UC Berkeley
Haas School of Business
Game Theory
(EMBA 296 & EWMBA 211)
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Bank runs, social learning and wrapping up!

Block IV

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Bank runs
The Diamond-Dybvig (1983) model

A simple Diamond-Dybvig (1983) model

"... bank runs are a common feature of the extreme crises that have played a prominent role in monetary history. During a bank run, depositors rush to withdraw their deposits because they expect the bank to fail...

... in fact, the sudden withdrawals can force the bank to liquidate many of its assets at a loss and to fail. In a panic with many bank failures, there is a disruption of the monetary system and a reduction in production..."

The mismatch of liquidity:

- Banks issue demand deposits that allow depositors to withdraw at any time and make loans that cannot be sold quickly even at a high price.

- Because the bank's liabilities are more liquid than its assets, it will face a problem when too many depositors attempt to withdraw at once.

$\Rightarrow$ A situation referred to as a bank run.

Consider the following asset on three dates, $T = \{0, 1, 2\}$:

– If one invests one unit at date 0, it will be worth r_2 at date 2, but only $r_1 < r_2$ at date 1.

– The lower r_1/r_2 is (holding constant market rates), the less liquid is the asset.

$\Rightarrow$ The lower the fraction of the present value of the future cash flow that can be obtained today, the less liquid is the asset.

Investors have an uncertain horizon:

- Each will need to consume either at date $T = 1$ or $T = 2$ but, as of date 0, does not know at which date s/he will need to consume.

- An investor a "type 1" if s/he needs to liquidate at $T = 1$ and a "type 2" otherwise.

- Each investor has a probability t of being of type 1 and $1 - t$ of being of type 2.

$\Rightarrow$ There is no aggregate uncertainty (there will be a fraction t of investors of type 1).

An investor who holds the asset (r_1, r_2), which gives a choice of r_1 at date 1 or $r_2 > r_1$ at date 2, consumes $c_1 = r_1$ if s/he is type 1 (with prob. t) or $c_2 = r_2$ if type 2 (with prob. $1 - t$).

The investor's expected utility is given by

$$tU(r_1) + (1 - t)U(r_2)$$

and we assume that the investors have the risk-averse utility function

$$U(c) = 1 - \frac{1}{c}.$$

Comparing more and less liquid assets

Consider the following two assets, both of which cost 1 at date 0:

illiquid	$(r_1 = 1, r_2 = R)$
more liquid	$(r_1 > 1, r_2 < R)$

We illustrate the demand for liquidity with the following numerical example for the case where the probability of being of type 1 is $t = \frac{1}{4}$ and

	r_1	r_2
illiquid	1	2
more liquid	1.28	1.813

(we will explain why these particular numerical values are used...).

The expected utility from holding the illiquid asset is

$$\frac{1}{4}U(1) + \frac{3}{4}U(2) = 0.375$$

where the expected utility from holding the more liquid asset is

$$\frac{1}{4}U(1.28) + \frac{3}{4}U(1.813) = 0.391 > 0.375$$

$\Rightarrow$ A risk-averse investor prefers holding the more liquid / less risky asset because s/he prefers its more 'smoother' pattern of returns.

But note that if investors were not risk averse $U(c) = c$, they would not prefer this particular liquid asset:

$$\frac{1}{4}U(1) + \frac{3}{4}U(2) = \frac{1}{4}(1) + \frac{3}{4}(2) = 1.75$$

where the expected utility from holding the more liquid asset is

$$\frac{1}{4}U(1.28) + \frac{3}{4}U(1.813) = \frac{1}{4}(1.28) + \frac{3}{4}(1.813) = 1.68 < 1.75.$$

$\Rightarrow$ An investor's demand for liquidity is greater the higher her/his (relative) risk aversion is.

The optimal amount of liquidity

1. Suppose that the bank receives \$1 from each of 100 investors at date $T = 0$ and in return offers to pay $r_1 = 1.28$ to those who withdraw at $T = 1$ or to pay $r_2 = 1.813$ to those who withdraw at $T = 2$.

2. If the bank invests in the illiquid asset, it will need to liquidate $25 \times 1.28 = 32$ assets (32% of the portfolio) at $T = 1$ to pay 1.28 to those who withdraw.

3. Then 68 assets will remain until $T = 2$, when they will be worth $R = 2$ each. The 75 depositors that will remain at $T = 2$ will receive

$$\frac{68 \times 2}{75} = 1.813$$

The optimal levels of r_1 and r_2 maximize the ex-ante expected utility of each investor at date 0. That is,

$$\max_{r_1,r_2} \quad tU(r_1) + (1-t)U(r_2)$$

$$\text{subject to} \quad r_1 \geq 0,\ r_2 \geq 0, r_2 \leq \frac{(1-tr_1)R}{1-t}$$

$\Rightarrow$ An investor needs all or none of his liquidity, while the bank knows that a fraction t of its depositors will need liquidity at date 1.

$\Rightarrow$ When long-term assets are even more illiquid, there is an additional way that banks can help investors and/or make profits...

Bank runs

- How much is left to pay depositors who wait until date 2 to withdraw if a fraction $f \geq t$ of initial depositors withdraw at date 1?

- Each depositor needs a forecast of f denoted by $\hat{f}$ upon which s/he chooses whether to withdraw at date 1.

- Recall that a Nash equilibrium is self-fulfilling, and in the good equilibrium (no bank run) $\hat{f} = f = t = \frac{1}{4}$.

- But there is also a bad equilibrium (bank run) in which <u>all</u> withdraw at date 1 because they all expect each other to do the same...

- The self-fulfilling prophecy of a bank run is $\hat{f} = f = 1$ where all rush to withdraw. That is, if a run is feared, it becomes a self-fulfilling prophecy.

- These two possible equilibrium beliefs (self-fulfilling forecasts of f) are so-called locally stable. The tipping point for a run is a forecast implying that

$$r_1 \geq r_2(\hat{f}) = \frac{(1 - \hat{f} \times r_1)R}{1 - \hat{f}}$$

or

$$\hat{f} > \frac{R - r_1}{r_1(R - 1)}$$

which in the example is 0.5625.

Takeaways

- Moving away from a good equilibrium requires a large change in beliefs $\Longrightarrow$ a run requires 'bad news' that many depositors see (and believe that others see).

- Not all depositors observe the same news so they do not have a way to tell if others are choosing to panic and run (no common knowledge).

- One way to stop and prevent runs is deposit insurance by government b/c it has taxation authority (ability to take resources without prior contracts).

- But there is also scope for banks to write more refined contracts, e.g. deposits with suspension of convertibility of to cash.

Social Learning
Herd behavior and informational cascades

Examples

Business strategy

– TV networks make introductions in the same categories as their rivals.

Finance

– The withdrawal behavior of small number of depositors starts a bank run.

Politics

– The solid New Hampshirites (probably) can not be too far wrong.

Crime

– In NYC, individuals are more likely to commit crimes when those around them do.

Why should individuals behave in this way?

Several “theories” explain the existence of uniform social behavior:

- benefits from conformity
- sanctions imposed on deviants
- network / payoff externalities
- social learning

Broad definition: any situation in which individuals learn by observing the behavior of others.

Informational cascades and herd behavior

Two phenomena that have elicited particular interest are *informational cascades* and *herd behavior*.

– Cascade: agents 'ignore' their private information when choosing an action.

– Herd: agents choose the same action, not necessarily ignoring their private information.

- While the terms informational cascade and herd behavior are used interchangeably there is a significant difference between them.

- In an informational cascade, an agent considers it optimal to follow the behavior of her predecessors without regard to her private signal.

- When acting in a herd, agents choose the same action, not necessarily ignoring their private information.

- Thus, an informational cascade implies a herd but a herd is not necessarily the result of an informational cascade.

A model of social learning

Signals

- Each player $n \in \{1, ..., N\}$ receives a signal θ_n that is private information.

- For simplicity, $\{\theta_n\}$ are independent and uniformly distributed on $[-1, 1]$.

Actions

- Sequentially, each player n has to make a binary irreversible decision $x_n \in \{0, 1\}$.

Payoffs

– $x = 1$ is profitable if and only if $\sum_{n \leq N} \theta_n \geq 0$, and $x = 0$ is profitable otherwise.

Information

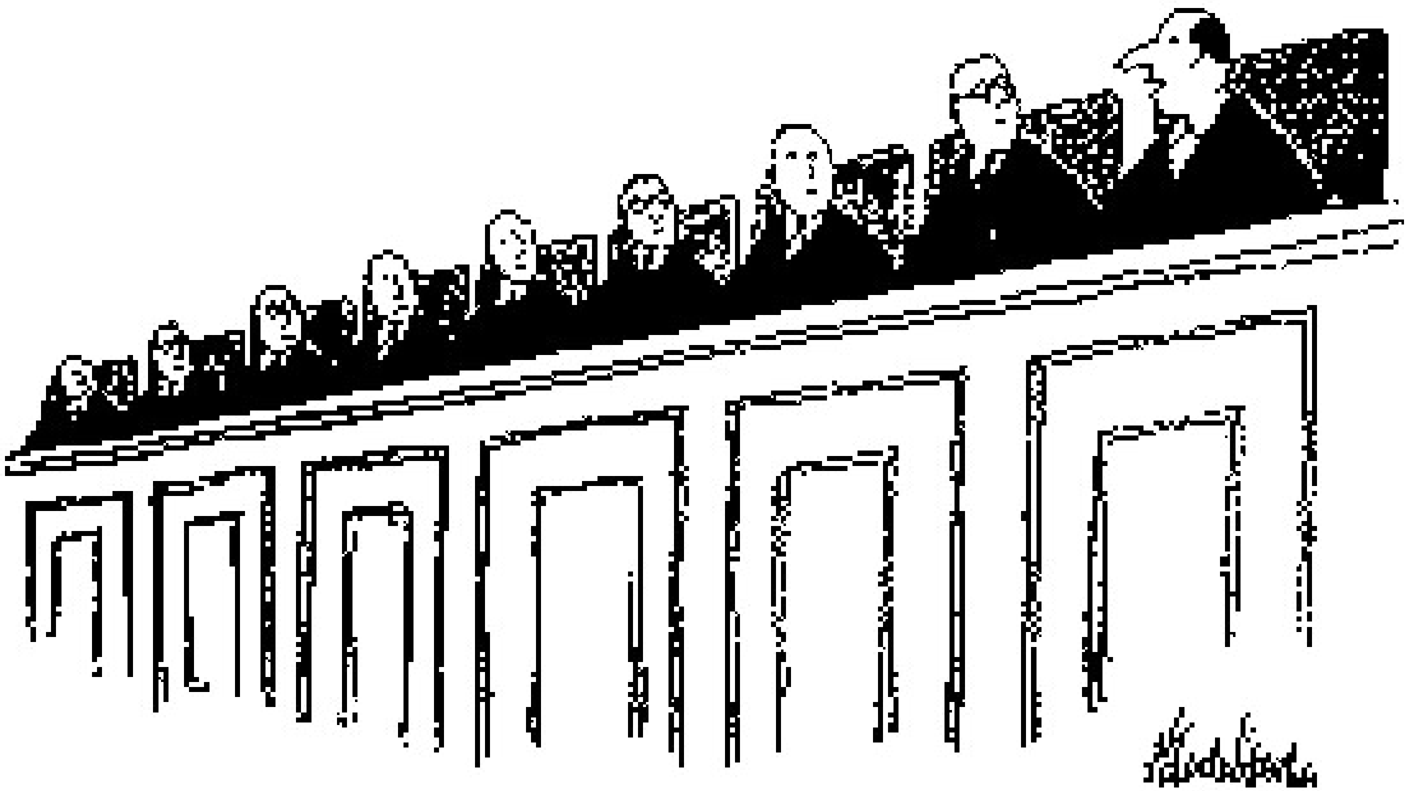
– Perfect information

$$\mathcal{I}_n = \{\theta_n, (x_1, x_2, ..., x_{n-1})\}$$

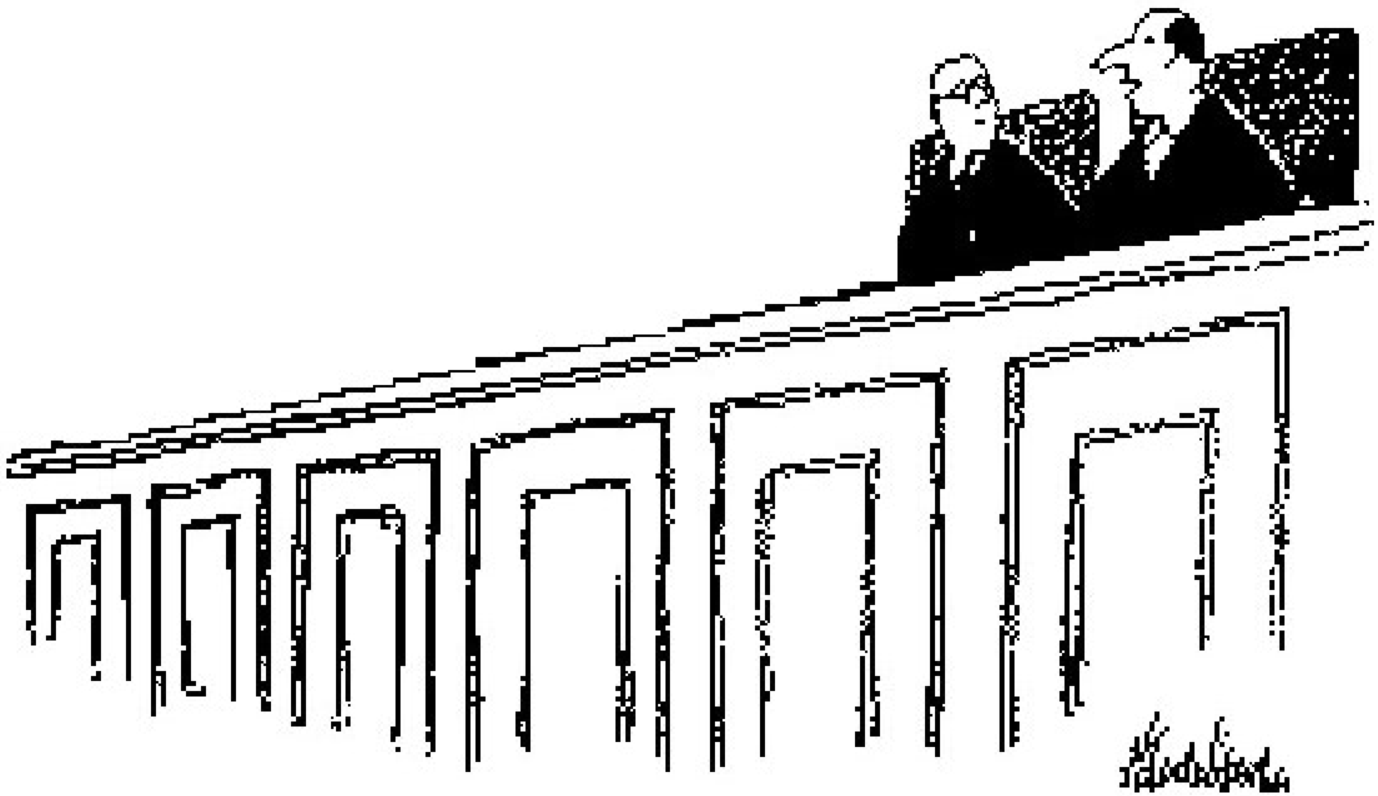
– Imperfect information

$$\mathcal{I}_n = \{\theta_n, x_{n-1}\}$$

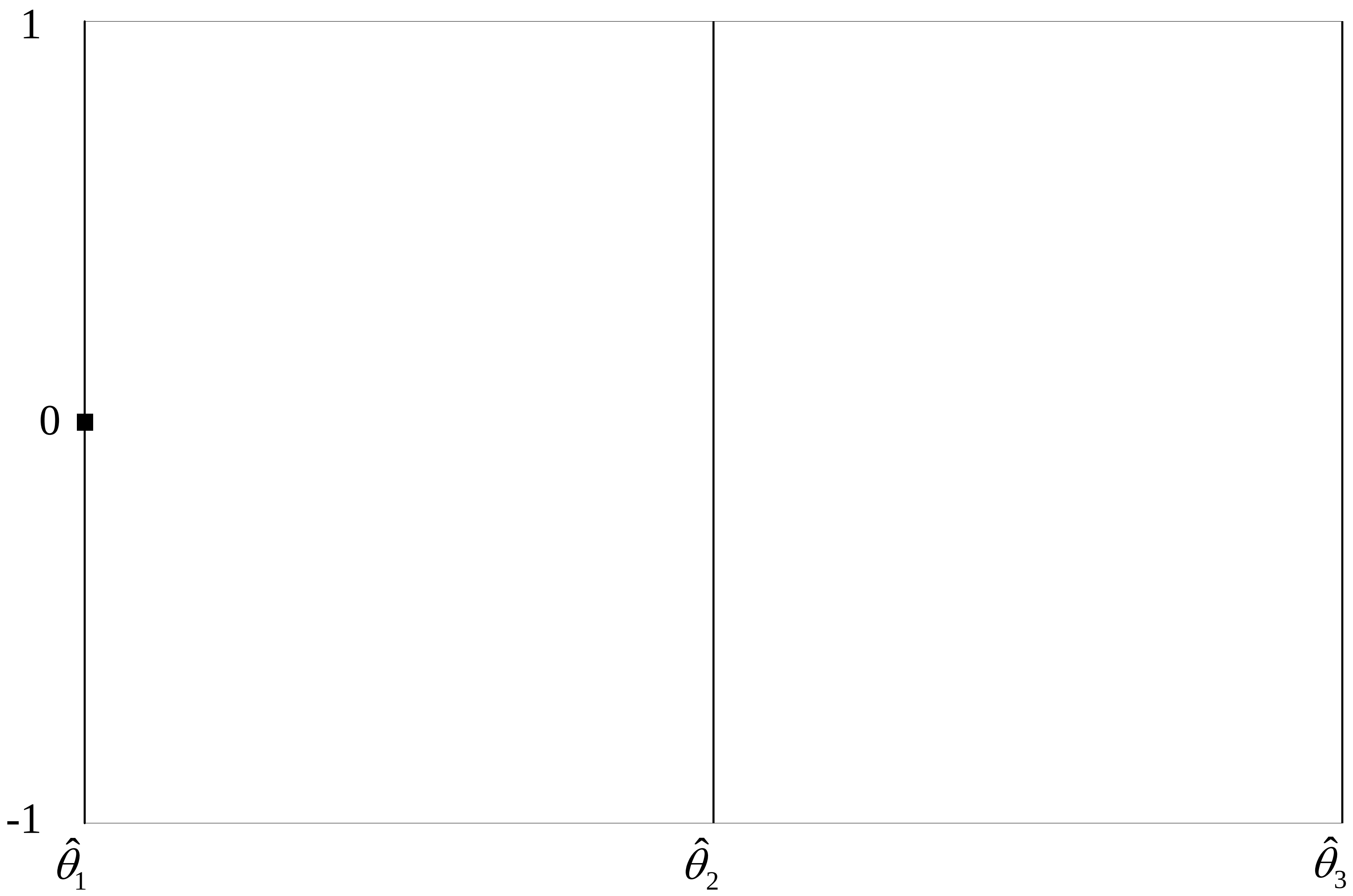
Sequential social-learning model:
Well heck, if all you smart cookies agree, who am I to dissent?



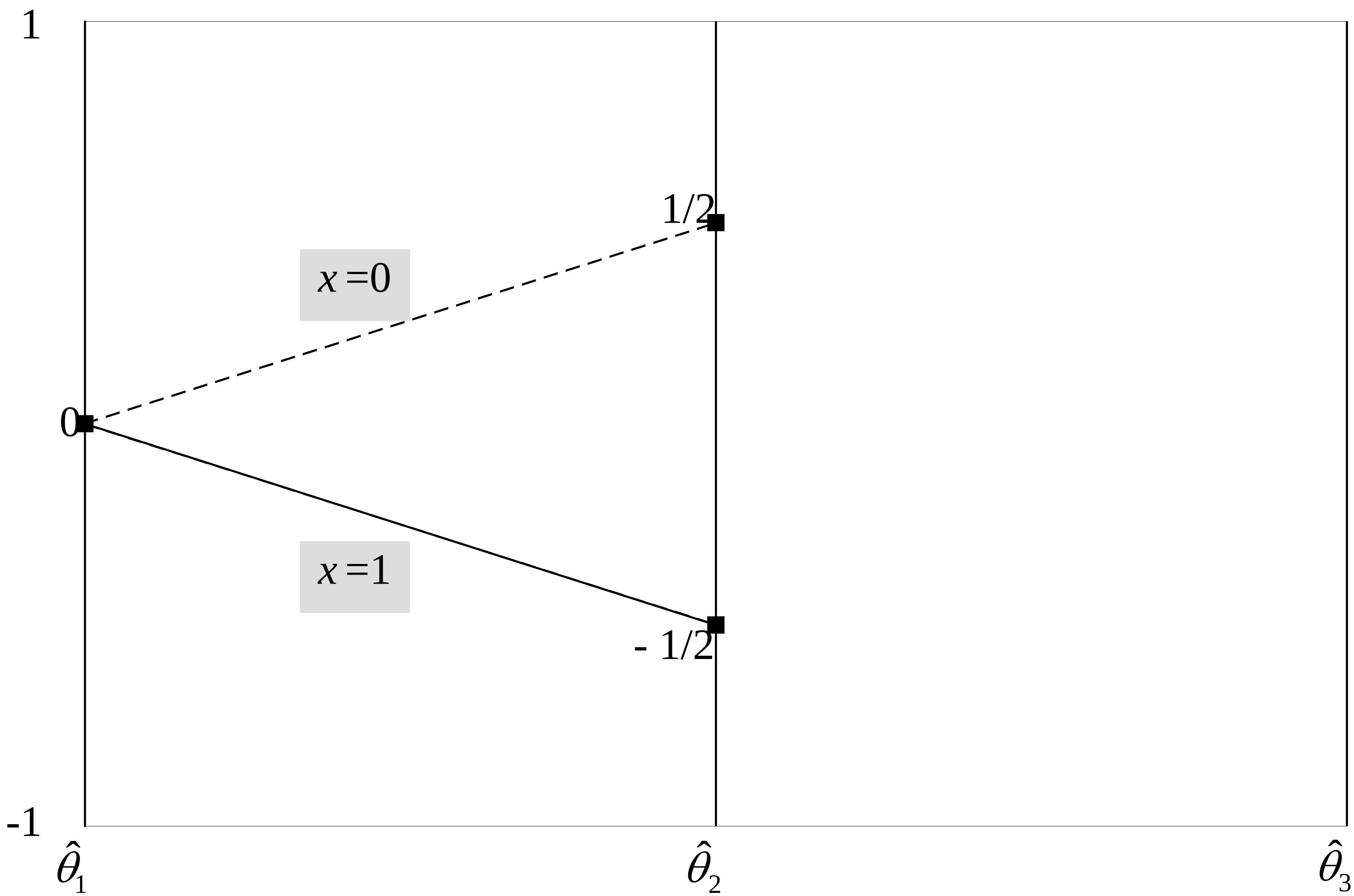
Imperfect information:
Which way is the wind blowing?!



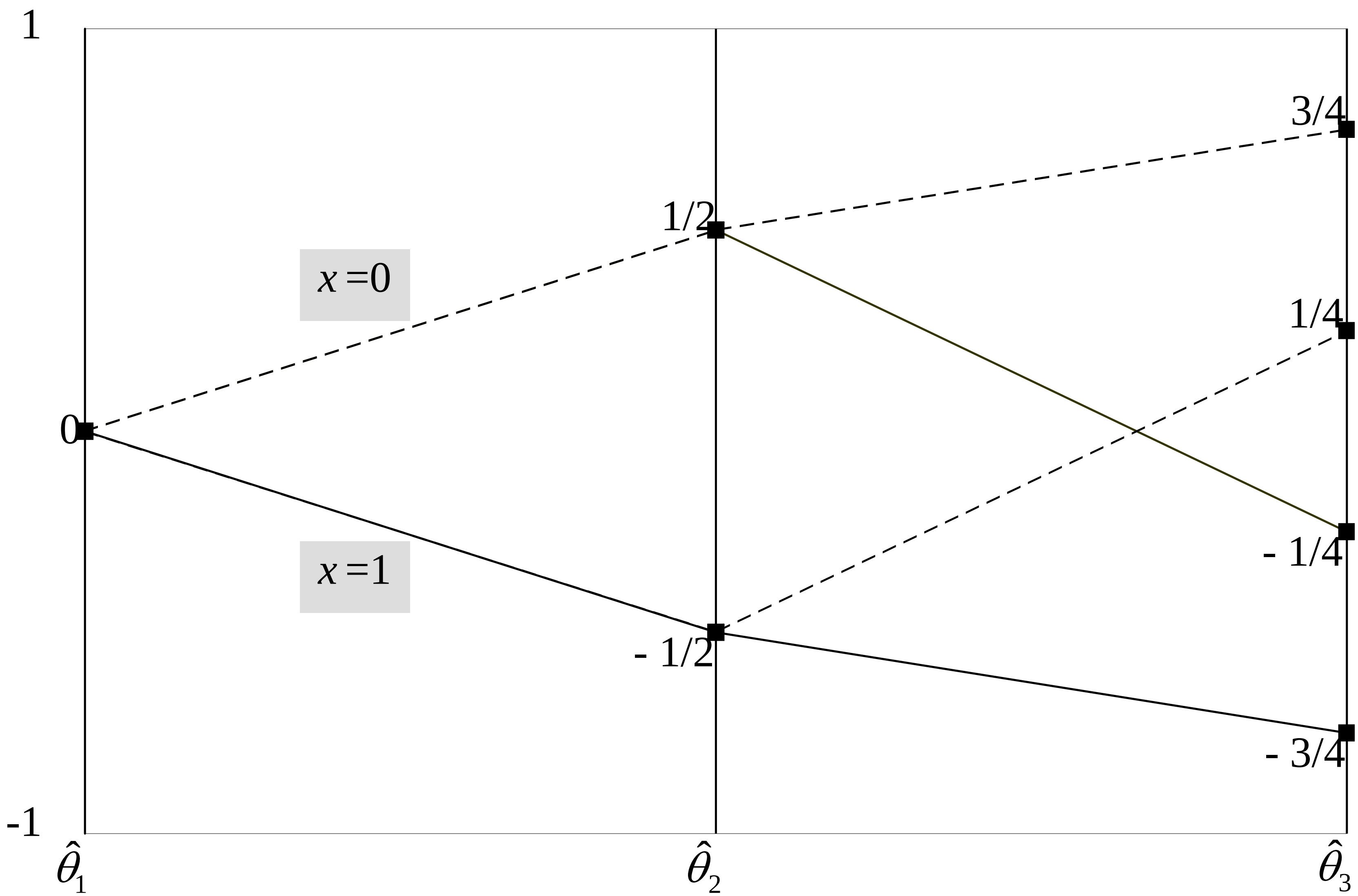
A three-agent example



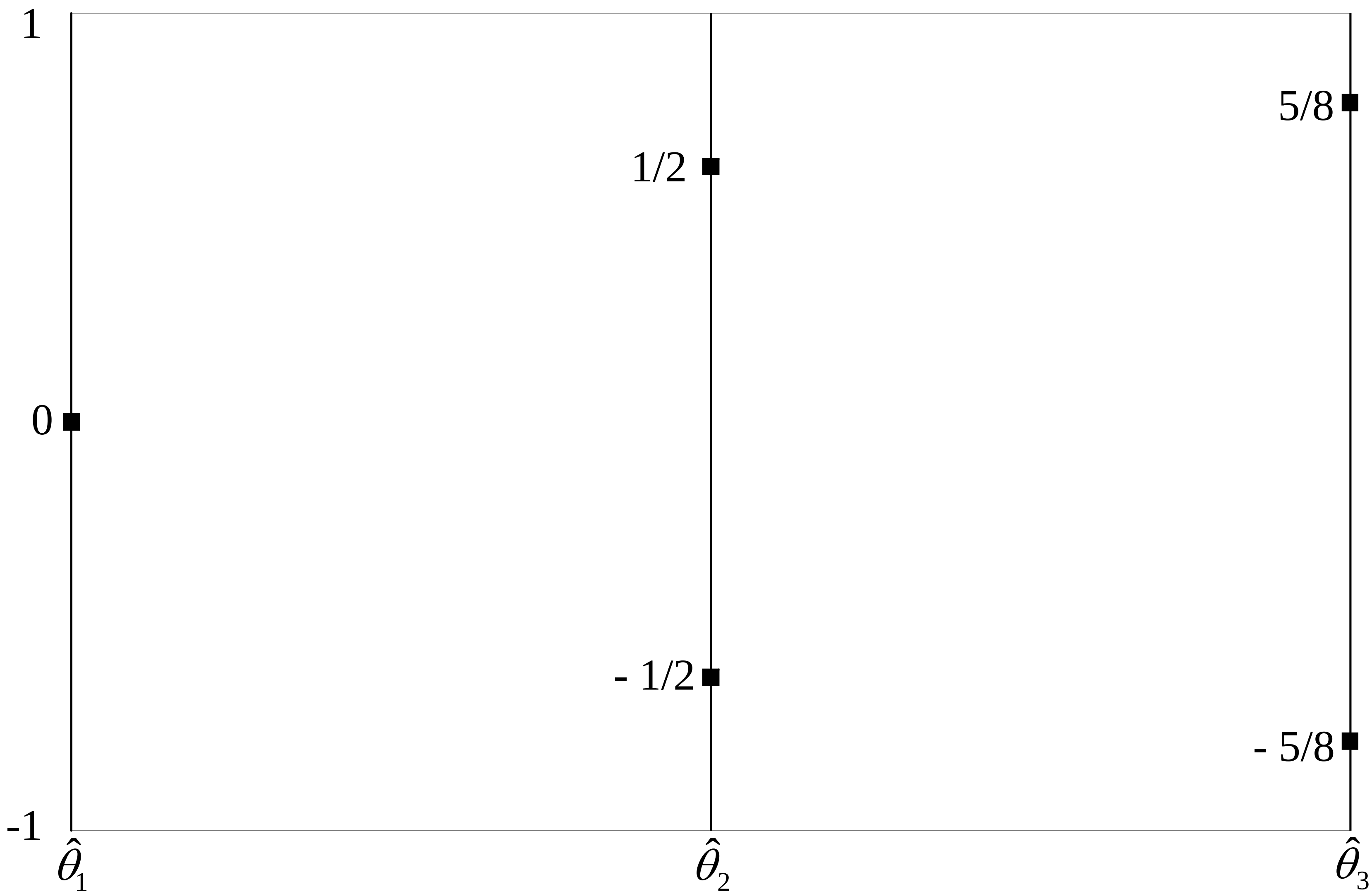
A three-agent example



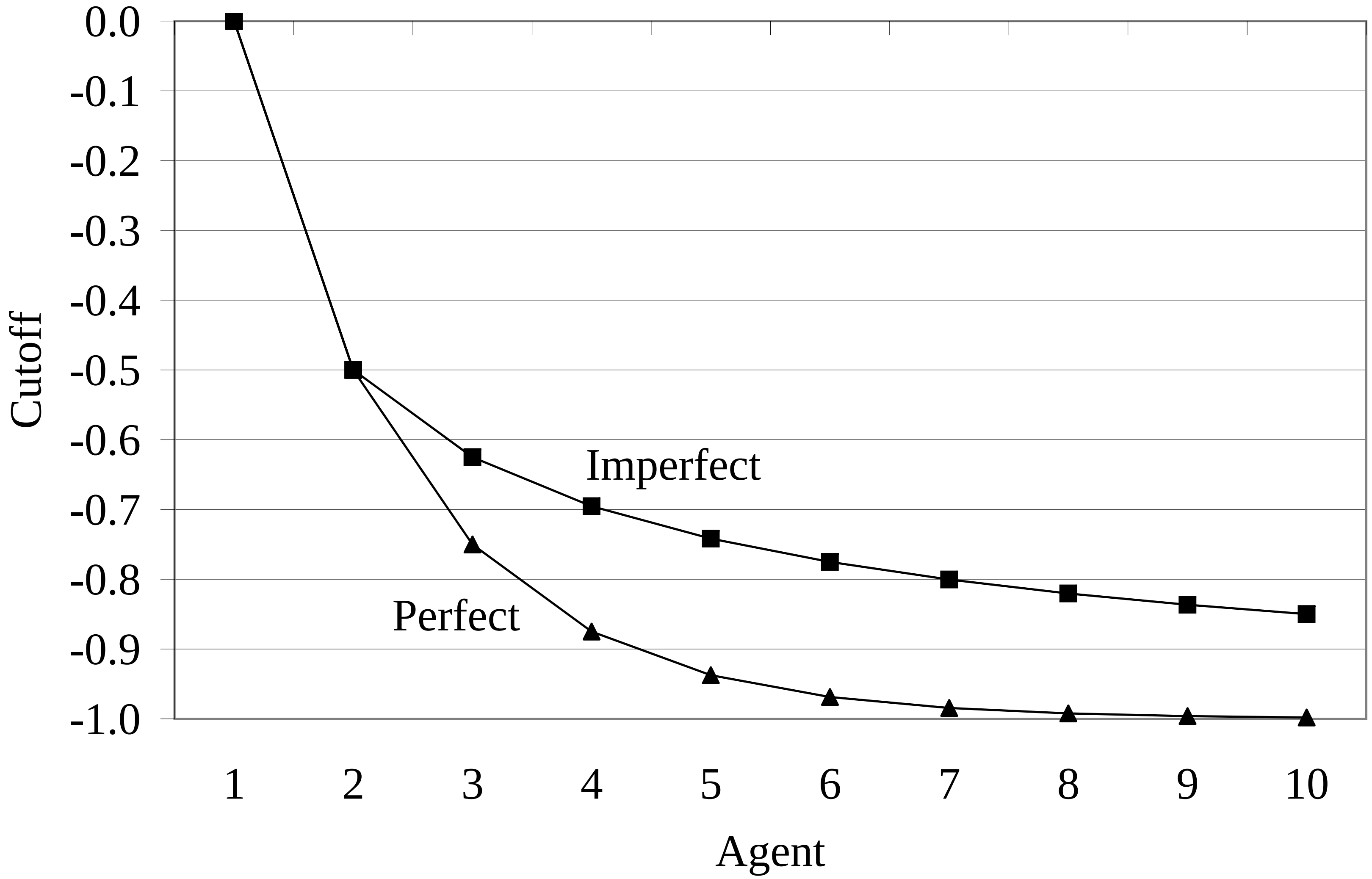
A three-agent example under perfect information



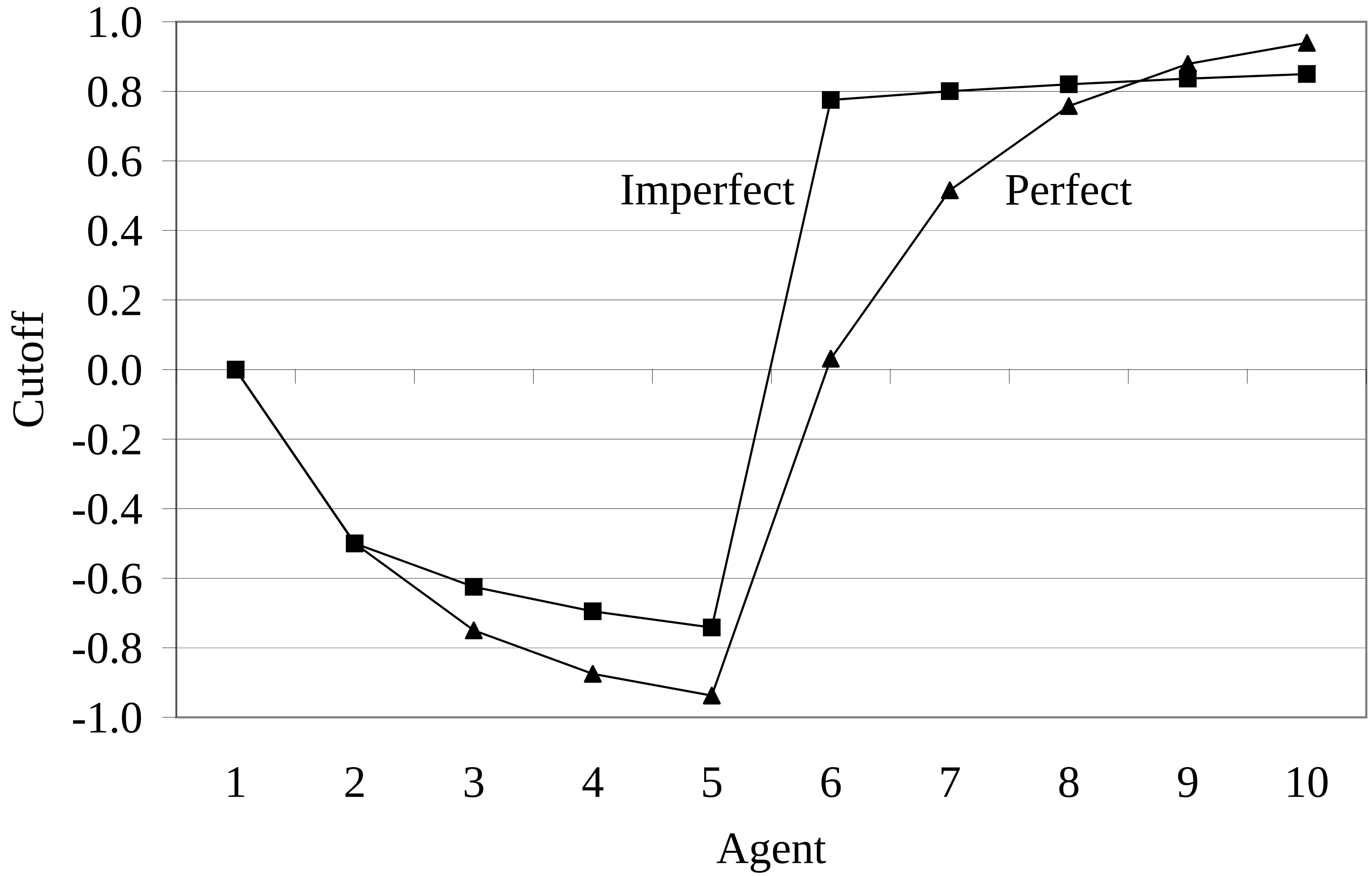
A three-agent example under imperfect information



A sequence of cutoffs under imperfect and perfect information



A sequence of cutoffs under imperfect and perfect information



The decision problem

– The optimal decision rule is given by

$$x_n = 1 \text{ if and only if } \mathbb{E}\left[\sum_{i=1}^{N} \theta_i \mid \mathcal{I}_n\right] \geq 0.$$

Since $\mathcal{I}_n$ does not provide any information about the content of successors' signals, we obtain

$$x_n = 1 \text{ if and only if } \mathbb{E}\left[\sum_{i=1}^{n} \theta_i \mid \mathcal{I}_n\right] \geq 0$$

Hence,

$$x_n = 1 \text{ if and only if } \theta_n \geq -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid \mathcal{I}_n\right].$$

The cutoff process

– For any n, the optimal strategy is the *cutoff strategy*

$$x_n = \begin{cases} 1 & if \quad \theta_n \geq \hat{\theta}_n \\ 0 & if \quad \theta_n < \hat{\theta}_n \end{cases}$$

where

$$\hat{\theta}_n = -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid \mathcal{I}_n\right]$$

is the optimal history-contingent cutoff.

– $\hat{\theta}_n$ is sufficient to characterize the individual behavior, and $\{\hat{\theta}_n\}$ characterizes the social behavior of the economy.

Overview of results

<u>Perfect information</u>

– A cascade need not arise, but herd behavior must arise.

<u>Imperfect information</u>

– Herd behavior is impossible. There are periods of uniform behavior, punctuated by increasingly rare switches.

- The similarity:
  - Agents can, for a long time, make the same (incorrect) choice.

- The difference:
  - Under perfect information, a herd is an absorbing state. Under imperfect information, continued, occasional and sharp shifts in behavior.

- The dynamics of social learning depend crucially on the extensive form of the game.

- The key economic phenomenon that imperfect information captures is a succession of fads starting suddenly, expiring rather easily, each replaced by another fad.

- The kind of episodic instability that is characteristic of socioeconomic behavior in the real world makes more sense in the imperfect-information model.

As such, the imperfect-information model gives insight into phenomena such as manias, fashions, crashes and booms, and better answers such questions as:

– Why do markets move from boom to crash without settling down?

– Why is a technology adopted by a wide range of users more rapidly than expected and then, suddenly, replaced by an alternative?

– What makes a restaurant fashionable over night and equally unexpectedly unfashionable, while another becomes the ‘in place’, and so on?

The case of perfect information

The optimal history-contingent cutoff rule is

$$\hat{\theta}_n = -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid x_1, ..., x_{n-1}\right],$$

and $\hat{\theta}_n$ is different from $\hat{\theta}_{n-1}$ only by the information reveals by the action of agent $(n-1)$

$$\hat{\theta}_n = \hat{\theta}_{n-1} - \mathbb{E}\left[\theta_{n-1} \mid \hat{\theta}_{n-1}, x_{n-1}\right],$$

The cutoff dynamics thus follow the cutoff process

$$\hat{\theta}_n = \begin{cases} \frac{-1+\hat{\theta}_{n-1}}{2} & \text{if} \quad x_{n-1} = 1 \\ \frac{1+\hat{\theta}_{n-1}}{2} & \text{if} \quad x_{n-1} = 0 \end{cases}$$

where $\hat{\theta}_1 = 0$.

Informational cascades

– $-1 < \hat{\theta}_n < 1$ for any n so any player takes his private signal into account in a non-trivial way.

Herd behavior

– $\{\hat{\theta}_n\}$ has the martingale property by the Martingale Convergence Theorem a limit-cascade implies a herd.

The case of imperfect information

The optimal history-contingent cutoff rule is

$$\hat{\theta}_n = -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid x_{n-1}\right],$$

which can take two values conditional on $x_{n-1} = 1$ or $x_{n-1} = 0$

$$\begin{aligned}
\overline{\theta}_n &= -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid x_{n-1} = 1\right], \\
\underline{\theta}_n &= -\mathbb{E}\left[\sum_{i=1}^{n-1} \theta_i \mid x_{n-1} = 1\right].
\end{aligned}$$

where $\overline{\theta}_n = -\underline{\theta}_n$.

The law of motion for $\overline{\theta}_n$ is given by

$$\begin{aligned}
\overline{\theta}_n = P(x_{n-2} = 1 | x_{n-1} = 1) &\left\{\overline{\theta}_{n-1} - \mathbb{E}\left[\theta_{n-1} \mid x_{n-2} = 1\right]\right\} \\
&+ P(x_{n-2} = 0 | x_{n-1} = 1) \left\{\underline{\theta}_{n-1} - \mathbb{E}\left[\theta_{n-1} \mid x_{n-2} = 0\right]\right\},
\end{aligned}$$

which simplifies to

$$\begin{aligned}
\overline{\theta}_n &= \frac{1 - \overline{\theta}_{n-1}}{2}\left[\overline{\theta}_{n-1} - \frac{1 + \overline{\theta}_{n-1}}{2}\right] \\
&\quad + \frac{1 - \underline{\theta}_{n-1}}{2}\left[\underline{\theta}_{n-1} - \frac{1 + \underline{\theta}_{n-1}}{2}\right].
\end{aligned}$$

Given that $\overline{\theta}_n = -\overline{\theta}_n$, the cutoff dynamics under imperfect information follow the cutoff process

$$\hat{\theta}_n = \begin{cases} -\frac{1+\hat{\theta}^2_{n-1}}{2} & \text{if} \quad x_{n-1} = 1 \\ \frac{1+\hat{\theta}^2_{n-1}}{2} & \text{if} \quad x_{n-1} = 0 \end{cases}$$

where $\hat{\theta}_1 = 0$.

Informational cascades

– $-1 < \hat{\theta}_n < 1$ for any n so any player takes his private signal into account in a non-trivial way.

Herd behavior

– $\{\hat{\theta}_n\}$ is not convergent (proof is hard!) and the divergence of cutoffs implies divergence of actions.

– Behavior exhibits periods of uniform behavior, punctuated by increasingly rare switches.