C103, Fall 20003, Problem Set 3 (due September 25)

1. In a Robinson Crusoe economy where the goods are leisure (H) and yams (Y), the feasible resource allocations lie on or below the curve $Y = [6(24\text{-}H)]^{½}$. Robinson has preferences that can be described by a utility function $u = \ln(Y) + H/12$, where ln denotes natural logarithm..

(A) Convert this to an unconstrained maximization problem by using the equality consraint $Y = [6(24\text{-}H)]^{½}$ to eliminate Y in the objective function, and then solve for the optimal levels of H, Y, and u.

(B) Maximize u subject to the constraints $Y \le [6(24\text{-}H)]^{½}$, $Y \ge 0$, and $0 \le H \le 24$ by setting up the Lagrangian and finding and solving the first-order conditions for a Lagrangian Critical Point.

(C) In the Lagrangian in part (B), give the Lagrangian multiplier and the LCP an economic market interpretation.

2. In the Robinson Crusoe economy, the firm operated by Mr. Friday is asked to maximize its profit $\pi = Y - w\cdot L$ over the combinations of output Y and labor input L satisfying the production possibility condition $Y = [6L]^{1/2}$, where profit π is measured in units of yams and w is the wage rate in yams per hour. Solve this problem and determine the labor demanded, yams supplied, and level of profit as functions of w.

3. Show that the profit function $\pi(w)$, giving the maximum profit obtainable as a function of w, that was obtained in Question 2 is a non-increasing, convex function of w. Show that the derivative of $\pi(w)$ with respect to w is the negative of the firm's demand for labor function.

4. Suppose the consumer Robinson faces the budget constraint $Y + wH = \pi + w\cdot 24$, or $Y = \pi + w\cdot L$ with $L = 24 - H$, where w is the wage rate in yams and π is non-wage (i.e., profit or dividend) income, and seeks to maximize utility $u = \log(Y) + H/12$.subject to this constraint, plus $Y \ge 0$ and $0 \le H \le 24$. Solve this problem by the Lagrangian method, and find yam demand, labor supply, and the maximized level of utility, called the *indirect utility function*, as functions of π and w. Be careful to consider the possibility of corner solutions. Show that the derivative of the indirect utility function with respect to w, divided by the derivative of the indirect utility function with respect to π, gives L, the supply of labor (Roy's identity).

5. Show that at the wage rate that equates supply and demand for labor, Robinson achieves the same utility level as the optimum in Question 1.