

Econ 201A. Part II.
 Due: Tuesday, November 22

Problem Set 2

Homework policy: Please try to do the following problems on your own first. If you get really stuck, feel free to discuss them with other people in the class, but acknowledge any discussion or ideas that you get on your homework, e.g. "I benefited from discussion with so-and-so on problem x." Please write your solutions clearly and concisely.

Problem 1. Consider a prisoner's dilemma repeated n times

	C	D			C	D			C	D	
C	1, 1	-1, 2	→	...	C	1, 1	-1, 2	→	C	1, 1	-1, 2
D	2, -1	0, 0			D	2, -1	0, 0		D	2, -1	0, 0

A grim-trigger strategy in a Prisoner's dilemma is a strategy that plays C as long as the opponent cooperates, and plays D for all subsequent periods after the opponent defects. For example if the opponent plays CCDCCDD, then a grim-trigger strategy will play CCCDDDD.

- (a) If two grim-trigger strategies are playing against each other, what path of play will happen?
- (b) If a fully rational player has a grim-trigger opponent, how will he play? In other words, what is a best response to a grim-trigger strategy?

For the next three parts, suppose that player 2 can be one of two types: *normal* with probability $1-p$ or *behavioral* with probability p . A normal type fully rational, but a behavioral type plays the grim-trigger strategy. Player 1 is normal. Of course, the probability that player 1 assigns to player 2 being behavioral can change depending on the path of play. For parts (c)-(e), assume that $n = 2$.

- (c) How will the normal type of player 2 play?
- (d) If $p = 3/4$, what happens in a PBE?
- (e) Describe what can happen in a PBE for different values of p .

Part (f) is a very hard bonus part.

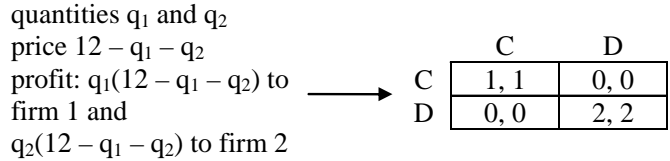
- (f) Assume that p and n are arbitrary. Describe a PBE.

Problem 2.

Consider a Cournot duopoly, in which two players simultaneously choose quantities q_1 and q_2 , sell at price $12 - q_1 - q_2$, so that the payoffs of firms 1 and 2 are $q_1(12 - q_1 - q_2)$ and $q_2(12 - q_1 - q_2)$ respectively.

- (a) Find the unique pure strategy Nash equilibrium.
- (b) Find the monopoly quantity, which maximizes $Q(12 - Q)$.

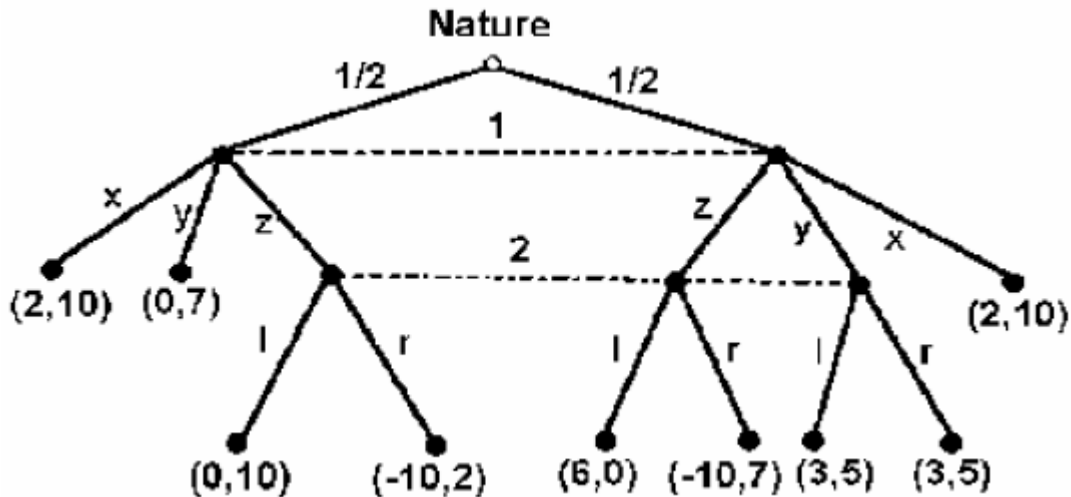
Now, consider a 2-period game, in which players play a Cournot duopoly in period 1 and a coordination game shown in the figure below in period 2:



For this problem, please focus exclusively on pure strategy SPE of this game.

- (c) Show that there is no pure strategy SPE, in which both firms are producing half the monopoly quantity in period 1.
- (d) Find the best SPE, in which both firms choose the same quantity in period 1? Hint: Firms want to produce less, closer to splitting the monopoly quantity, but they are constrained by rewards and punishments available in the second period to provide them with incentives against deviations. Find the smallest pair of quantities that can be enforced by the available rewards and punishments in the second period.

Problem 3. Consider the following two player game in extensive form:

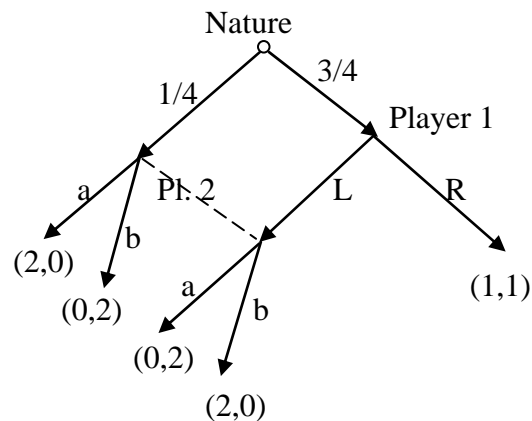


- A. Characterize the set of Perfect Bayesian Equilibria.
- B. Characterize the set of Sequential Equilibria. Hint: What does Sequential Equilibrium imply about player 2's beliefs in his information set?

Problem 4. Consider the following game played by two parties, A and B. First, “nature” chooses either C or D. C is chosen with probability 0.7, and D is chosen with probability 0.3. Second, party A chooses either E or F. Party A does not observe nature’s choice when it makes this choice. Next, party B chooses either G or H. Prior to making this choice, party B observes the choice of party A; party B also observes nature’s choice if A has chosen E, but does not observe nature’s choice if A has chosen F. Payoffs are determined as follows: A and B always receive the same payoff; the payoff is 0 if A chooses E and B chooses G, regardless of nature’s choice; the payoff is 5 if A chooses E and B chooses H, regardless of nature’s choice; the payoff is 0 if nature chooses C, A chooses F, and B chooses G; the payoff is 10 if nature chooses C, A chooses F, and B chooses H; the payoff is 10 if nature chooses D, A chooses F, and B chooses G; and the payoff is 0 if nature chooses D, A chooses F, and B chooses H.

- Draw the extensive form of this game.
- Please write a list of all strategies for each player.
- Identify all pure strategy Subgame Perfect Equilibria for this game.
- Identify all pure strategy Perfect Bayesian Equilibria for this game.

Problem 5. In the game below, please find all Perfect Bayesian Equilibria.



Problem 6.

- Consider a monopolist with marginal cost $c = 0$ or 3 who faces a demand of $12 - Q$. The monopolist’s profit is given by $(12 - Q - c)Q$. Find the monopoly quantity and profit.
- Consider a two firms, who play Cournot duopoly. Firm 1 has marginal cost 0 and firm 2 has marginal cost c . They face a demand of $12 - q_1 - q_2$ and the profits are given by $q_1(12 - q_1 - q_2)$ and $q_2(12 - q_1 - q_2 - c)$. Find the Nash equilibrium (as a function of c , assuming that $c = 0$ or 3).
- Consider the following two-period game. The market demand is given by $12 - Q$ in each period. In period 1 firm 1, an entrant, watches the behavior of firm 2, an incumbent, which has cost $c = 0$ or 3 . The entrant does not know the incumbent’s

cost, but can see the quantity chosen by incumbent in period 1. Between periods 1 and 2, the entrant decides whether to enter or not. Entering involves a fixed cost of 20. If the entrant does not enter, she gets a payoff of 0 and the incumbent gets to be a monopolist in period 2. If the entrant enters, then she (with marginal cost 0) and the incumbent (with marginal cost c) play Cournot duopoly in period 2. Find all separating PBE, in which two types of the incumbent choose different quantities in period 1, and the entrant enters only if the incumbent is high-cost.