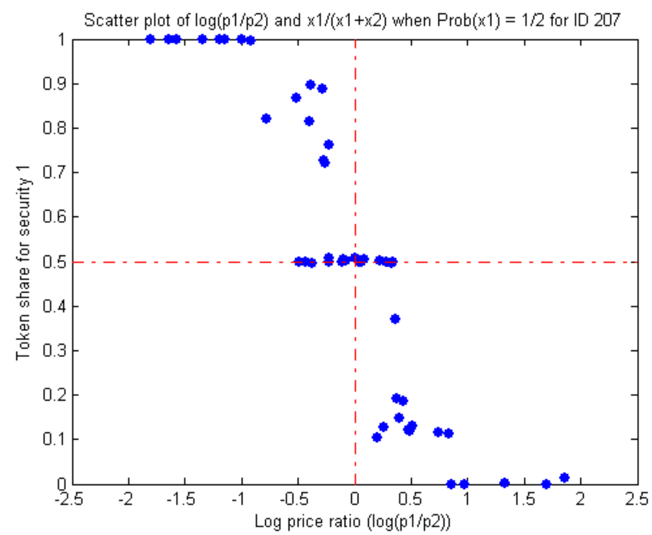

Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 303
Demand for security 2 (x2)
Demand for security 1 (x1)
0
10
20
30
40
50
60
70
80
90
100

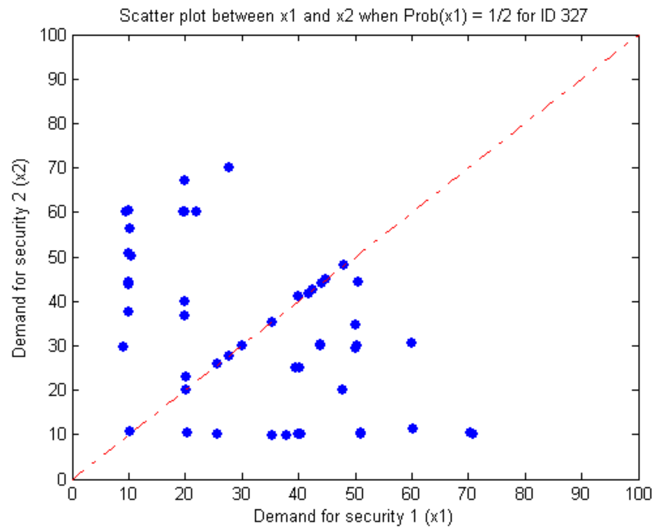

Scatter plot of log(p1/p2) and x1/(x1+x2) when Prob(x1) = 1/2 for ID 303
Token share for security 1
Log price ratio (log(p1/p2))
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

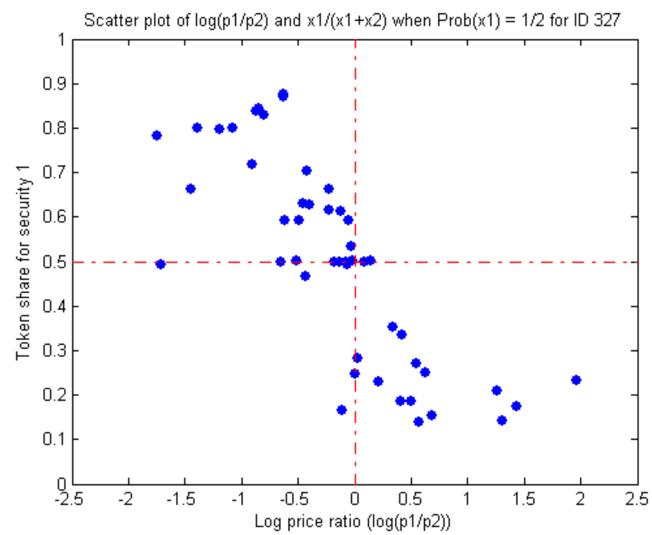

Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 307
Demand for security 2 (x2)
Demand for security 1 (x1)

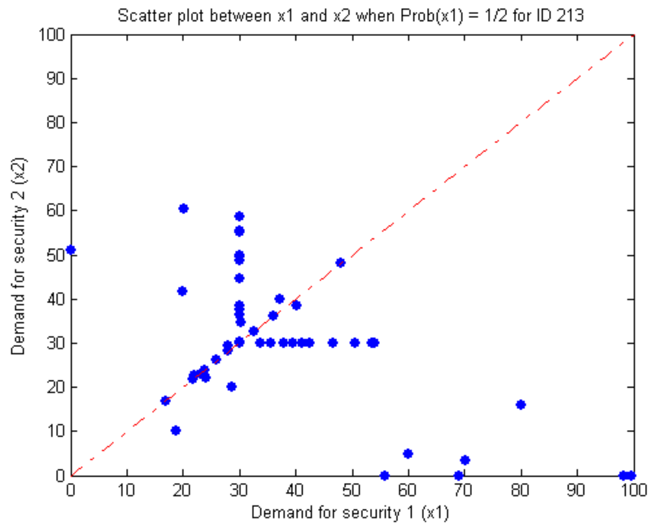

Scatter plot of log(p1/p2) and x1/(x1+x2) when Prob(x1) = 1/2 for ID 307
Token share for security 1
Log price ratio (log(p1/p2))


Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 207
Demand for security 2 (x2)
Demand for security 1 (x1)
0
10
20
30
40
50
60
70
80
90
100

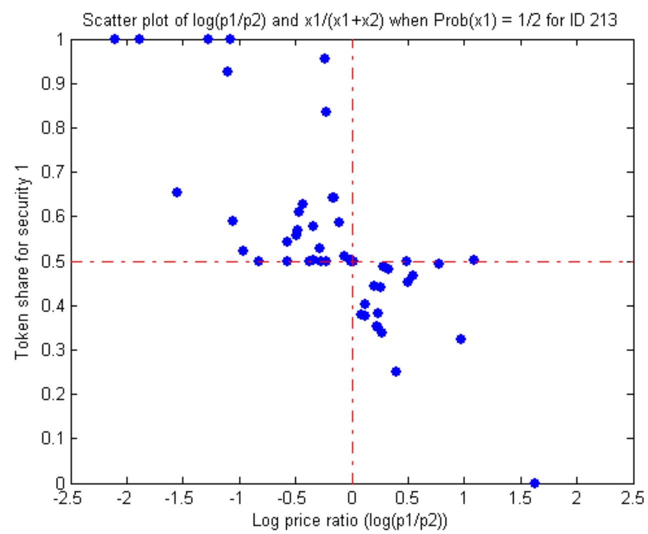

Scatter plot of log(p1/p2) and x1/(x1+x2) when Prob(x1) = 1/2 for ID 207
Token share for security 1
Log price ratio (log(p1/p2))
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9


Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 327
Demand for security 2 (x2)
Demand for security 1 (x1)
0
10
20
30
40
50
60
70
80
90
100


Scatter plot of log(p1/p2) and x1/(x1+x2) when Prob(x1) = 1/2 for ID 327
Token share for security 1
Log price ratio (log(p1/p2))
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

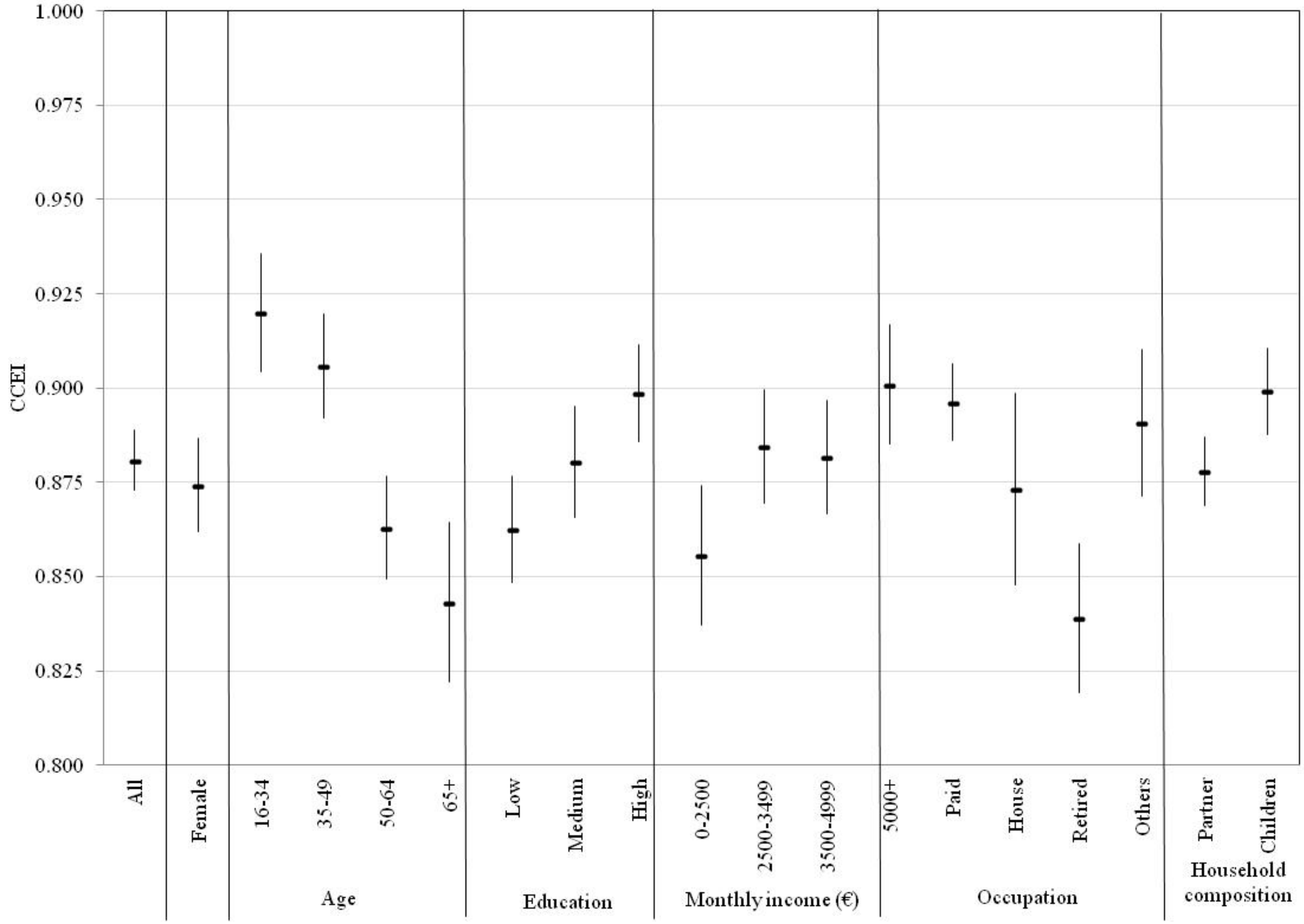


Scatter plot between x1 and x2 when Prob(x1) = 1/2 for ID 213
Demand for security 2 (x2)
Demand for security 1 (x1)



Scatter plot of log(p1/p2) and x1/(x1+x2) when Prob(x1) = 1/2 for ID 213
Token share for security 1
Log price ratio (log(p1/p2))

<u>Homo Economicus</u>: equiprobable lotteries



Wealth differentials

$\Longrightarrow$ The heterogeneity in wealth is not well-explained either by standard observables (income, education, family structure) or by standard unobservables (intertemporal substitution, risk tolerance).

$\Longrightarrow$ If consistency with utility maximization in the experiment is a good proxy for (financial) DMQ then the degree to which consistency differ across subjects should help explain wealth differentials.

The relationship between CCEI scores and wealth

	(1)	(2)	(3)
CCEI	1.351**	1.109**	101888.0*
	(0.566)	(0.534)	(52691.9)
Log 2008 household income	0.584***	0.606***	
	(0.132)	(0.126)	
2008 household income			1.776***
			(0.4)
Female	-0.313*	-0.356**	-32484.3*
	(0.177)	(0.164)	(17523.9)
Partnered	0.652***	0.595***	46201.9***
	(0.181)	(0.171)	(17173.7)
# of children	0.090	0.109	14078.6*
	(0.093)	(0.086)	(8351.5)
Age	Y	Y	Y
Education	Y	Y	Y
Occupation	Y	Y	Y
Constant	6.292	0.469	76214.4
	(6.419)	(3.598)	(559677.5)
R^2	0.179	0.217	0.188
# of obs.	517	566	568

The robustness of the correlation -- controls for constraints

	(1)	(2)	(3)	(4)	(5)
CCEI	1.322**	1.318**	1.925***	1.888***	1.441**
	(0.570)	(0.574)	(0.672)	(0.652)	(0.578)
Log household income					
2008	19.770	1.000	0.544***	0.285*	0.616***
	(14.629)	.	(0.137)	(0.165)	(0.128)
2008^2	-2.194				
	(1.533)				
2008^3	0.082				
	(0.053)				
2006				0.232	
				(0.231)	
2004				0.215	
				(0.174)	
Female	-0.291	-0.201	-0.337*	-0.296	-0.321*
	(0.181)	(0.173)	(0.185)	(0.186)	(0.176)
Partnered	0.598***	0.561***	0.734***	0.707***	0.641***
	(0.181)	(0.178)	(0.192)	(0.193)	(0.179)
# of children	0.091	0.101	0.018	0.031	0.088
	(0.092)	(0.096)	(0.099)	(0.095)	(0.093)
Age	Y	Y	Y	Y	Y
Education	Y	Y	Y	Y	N
Occupation	Y	Y	Y	Y	Y
Constant	-47.059	0.864	5.354	3.016	6.398
	(46.275)	(6.545)	(6.93)	(7.109)	(6.484)
R^2	0.187		0.205	0.217	0.177
# of obs.	517	517	449	449	517

The robustness of the correlation -- controls for preferences and beliefs

	(1)	(2)	(3)	(4)	(5)
CCEI	1.379**	1.396**	1.404**	1.214*	1.237**
	(0.568)	(0.568)	(0.569)	(0.625)	(0.623)
Risk tolerance					
Quantitative (experiment)	-0.768	-0.808	-0.766		
	(0.714)	(0.711)	(0.718)		
Qualitative (survey)		0.017	0.023		
		(0.074)	(0.076)		
Qualitative (survey) missing		-0.190	-0.162		
		(0.335)	(0.482)		
Conscientiousness			0.089		
			(0.072)		
Conscientiousness missing			-0.040		
			(0.668)		
Longevity expectations					-0.034
					(0.040)
Log 2008 household income	0.589***	0.578***	0.572***	0.443***	0.434***
	(0.132)	(0.131)	(0.133)	(0.123)	(0.123)
Female	-0.316*	-0.310*	-0.323*	-0.415**	-0.417**
	(0.177)	(0.181)	(0.181)	(0.186)	(0.186)
Partnered	0.655***	0.658***	0.642***	0.686***	0.687***
	(0.181)	(0.181)	(0.182)	(0.204)	(0.205)
# of children	0.086	0.087	0.083	0.075	0.083
	(0.093)	(0.093)	(0.093)	(0.102)	(0.102)
Age	Y	Y	Y	Y	Y
Education	Y	Y	Y	Y	Y
Occupation	Y	Y	Y	Y	Y
Constant	6.840	6.883	6.496	3.777	4.411
	(6.361)	(6.357)	(6.395)	(15.258)	(15.256)
R^2	0.179	0.176	0.176	0.163	0.163
# of obs.	517	517	517	414	414

Evaluating alternative measures of *DMQ*

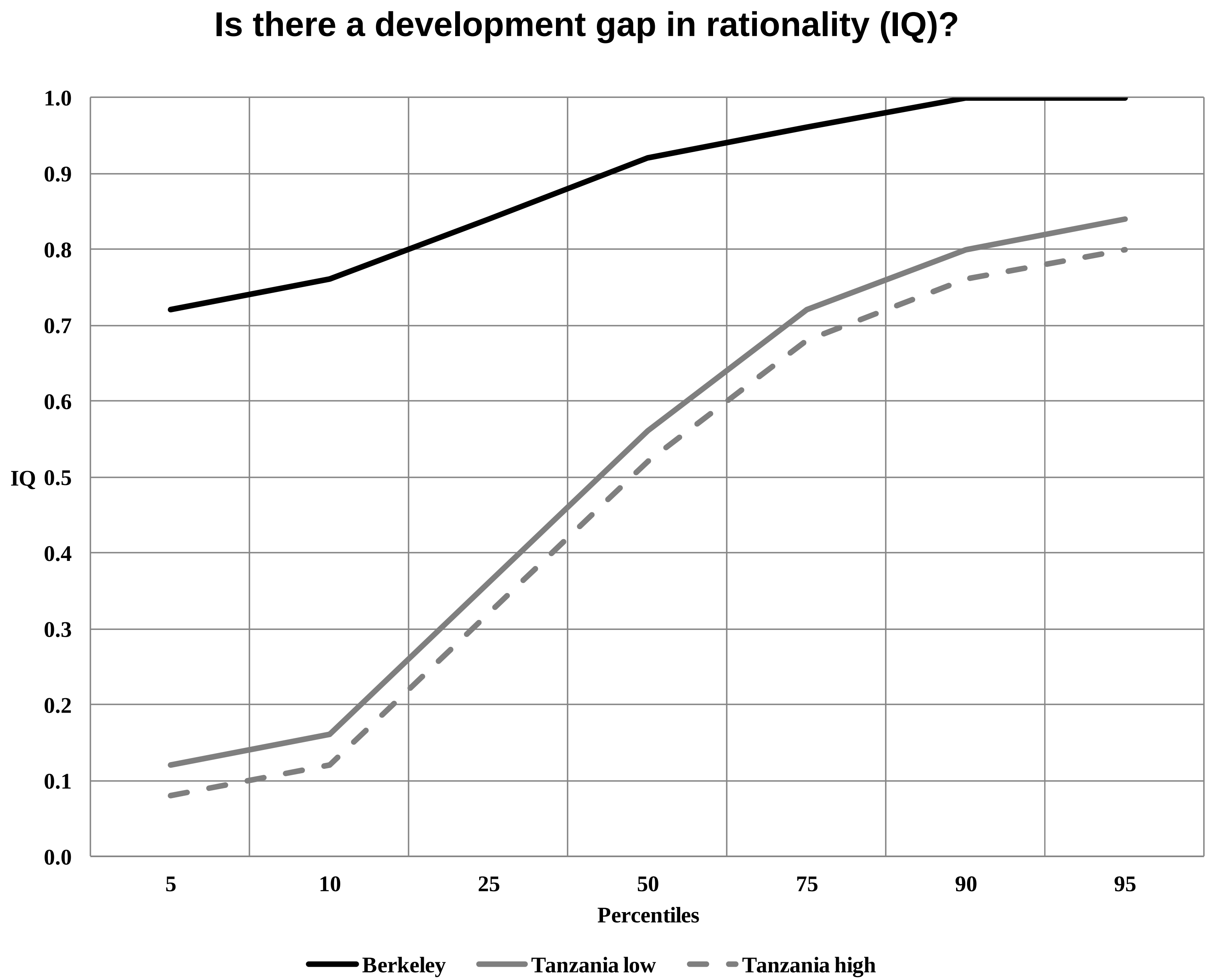
	(1)	(2)	(3)	(4)
CCEI	1.253*	1.401*	1.269*	1.177**
	(0.712)	(0.729)	(0.729)	(0.583)
CCEI (combined dataset)	0.099			
	-0.38			
von Gaudecker et al. (2011)			0.927*	
			(0.485)	
Cognitive Reflection Test (CRT)				0.120*
				(0.071)
CRT missing				-0.203
				(0.237)
Log 2008 household income	0.586***	0.388*	0.383*	0.577***
	(0.132)	(0.155)	(0.154)	(0.132)
Female	-0.314*	-0.218	-0.207	-0.292*
	(0.177)	(0.212)	(0.211)	(0.176)
Partnered	0.653***	0.907***	0.926***	0.690***
	(0.181)	(0.230)	(0.228)	(0.181)
# of children	0.089	0.105	0.096	0.091
	(0.093)	(0.114)	(0.113)	(0.092)
Age	Y	Y	Y	Y
Education	Y	Y	Y	Y
Occupation	Y	Y	Y	Y
Constant	6.237	10.056	8.355	6.855
	(6.424)	(6.976)	(6.990)	(6.464)
R^2	0.177	0.225	0.232	0.181
# of obs.	517	326	326	517

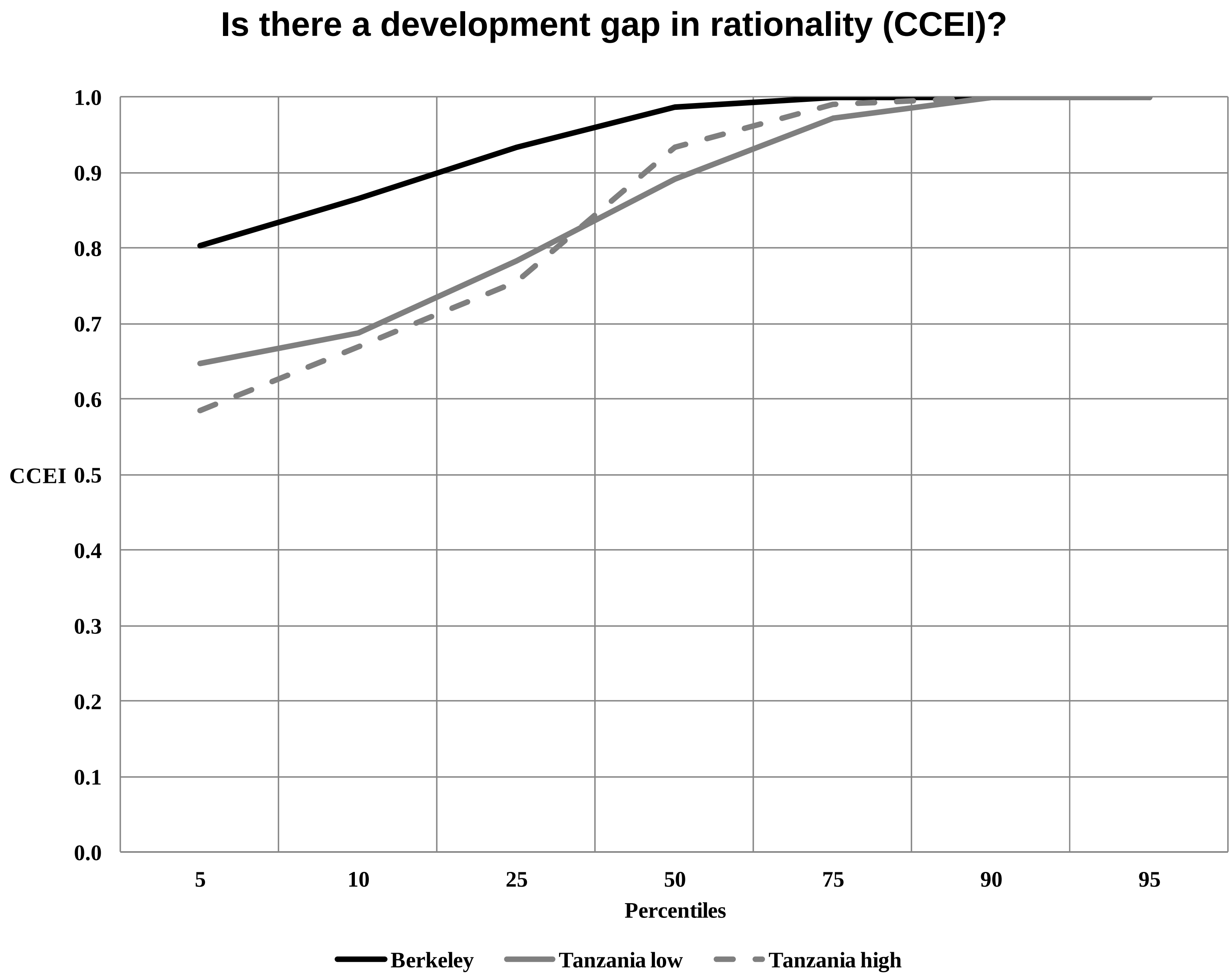
The sources of the relationship

	(1) Have checking	(2) Fraction in checking	(3) Have saving	(4) Fraction in saving
CCEI	0.03	-0.098*	-0.047	-0.162*
	(0.032)	(0.057)	(0.053)	(0.097)
Log 2008 household income	0.001	-0.029**	0.003	-0.068***
	(0.002)	(0.013)	(0.010)	(0.021)
Female	0.007	0.023	0.014	0.038
	(0.005)	(0.020)	(0.019)	(0.033)
Partnered	-0.005	-0.031	0.017	-0.054
	(0.004)	(0.020)	(0.022)	(0.033)
# of children	0.000	-0.004	-0.025*	-0.043***
	(0.001)	(0.010)	(0.014)	(0.013)
Age	Y	Y	Y	Y
Education	Y	Y	Y	Y
Occupation	Y	Y	Y	Y
Constant	0.998***	0.106	1.126	1.448
	(0.172)	(0.822)	(0.848)	(1.288)
R^2	-0.007	0.021	-0.011	0.083
# of obs.	512	512	502	502

The sources of the relationship (cont.)

	(5) Have stocks	(6) Fraction in stocks	(7) Have a house	(8) Fraction in house
CCEI	0.167	0.001	0.352**	0.324**
	(0.163)	(0.050)	(0.152)	(0.129)
Log 2008 household income	0.148***	0.013	0.134***	0.096***
	(0.031)	(0.009)	(0.029)	(0.024)
Female	0.007	0.009	-0.038	-0.066
	(0.050)	(0.013)	(0.050)	(0.043)
Partnered	0.005	-0.007	0.207***	0.127***
	(0.049)	(0.014)	(0.051)	(0.044)
# of children	0.003	0.000	0.048**	0.063***
	(0.026)	(0.007)	(0.020)	(0.019)
Age	Y	Y	Y	Y
Education	Y	Y	Y	Y
Occupation	Y	Y	Y	Y
Constant	-3.152*	-0.317	-1.047	-1.151
	(1.856)	(0.398)	(1.760)	(1.419)
R^2	0.079	0.002	0.148	0.123
# of obs.	514	514	479	479


Is there a development gap in rationality (IQ)?
1.0
0.9
0.8
0.7
0.6
IQ 0.5
0.4
0.3
0.2
0.1
0.0
5
10
25
50
75
90
95
Percentiles
Berkeley
Tanzania low
Tanzania high


Is there a development gap in rationality (CCEI)?
CCEI
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
5
10
25
50
75
90
95
Percentiles
Berkeley
Tanzania low
Tanzania high

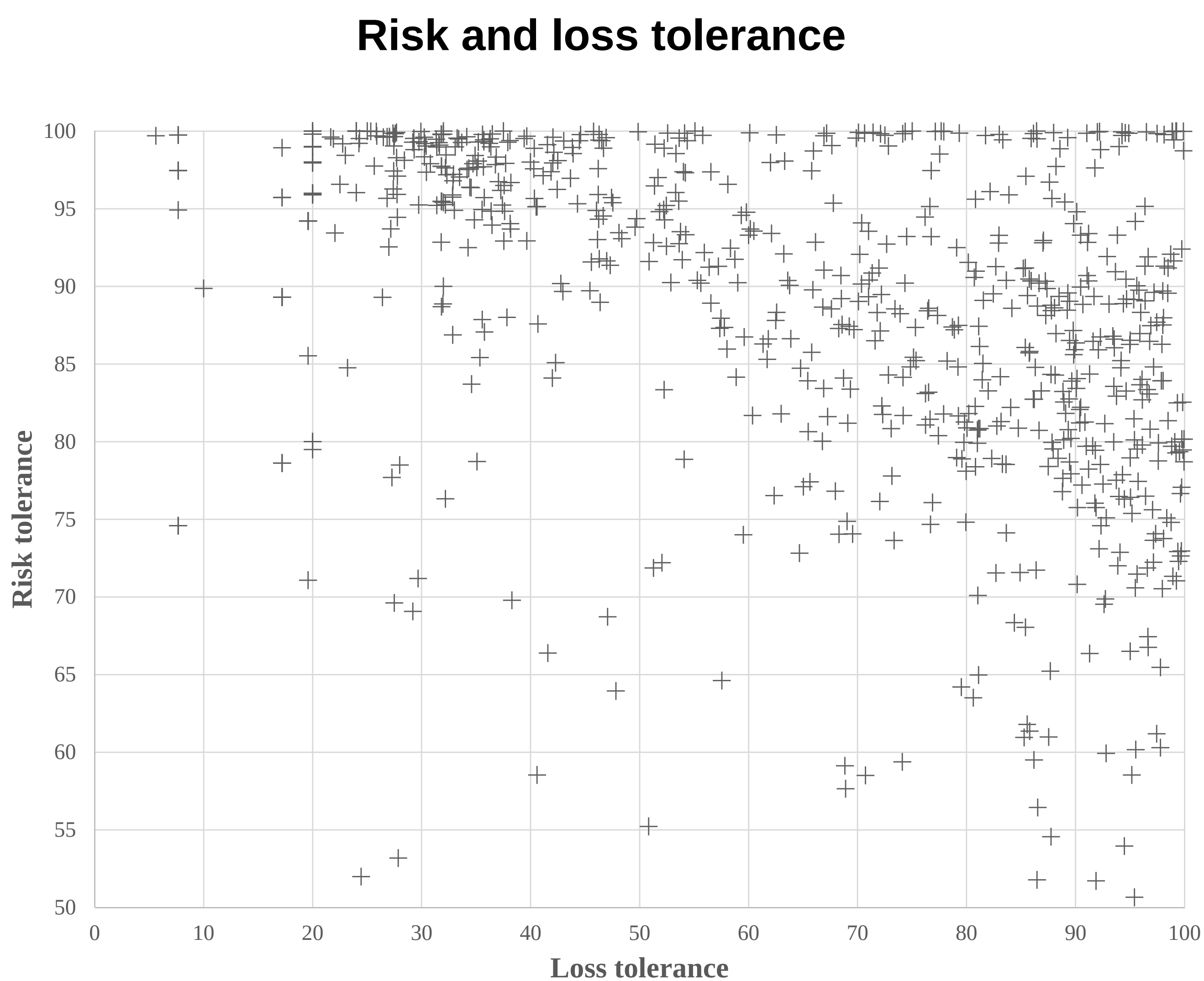
Loss aversion/tolerance

Suppose the underlying utility function over portfolios takes the form

$$\min \{\alpha u(x) + u(y), u(x) + \alpha u(y)\},$$

where $\alpha \geq 1$ measures loss aversion and $u(\cdot)$ measures risk aversion using CRRA or CARA.

If $\alpha > 1$ there is a kink at the point where $x = y$ and if $\alpha = 1$ we have loss neutrality (standard EUT representation).


Risk and loss tolerance
Risk tolerance
100
95
90
85
80
75
70
65
60
55
50
0
10
20
30
40
50
60
70
80
90
100
Loss tolerance