Econ 240B Spring 2009
Optional Problem Set 6

This problem set is due in class on Monday May 4th, 2009

1. Consider the structural equation

$$y_{1t} = y_{2t}^2\gamma + x_{1t}'\beta + \epsilon_t$$

where $\epsilon_t \sim i.i.d \quad (0, \sigma^2)$, y_{2t} is a scalar endogenous variable, and x_{1t} is a vector of exogenous variables. Suppose we have a set of instruments x_t such that x_t contains all the elements of x_{1t} plus several other variables. ϵ_t is independent of x_t. Evaluate the following statements:

   - Regressing y_{2t} on x_t and substituting the predicted values $\hat{y}_{2t} = x_t'\hat{\pi}$ (where $\hat{\pi}$ is the LS estimate of π) for y_{2t} in the structural equation and applying LS to the resulting equation yields consistent estimates for γ and β, but it produces the wrong standard errors for these estimates.

2. True or False, explain. Consider the two-equation system

$$y_1(t) = x_1(t)\beta + y_2(t)\gamma + e(t)$$
$$y_2(t) = x_2(t)\pi + u(t)$$

where $x_1(t)$ and $x_2(t)$ are exogenous variables and

$$(e(t), u(t)) \sim iid\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{pmatrix}\right) \tag{1}$$

   (a) Even though $y_2(t)$ is endogenous in this model, LS applied to the first equation yields consistent estimates of all coefficients.

   (b) Applying three stage least square to this system of equation yields estimators that are numerically identical to those obtained by applying LS to each equation individually.

   (c) Applying three stage least square to this system of equation yields estimators that are asymptotically equivalent to those obtained by applying LS to each equation individually.

3. Consider the following simultaneous equation model of the price elasticity of electricity:

$$\log Q = \alpha_1 + \alpha_2 \log P + \alpha_3 \log Y + \alpha_4 \log J + \epsilon$$
$$\log P = \beta_1 + \beta_2 \log Q + \beta_3 \log L + \beta_4 \log F + u$$

where

$$\begin{aligned} Q &= \text{average annual residential electricity sales per customer} \\ P &= \text{marginal price of residential electricity} \\ Y &= \text{annual per capita income} \\ J &= \text{average July temperature} \\ L &= \text{cost of labor} \\ F &= \text{cost of fuel per kilowatt-hour of generation} \end{aligned} \tag{2}$$

(a) Which equation is the supply equation and which equation is the demand equation? What do you expect to be the signs of α_2 and β_2?

(b) Assume that Y, J, L, F are exogenous variables that are independent of the error terms ϵ and u. You want to use $\log L$ and $\log F$ as instrument variables in a 2SLS estimation of the first equation. What conditions do you need to impose on β_3 and β_4 in order for $\log L$ and $\log F$ to be valid instruments? How can you test these conditions statistically? (Write down the stata command that you will use to test these conditions.)

(c) Suppose the conditions you impose on β_3 and β_4 are satisfied, but you are not entirely sure whether L and F are both uncorrelated with ϵ. How would you test this statistically? (Describe the stata commands you will use to perform the test. State the null and the alternative hypotheses clearly.)

(d) What are the reduced form equations for this simultaneous equation system? Can you give a set of conditions under which ordinary least square estimation of the first equation will give consistent estimates of α_2? You can impose your conditions on the β coefficients and on the relation between ϵ and u.

4. Consider the model

$$\begin{aligned} q_t =& \alpha + \beta p_t + u_t && \text{Demand Equation} \\ q_t =& \gamma + \delta\left(p_t + s_t\right) + v_t && \text{Supply Equation} \end{aligned}$$

where q_t is the amount of the commodity produced and sold in year t, p_t is the average price of the commodity in year t and s_t is the rate of subsidy in the year t. u_t and v_t are random disturbances which are serially independent and are distributed with zero mean and finite second moments. The variables s_t is assumed to have finite mean and variance.

(a) Which of the parameters α, β, γ and δ are identified?

(b) Obtain consistent estimates of the identified parameters from the sample means $\bar{p} = 4$, $\bar{q} = 5$, $\bar{s} = 1/3$ and the following sample moments:

$$\frac{1}{n}\sum_{t=1}^{n} p_t^2 = 6,\ \frac{1}{n}\sum_{t=1}^{n} p_t q_t = 1,\ \frac{1}{n}\sum_{t=1}^{n} p_t s_t = 0,$$
$$\frac{1}{n}\sum_{t=1}^{n} q_t^2 = 5,\ \frac{1}{n}\sum_{t=1}^{n} q_t s_t = 2,\ \frac{1}{n}\sum_{t=1}^{n} s_t^2 = 4.$$