Chapter IV.4

FACTOR SUBSTITUTION IN ELECTRICITY GENERATION: A TEST OF THE PUTTY–CLAY HYPOTHESIS*

MELVYN FUSS

University of Toronto

1. Introduction

Recently I introduced a model of production within which it is possible to test hypotheses concerning the nature of the structure of technology [Fuss (1970 and 1977b)]. This paper presents an example of the empirical usefulness of the basic model and provides evidence in support of the hypothesis that the "putty–clay" model is the most appropriate one for steam-electric power generation.

Surprisingly, this hypothesis has remained in dispute despite the large number of production function studies which have utilized data drawn from the electricity generation industry. Barzel (1964) concludes from his study that:[1]

> "Factor substitution, contrary to prevalent notions, is not absent. It operates not only in the long run, when it is possible to incorporate price information in the design of equipment, but also for existing plants."

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[1]Barzel (1964, p. 148).

Galatin (1968), in reviewing Barzel's study, states:[2]

> "It is difficult to see what meaning can be given to x_5 in the fuel-and labour-input functions. x_5 is defined as x_{4t}/x_4 where x_{4t} is the factor price ratio for the plant at time t. If fuel or labour input were affected by changes in the factor price ratio over time, this would imply that there is ex-post substitution between labour and fuel in electric-power generation which is distinctly implausible."

Galatin then imposes the "putty–clay" hypothesis as a maintained hypothesis and analyzes the *ex post* production technology. Komiya (1962) assumes that the technology is "clay–clay" since he utilizes a modified version of the Leontief fixed coefficients (both *ex ante* and *ex post*) model. Nerlove (1963) and Dhrymes and Kurz (1964) implicitly adopt the "putty–putty" model, or the "putty–clay" model with a myopic planning horizon, since they assume producers only take into account current prices when choosing factor proportions. Neither of these last two assumptions is tenable for a production process like electricity generation which is dominated by substantial investment in long-lived physical capital. Belinfante (1969) utilizes a "putty–clay" model in his study of returns to scale, technical change and factor substitution. An attempt to test the maintained hypothesis of "putty–clay" by grafting on *ex post* substitution terms to the basic model proved unsuccessful. The procedure led to a rejection of the basic underlying model (no *ex post* factor substitution) and in one case produced an *ex post* elasticity of substitution greater than the *ex ante* elasticity.[3]

I conclude from this brief survey of previous studies that the nature of the structure of technology needs to be tested. In most cases very special structures have been imposed. In the one case where a test was attempted, the model used was inappropriate.

The basic model is reviewed in Section 2. In Section 3, the model is specialized to take into account the important features of electricity generation. Section 4 presents the results of testing the structure. In Section 5, estimates of the underlying production parameters and substitution characteristics for the non-rejected structure are presented. Section 6 contains concluding remarks.

[2]Galatin (1968, p. 70).

[3]While this is not impossible [see Syrquin (1970)] it is highly unlikely in the case of electricity generation.

2. The Basic Model

In this section a summary of the *ex ante – ex post* model of production is presented. A detailed discussion can be found in Fuss (1977b), and a generalization of the model in Chapter II.4 written by Fuss and McFadden.

Suppose a producer expects to use n variable factors to produce y_t units of output at time t. Using the duality relationship between cost and production, and assuming cost-minimizing behavior, we can specify the nature of expected *ex post* production indirectly by the cost function

$$
{}_EC_t^v(p_{1t}^v,\ldots,p_{nt}^v,{}_Ey_t^v) = \sum_{i=1}^{n} b_{ii}^v p_{it}^v h_i({}_Ey_t^v) + \sum_{i\neq j}\sum b_{ij}^v (p_{it}^v p_{jt}^v)^{1/2} h({}_Ey_t^v), \qquad i,j = 1,\ldots,n, \quad (1)
$$

where p_{it}^v is the expected price at time t of the ith factor (expectations formed at time v), and $h({}_Ey_t^v)$, $h_i({}_Ey_t^v)$ are functions of output that specify the *ex post* output expansion path. The parameters b_{ii}^v, b_{ij}^v are *ex post* parameters specifying production possibilities. They are functions of the *ex ante* choice of technique.

Equation (1) has the following advantages as a description of technology: (1) Allen–Uzawa partial elasticities of substitution are not necessarily all equal or constant, and (2) the underlying production function is not constrained to be homothetic. The homotheticity constraint is particularly inappropriate in the ease of electricity generation.

Define the present value of expected cost as

$$
V = \sum_{t=v+1}^{v+L} \rho_{t-v}\, {}_EC_t^v = \sum_{i,j} b_{ij}^v q_{ij}^v,
$$

where ρ_{t-v} is an expected discount rate, L is the planning period, and

$$
\begin{aligned}
q_{ij}^v &= \sum_t \rho_{t-v} p_{it}^v \cdot h_i({}_Ey_t^v), && i = j,\\
&= \sum_t \rho_{t-v} (p_{it}^v p_{jt}^v)^{1/2} h({}_Ey_t^v), && i \neq j. \quad (2)
\end{aligned}
$$

It has been shown in Fuss (1977b) that if we specify, as a functional form for b_{ij}^v,

$$
b_{ij}^v = \sum_k \sum_l (s_{kl}/s_{ij})^{1/2} a_{ij,kl}, \qquad i,j,k,l = 1,\ldots,n,
$$

where s_{ij}, s_{kl} are arbitrary variables $\geqq 0$, and $a_{ij,kl}$ are fixed *ex ante* parameters, V is minimized when

$$b_{ij}^{v} = \sum_{k} \sum_{l} (q_{kl}^{v}/q_{ij}^{v})^{1/2} a_{ij,kl}, \qquad i,j,k,l = 1,\ldots,n.$$

The observed (realized) *ex post* cost function at time t is

$$\begin{aligned} C_t^v &= \sum_{i=1}^{n} b_{ii}^{v} p_{it} h_i(y_t^v) + \sum_{i} \sum_{j} b_{ij}^{v} (p_{it} p_{jt})^{1/2} h(y_t^v) \\ &= \sum_{i} \sum_{k,l} [(q_{kl}^{v}/q_{ii}^{v})^{1/2} p_{it} h_i(y_t^v)] a_{ii,kl} \\ &\quad + \sum_{ij} \sum_{kl} [(q_{kl}^{v}/q_{ij}^{v})^{1/2} (p_{it} p_{jt})^{1/2} h(y_t^v)] a_{ij,kl}, \qquad i \neq j. \end{aligned}$$

Using the derivative property of cost functions, the observed (*ex post*) system of factor demand functions is

$$\begin{aligned} X_{it}^{v} &= \sum_{kl} [(q_{kl}^{v}/q_{ii}^{v})^{1/2} h_i(y_t^v)] a_{ii,kl} \\ &\quad + \sum_{j} \sum_{kl} [(q_{kl}^{v}/q_{ij}^{v})^{1/2} (p_{jt}/p_{it})^{1/2} h(y_t^v)] a_{ij,kl}, \qquad j \neq i, \end{aligned} \tag{3}$$

where X_{it}^{v} is the quantity demanded of the ith factor at time t for use in a production process of vintage v.

In Fuss (1970, 1977b) it was shown that with appropriate restrictions on the *ex ante* parameters of (3), this system of demand functions describes various structures of technology. The appropriate restrictions are given in Table 1.

TABLE 1

Structure of technology	Related hypotheses
putty–clay	$a_{ij,kl} = 0$ unless $i = j$ and $k = l$
putty–putty	$a_{ij,kl} = 0$ unless $ij = kl$
clay–clay	$a_{ij,kl} = 0$ unless $i = j = k = l$

3. The Model Applied to Electricity Generation

In this section the basic model is simplified and specialized so that it can be applied to data drawn from a sample of pooled cross-section and time series observations on steam-electric generating plants. These data are discussed in Section 7, Appendix A. Only those features of electricity generation which bear directly on the final form of the estimated demand equations will be analysed. The reader is referred elsewhere for a detailed description of the industry.[4]

The production process analysed is the case in which four factors (structures, equipment, fuel and operating labor) are used to produce electricity. All four factors are assumed to be variable *ex ante*. *Ex post* production is characterized by the addition of variable flows of fuel and operating labor services to the fixed capital stock (structures and equipment). Since there is no *ex post* addition to capacity output,[5] it is reasonable to specify, for the electricity generation industry, that there is no *ex post* addition to capital services.

This assumption of a fixed capital stock can be employed to reduce the number of parameters which need to be estimated.

3.1. *The Ex Post Specification*

From Section 2 the *ex post* demand function for factor i is

$$X^v_{it} = b^v_{ii}h_i(y^v_t) + \sum_{j\neq i} b^v_{ij}(p_{jt}/p_{it})^{1/2}h(y^v_t), \qquad i,j = 0,1,2,3,$$

where X^v_{it} is the cost-minimizing quantity of service flow during period t associated with an electricity generation plant of vintage v; and $i,j =$ 0,1,2,3 index structures, equipment, fuel and labor, respectively. Suppose factor i is a fixed factor *ex post*. Then the service flow utilized from the fixed stock during period t will depend on the rate of utilization of the production process (the turbine–generator unit), and not on the current price of capital services imputed from the current price of the stock of capital.[6] If the utilization rate is constant over time, X^v_{it} can be

[4]There are numerous detailed descriptions of electricity generation as a production process since data drawn from this industry have been used extensively. One such study is Galatin (1968). Descriptions of the industry which focus more closely on factor substitution can be found in Belinfante (1969, and in Chapter IV.3), and Cowing (1969). An elaboration of this section is contained in Fuss (1970).

[5]This is true for all plant observations used in this study.

[6]The actual period t imputed service price is a function of the period t asset price, interest rate and change in asset price. Assuming a perfect capital market, no taxes or depreciation, $p_{it} = q_t \cdot r_t - (\Delta q_t/\Delta t)$, where q_t is the period t asset price, and r_t is the period t interest rate. If a firm were able to adjust the stock of the fixed factor *ex post* to obtain the cost-minimizing service flow, p_{it} would be the service price which enters into the determination of the minimum cost input proportions. For a derivation of the above formula and a similar interpretation, see Jorgenson (1963).

assumed constant and proportional to the stock of the asset. Then X^v_{it} can be replaced by a measure of the stock, X_{iv}. It is more reasonable to assume that the utilization rate will decline over time as the plant ages. If the decline is assumed smooth and continuous, a weighted proportionality (weighted by the rate of decline) can be used to maintain a simple relationship between the stock and flow variables.

If factor i is a fixed factor, it will also be the case that the quantity demanded, *ex post*, of the variable factors and the other fixed factors at time t will be independent of the actual period t imputed price of the services of factor i. The effect of a change in the *ex post* price of input i on the quantity demanded of input j, all other prices held constant, is given by

$$\partial X^v_{jt}/\partial p_{it} = \partial^2 C^v_t(y^v_t,\mathbf{p})/\partial p_{jt}\partial p_{it} = \frac{1}{2}\sum_{i\neq j} b^v_{ji}(p_{jt}p_{it})^{-1/2}\cdot h(y^v_t).$$

If input i is a fixed factor, from the preceding argument we have $\partial X^v_{jt}/\partial p_{it} = 0$, $j\neq i$. This set of restrictions must hold for all configurations of factor prices, which implies $b^v_{ji} = 0$ for all $j\neq i$. By symmetry we also have $b^v_{ij} = 0$ for all $j\neq i$.

Utilizing the above restrictions with structures and equipment as the fixed factors, the *ex post* cost function becomes

$$\begin{aligned} C^v_t(y^v_t,\mathbf{p}) &= \left[\sum_{i=0,1} b^v_{ii}p_{it}h_i(y^v_t)\right] \\ &\quad + \left[\sum_{i\neq 0,1} b^v_{ii}p_{it}h_i(y^v_t) + \sum_{i,j\neq 0,1}\sum b_{ij}(p_{it}p_{jt})^{1/2}h(y^v_t)\right] \\ &= FC_t + VC_t(y^v_t,\mathbf{p}). \end{aligned}$$

The term inside the first set of brackets is the proportion of fixed cost attributable to period t. The term inside the second set of brackets is the *ex post* variable (restricted) cost function. Given the stock of capital, $VC_t(y^v_t,\mathbf{p})$ is the minimum cost of utilizing the variable factors of production to product y^v_t. The fixed charges are evaluated using the period t services prices p_{it}. These prices would represent the opportunity cost, during period t, of using the capital stock if a rental market existed. The *ex post* variable input demand functions become

$$\begin{aligned} X^v_{it} &= \partial VC_t(y^v_t,\mathbf{p})/\partial \mathbf{p}_{it}, \qquad i = 2,3, \\ &= b^v_{ii}h_i(y^v_t) + \sum_{j\neq 0,1} b^v_{ij}(p_{jt}/p_{it})^{1/2}h(y^v_t). \end{aligned}$$

To complete the specialization of the *ex post* technology of electricity generation we need to specify the output functions $h_i(y_t^v)$, $h(y_t^v)$. We have chosen the specification

$$h_i(y_t^v) = (l_t)^{\gamma_i} Y_t^v = (y_t^v / Y_t^v)^{\gamma_i} Y_t^v,$$

and

$$h(y_t^v) = (l_t)^{\gamma} Y_t^v,$$

where Y_t^v is the "designed" output[7] at time t, y_t^v is the actual output at time t, l_t is the plant load factor,[8] and γ_i, γ are parameters to be estimated.

If $\gamma_i = \gamma = 1$, *ex post* production, given the fixed stock of capital is subject to constant returns to scale (capacity utilization). If $\gamma_i, \gamma < 1$, increasing returns to scale result. If $\gamma_i, \gamma > 1$, decreasing returns to scale result. If $\gamma_i < 1$, $\gamma > 1$, or vice versa, returns to scale are indeterminate *a priori* but can be determined from the estimates of the parameters. Returns to scale are classified by the effect on average cost of changes in the level of output when factor prices are held constant. It is more usual in production theory to classify returns to scale by the effect on output of proportionate changes in inputs. That method is really only appropriate for homothetic production functions, where expansion along any ray from the origin of the input space is unambiguous. The definition used in this paper has an easily recognized economic interpretation, regardless of the nature of the production structure. For an elaboration of this point, see Fuss (1970) and Hanoch (1975b).

The fixed cost function can be analyzed most conveniently within the context of the *ex ante* choice of technique. We shall now turn to the *ex ante* specification of production for the electricity generation industry.

3.2. *The Ex Ante Specification*

The expected present value function can be written

$$V = \sum_{i=0,1} b_{ii}^v q_{ii}^v + \sum_{i,j\neq 0,1} \sum b_{ij}^v q_{ij}^v,$$

where q_{ii}^v, q_{ij}^v are defined by equation (2).

[7]The designed output will be assumed to be the normal expected output. The chosen capacity of the boiler-generator unit will be determined by this expected output.

[8]The load factor is defined as the actual yearly output divided by the (capacity) output which could have been produced while the turbine generator was "hot and connected to load".

The second term is the expected present value of the variable costs of production. The first term is the expected present value of the fixed costs of production. But the fixed costs are expenditures which are committed when the plant is being constructed. Therefore $\sum_{i=0,1} b_{ii}^v q_{ii}^v$ represents the original cost (at time v) of building the plant; that is,

$$q_i^v \cdot X_i^v = b_{ii}^v q_{ii}^v = b_{ii}^v \sum_{t=v+1}^{v+L} {}_E\rho_{t-v} p_{it}^v l_t^{\gamma_i} \, {}_E Y_t^v, \qquad i = 0,1,$$

where q_i^v is the asset price per unit of the stock of fixed factor i and X_i^v is a measure of the stock of factor i embodied in the plant.[9] The expected yearly output at time t, ${}_E Y_t^v$, is assumed to be equal to the rated capacity (on a yearly basis) times the expected proportion of the year the turbine-generator is hot and connected to load. This proportion will decline as the plant ages since the plant will be subject to physical deterioration and economic obsolescence. Physical deterioration results in an increase in forced outage and economic obsolescence causes an increase in desired outage. Suppose these two economic phenomena can be represented by a depreciation function d_{t-v}, so that

$${}_E Y_t^v = {}_E d_{t-v} \cdot Y_v^v,$$

where ${}_E d_0 = 1$. Then

$$q_i^v X_i^v = [b_{ii}^v Y_v^v] \left[\sum_{t=v+1}^{v+L} {}_E\rho_{t-v} \, {}_E d_{t-v} \, p_{it}^v \, l_t^{\gamma_i} \right], \qquad i = 0,1. \tag{4}$$

Recalling the definitions of the load factor (from footnote 8) and ${}_E Y_t^v$, we may reasonably assume that the expected load factor is 1 in each period t. This assumption is equivalent to specifying that the plant is expected, *ex ante*, to produce output at some normal rate whenever the turbine-generator is hot and connected to load. Adding this assumption, (4) becomes

$$q_i^v X_i^v = [b_{ii}^v Y_v^v] \left[\sum_{t=v+1}^{v+L} \rho_{t-v} d_{t-v} p_{it}^v \right],$$

where the expectation notation (E) has been removed for simplicity.

I have assumed that depreciation (physical plus economic) causes a decline in the utilization rate of the capital stock rather than a decline in the productive quality of capital. A companion assumption is that the *expected* imputed price of a unit of capital services is constant over

[9] I shall ignore the difficulties involved in trying to combine the components of capital into aggregate inputs. This problem is discussed in Appendix E of Fuss (1970).

time.[10] Then $p_{it}^{v} = p_{iv}^{v}$ for all t, and

$$q_i^v X_i^v = [b_{ii}^v Y_v^v]\left[\sum_{t=v+1}^{v+L} \rho_{t-v} d_{t-v} p_{iv}^v\right], \qquad i = 0,1.$$

The term in the second set of brackets is the expected present value of the capital services embodied in 1 unit of the capital asset.[11] Assuming a perfect capital market, this sum equals the unit price of the asset, q_i^v. As a result we have the following two relationships:

$$q_i^v = \sum_{t=v+1}^{v+L} \rho_{t-v} d_{t-v} p_{iv}^v, \qquad i = 0,1, \tag{5}$$

$$X_i^v = b_{ii}^v Y_v^v. \tag{6}$$

Equation (5) can be used to calculate the expected price of capital services,

$$p_{iv}^v = q_i^v \Big/ \sum_t \rho_{t-v} \cdot d_{t-v}.$$

Equation (6) is the *ex post* demand function for fixed factor i which is part of the system of input demand functions to be estimated.

3.3. *Ex Ante Returns to Scale and Embodied Technical Progress*

In Fuss (1970), I stressed the fact that the basic model is constructed under the assumption that all production units are formed subject to the same *ex ante* production possibility set. However, over time the "best practice" technique embodied in a plant will reflect advances in technical knowledge and the exploitation (in an expanding market) of any *ex ante* available economies of scale, as well as changes in relative factor prices.

It is difficult to determine to what extent each of these effects, taken separately, are responsible for observed differences in *ex ante* design. Post-World War II production of electricity has been characterized: (i)

[10]I have assumed that production is planned under the belief that capital will be maintained at a constant quality level throughout its lifetime. If we further assume that variations in the outage rate do not affect the marginal product of capital services but only the quantity, then the *ex post* marginal and average products of a unit of capital service would be expected to remain constant over time.

[11]The stock–service flow proportionality assumption and the assumption of constant expected average product are invoked here to obtain the result.

by increased efficiency in the use of factors of production for a given size boiler-plant, turbine–generator unit; (ii) by the continual introduction of larger scale units; and (iii) by the substitution of capital services for the services of other inputs. Characteristic (i) is a clear-cut example of technical progress in electricity generation. With regard to characteristic (ii), larger-scale units are more efficient units. If large-scale units are introduced as soon as they become technically feasible, the increased efficiency can be attributed to scale-augmenting technical progress. However, the technology needed to build the larger unit may be available years before the unit is actually installed. If the introduction of the more efficient larger unit is conditional on the expansion of markets, the effect of this introduction is the exploitation of economies of scale; and this exploitation is not directly attributable to technical progress.

Technical progress, economies of scale, and factor substitution can also be confounded. Suppose technical progress in the equipment supplying industry results in a lower supply price of constant quality equipment. This should lead to a substitution of the services of capital for the services of other factors of production. If there is an overall decline in the cost of producing a unit of output,[12] this cost saving will eventually be reflected in a rate reduction which leads to an increase in the quantity of output demanded. In addition, many elements of technical change can be embodied only in the largest units and these units will be installed only when the potential market is sufficiently extensive. The economies of scale which result from exploiting the expanded market are an example of the confounded effect.

The difficulty inherent in attempting to distinguish between pure economies of scale and scale-augmenting technical change should be apparent from the above examples. I will not attempt to make this distinction, and will group all effects due to larger scale under the heading of economies of scale.

Efficiency in the electricity generation industry is characterized by higher temperature and pressure conditions which decrease the demand for fuel per unit of output. Since the fuel input represents about 50% of the total generation costs, reducing the heat rate is of primary concern. More efficient steam conditions are generally characteristic of large turbine–generator units. "High steam pressure and temperature, and large scale of unit are mutually related in joint application for most

[12]There need not be a decline in unit costs, since the nominal prices of the factors being substituted for may increase faster than the rate of substitution.

favourable results in heat rate gains."[13] In addition, labor and capital inputs per unit of capacity tend to be lower in plants which install larger units.[14,15]

For the above reasons I have taken the size of the turbine–generator, boiler-plant unit as the main component of the index of efficiency. The measure of efficiency is introduced into the model in the following way. Suppose E is the measure of efficiency. Then the cost function may be written as

$$C_t^v = \sum_i p_{it} \cdot a_{ii,ii}(E) l_t^{\gamma_i} Y_t^v + \sum_{\substack{i,j,k,l \\ \text{not all}=i}} (q_{kl}^v / q_{ij}^v)^{1/2} (p_{it} p_{jt})^{1/2} a_{ij,kl}(E) \cdot l_t^{\gamma} Y_t^v, \qquad i \neq j \quad \text{unless} \quad i = j = 0,1,$$

where $a_{ij,kl}(E)$ indicates that the *ex ante* parameters are functions of the efficiency measure. The number of parameters which must be estimated under this specification is quite formidable. Therefore, I have made the simplifying assumption that only the diagonal elements $a_{ii,ii}$ of the parameter matrix $\mathbf{A} = (a_{ij,kl})$ differ from plant to plant. This assumption has the effect of restricting the way in which efficiency characteristics can influence the *ex ante* and *ex post* elasticities of substitution. However these elasticities are not independent of the efficiency measure since C_t^v and X_{it}^v are still functions of E and they appear in the elasticity formulae. Subject to the imposed specification, it is possible to estimate

[13] Ling (1964, p. 30).

[14] *Electrical World*. This fact is corroborated by my empirical results.

[15] Despite the apparent overall gain in efficiency derived from the introduction of larger units, this technical advance is not uniformly implemented. It is observed that firms, having access to the same technology, build plants with the same capacity, but with different efficiency characteristics. (That is, one plant may be built with one generator of 60 megawatts capacity, while at the same time a plant is built which uses two units, each of 30 megawatts capacity.) There are three possible explanations for this behavior. First, plants which are not built using the largest unit available (in accordance with the designed capacity) are not characterized by cost minimization and thus are "off" the *ex ante* production frontier. Second, the capital investment in additional reheat cycles and mechanization is uneconomical; that is, the observed differences in efficiency are the result of *ex ante* substitution between capital and other inputs that is capital-saving. Third, the price of capital services calculated from the market price of the assets is not the correct measure of the expected cost of capital services for new vintage equipment. The prevalence of "shake-down" expenses which accompany the installation of untried equipment indicates that the expected cost of maintenance services should be capitalized. If this is the case, the price of capital services from new vintage equipment is understated relative to that of previous vintages; and the apparent discrepancy in efficiency is due to *ex ante* capital input–maintenance input substitution.

the effect of economies of scale and technical progress on the elasticities of substitution.

An investigation of the cost survey literature[16] indicates that the decline in the input–output ratio with increasing size can best be represented by a hyperbolic function of size. In addition there may exist embodied technical change which is independent of scale. One of the ways this phenomenon could occur is for firms to benefit from a "learning by doing" process in the construction of new plants. Finally, technical change, as represented by a time trend, and scale may interact to change the shape of the hyperbolic function. The functional form chosen was

$$a_{ii,ii}(E) = \alpha_{i1} + \alpha_{i2}(1/AC) + \alpha_{i3}(V/AC) + \alpha_{i4}(V), \qquad i = 1,2,3, \tag{7}$$

where α_{ij}, $j = 1,\ldots,4$, are parameters to be estimated, AC (average operating capacity of the units installed in a plant) is the index of size of the boiler–generator unit, and $V = 0,1,2$, etc. is an index of embodied technical progress. The structures equation ($i = 0$) is treated somewhat differently and is discussed below.

The system of input demand functions takes the form

$$\begin{aligned} X^v_{it}/y^2_t = &\left[a_{ii,ii}(E) + \sum_{\substack{k\neq i\\ l\neq i}} (q^v_{kl}/q^v_{ii})^{1/2} a_{ii,kl}\right] l_t^{\gamma_i-1} \\ &+ \left[\sum_{j\neq i} \sum_{\substack{k\neq i\\ l\neq i}} (q^v_{kl}/q^v_{ij})^{1/2} (p_{jt}/p_{it})^{1/2} a_{ij,kl}\right] l_t^{\gamma-1}, \\ & \qquad i = 2,3, k,l \neq 1 \quad \text{unless} \quad k = l = 1, \end{aligned} \tag{8}$$

$$X^v_1/Y^v_v = a_{11,11}(E) + \sum_{k,l\neq 1} (q^v_{kl}/q^v_{11})^{1/2} a_{11,kl},$$

$$X^v_0/Y^v_v = b^v_{00},$$

where $a_{ii,ii}(E)$ is given by (7).[17]

We need to add one final assumption to obtain the system of demand equations actually estimated.

Apparently there is little scope for *ex ante* or *ex post* substitution between the structures component of capital and the other factors of

[16] *Electrical World.*

[17] The dependent variables used in the regression analysis are input–output ratios. They are obtained by assuming that the standard deviations of the error terms are proportional to designed capacity, $i = 0,1$; and actual output, $i = 2,3$; and making the appropriate variable transformations to reduce heteroscedasticity.

production.[18] I have assumed that the choice of the structures component is independent, both *ex ante* and *ex post*, of the prices of the other factors of production. This implies that the relevant input–output ratio can be represented by the demand equation

$$X_0^v/Y_v^v = b_{00}^v = a_{00,00}(s),$$

where s represents characteristics such as size of plant, vintage of plant, geographical location, etc. The form actually chosen for the structures demand equation was

$$X_0^v/Y_v^v = \alpha_{01} + \alpha_{02}(1/OC) + \alpha_{03}(V/OC) + \alpha_{04}V + \sum_{i=5}^{9} \alpha_{0i}d_i, \tag{9}$$

where OC is the total operating capacity of the plant, V is an index of vintage as before, and d_i is a dummy variable representing the ith geographical region.

Interpreting factor 1 in the system of equations (8) as equipment, and adding (9) as the demand for structures we obtain the set of input demand equations which was estimated.

3.4. *Summary*

The putty–semi-putty model developed in Fuss (1970, 1977b) is a relatively general model of production. However, along with the advantage of generality, we are confronted with the disadvantage of needing to estimate the large number of parameters contained in a four-factor version of the model. The major portion of this section has been devoted to reducing the number of parameters to be estimated by utilizing *a priori* information about the nature of electricity generation. Of particular importance was the *ex post* fixity of structures and equipment. In addition, information obtained from technical sources was utilized to provide simple functional representations of embodied technical change and *ex ante* and *ex post* returns to scale. A reduction from the general to the particular may bias the results in unknown ways; but such a reduction is unavoidable. Since the estimation results which are reported in the next section are generally in agreement with *a priori* expectations, we conclude, somewhat cautiously, that the numerous assumptions set forth in this section are a reasonable, limited, distortion of reality.

[18]The other factors being considered are fuel, operating labor and the equipment component of capital.

4. Tests of the Structure of Technology

In this section the putty–clay and clay–clay hypotheses are tested against the maintained hypothesis of putty–semi-putty. The putty–putty model is rejected by assumption, since the capital input is assumed fixed *ex post*.

4.1. *The Specification of Expectations*

The first problem which must be confronted in an empirical implementation of the model is the specification of expectations. The best practice technique depends on future factor prices and output requirements that are unknown to the producer; and which he must estimate. No data are available to the *ex post* observer from which he can infer the producer's estimation procedure. Therefore some restrictive assumptions must be introduced.

Since installed capacity is known, the specification of output expectations reduces to the problem of specifying expected future outage rates. The assumption that outage rates are expected to increase smoothly over time was introduced in Section 3 and will be retained here.

Price expectations are a more troublesome problem. In most econometric studies dealing with models of production for which theory tells us expectations are important, one of two simplifying assumptions have been made. The first is to assume the producer has "zero foresight" so that all decisions are based on current relative prices.[19] In the electricity generation industry large capital expenditures are frozen for many years once a decision has been made. Thus an overall application of this first assumption is unreasonable.[20] The second assumption is one of "perfect certainty". In this case the actual prices observed are assumed to be the expected prices, so expectations need not be estimated.[21] Of course this assumption is unlikely to be strictly correct, but may not be an unreasonable assumption for the particular industry in question.

[19]For example, see Jorgenson (1963).

[20]When the zero foresight assumption was employed, the results were far inferior to the results obtained when the less myoptic rule discussed below was used. I consider this to be an encouraging feature of the model.

[21]This is the assumption employed by Belinfante in Chapter IV.3.

The specification which was finally chosen is a hybrid combination of the two possibilities outlined above. For prices which have no strong historical trends and/or are often fixed by long-term contracts, the assumption of "zero foresight" or "static expectations" has been used. The price of fuel and the interest rate come under this category.[22] The price of labor has a strong upward trend component and producers of electricity are presumably aware of this fact. While the producer is unable to predict the *exact* price he will pay for labor services in the future, he can estimate, fairly accurately, the trend prevailing in the industry in his geographic region. He applies this trend to the initial price to form the expected future price. If the actual prices observed in each region are used to estimate the trends, the following expectations hypothesis is implied: expectations are realized on the average, even though no individual producer's expectations are necessarily realized.

The results of this estimation procedure are presented in Section 7, Appendix B. The main characteristic of the results is the precision with which the trends are estimated. In all cases the standard errors are quite small so that the usual confidence intervals (e.g., 95%) will tightly bracket the estimates. This fact is very important since these estimated trends will be used as if they were the actual expected trends.[23]

With the specification of expectations completed, the variables $(q^v_{kl}/q^v_{ij})^{1/2}$ can be calculated and hence the system of demand functions (8), (9) can be estimated.

4.2. *The Estimation Procedure*

If we add a stochastic specification[24] to the deterministic specification,

[22]The period covered by the data was 1948–61. For recent years this assumption would be inappropriate since interest rates have been at historically high levels and fuel prices have been rising rapidly.

[23]It cannot be claimed that this procedure is identical to the standard theory of certainty equivalents, where the use of minimum variance estimates from the first stage as summary data in the second stage leads to overall minimum variance estimation. This lack of equivalence results from the fact that the λ^E_i enter the input demand equations in a non-linear way. However from a Bayesian point of view, the results of Table 5 in Section 7 provide strong evidence that the posterior distribution of λ^E_i (viewed as a random variable) is "tight", so that a second-stage decision (expected cost minimization) based on the summary statistic $\hat{\lambda}^E_i$ should not deviate substantially from the decision based on the true λ^E_i. For an analysis of this form of approximation to certainty equivalence, see Raiffa and Schlaifer (1968, Ch. 6).

[24]The stochastic specification for this model is discussed at length in Fuss (1970, Ch. III) where a suitable autocorrelation adjustment procedure is developed.

the system of equations (8), (9) written in more compact form become

$$y_i = g_i(\mathbf{z}_i;\boldsymbol{\theta}_i) + \boldsymbol{\epsilon}_i, \qquad i = 0,1,2,3, \tag{10}$$

where y_i is the dependent variable of the ith equation; $\mathbf{z}_i$ is the vector of predetermined variables of the ith equation; $\boldsymbol{\theta}_i$ is the vector of parameters of the ith equation, $\boldsymbol{\theta}_i = (\alpha_{ij}, a_{ijkl}, \gamma, \gamma_i)$; g_i is a non-linear function; and $\boldsymbol{\epsilon}_i$ is a vector of error terms.

The system (10) can also be written in the full vector form

$$\mathbf{y} = g(\mathbf{z};\boldsymbol{\theta}) + \boldsymbol{\epsilon},$$

where

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_0 \\ \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix}, \quad g(\mathbf{z};\boldsymbol{\theta}) = \begin{bmatrix} g_0(\mathbf{z}_0;\boldsymbol{\theta}_0) \\ \vdots \\ g_3(\mathbf{z}_3;\boldsymbol{\theta}_3) \end{bmatrix}, \quad \boldsymbol{\epsilon} = \begin{bmatrix} \boldsymbol{\epsilon}_0 \\ \vdots \\ \boldsymbol{\epsilon}_3 \end{bmatrix},$$

and $V(\boldsymbol{\epsilon}) = \boldsymbol{\Omega}$.

The class of parameter estimators chosen for this nonlinear multivariate regression system is Malinvaud's minimum distance estimator [Malinvaud (1966, Ch. 9)]. The minimum S-distance estimator, say $\hat{\boldsymbol{\theta}}(\mathbf{S})$, is that estimator which minimizes

$$Q(\mathbf{S},\boldsymbol{\theta}) = [\mathbf{y} - g(\mathbf{z};\boldsymbol{\theta})]'\mathbf{S}[\mathbf{y} - g(\mathbf{z};\boldsymbol{\theta})],$$

for the observed sample, where $\mathbf{S}$ is a positive definite matrix. If $\mathbf{S}$ is a consistent estimator of $\boldsymbol{\Omega}^{-1}$, then under a set of relatively mild assumptions $\hat{\boldsymbol{\theta}}$ is a consistent estimator of $\boldsymbol{\theta}$ and is asymptotically normal. If in addition, $\boldsymbol{\epsilon}$ is normally distributed $\hat{\boldsymbol{\theta}}$ is asymptotically efficient.[25]

In the data sample there is only one observation per plant for the fixed factor demands (structures and equipment). To simplify the structure of $\boldsymbol{\Omega}$ we will assume that the error terms corresponding to these equations are distributed over time independently of the error terms associated with the variable factors. Then the variance–covariance matrix takes the form

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\Omega}_{00} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Omega}_{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Omega}_{22} & \boldsymbol{\Omega}_{32} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Omega}_{23} & \boldsymbol{\Omega}_{33} \end{bmatrix},$$

[25]The necessary assumptions and the proof of this proposition can be found in Malinvaud (1966, pp. 290–299).

where $V(\boldsymbol{\epsilon}_0) = \boldsymbol{\Omega}_{00}$, $V(\boldsymbol{\epsilon}_1) = \boldsymbol{\Omega}_{11}$, $\text{cov}(\boldsymbol{\epsilon}_2,\boldsymbol{\epsilon}_3) = \boldsymbol{\Omega}_{23} = \boldsymbol{\Omega}_{32}$.

Since the demand for structures has been assumed deterministically independent of the demand for the other factors,

$$\begin{aligned} Q(\boldsymbol{\Omega},\boldsymbol{\theta}) &= [\mathbf{y}_0 - g_0(\mathbf{z};\boldsymbol{\theta}_0)]'\boldsymbol{\Omega}_{00}^{-1}[\mathbf{y}_0 - g_0(\mathbf{z}_0;\boldsymbol{\theta}_0)] \\ &\quad + [\mathbf{y} - g(\mathbf{z};\boldsymbol{\theta})]'\boldsymbol{\Omega}_{123}^{-1}[\mathbf{y} - g(\mathbf{z};\boldsymbol{\theta})], \end{aligned} \tag{11}$$

where

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \end{bmatrix}, \qquad \boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\theta}_1 \\ \boldsymbol{\theta}_2 \\ \boldsymbol{\theta}_3 \end{bmatrix},$$

and

$$\boldsymbol{\Omega}_{123} = \begin{bmatrix} \boldsymbol{\Omega}_{11} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Omega}_{22} & \boldsymbol{\Omega}_{32} \\ \mathbf{0} & \boldsymbol{\Omega}_{23} & \boldsymbol{\Omega}_{33} \end{bmatrix}.$$

From (11) it is clear that the parameters of the demand for structures equation can be estimated independently of the rest of the system. Since this equation is linear in the parameters and

$$\boldsymbol{\Omega}_{00} = \begin{bmatrix} \sigma_{00} & & 0 \\ & \ddots & \\ 0 & & \sigma_{00} \end{bmatrix},$$ [26]

the minimum distance estimators are the ordinary least-squares estimators.

The remaining three equations form an interrelated non-linear factor demand system. However, conditional on estimated values of γ, γ_i, this system is linear in the parameters and the across-equations constraints are also linear.[27] To take advantage of this fact, the following two-stage estimation procedure was employed:

(1) Estimates of γ, γ_2, γ_3 and $\boldsymbol{\Omega}_{123}$ were obtained by minimizing the second term of (11) ignoring all the across-equations constraints except the one which implies that the same γ appears in both the fuel and labor

[26]The assumptions of homoscedasticity and interplant independent error terms are used to obtain this result.

[27]The assumption of cost-minimizing behavior imposes certain symmetry restrictions on the *ex ante* parameters. These restrictions are discussed in Fuss (1977b).

equations. This non-linear problem was solved by a search procedure – searching through values of γ, γ_2, γ_3. From the theory of minimum-distance estimators, the resultant estimates of γ, γ_2, γ_3, and $\mathbf{\Omega}_{123}$ are consistent estimates but are not (even asymptotically) minimum variance since some constraints were ignored.[28]

(2) After substituting the estimates obtained in (1) into (11) Q was minimized using the second stage of Zellner's unrelated regression procedure;[29] this time incorporating all the across-equations constraints. *Conditional* on the first-stage estimates of γ, γ_2, γ_3 and $\mathbf{\Omega}_{123}$, the estimates of the remaining parameters are consistent and asymptotically efficient.

4.3. *Hypothesis Testing*

Suppose that under the maintained hypothesis of putty–semi-putty the vector of parameters $\boldsymbol{\alpha} = (a_{ij,kl})$ lies in the linear subspace Π_{ω_0}. Suppose further that the hypotheses of putty–clay and clay–clay constrain the vector $\boldsymbol{\alpha}$ to lie in the linear subspaces $\Pi_{\omega_1}$and Π_{ω_2}, respectively. Writing the hypotheses in the usual notation, we have

H_{ω_0}: maintained hypothesis,

H_{ω_1}: $a_{ij,kl} = 0$, unless $i = j$ and $k = l$,

H_{ω_2}: $a_{ij,kl} = 0$, unless $i = j = k = l$.

It is evident from the form of the hypotheses that

$$\Pi_{\omega_0} \supset \Pi_{\omega_1} \supset \Pi_{\omega_2},$$

so that the hypotheses form a sequence of "nested" hypotheses. This sequence can be represented schematically by Figure 1.

The fact that the hypotheses are nested allows us to avoid the statistically difficult problem of making multiple comparisons of different competing models.[30] The hypotheses can be tested in the following sequence:

[28]Malinvaud (1966, Ch. 9). At the time these results were obtained (1969), currently available computer programs which efficiently estimate nonlinear systems of equations had not been developed.

[29]Zellner (1962).

[30]However we need to be careful about the way in which significance levels are handled. If H_{ω_1} is tested at level α_1 and H_{ω_2} at level α_2 then the probability of a Type I error at the second stage is $\alpha_1 + \alpha_2 - \alpha_1 \cdot \alpha_2$. For an explanation of the theory of nested hypothesis testing, see Scheffé (1959, pp. 44–45).

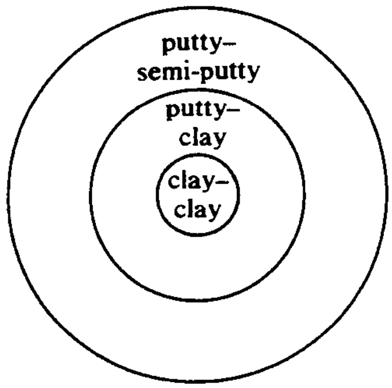

FIGURE 1

(1) We test the null hypothesis of putty–clay against the alternative hypothesis of putty–semi-putty. If the null hypothesis is rejected, the sequence of testing is ended since a necessary condition that the technology be clay–clay is that it at least be putty–clay. If the putty–clay hypothesis cannot be rejected, we proceed to the second step in the sequence.

(2) We impose the putty–clay hypothesis as the maintained (alternative) hypothesis. This involves a re-estimation of the parameters of the model. We then test the null hypothesis of a clay–clay structure against the alternative hypothesis.

In order to proceed with stage one of the sequence of tests, we form the test statistic

$$\mathcal{F} = \frac{(R_1 - R_0)/q}{R_0/(n-k)}, \tag{12}$$

where $R_0 = \mathbf{e}'\mathbf{\Omega}^{-1}\mathbf{e}$ is the weighted total (unconstrained) residual sum of squares under the maintained H_{ω_0}, $R_1 = \mathbf{e}'\mathbf{\Omega}^{-1}\mathbf{e}$ under the null hypothesis H_{ω_1}, q is the number of restrictions imposed to obtain H_{ω_1}, and $n - k$ is the total degrees of freedom for the unconstrained regression. When $\mathbf{\Omega}$ is known, $\mathcal{F}$ is distributed as Snedecor's F with q and $n - k$ degrees of freedom. But in our case $\mathbf{\Omega}$ is unknown. However Zellner (1962) has shown that replacing $\mathbf{\Omega}$ by a consistent estimate in (12) results in a statistic which converges to the optimal statistic as $n \to \infty$. Such a statistic is thus valid for the large sample which we possess. The test of the putty–clay hypothesis reported below is based on a "consistent" statistic of the form (12).[31] The consistent estimate of $\mathbf{\Omega}$ used was the

[31]This form of the statistic was also used to test the clay–clay hypothesis.

TABLE 2
A test of the putty–clay hypothesis.

Description	Residual sum of squares	Degrees of freedom	Test statistic	Critical value of F
Putty–clay (H_{ω_1})	524.22	424		
Putty–semi-putty (H_{ω_0})	512.16	420		$F(4,420)$ =
$R_1 - R_0$	12.6	4	2.47	$\begin{cases} 2.39\,(5\%) \\ 3.36\,(1\%) \end{cases}$

one obtained from the first stage of the two-stage estimation of the parameters of the putty–semi-putty model.[32] Table 2 presents the results of the test.

We observe from inspection of Table 2 that no clear-cut decision is readily apparent. Significance levels of 5% or 1% are most frequently incorporated into the decision rule when testing economic hypotheses. At the 5% level, the putty–clay hypothesis is rejected ($2.47 > 2.39$). But at the 1% level, the hypothesis cannot be rejected ($2.47 < 3.36$). Another way of stating this result is to say that the hypothesis cannot be rejected at any significance level $\leqq 4\%$. Any numerical choice of a significance level as part of the decision rule is arbitrary, and represents a judgment concerning the trade-off between the probabilities of committing Type I and Type II errors. This judgment should be based on two criteria: the relative importance of avoiding Type I or Type II errors, and the strength of outside evidence concerning the validity of the hypothesis being tested. This latter argument is presented by E.L. Lehmann in the following way:

> "A consideration that frequently enters into the specification of a significance level is the attitude toward the hypothesis before the experiment is performed. If one firmly believes the hypothesis to be true, extremely convincing evidence will be required before one is willing to give up this belief: and the significance level will accordingly be set very low."[33]

The bulk of the *a priori* information available (from the engineering literature and actual plant operation) supports the view that *ex post*

[32]This estimate corresponds to the "unrestricted" residual sum of squares used as the denominator of the analogous test statistic in the case of a single equation whose error terms satisfy the Gauss–Markov Theorem. The resultant test statistic is known as the Wald statistic [Berndt and Savin (1977)].

[33]Lehmann (1959, p. 62).

factor substitution (capital input held constant) is highly unlikely. In view of this fact, we must conclude that we are unable to reject the putty–clay hypothesis on the basis of the results presented in Table 2.[34] Therefore we impose the putty–clay hypothesis as the maintained hypothesis and proceed to test the clay–clay hypothesis.

In order to estimate the parameters of the putty–clay model we use the two-stage estimation procedure outlined earlier, imposing the constraints $a_{ij,kl} = 0$ unless $i = j$ and $k = l$.

The testing procedure is identical to that used to test the putty–clay hypothesis. We calculate the residual sum of squares, R_1, under the maintained hypothesis (putty–clay) and the residual sum of squares, R_2, under the null hypothesis (clay–clay) and form the statistic

$$\frac{(R_2 - R_1)/q}{R_1(n-k)} = \frac{\{(\mathbf{e}'\mathbf{\Omega}^{-1}\mathbf{e})\omega_2 - (\mathbf{e}'\mathbf{\Omega}^{-1}\mathbf{e})\omega_1\}/q}{\{(\mathbf{e}'\mathbf{\Omega}^{-1}\mathbf{e})\omega_1\}/(n-k)},$$

which is approximately distributed as $F(q,n-k)$ under the null hypothesis. The estimate of $\mathbf{\Omega}$ was obtained from the residuals of the first-stage estimation of the putty–clay model. The results of the test are reported in Table 3.

It is evident from Table 3 that the null hypothesis of a clay–clay structure is rejected at any reasonable significance level. Combining this result with the result from Table 2, we have narrowed the possible descriptions of technology for the electricity generation industry. The results of this section lend support, in the form of statistical inference, to the *a priori* belief that this industry is characterized by a putty–clay

TABLE 3
A test of the clay–clay hypothesis.

Description	Residual sum of squares	Degrees of freedom	Test statistic	Critical value of F
Clay–clay (H_{ω_2})	579.34	427		
Putty–clay (H_{ω_1})	459.28	424		
				$F(3,424) =$
$R_2 - R_1$	120.06	3	36.95	2.62 (5%) 3.83 (1%)

[34]This decision needs to be tempered by the fact that there was substantial multicollinearity between the actual and expected cost variables; a problem which reduces the power of the test. On the other hand, with such a large sample size, it is difficult not to reject *any* hypothesis at conventional significance levels since the trade-off curve between the probabilities of committing Type I and Type II errors shifts towards the origin as the sample size increases. This fact is illustrated by the large test statistic obtained for the test of the clay–clay hypothesis.

production technology. Producers choose their technique *ex ante* by substituting among equipment, fuel and labor to obtain that technique which is of minimum expected cost. *Ex post*, no similar adjustment is possible. Producers using a production unit already installed do not respond to period-by-period changes in relative factor prices.[35]

5. Estimation of the Putty–Clay Model

The estimated structure of the system of factor demand equations when the putty–clay hypothesis is imposed is given by equations (13)–(16). The numbers in brackets are approximate standard errors.

Structures

$$\begin{aligned} X_0^v/Y_v^v = {} & \underset{(0.0195)}{0.0977} + \underset{(0.552)}{0.473}\,(1/OC) + \underset{(0.148)}{0.142}\,(V/OC) - \underset{(0.00285)}{0.00452}\,V \\ & - \underset{(0.0182)}{0.0154}d_5 + \underset{(0.0232)}{0.0052}d_6 + \underset{(0.0268)}{0.0165}d_7 - \underset{(0.0232)}{0.0456}d_8 - \underset{(0.0406)}{0.0150}d_9, \end{aligned} \tag{13}$$

$R^2 = 0.457$, SER $= 0.0380$ (standard error of regression), D.W. $= 2.07$ (Durbin–Watson statistic), $N_0 = 34$ (number of observations).

Equipment

$$\begin{aligned} X_1^v/Y_v^v = {} & \underset{(1.649)}{0.519} + \underset{(12.962)}{38.930}\,(1/AC) + \underset{(3.503)}{4.245}\,(V/AC) - \underset{(0.083)}{0.152}\,V \\ & + \underset{(0.564)}{1.437}\,(q_{22}/q_{11})^{1/2} + \underset{(0.103)}{0.403}\,(q_{33}/q_{11})^{1/2}, \end{aligned} \tag{14}$$

$R^2 = 0.721$, SER $= 1.25$, D.W. $= 1.80$, $N_1 = 34$.

Fuel

$$\begin{aligned} X_{2t}^v/y_t^v = {} & \left[\underset{(0.399)}{8.530} + \underset{(6.740)}{27.818}\,(1/AC) + \underset{(2.013)}{5.485}\,(V/AC) - \underset{(0.044)}{0.267}\,V\right] l_t^{\gamma_2-1} \\ & + \left[\underset{(0.564)}{1.437}\,(q_{11}/q_{22})^{1/2} + \underset{(0.039)}{0.257}\,(q_{33}/q_{11})^{1/2}\right] l_t^{\gamma_2-1}, \end{aligned} \tag{15}$$

$\gamma_2 = \underset{(0.01)}{0.87}$, $R^2 = 0.970$, SER $= 0.270$, D.W. $= 2.45$, $N_2 = 190$.

[35]The degree of utilization of a plant which is part of a larger system may depend on *ex post* factor prices and to this extent is not exogenously determined, as assumed.

Labor

$$X^v_{3t}/y^v_t$$

$$= \begin{bmatrix} -\ 0.0594 & +\ 1.105\ (1/AC) & +\ 0.132\ (V/AC) & -\ 0.00205\ V \\ (0.0107) & (0.118) & (0.036) & (0.00077) \end{bmatrix} l_t^{\gamma_3 - 1}$$

$$+ \begin{bmatrix} 0.403\ (q_{11}/q_{33})^{1/2} & +\ 0.257\ (q_{22}/q_{33})^{1/2} \\ (0.103) & (0.039) \end{bmatrix} l_t^{\gamma_3 - 1}, \qquad (16)$$

$\underset{(0.057)}{\gamma_3 = 0.047}$, $R^2 = 0.990$, SER $= 0.00753$, D.W. $= 2.00$, $N_3 = 190$.

The responsiveness of factor demands to changes in relative prices can be measured by price elasticities. Since the putty–clay hypothesis was not rejected, the *ex post* price elasticities are zero. The *ex ante* elasticities are presented in Table 4.[36]

All the own-price elasticities are non-positive as required by concavity of the cost function. All cross-price elasticities are non-negative which indicates that there are no complementary factors of production. Both of the above results are consequences of the fact that all $a_{ij,kl} \geqq 0$, $i = j \neq k = l$. In Fuss (1977b) it was shown that non-negativity of the substitution parameters implies a production structure which is globally well-behaved in the sense that the cost function is everywhere concave. Our estimated structure has this desired property.

It is to be expected that, even *ex ante*, substitution possibilities in the production of electricity would be somewhat limited. Table 4 confirms this hypothesis. The elasticities are generally low. Only the own-price elasticity of labor numerically exceeds unity. Also, some of the un-

TABLE 4
Mean values of *ex ante* price elasticities of factor demand.[a]

	Structures	Equipment	Fuel	Labor
Structures	0	0	0	0
Equipment	0	−0.460	0.170	0.295
Fuel	0	0.046	−0.246	0.076
Labor	0	0.618	0.576	−1.194

[a]All zero elasticities are by assumption.

[36]The analytic expression for these elasticities is developed in Fuss (1977b) and Fuss (1970, Ch. V). Allen–Uzawa *ex ante* elasticities of substitution were also developed and estimated in Fuss (1970).

constrained cross-elasticities are close to zero from an economic point of view, even though they are statistically different from zero (at a 5% significant level).[37]

6. Conclusion

The results of this paper are consistent with the view that the "putty–clay" model is applicable to electricity generation. Capital durability was explicitly taken into account by the specification of a non-myopic planning horizon, and this allowed us to distinguish between different structures of technology.

One particular extension of the basic model needs to be explored. In its present form the model abstracts from uncertainty since estimated expectations are assumed to hold with probability one. The effects of explicit consideration of uncertainty are analyzed by Fuss and McFadden in Chapter II.4.

7. Appendixes

7.1. Appendix A: The Data

In this appendix I shall outline the assumptions and methods used to construct the variables upon which the results of Section 5 are based. The data correspond primarily to that information collected by Belinfante (1969). A detailed description of the source material and key calculations are contained in the statistical appendix to Belinfante's thesis. These data consist of a sample of 457 sets of observations covering the period 1948–61 on the relevant cost and production variables for 79 new steam-electric power plants put into operation between 1947 and 1959. Plant observations begin with the first full year of operation following the initial installation and continue as long as no additional capacity is installed.

[37]While the purpose of this paper is to investigate factor substitution, the more common (for this industry) investigations of scale economics and technical change can be carried out by analyzing the estimated structure (13)–(16). The results of this analysis are consistent with previous studies, adding support to the efficacy of the model. For the empirical details, once again see Fuss (1970, Ch. V).

Actual and Capacity Output

Actual and capacity output are measured in millions of kilowatt-hours per year. Capacity output should be measured as the capacity output attainable during that portion of the year the generators were hot and connected to load.[38] Unfortunately this information is unavailable for the time period covered by the data we are using. Instead, the output which could have been produced if the generators had been connected to load for the full year is used as capacity output. This measure overstates capacity, or normal output as defined earlier. Consequently, the plant load factor is underestimated, and this downward bias increases with the age of the plant. However, since most of the plants in the sample are relatively new plants, this bias is expected to be a small one.

The measure of average capacity used in estimating *ex ante* returns to scale is calculated as the installed generator capacity (in megawatts) divided by the number of turbine–generator units installed in the plant. For those plants which contain a single generator unit, or multiple units of equal capacity, the average capacity variable is an accurate measure of size. This situation is found in a considerable majority of the plants in the sample. For the remaining plants the average capacity variable will only approximate the characteristics of size.

Capital Service Quantity and Price

For a subsample of 34 plants containing 190 observations disaggregated initial capital expenditure data were available. The individual components available correspond to the structure, turbine–generator, boiler plant, and accessory equipment categories. For this subsample the last three were aggregated into indices of equipment service flow and equipment service price by a method which is explained in detail in Fuss (1970). The structures component of the capital input was retained as a separate input into the production process. The results reported in this paper are based on this subsample of 190 observations. For the entire sample of 79 plants and 457 observations the "quantity of capital" variable is a simple (constant) dollar aggregate of all equipment and structures since no disaggregated data were available. Given the belief that structures and equipment enter the production process in fundamentally different ways, this capital measure is inadequate. A

[38]Galatin (1968) has an extensive discussion of this point.

restricted set of results for the entire sample is reported in Fuss (1970).

The expected price of capital services is a function of the initial cost of the capital asset, the expected cost of financing the investment (the cost of capital), and the expected depreciation. Handy–Whitman component indices were utilized as asset price indices. A detailed examination of these indices is provided in Belinfante (1969) and in Chapter IV.3.[39] Utility corporation bond yields taken from Moody's *Industrial Manual*, July 1968, were used to estimate the cost of capital. This procedure corresponds to Miller and Modigliani's (1966) conclusions that, although electric utility companies raise varying proportions of necessary financial capital by debt issue, preferred stock and common stock, variations in the cost of capital can be reasonably approximated by the yield on high-grade bonds. Estimates of depreciation used are those obtained by Komiya (1962) in his extensive study.

Labor and Fuel Variables

Quantity and price measurements for fuel and operating labor are taken directly from Belinfante (1969). The reader is referred to his statistical appendix for a detailed account of these data.

7.2. *Appendix B: Calculation of Labor Price Expectations*

The assumptions used in the main body of the paper to obtain operating labor price expectations imply that the expectations model takes the form

$$\omega^{v}_{ijt} = \omega_{ijv}(1 + \lambda^{E}_{j})^{t-v},$$

where ω^{v}_{ijt} is the expected wage rate for the ith plant of vintage v in the jth geographical region at time t; ω_{ijv} is the actual initial wage rate for the ith plant of vintage v in the jth region; λ^{E}_{j} is the expected trend in wages in region j; $t-v$ is the number of time periods in the future for which the expectation is being formed.

[39]The market for turbine–generator units cannot reasonably be assumed to be a competitive factor market since there is a very small number of producers. One of the conditions under which the duality theorem holds is the presence of competitive factor markets. The existence of competitive markets provides the simplest form of the theorem but is not a necessary condition. The theorem will be satisfied if the electricity-generating firms face exogenously determined prices which they are unable to influence (say by volume buying), even if these prices represent the exploitation of monopolistic power. This could occur if the turbine–generator units were priced by a "uniform percentage markup" rule – a not unreasonable assumption.

TABLE 5
Estimation of wage rate trends.

Geographical region	$\widehat{\log(1 + \lambda_j^E)}$	$\hat{\lambda}_j^E$	K_j	Number of observations	R^2	D.W.
(1) *North Atlantic* ME, NH, VT, MA, RI, CT, NY, PA, NJ, WV, MD, DE	0.0439 (0.0026)	0.045	18	92	0.97	1.3
(2) *South Atlantic* VA, NC, SC, GA, FL, KY, TN, MS, AL	0.0410 (0.0017)	0.042	19	98	0.97	1.5
(3) *North Central* OH, MI, IN, IL, WI, MN, IA, MO, ND, SD, NB, KA	0.0576 (0.0022)	0.059	24	157	0.82[a]	1.1
(4) *South Central* AR, LA, OK, TX	0.0441 0.0030	0.045	6	45	0.96	2.2
(5) *Plateau* MO, ID, WY, CO, UT, NV, NM, AZ	0.0514 (0.0044)	0.053	10	59	0.90	1.6
(6) *Pacific Coast* WA, OR, CA	0.0462 (0.0051)	0.047	2	10	0.96	2.2

[a]Regression 3 was obtained from the covariance analysis technique since the regression program used would not handle the number of variables required for the dummy variable approach. This change in procedure is reflected in the relatively low R^2 since much of the variation in the original data is removed by the preprocessing.

To estimate λ_j^E, the appropriate regression model is

$$\log \omega_{ijt} = \sum_{k=1}^{K_j} \alpha_{kj} D_{kj} + [\log(1 + \lambda_j^E)]\cdot(t - v) + \epsilon_{ijt},$$

where ω_{ijt} is an *actual* wage rate; D_{kj} is a dummy variable equal to one when the observation is taken from the ith plant ($k = i$) and zero otherwise ($k \neq i$); K_j is the number of plants in the jth region; ϵ_{ijt} is a random disturbance term assumed to satisfy the assumptions of the Gauss–Markov Theorem; α_{kj} and $\log(1 + \lambda_j^E)$ are parameters to be estimated.

The dummy variables are added to reflect the fact that although there may be a common trend in each region, the initial wage rates will vary from plant to plant. In this case an estimate of ω_{ijv} is given by the estimate of α_{ij}. The actual ω_{ijv} was not used as the intercept in order not to give undue weight to the first observation on each plant. The geographical regions correspond to those used in the construction of the Handy–Whitman Indices and are presented, along with the regression results in Table 5.