Chapter III.3

HOMOTHETICITY AND REAL VALUE-ADDED IN CANADIAN MANUFACTURING

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1. Introduction[1]

The estimation of production technologies and factor demand equations requires some measure of the output or activity level. Government statistical agencies have developed a variety of measures of real net output such as Real Domestic Production in Canada or Gross Product Originating in the United States. At the present time price and quantity data on gross output and intermediate materials is either not readily available or is not systematically collected. In a wide variety of practical applications the available real output measures are used without enough consideration of the consequences. The use of real net output measures have strong implications concerning the underlying production technology.

Arrow (1974), Bruno (Chapter III.1) and Diewert (Chapter III.2) have extensively developed the theoretical implications of using real net output or real value-added. Provided the sample data do not satisfy the restrictive aggregation condition that the ratio of the price of materials to the price of output is constant, any particular measure of net output will impose at least weak separability on the technology. In this paper we will test the hypotheses on separability that are implicit in a number of cases of real value-added.

[1]The extensive assistance of Melvyn Fuss is gratefully acknowledged. Cheryl Pinto was responsible for the econometric estimation.

Using a set of data on Canadian manufacturing from 1950–70 and a non-homothetic translog approximation to the cost function, we estimate the set of factor share equations. We are unable to accept any of the constraints required for real value-added.

2. Real Value-Added and the Production Structure

Bruno (Chapter III.1) and Diewert (Chapter III.2) provide extensive discussions of the theoretical restrictions involved in the use of real value-added in studies of production. We will simply consider the particular theoretical issues that pertain to our empirical investigation. The maintained hypothesis in our study is that we may approximate the production technology by a five-input production function,

$$Q = f(P,N,S,E,M). \tag{1}$$

Q = gross output,
P = production workers,
N = non-production workers,
S = structures,
E = equipment,
M = materials.

If the production function is weakly separable in materials from all other inputs,

$$Q = F(G(P,N,S,E),M). \tag{2}$$

Real value-added (VQ) may be defined as

$$VQ = G(P,N,S,E), \tag{3}$$

and the production function may be written as

$$Q = F(VQ,M). \tag{4}$$

If real value-added is defined by (3), there are no direct observations on VQ independent from the primary inputs.[2] An aggregation formula can

[2]The concept of real value-added is not well-defined in the literature. There are ambiguities about the relationship of real value-added to the production function inclusive of all inputs. We are asserting that for our purposes real value-added is a function of the primary inputs and perhaps gross output. If gross output is included, equation (2) may be written $H(G(P,N,S,E,Q),M,Q) = 0$. It is difficult to answer the question, what is the contribution of specific inputs to production, except under very restrictive conditions.

be selected and real value-added defined as aggregate primary input. The failure of national statistical agencies to develop measures of real capital inputs prohibits the aggregation of primary inputs. The method adopted by the U.N. and by most countries is called "double deflation". The value of gross output and materials are separately deflated and the difference in the deflated figures is defined as real value-added or real net output,

$$VQ^* = Q - M. \tag{5}$$

It is implicitly assumed[3] that the production function is additively separable of the form

$$Q = VQ^* + M. \tag{6}$$

This method of measuring real net output is convenient and practical for providing detailed industrial statistics. However, it unfortunately introduces further assumptions into the output data that are not particuarly desirable for studies of the production technology. Specifications of demand equations or alternative representations of the technology may lead to erroneous estimates unless the role of materials is explicitly introduced.

If the data sample does not satisfy the highly restrictive conditions required for direct aggregation referred to in the introduction, there are only a small number of interesting cases of real value-added. Some form of separability is required for all cases of real value-added. Weak separability of the material inputs from the non-material inputs is implied in all of the special cases.

If materials are perfect substitutes in production, then the production function can be written as

$$Q = G(P,N,S,E) + H(M). \tag{7}$$

The additively separable form can be specialized further by letting $H(M) = M$. With this restriction, the production technology has the form directly implied by the double deflation technique.

If perfect substitutability between materials and other inputs is unlikely, then the opposite extreme of perfect complementarity has often been suggested. If materials are used in fixed proportion to gross output, the production technology becomes

$$Q = \min[G(P,N,S,E),aM], \tag{8}$$

[3]Sims (1969) provides a justification for this procedure as an approximation to a production function weakly separable in primary inputs and materials.

where a is a constant. If the producer is efficient,

$$Q = G(P,N,S,E) \quad \text{and} \quad Q = aM.$$

Gross output equals a function of the primary inputs and simultaneously equals a homogeneous linear function of materials. It is still possible to define a double deflated real value-added function,

$$VQ^* = Q - M = (1 - a)\cdot G(P,N,S,E).$$

In this case, real value-added is not simply a function of the primary inputs but depends on the input–output coefficient for materials.

In the section presenting the empirical results we will test for weak separability and the special types of separability required for "double deflation". Finally, we will test the hypothesis that the sample data satisfy the conditions for Hicks aggregation discussed by Diewert in Chapter III.2. If they do a real value-added function may be defined.

3. The Approximate Translog Cost Function

The production structure is represented by the five-input cost function,

$$C = g(p_S,p_E,p_P,p_N,p_M,Q), \tag{9}$$

where the subscripts on the prices, p, represent the inputs defined above. We will assume that the cost function can be approximated up to the second-order by a non-homothetic translog cost function.[4] The cost function can be written as

$$\log C = \log \alpha_0 + \sum_i \alpha_i \log p_i + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \log p_i \log p_j + \sum_i \gamma_{iQ} \log Q \log p_i + \alpha_Q \log Q + \frac{1}{2}\gamma_{QQ}(\log Q)^2, \qquad i,j = S,E,P,N,M. \tag{10}$$

The share equations for each input of the translog cost function are

$$S_i = \alpha_i + \gamma_{iQ} \log Q + \sum_j \gamma_{ij} \log p_j, \qquad i = S,E,P,N,M, \tag{11}$$

[4]Berndt and Wood (1975) discuss the properties of the translog cost function. Jorgenson and Lau (1975a) have developed the approximate approach to testing for the translog utility function.

and

$$S_i = p_i X_i \Big/ \sum_j p_j X_j.$$

where X_i is the quantity demanded of input i.

Since the shares must add to one, the following parameter restrictions hold:

$$\sum_i \alpha_i = 1,$$

$$\sum_j \gamma_{ij} = 0, \qquad i,j = S,E,P,N,M,$$

$$\sum_i \gamma_{iQ} = 0.$$

These restrictions will be imposed throughout the paper.

The approximate translog production and cost function have been extensively discussed by Denny and Fuss (1977). In that paper, their Proposition 1 states that the translog cost function may be viewed as a quadratic approximation to an arbitrary cost function. For this to be true the additional constraints,

$$\gamma_{ij} = \gamma_{ji}, \qquad i \neq j,$$

must hold.[5]

The distinction between the translog as an exact functional form and as an approximation to a functional form at a point can be summarized quickly. As an exact functional form, the translog cannot adequately represent a separable technology as a flexible second-order approximation.[6] The set of constraints required for weak separability impose strong restrictions on either the micro aggregator functions or the macro function.[7,8] In order to avoid these restrictions, the weaker notion of a second-order approximation *at a point* has been adopted. There is a trade-off in the adoption of this viewpoint. A set of restrictions for weak separability of the translog cost function viewed as an

[5]If they do not, the cross partial derivatives will not be equal and this is required.

[6]Blackorby, Primont and Russell (1977) and Denny and Fuss (1977) discuss the restrictions.

[7]For a general discussion of aggregation, see Diewert (1976), from whom this terminology was borrowed.

[8]The constraints imply that the separable form of a translog function is either a Cobb–Douglas function of translog aggregates or else a translog function of Cobb–Douglas aggregates.

approximation at a point exist. However, in order to avoid the restrictions on the separable form imposed in the exact case, the approximation had to be weakened to hold only at a point. It is not clear that this loss is trivial since the behavior of the approximation away from the point of approximation will depend on the data.

4. Homotheticity and Homogeneity

The translog approximation to a non-homothetic cost function is represented by the model discussed in the previous section. Since most of the studies using flexible second-order approximations have assumed a linear homogeneous cost or production function, it will be useful to test this result. Suppose we write the non-homothetic cost function as

$$C = C(\mathbf{w}, Q), \tag{12}$$

where $\mathbf{w}$ is a vector of factor prices and Q is gross output. If the production technology is homothetic, Shephard (1970) has shown that (12) may be written as

$$C = c(\mathbf{w}) \cdot g(Q). \tag{13}$$

If we wish to use the translog approximation to (13) what restrictions are involved?

Taking logarithms of both sides of equation (13), we obtain

$$\log C = \log c(\mathbf{w}) + \log g(Q). \tag{14}$$

The translog approximation to the non-homothetic cost function (10), will be homothetic if we impose the conditions

$$\gamma_{iQ} = 0, \qquad i = P,N,S,E,M.$$

This can be readily seen by remembering that the second partial derivatives of (14),

$$\frac{\partial^2 \log C}{\partial \log p_i \, \partial \log Q} = 0, \qquad i = P,N,S,E,M.$$

If the homothetic cost function is homogeneous,

$$C = c(\mathbf{w}) \cdot Q^{\delta},$$

$$\log C = \log c(\mathbf{w}) + \delta \log Q. \tag{15}$$

This requires that we impose the constraint

$$\gamma_{QQ} = 0$$

on equation (10) since the second-order partial derivative of (15),

$$\frac{\partial^2 \log C}{\partial (\log Q)^2} = 0 = \gamma_{QQ}.$$

Finally, if we wish to impose the condition that the homogeneous cost function is linear homogeneous,

$$C = C(\mathbf{w})\cdot Q, \qquad \frac{\partial \log C}{\partial \log Q} = 1 = \alpha_Q.$$

5. The Canadian Manufacturing Data

The data used in this study were developed with considerable assistance from various members of Statistics Canada. For the years 1950–70, we have managed to construct a relatively satisfactory set of data on the prices and quantities of inputs.[9]

The output variable is gross output. It is constructed as an aggregate of shipments and changes in inventories of finished goods and work in progress. The industry selling price and the wholesale price index were linked to provide a price series for total manufacturing output. The constant dollar gross output series implicitly defined by this price series is the real gross output measure.

Since data on the use of materials are not published in detailed form, our method of calculating the price and quantity of materials is only an approximation. The materials inputs were derived by reversing the procedures used by Statistics Canada to achieve measures of real domestic product. Statistics Canada deflates the value of gross output and materials separately. The difference in the deflated series is real domestic product. We have defined current and constant dollar materials as the difference between current and constant dollar gross output and domestic product. The price of materials is the implicit price index defined by the current and constant dollar materials data. The restrictive

[9]A more complete description of the data may be found in Denny and May (1975).

assumptions implicit in Statistics Canada's calculation of real domestic product are removed by this procedure.

An alternative method of calculating material inputs has been used recently by Berndt and Wood (1975) and Star (1974). It relies on information from input–output tables. There are two serious problems with this approach. First, the input–output tables themselves are based on rather limited data, some of which depend on assumptions about a fixed input–output ratio. Second, since input–output tables are only available at infrequent intervals, some form of interpolation must be used during intervening years. This will tend to create artificial correlations amongst series and may involve further assumptions about fixed or smoothly changing input–output coefficients. To the extent that some variant of the fixed input–output coefficient assumption is utilized in the derivation, the resulting materials series are biased in favour of a separable technology.

The capital stock data are calculated from the Statistics Canada data on investment in manufacturing. The two-digit SIC industrial data by asset type were aggregated across industries to provide series on three capital stocks in manufacturing. The assets are building construction, engineering construction, and machinery and equipment. The two construction stocks were aggregated to provide a Divisia index of structures.

The price of capital services is the service price (p_s) based on the equation

$$p_s = p_A(r + \delta),$$

where p_A is the asset price, r is an interest rate, and δ a rate of replacement. Following the procedures outlined in Hall and Jorgenson (1967) we have included the corporate tax system in the capital service price estimates.

The labour series are man-hours in constant dollars and a wage index for production and non-production workers. The man-hours data were supplied by Statistics Canada for both production and non-production workers. Extensive adjustments have been made to the data published by Statistics Canada to correct for fringe benefits, hours paid and not worked, and other smaller problems. The best published description of these series can be found in *Statistics Canada* (1963). The wage rates are defined implicitly by using total earnings of production and non-production workers from the *General Review of the Manufacturing Industries in Canada.*

6. The Estimation of the Production Structure

The set of factor share equations for the non-homothetic cost function was estimated for the years 1950–70. There are a number of cross-equation parameter restrictions and the errors in the equations are probably not independently distributed. The equations were jointly estimated using generalized least squares (GLS) in the particular form developed by Zellner (1962). The share equations are of the form

$$S_i = \alpha_i + \gamma_{iQ} \log Q + \sum_j \gamma_{ij} \log p_j, \qquad j = P,N,S,E,M.$$

They are estimated[10] subject to the constraints $\sum \alpha_i = 1$, $\sum \alpha_{iQ} = 0$, $\gamma_{ij} = \gamma_{ji}$ and $\sum \gamma_{ij} = 0$ for $i,j = P,N,S,E,M$.

Table 1 presents our estimates of the parameters of the cost function. There is very little that can be inferred directly from the table and we will turn to the tests.

Separability, Homotheticity and Real Value-Added

We have begun with the maintained hypothesis that the production technology is non-homothetic. In Section 4 it was shown that if the technology is *homothetic in all inputs*,

$$\gamma_{iQ} = 0, \qquad i = P,N,S,E,M.$$

The test statistic for this hypothesis is

$$F(4,62) = 43.30, \qquad F_{0.05} = 2.53.$$

The hypothesis is rejected. If the technology is not homothetic, it is not homogeneous of any degree. We will proceed with the maintained hypothesis that the technology is non-homothetic in all inputs.

Assume that we have a non-homothetic production function,

$$Q = F(P,N,S,E,M),$$

which is weakly separable,

$$Q = F(G(P,N,S,E),M).$$

[10]The estimation procedure involves two stages. The first stage provides estimates of the variance–covariance matrix without the symmetry constrainst. In the second stage the variance–covariance matrix is held constant and the parameters are estimated with the symmetry constraints imposed. All testing is done holding the estimated variance-covariance matrix constant.

TABLE 1
Non-homothetic translog parameter estimates.

	Two types of capital – Two types of labour – Materials X_i = structures and X_2 = equipment; X_3 = production workers and X_4 = non-production workers; X_5 = materials.	
Parameter	GLS estimates	Standard errors
α_1	0.0229	0.0003
α_2	0.0298	0.0002
α_3	0.1492	0.0009
α_4	0.0921	0.0008
α_5	0.7059	0.0012
α_{1Q}	−0.0130	0.0017
α_{2Q}	−0.0086	0.0016
α_{3Q}	−0.0322	0.0054
α_{4Q}	−0.0623	0.0051
α_{5Q}	0.1160	0.0085
γ_{11}	0.0237	0.0024
γ_{22}	0.0336	0.0034
γ_{33}	0.1157	0.0286
γ_{44}	0.1231	0.0153
γ_{55}	0.1764	0.0123
γ_{12}	0.0066	0.0019
γ_{13}	0.0063	0.0064
γ_{14}	0.0009	0.0044
γ_{15}	−0.0375	0.0028
γ_{23}	−0.0200	0.0073
γ_{24}	0.0166	0.0058
γ_{25}	−0.0369	0.0028
γ_{34}	−0.0703	0.0199
γ_{35}	−0.0317	0.0094
γ_{45}	−0.0703	0.0083

If G is homothetic in the primary inputs, then the dual cost function will be weakly separable. The non-homothetic cost function (9) will be weakly separable into a real value-added aggregate price, p_v,

$$p_v = h(p_S, p_E, p_N, p_P),$$

and a materials price, p_M, such that,

$$C = g(p_v, p_M, Q). \tag{16}$$

Real value-added, VQ, equals nominal value-added deflated by the aggregate price, p_v.

The translog cost function (10) will be approximately weakly separable if

$$\alpha_i/\alpha_j = \gamma_{iM}/\gamma_{jM}, \quad \alpha_i/\alpha_j = \gamma_{iQ}/\gamma_{jQ}, \qquad i,j = P,N,S,E. \tag{17}$$

This may be demonstrated by direct application of the methods used in Denny and Fuss (1977). The partial derivations that are needed are

$$\frac{\partial \log C}{\partial \log p_i} = \frac{\partial \log g}{\partial \log h} \cdot \frac{\partial \log h}{\partial \log p_i} = \alpha_i, \qquad i = P,N,S,E,$$

$$\frac{\partial^2 \log C}{\partial \log p_i \, \partial \log p_M} = \frac{\partial \log h}{\partial \log p_i} \cdot \frac{\partial^2 \log g}{\partial \log h \, \partial \log p_M} = \gamma_{iM},$$

$$\frac{\partial^2 \log C}{\partial \log p_i \, \partial \log Q} = \frac{\partial \log h}{\partial \log p_i} \cdot \frac{\partial^2 \log g}{\partial \log h \, \partial \log Q} = \gamma_{iQ}.$$

The restrictions, equation (17), follow directly. Applying these restrictions, the test statistic is

$$F(6,60) = 103.12, \qquad F_{0.05} = 2.25.$$

The approximate weak separability of the cost function is rejected.

7. An Alternative Real Value-Added Formulation

There is a certain ambiguity concerning the notion of real value-added that will be explored here in one special case. The weak separability imposed on the non-homothetic translog cost function in equation (16) implies that the aggregate price index of real value-added is homogeneous in the primary input prices[11] and independent of the output level. This has several advantages. It is useful to relate the particular problems with real value-added to theoretical knowledge about two-stage optimization and to index number theory. Consistent two-stage optimization requires that the first stage or micro functions be homothetic in the prices. Most index number formulae are linear homogeneous in the variables. The implied micro function, p_v, in the estimated translog cost function will satisfy the conditions for consistent two-stage optimization. It will not be linear homogeneous, but it certainly can be thought of as an estimated homogeneous aggregation formula.

Consider an alternative formulation that does not have the same properties but may be extremely useful for many applied problems.

[11]An extension of Proposition 7 in Denny and Fuss (1977) implies that the micro functions will be homogeneous for the translog form.

Suppose the non-homothetic cost function (9) was weakly separable in the form

$$C = g(h(p_S,p_E,p_N,p_P,Q),p_M,Q). \tag{18}$$

The primary input prices are now separable from only the price of materials and not from output. The micro function or real value-added cost function,

$$C_v = h(p_S,p_E,p_N,p_P,Q), \tag{19}$$

is non-homothetic in terms of the primary inputs. That is, the production function dual to C_v will not be homothetic. If the overall cost function (18) is non-homothetic, it may be more natural to consider a less restrictive real value-added separability that is also non-homothetic.[12] If a real value-added quantity index was desired, nominal value-added could be deflated by average real value-added cost, C_v/Q.

To clarify this alternative formulation, consider the practical problems that confound empirical studies of the production technology. Two broad classes of data problems are of special interest to us. Real value-added is of interest because official statistical agencies produce disaggregated industrial output measures of this general type. Information on material inputs is very limited and even gross output data are weak. Consequently studies must assume that real value-added technologies exist. In the alternative presented in this section, we have tried to extend the notion of real value-added to cases in which gross output data are available or can be constructed. There will be many practical situations in which estimation of a non-homothetic real value-added cost function (19) would be useful. A weaker set of assumptions on the overall technology is assumed and this may lessen the errors arising from missing data on material inputs. A two-stage procedure may be chosen even in cases when a complete set of data are available. Studies that attempt to use a large number of disaggregated inputs may be forced to adopt a two-stage procedure. The quality of the data may be low due to either poor collection procedures or the small range of variability. To imperfectly overcome these limitations it may be useful to estimate a first stage involving a micro function of a particular set of disaggregated inputs and a second stage using aggregate inputs. An

[12]Basically we are proposing a model to be used when information on the complete list of inputs is unavailable but data on gross output can be approximated, at least. In that case, assuming that real value-added is homogeneous is unnecessarily restrictive.

excellent example is the work by Fuss (1977a) on energy. Fuss used a two-stage procedure that assumed weak separability such that the micro function was independent of output. However a less restrictive version might have used the alternative form presented here.[13]

What are the constraints on the translog cost function (10) such that it will be approximately weakly separable of the form given in (18)? The parameter restrictions are

$$\alpha_i/\alpha_j = \gamma_{iM}/\gamma_{jM}, \qquad i,j = P,N,S,E. \tag{20}$$

These restrictions follow from the fact that the partial derivatives of the expansion are

$$\frac{\partial \log C}{\partial \log P_i} = \frac{\partial \log h}{\partial \log g} \cdot \frac{\partial \log g}{\partial \log p_i} = \alpha_i, \qquad i = P,N,S,E,$$

$$\frac{\partial \log^2 C}{\partial \log p_i \, \partial \log p_M} = \frac{\partial \log g}{\partial \log p_i} \cdot \frac{\partial^2 h}{\partial \log g \, \partial \log p_M} = \gamma_{iM}.$$

The test statistic for the hypothesis that the translog cost function is weakly separable in the form given by (18) is

$$F(3,63) = 84.60, \qquad F_{0.05} = 2.76.$$

The hypothesis is rejected.

8. Weak Homotheticity

Production theorists have used a particular definition of homotheticity introduced by Shephard (1970). He defined a homothetic function as a function that is a positive monotonic transformation of a linear homogeneous function. If the production function is homothetic then the cost function has the separable form given in equation (13). There are two special economic properties implied by this definition. First, the ratio of any two factor demand equations is independent of the output level. Second, the elasticity of total or average cost with respect to output is independent of factor prices. We have used homotheticity in earlier sections of this paper to indicate that both these properties were implied. For this section, a production function which satisfies both these condi-

[13]Fuss explicitly assumes a separable aggregate technology. His micro energy technology is linear homogeneous in the energy input prices. It can be considered a special version of case three with translog functions at both stages.

tions will be called *strongly homothetic*. If only the first condition is satisfied then the production function will be *weakly homothetic*. The dual cost function to a weakly homothetic production function may be written

$$C = C(g(\mathbf{w}),Q). \tag{21}$$

The input price vector, $\mathbf{w}$, is weakly separable from output Q. The factor demand equations have the form,

$$X_i = \partial C/\partial g \cdot \partial g/\partial w_i, \qquad i = 1,\ldots,n.$$

Since the first term is independent of i and the second term independent of output, the ratio of any two factor demands is independent of output. Thus the first condition will hold but not the second, and it is no longer true that the dual production function is a positive monotonic transformation of a linear homogeneous production function.[14]

If we wish to approximate a weakly homothetic production function, the translog cost function (10) must satisfy the following constraints:

$$\alpha_i\gamma_{jQ} = \alpha_j\gamma_{iQ}, \qquad i,j = P,N,S,E,M. \tag{22}$$

The partial derivatives of the approximation to the separable cost function are

$$\frac{\partial \log C}{\partial \log p_i} = \frac{\partial \log C}{\partial \log g} \cdot \frac{\partial \log g}{\partial \log p_i} = \alpha_i, \qquad i = P,N,S,E,M,$$

$$\frac{\partial^2 \log C}{\partial \log p_i \, \partial \log Q} = \frac{\partial^2 \log C}{\partial \log g \, \partial \log Q} \cdot \frac{\partial \log g}{\partial \log p_i} = \gamma_{iQ}.$$

Conditions (22) follow directly.

Weak homotheticity is sufficient for properties such as linear expansion paths and in many uses strong homotheticity need not be assumed. International trade would provide some examples. Other cases may exist in which the first condition and not the second is required for particular applications.

Weak homotheticity forges a link between the two cases of real value-added weak separability. Table 2 provides a summary of the analytic forms of three cases of weak separability. It also includes the parameter restrictions for approximating these cases with the non-homothetic translog cost function. The conventional aggregate price of real value-added, $p_v = g(p_S,p_E,p_N,p_P)$, of case 3 requires parameter restrictions in the approximate translog case that are the sum of the other two cases. Consequently, case 3 may be thought of as the

[14]The dual production function will have the implicit form $H(F(\mathbf{X},Q),Q) = 0$ where $\mathbf{X}$ is a vector of inputs. McFadden (Chapter I.1) discusses this case.

TABLE 2
Analytic forms for weak homotheticity and weak separability.

(1)	*Weak homotheticity*
	$C = g(h(p_S, p_E, p_N, p_P, p_M), p_M, Q)$,
	$\alpha_i/\alpha_j = \gamma_{iQ}/\gamma_{jQ}$, $\quad i,j = P,N,S,E.$
(2)	*Real value-added: non-homothetic cost function*
	$C = g(h(p_S, p_E, p_N, p_E, Q), p_M, Q)$,
	$\alpha_i/\alpha_j = \gamma_{iM}/\gamma_{jM}$, $\quad i,j = P,N,S,E.$
(3)	*Real value-added: homogeneous price aggregate*
	$C = g(h(p_S, p_E, p_N, p_E), p_M, Q)$,
	$\alpha_i/\alpha_j = \gamma_{iQ}/\gamma_{jQ}$, $\alpha_i/\alpha_j = \gamma_{iM}/\gamma_{jM}$, $\quad i,j = P,N,S,E.$

simultaneous imposition of the constraints for weak homotheticity in the primary inputs and a real value-added cost function, $C_v = g(p_S, p_E, p_N, p_P, Q)$.

To test for weak homotheticity within our framework requires the imposition of the constraints given in equation (22). An unfortunate limitation of the translog share equations is that the constraints for weak homotheticity in all inputs and the adding up constraints ($\sum_i S_i = 1$) cannot be imposed simultaneously without imposing strong homotheticity. For example, if there are only two inputs, the condition for weak homotheticity is

$$\alpha_1 \gamma_{2Q} = \alpha_2 \gamma_{1Q}.$$

However, the adding up constraints $\alpha_1 + \alpha_2 = 1$ and $\gamma_{1Q} + \gamma_{2Q} = 0$ cannot simultaneously hold with this constraint unless $\gamma_{1Q} = \gamma_{2Q} = 0$.

In Table 2, weak homotheticity in the primary inputs alone is required. It can easily be shown that the conditions for weak homotheticity can be applied to the share equations for any subset of the inputs. Since weak homotheticity links the two special cases of real value-added weak separability, this hypothesis is tested. The test statistic is

$$F(3,63) = 16.02, \qquad F_{0.05} = 2.76.$$

Weak homotheticity of the technology in the primary inputs is rejected.

Although weak homotheticity has been rejected the imposition of this constraint is less damaging than the constraint for strong separability. The test statistics are $F = 43.3$ and $F = 16.0$. Since data on gross output can often be located in cases when information on materials is absent this model may be of some practical use.

9. Double Deflation and Real Value-Added

The use of double deflation in the construction of real value-added requires a special set of assumptions. Double deflation assumes that the production function is separable in the form

$$Q = f(P,N,S,E) + H(M).$$

The elasticity of substitution between materials and the other inputs must be infinite. That is,

$$\sigma_{iM} = \infty, \quad \text{for} \quad i = P,N,S,E.$$

The Allen–Uzawa elasticity of substitution between inputs i and j for the cost function is

$$\sigma_{ij} = \frac{CC_{ij}}{C_i C_j},$$

where

$$C_i = \partial C/\partial p_i \quad \text{and} \quad C_{ij} = \partial^2 C/\partial p_i \partial p_j.$$

For the translog cost function, it can be shown [Berndt and Wood (1975)] that

$$\sigma_{ij} = \frac{\gamma_{ij} + S_i S_j}{S_i S_j}. \tag{23}$$

At the point of approximation, $p_i = 1$ for all i, and

$$\sigma_{ij} = 1 + \frac{\gamma_{ij}}{\alpha_i \alpha_j}. \tag{24}$$

As a limit, the elasticities will approach infinity as either α_M *or* α_i, $i = P,N,S,E$, approach zero. The tests for additive separability implicit in the use of double deflation are

(a) $\alpha_M = 0$,

(b) $\alpha_i = 0, \quad i = P,N,S,E.$

The test statistics for these two hypotheses are

(a) $F(1,65) = 22.99, \quad F_{0.05} = 4.0,$

(b) $F(4,65) = 27.41, \quad F_{0.05} = 2.53.$

Neither hypothesis is accepted. The technology is not separable in the form required for double deflation.

Leontief Technology in Materials

The notion that materials are used in fixed proportion with gross output might be a reasonable assumption over short time periods and with very disaggregated data. Neither of these is satisfied by our attempt to approximate the technology for total manufacturing over two decades.[15] The fixed proportion assumption assumes that the elasticities of substitution between materials and all other inputs are zero, i.e.,

$$\sigma_{iM} = 0, \qquad i = P,N,S,E.$$

The parameters of the translog approximation must satisfy the constraints[16]

$$\gamma_{iM} = \alpha_i \cdot \alpha_M, \qquad i = P,N,S,E.$$

The test statistic for perfect complementarity of materials and the other inputs is

$$F(4,62) = 37.40, \qquad F_{0.05} = 2.53.$$

The hypothesis is rejected.

Hicks' Aggregation and Real Value-Added

It is possible to define real value-added as the deflated value of nominal value-added if certain conditions obtain. Suppose a nominal value-added function is defined as the maximum profits available from any fixed bundle of primary inputs. If the prices of output and materials vary in fixed proportion, real value-added is nominal value-added deflated by the price of output. Consider the following direct statistical test. The equation

$$\log p_Q = \alpha + \beta \log p_M$$

was estimated. If the conditions for Hicks' aggregation are to hold then $\beta = 1$. The estimated coefficient was $\beta = 0.75$ with a standard error of 0.038. We can reject the hypothesis that $\beta = 1$ and that prices moved in fixed proportion.[17]

[15]The use of an aggregate technology to represent total manufacturing will eliminate through aggregation any fixed input–output coefficients that may well exist at very disaggregated levels.

[16]This can be derived directly from equation (24).

[17]This hypothesis was also tested with the nonlinear equation $p_Q = \alpha p_M^{\beta}$. The test statistic still led to the rejection of the hypothesis that $\beta = 1$.

10. Conclusion

For Canadian manufacturing during the period 1950–70, there is no evidence that real value-added technologies are acceptable. Every hypothesis that we tested was rejected. This suggests that improved data on material inputs and gross output would be very useful. Further empirical tests of these hypotheses would be useful for more disaggregated industries. In addition, when inputs are available in highly disaggregated form the possibility of aggregation errors affecting the tests may be considered. Finally, for the United States and Canada, it is often possible to approximate a series on gross output. When this can be done, the use of the alternative real value-added function may be useful even if only the first stage can be completed.