Ecom 240B Spring 2009
Problem Set 1

This problem set is due in class on Monday, April 6th

1. Prove consistency of the least absolute deviation estimator:

$$\hat{\theta} = \text{argmin} \sum_{i=1}^{n} |y_i - x_i'\theta|$$

Assume that for $\epsilon_i = y_i - x_i'\theta_0$. $P(\epsilon_i \leq 0|x_i) = \frac{1}{2}$ for all x_i. $Ef_\epsilon(0|x_i)\, x_i x_i'$ is nonsingular. Also specify the regularity conditions you need to show consistency.

2. The density of X is given by

$$f(x) = \begin{cases} 3/(4\theta) & \text{for} \quad 0 \leq x \leq \theta, \\ 1/(4\theta) & \text{for} \quad \theta < x \leq 2\theta, \\ 0 \quad \text{otherwise.} \end{cases}$$

Assuming that a sample of size 4 from this distribution yielded observations 1, 2.5, 3.5 and 4, calculate the maximum likelihood estimator of θ.

3. Suppose $X \sim N(\mu, 1)$ and $Y \sim N(2\mu, 1)$, independent of each other. Obtain the maximum likelihood estimator of μ based on N_x i.i.d. observations on X and N_y i.i.d. observations on Y and show that it is the best unbiased estimator in the sense of having the smallest variance.

4. Let $\{X_i\}, i = 1, 2, \ldots, n$ be a random sample on $N(\mu, \mu)$, where we assume $\mu > 0$.

   (a) Obtain the maximum likelihood estimator of μ and prove its consistency.

   (b) Also obtain its asymptotic variance and compare it with the variance of the sample mean (If you think one of them is larger than the other, prove it explicitly).

5. Suppose that $X_1, \ldots, X_n$ are independent and that it is known that $(X_i)^\lambda - 10$ has a standard normal distribution, $i = 1, \ldots, n$. This is called the *Box-Cox transformation.*

   - Derive the second-round estimator $\hat{\lambda}_2$ of the Newton-Raphson iteration, starting from an initial guess that $\hat{\lambda}_1 = 1$.
   - For the following data, compute $\hat{\lambda}_2$:
     96, 125, 146, 76, 114, 69, 130, 119, 85, 106