Economics 201A, Fall 2005
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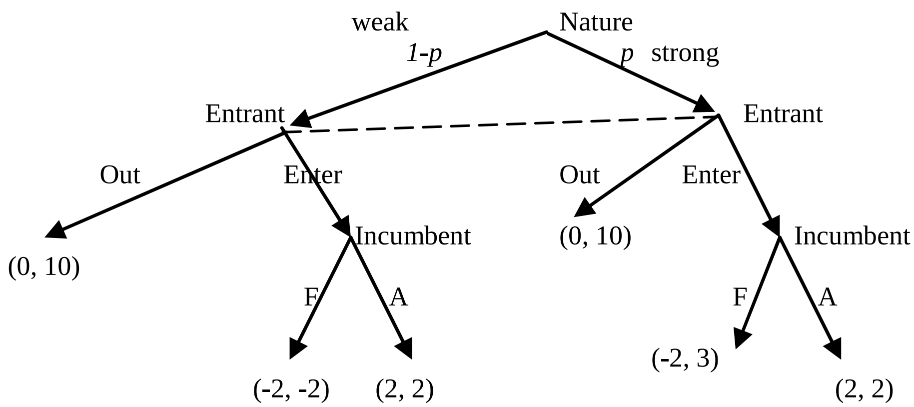
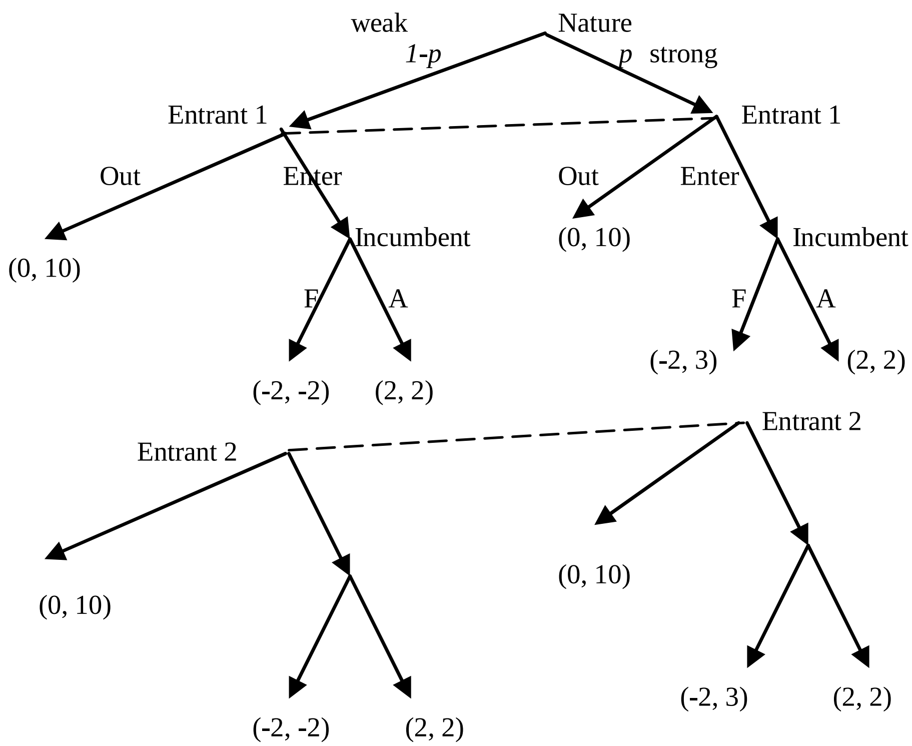
These notes cover the analysis behind the entrant-incumbent game that we discussed in class. First, we considered a one-stage version of the game:



For this example, we have concluded that the entrant will enter if $p < ½$, not enter if $p > ½$ and may enter with any probability if $p = ½$. The incumbent always fights if he is strong, and never fights if he is weak.

The answer for what happens in the one-stage version will be useful as a building block in the analysis of a two-stage version. Here is the two-stage entrant-incumbent game:



The question is to find what happens on the equilibrium path in a PBE of this game when p = 3/8.

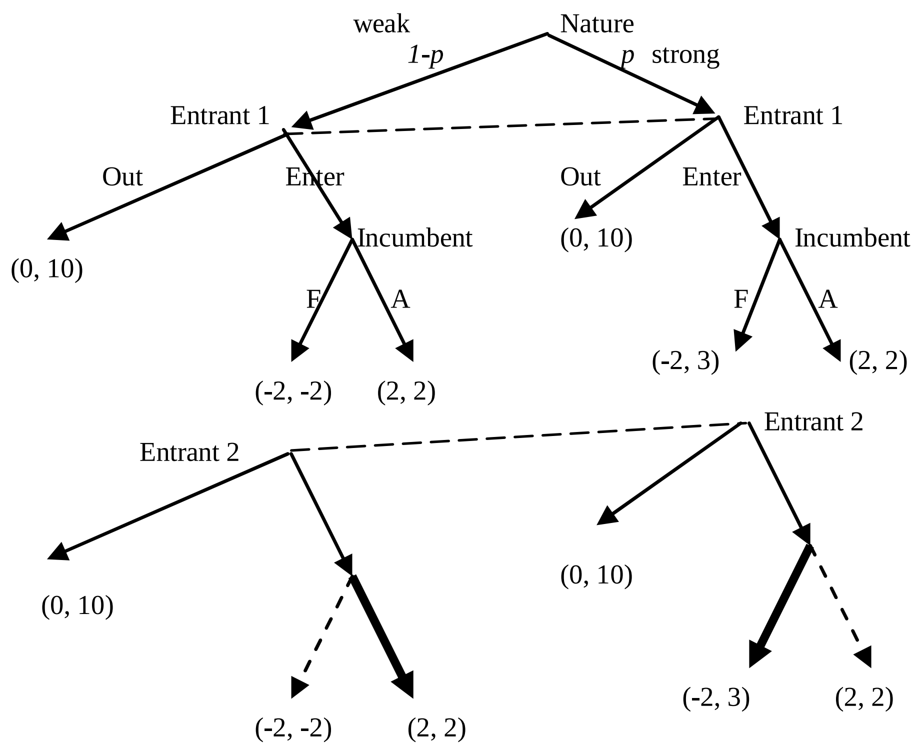
If you face a game like this on a problem set (or a simpler game on the final), then a good strategy to approach it is to

I. Identify some immediate conclusions of what must happen in the game.

II. After that, pick a point in the game to start analysis, and consider different cases.

Let's go…

I. One can immediately conclude that the strong incumbent always fights and the weak incumbent always accommodates in period 2.



II. Let us look at the action of entrant 1 in period 1, and consider two cases:

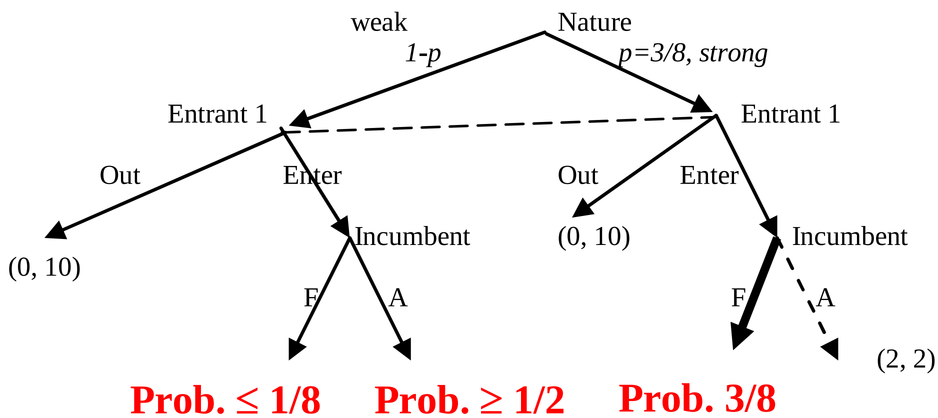
Case 1: Entrant 1 enters with positive probability. Let us see what happens:

A. The strong incumbent will always fights in period 1. Let us prove it:

- The strong incumbent prefers fighting, so he would accommodate only if doing so deters entry (that is, sufficiently reduces the probability that entrant enters in period 2). If accommodating deterred entry, then the weak incumbent would always accommodate in period 1. But then Entrant 2 must place probability at least 5/8 on the incumbent being weak if he sees accommodating at the end of period 1, so he would enter in period 2 for sure. We conclude that accommodation in period 1 cannot deter entry, and so the strong incumbent will fight.

B. The total probability that the incumbent accommodates given entrant 1 enters is at least ½ (because otherwise it would be strictly better for the entrant 1 to stay out).

From A and B we conclude the following about total probabilities of reaching different nodes at the end of period 1 conditional on entrant 1 entering:



Let me emphasize that these are **total probabilities conditional on Entrant 1 entering.**

Given these probabilities, Entrant 2 must believe that the incumbent is weak for sure if he sees accommodating in period 1 **(and will enter)**, and that the incumbent is strong with probability ≥ ¾ if he sees fighting in period 1 **(and will not enter –** here we use our analysis of the one-period game to reach this conclusion).

Let us summarize what we found so far: for this case (when Entrant 1 enters with positive probability in period 1), the total probabilities conditional on entrant 1 entering are shown in the figure above, and entrant 2 will enter if he sees accommodating and will stay out if he sees fighting. Now, let us compute total payoff of a weak incumbent from fighting and from accommodating. The payoff from fighting is -2 + 10 (since entrant 2 stays out in period 2) and the payoff from accommodating is 2 + 2 (since entrant 2 enters in period 2, and the incumbent accommodates). We conclude that it is strictly better for the weak incumbent to fight in period 1, which contradicts the probabilities in the figure above.

A contradiction means that there is no PBE in this case.

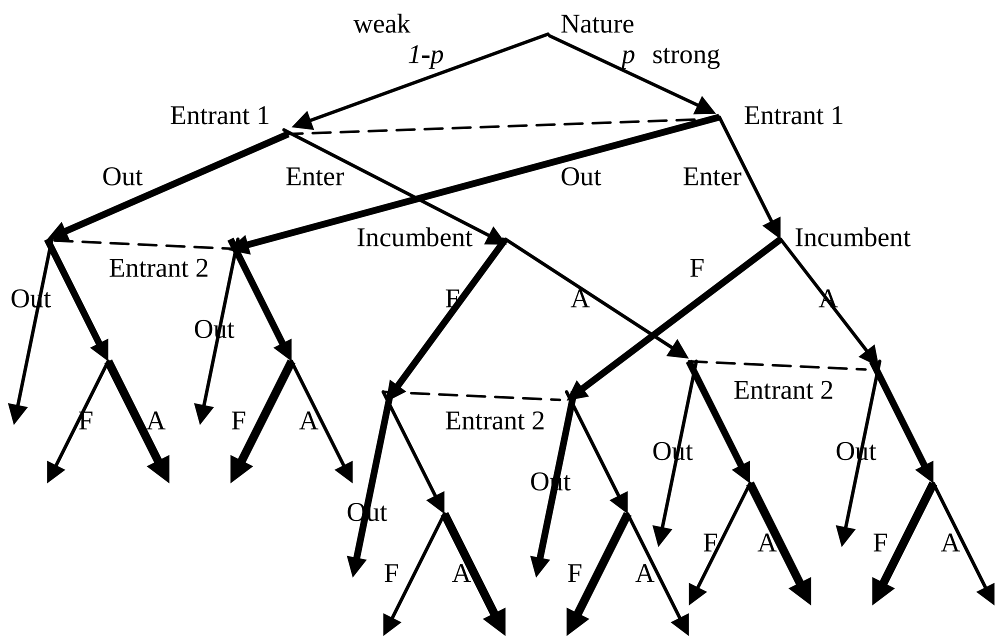
Case 2: Entrant 1 stays out for sure.

Let us recall the question we were set to answer: The question is to find what happens on the equilibrium path in a PBE of this game when p = 3/8.

On the equilibrium path entrant 2 will see Entrant 1 staying out, and will maintain old beliefs (weak with probability 5/8 and strong with probability 3/8). For these beliefs, our one-period "building block" says that entrant 2 will enter, the strong incumbent will fight, and the weak incumbent accommodates.

Let us summarize the answer we have found: **On the equilibrium path, entrant 1 stays out, entrant 2 enters, weak incumbent accommodates in period 2 and strong incumbent fights in period 2. The beliefs of entrant 1 are (5/8, 3/8). The beliefs of entrant 2 in case he sees entrant 1 staying out are also (5/8, 3/8).**

To be proper, we must justify this answer by demonstrating some off-equilibrium path strategies and beliefs consistent with PBE. Let's say both incumbent types fight in period 1 if entrant 1 enters. Let's say that entrant 2 holds beliefs (0, 1) in case he sees entrant 1 entering and incumbent fighting and stays out in period 2 (recall that we can specify beliefs arbitrarily off the equilibrium path). Let's say that entrant 2 holds beliefs (1, 0) in case he sees entrant 1 entering and incumbent accommodating, and enters in period 2. In period 2, the strong incumbent always fights and the weak incumbent always accommodates. Let me summarize this PBE graphically:



The beliefs, although not displayed in the picture, are (5/8, 3/8) in period 1, and (5/8, 3/8), (0,1) and (1,0) in the three information sets in period 2 respectively. Let's verify that this is a PBE. We need to check that beliefs are formed by Bayes rule whenever possible, and all actions in all information sets (on and **off** equilibrium path) are optimal given beliefs and strategies.

- beliefs are formed by Bayes rule on the equilibrium path
- given both types of incumbent would fight in period 1, the action of entrant 1 to stay out is optimal
- given entrant 2 enters if he sees accommodation in period 1 and stays out if he sees fighting, it is optimal for both incumbent types to fight in period 1 if entrant 1 enters
- in period 2, starting from all three information sets we have a PBE in the continuation game: this follows from our “building block” one-period game

Thus, we conclude that we found a PBE. If you understand this far, you know enough about this game for the class.

What follows is extra.

Question: Is the equilibrium we found sequential?

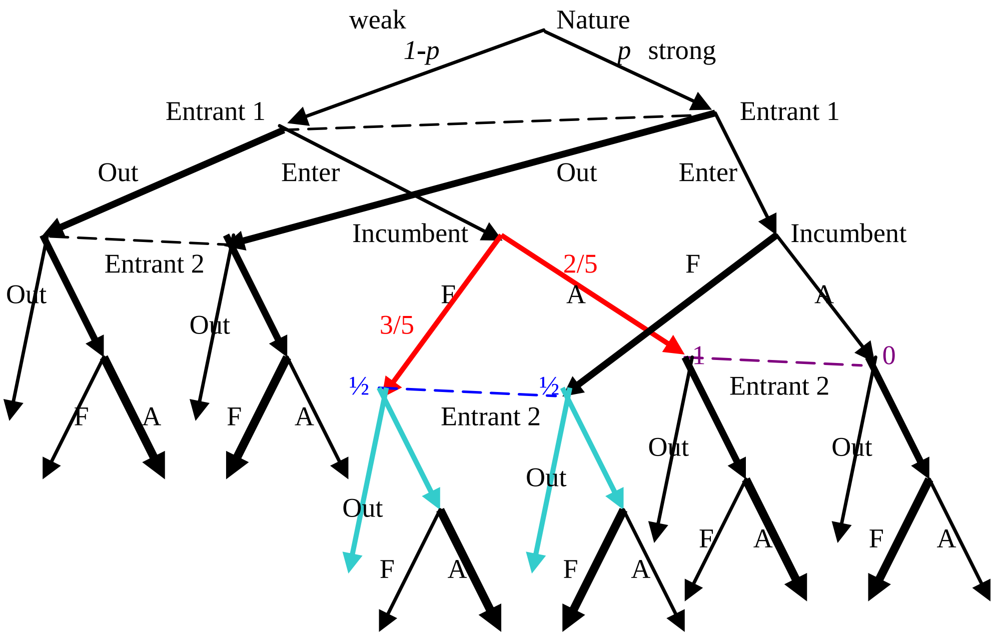
To answer this question, we need to check the consistency of beliefs. They are consistent on the equilibrium path, and in the information set that follows incumbent one accommodating in period 1 (entrant 2 could believe that the weak incumbent is infinitely more likely to accommodate than the strong incumbent, even though neither type is supposed to do so in the suggested PBE). However, beliefs in the information set that follows incumbent fighting in period 1 are *inconsistent*: the only consistent beliefs in that information set are (5/8, 3/8), because both types of the incumbent fight with probability 1 if entrant 1 enters. Even if we adjusted the beliefs to (5/8, 3/8) in that information set, we still would not get a sequential equilibrium, because now entrant 2 would not want to stay out if he sees fighting.

Answer: No, the PBE we found is not sequential.

If we wanted to construct a sequential equilibrium, we would need to figure out what happens off the equilibrium path. The argument could proceed via the following steps:

- Argue that the strong incumbent must fight in period 1 if entrant 1 enters, by an argument similar to what we had above.
- Consider three cases for what the weak incumbent would do in period 1: always fight, always accommodate, mix. Get a contradiction in the first two cases and conclude that the weak incumbent must mix.

Eventually, we would arrive at the following sequential equilibrium:



Let me summarize some important points about what happens in this sequential equilibrium:

- The probabilities with which the weak incumbent mixes in period 1 are chosen to generate beliefs (½ , ½) of entrant 2 in the middle information set in period 2.
- The beliefs (½ , ½) are exactly the beliefs that allow entrant 2 to mix between entering and staying out if he had observed (Enter, Fight) in period 1.
- I have not computed the mixed action of entrant 2 if he had observed (Enter, Fight) in period 1, but the probabilities with which he mixes are pinned down uniquely by the requirement that the weak incumbent must be indifferent between Fighting and Accommodating in period 1.

This sequential equilibrium is also a PBE (because any sequential equilibrium is a PBE). Note that the same thing happens on the equilibrium path as in the PBE we found earlier.