Econ 240B Spring 2009
Problem Set 3

This problem set is due in class on Monday April 20th, 2009

1. Suppose that there are two parameter tests given by smooth functions $g : \mathbb{R}^p \mapsto \mathbb{R}^k$ and $r : \mathbb{R}^p \mapsto \mathbb{R}^k$ (i.e. the sets of values of g and r have the same dimensionality). Suppose also that there exists a smooth and invertible function $\varphi(\cdot)$ such that $r(\theta) = \varphi(g(\theta))$. The tests are defined by the null hypothesis regarding the parameter restrictions (with a standard composite alternative):
$$H_0^1 \ : \ g(\theta) = \gamma_0,$$
and
$$H_0^2 \ : \ r(\theta) = \varphi(\gamma_0).$$
We assume that the distribution of the data satisfies the standard regularity conditions and use the likelihood ratio, the Wald and the score test statistics to test H_0^1 and H_0^2. For each choice of the test statistic, find all smooth functions $\varphi(\cdot)$ such that the test power for H_0^1 and H_0^2 is exactly the same.

2. Consider a general binary response model with observable variables d and x such that
$$\Pr(d = 1 \,|\, x) = F(x'\beta_0).$$
Suppose that
   - There is an i.i.d. sample (d_t, x_t) for $t = 1, \ldots, T$ where x has a continuous density $f_X(\cdot)$ such that $f_X(x) > 0$ for all $x \in \mathbb{R}^p$
   - $F(\cdot)$ is a c.d.f. of some continuous random variable with the median equal to zero, and $F(z) = \frac{1}{2}$ iff $z = 0$.
   - Parameter space is a unit sphere $\mathcal{B} = \left\{b \,\middle|\, b \in \mathbb{R}^p,\ b'b = 1\right\}$

   Consider the following objective function:
$$\widehat{Q}_T(\beta) = \frac{1}{T}\sum_{t=1}^{T}\left[d_t\,\mathbf{1}\{x_t'\beta \geq 0\} + (1 - d_t)\,\mathbf{1}\{x_t'\beta < 0\}\right],$$
   where $\mathbf{1}\{\cdot\}$ is an indicator function. Let $\widehat{B}_T$ be the set of maximizers of the chosen objective function, that is
$$\widehat{B}_T = \left\{b \,\middle|\, \widehat{Q}_T(b) \geq \widehat{Q}_T(b'),\ b' \neq b,\ b, b' \in \mathcal{B}\right\}.$$
   Prove that for any chosen sequence of points in the set of maximizers of the sample objective $\left\{\widehat{\beta}_T \in \widehat{B}_T\right\}_{T=1}^{\infty}$
$$\widehat{\beta}_T \xrightarrow{p} \beta_0, \text{ as } T \to \infty.$$
   This means that all points in $\widehat{B}_T$ are consistent estimates for β_0.