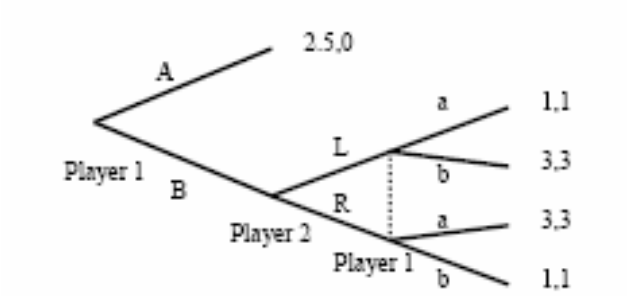
Professor Yuliy Sannikov
Economics 201A
Game Theory

Practice Final Exam

Instructions. The exam is open book and open notes. Please answer all four questions. Be clear but concise. Please box your answers. You have three hours to complete the exam. Good luck!

Problem 1.

What are the Subgame Perfect Equilibria of the game in the figure below?



Problem 2.

Consider a buyer and a seller. At date one, the seller chooses an investment x in his product. The investment has cost x. At date two, the seller can produce one unit. His cost will be c(x) and the buyer's value will be v(x). Both are functions of the investment at date one. Assume that for all x, $v(x) > c(x)$, that $c' < 0$; $c'' > 0$; and that $v' > 0$; $v'' < 0$. Also, assume that $v'(0) - c'(0) > 1$ and $v'(x) - c'(x) < 1$ as $x \to \infty$.

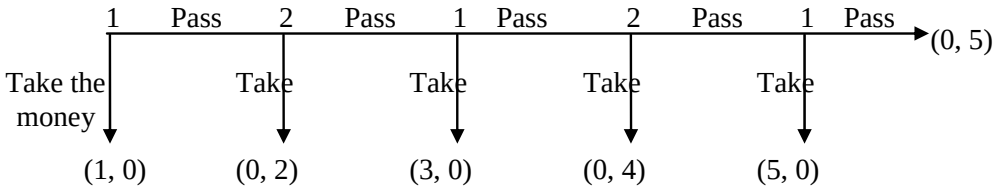
(a) What is the socially optimal level of investment?

(b) Suppose at date two, the buyer can make a take-it-or-leave-it offer to the seller concerning the price. What offer will the buyer make? How much will the seller invest as a result in the subgame perfect equilibrium?

(c) What would happen if the seller could make the offer?

Problem 3.

Consider a centipede game, except with a modification that player 1 may be behavioral with probability $p > 0$ and normal with probability $1-p$. The following game shows the payoffs of player 2 and the normal type of player 1:



A behavioral player always chooses Pass. For the following questions, let us number the nodes from 1 to 5 from left to right.

A. Is there a PBE where the normal type of player 1 always takes the money in node 1?
B. Suppose that in node 4 player 2 believes that player 1 is behavioral with probability p_4. For what values of p_4 will player 2 take the money? For what values will player 2 pass? For what value is player 2 indifferent? Denote the value that makes player 2 indifferent by p_4^*.
C. Denote by p_3 the probability with which player 3 is behavioral when node 3 is reached. Suppose that $p_3 < p_4^*$. Show that the normal type of player 1 must mix in a PBE. Conditional on player 1 passing, what is the probability that he is behavioral? With what probability will player 1 pass?
D. Suppose that in node 2 player 2 believes that player 1 is behavioral with probability p_2. For what values of p_2 will player 2 take the money? For what values of p_2 will player 2 pass?
   Hint: If player 2 passes, then $p_3 = p_2$. In part C you found the probabilities with which player 1 mixes in node 3 as a function of p_2.
E. Suppose $p = ½$. How will player 1 act in node 1?

Problem 4.

Consider an education signaling model with three types of workers, who have productivities $x_L < x_M < x_H$. Suppose that education does not add to productivity and has per-unit costs of $c_L > c_M > c_H$ to the three types. Each worker knows his type, but the market has initial belief (p_L, p_M, p_H) with $p_L + p_M + p_H = 1$.

A. For what range of education levels is there a pooling equilibrium in which all types choose the same education level. Does the pooling equilibrium satisfy the intuitive criterion?
B. Find the least-cost fully separating equilibrium, i.e. a separating equilibrium in which each type chooses a different level of education. Does it satisfy the intuitive criterion?
C. Are there any other fully separating equilibria that satisfy the intuitive criterion?